Title: Learning and testing quantum states of fermionic systems

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Abstract:

Abstract: The experimental realization of increasingly complex quantum states in quantum devices underscores the pressing need for new methods of state learning and verification. Among the various classes of quantum states, fermionic systems hold particular significance due to their crucial roles in physics. Despite their importance, research on learning quantum states of fermionic systems remains surprisingly limited. In our work, we aim to present a comprehensive rigorous study on learning and testing states of fermionic systems. We begin by analyzing arguably the simplest important class of fermionic states—free-fermionic states—and subsequently extend our analysis to more complex fermionic states. We meticulously delineate scenarios in which efficient algorithms are feasible, providing experimentally practical algorithms for these cases, while also identifying situations where any algorithm for solving these problems must be inherently inefficient. At the same time, we present novel fundamental results of independent interest on fermionic systems, with additional applications beyond learning and characterizing quantum devices, such as many-body physics, resource theory of non-Gaussianity, and circuit compilation strategies. (Talk based on https://arxiv.org/pdf/2409.17953 , https://arxiv.org/pdf/2402.18665)

Learning and testing quantum states of fermionic systems

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arXiv > quant-ph > arXiv:2409.17953

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Quantum Physics

[Submitted on 26 Sep 2024]

Optimal trace-distance bounds for free-fermionic states: Testing and improved tomography

Lennart Bittel, Antonio Anna Mele, Jens Eisert, Lorenzo Leone

arXiv > quant-ph > arXiv:2402.18665

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Quantum Physics

[Submitted on 28 Feb 2024 (v1), last revised 18 Aug 2024 (this version, v2)]

Efficient learning of quantum states prepared with few fermionic non-Gaussian gates

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Outline

- Introduction
- Learning fermionic Gaussian states
- Learning t-doped fermionic Gaussian states

Introduction

- Advances in quantum technologies have inspired a new field: *Quantum Learning* [1].

- Problem 1: **Learning** quantum states ('tomography').
  - Without any prior assumption, this task is hard. [1] ❌
  - But, if the unknown state belongs to a specific class, efficient learning may be possible. ✓

    (e.g., MPS [2], stabilizers [3], t-doped stabilizer states [4,5], …)

- Problem 2: **Testing** quantum states [6].

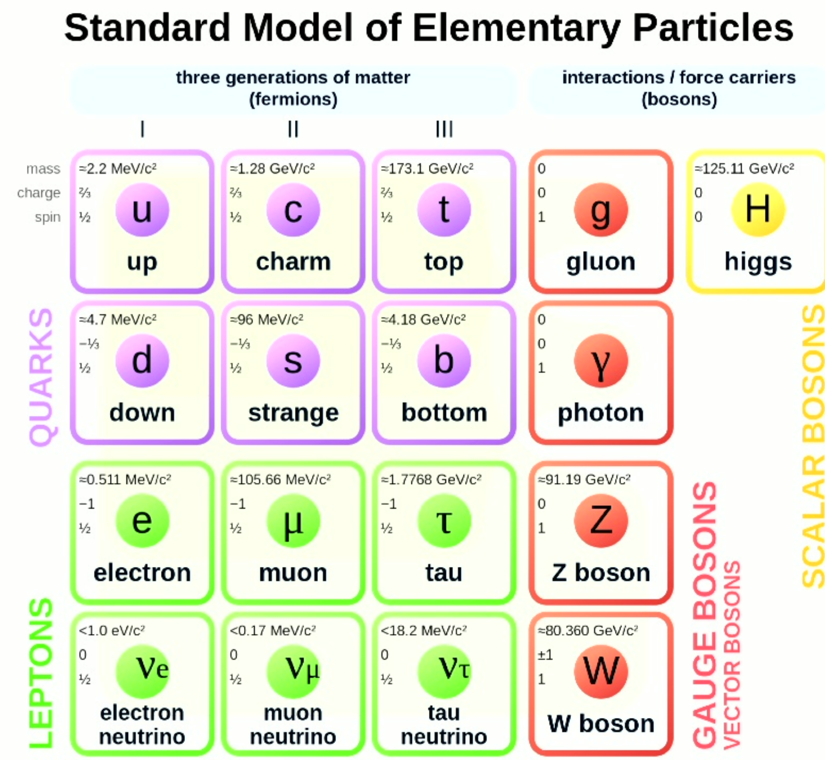
  ("Decide if a state is close to or far from a given class").

  (e.g., Is this state a stabilizer state or not? [7-11])

[1] Anshu et al, A survey on the complexity of learning quantum states, Nature Physics (2024)
[2] Lanyon et al, Efficient tomography of a quantum many-body system, Nature Physics (2017)
[3] Montanaro, Learning stabilizer states by Bell sampling (2017)
[4] Grewal et al, Efficient learning of quantum states prepared with few non-clifford gates (2023)
[5] Leone et al, Learning t-doped stabilizer states, Quantum (2023)
[6] Montanaro et al, A Survey of Quantum Property Testing, Theory of Computing (2013)
[7] Gross et al, Schur-Weyl Duality for the Clifford Group, Comm. in Math. Phys. (2023)
[8] Arunachalam et al, Polynomial-time tolerant testing stabilizer states, (2024)
[9] Hinsche et al, Single-copy stabilizer states, (2024)
[10] Bao et al, Tolerant testing of stabilizer states, (2024)
[11] Liang et al, Tolerant Testing of Stabilizer States with Mixed State Inputs, (2024)

Fermions are ubiquitous in physics

- Fermions are a type of quantum particle.

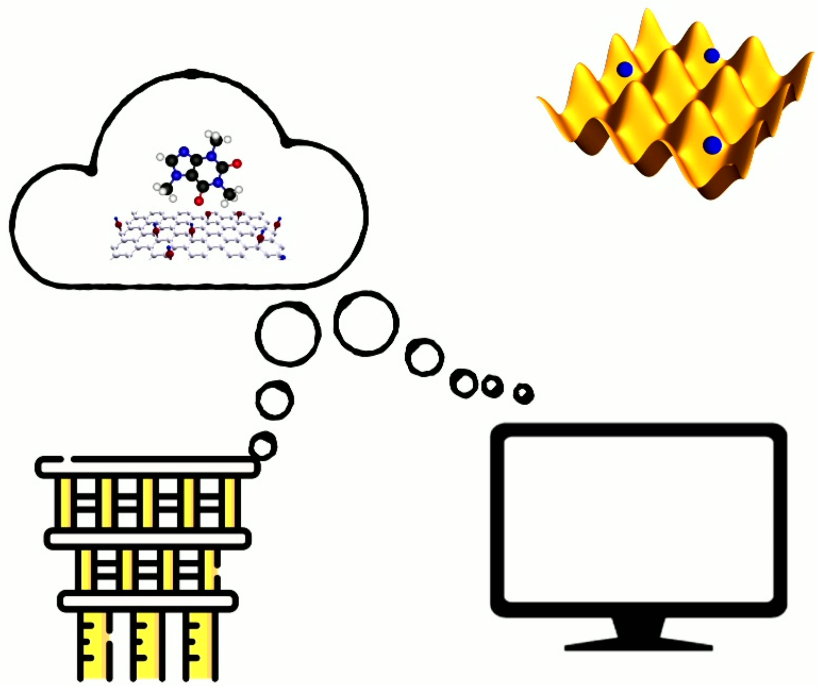


Fermions are ubiquitous in physics

- Fermions are a type of quantum particle. They make up all the matter!

- In all "quantum technologies" (chemistry, semiconductors, etc) of today, fermions —electrons—play a key role.

- Designing materials and chemicals = hard computational problems about fermions.



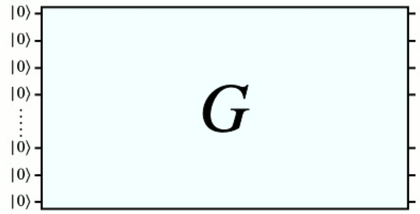
Introduction

- Despite their importance, research on learning fermionic states remains limited.

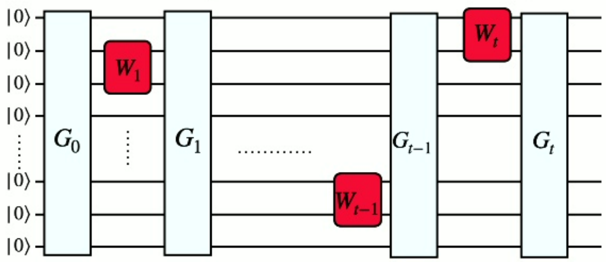
[11] Aaronson et al, Efficient tomography of non-interacting fermion states (2023)
[12] O'Gorman. Fermionic tomography and learning, (2022), …

<u>Our work</u> aims to provide a comprehensive study on Learning and Testing fermionic states.

✓ We start with the *simplest* fermionic states: 'Gaussian states'.



✓ We then analyze more *complex* states: 't-doped Gaussian states'.



We design *practical* efficient algorithms, while also showing cases where *any* algorithm must be inefficient.

Along the way, we uncover **fundamental properties** of these states.

Fermionic Gaussian states

(also called free-fermionic states, non-interacting fermions, states prepared by 1D-matchgates circuits, …)

- Fermionic Gaussian states = Gibbs states of “Free-fermions” Hamiltonians

$$\rho = \frac{e^{-\beta H_{\mathrm{free}}}}{\mathrm{Tr}(e^{-\beta H_{\mathrm{free}}})}, \qquad H_{\mathrm{free}} = i \sum_{\mu<\nu\in[2n]} h_{\mu,\nu}\gamma_\mu\gamma_\nu$$

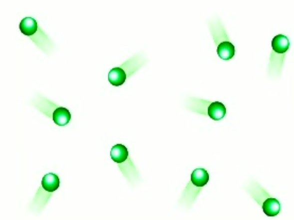
Majorana operators

- Majorana operators:
  (They are just some Pauli strings)

$$\gamma_{2k-1} := \left(\prod_{j=1}^{k-1} Z_j\right) X_k, \quad \gamma_{2k} := \left(\prod_{j=1}^{k-1} Z_j\right) Y_k, \qquad \text{for } k \in \{1,\dots,n\}$$

- Gaussian unitaries: $U = e^{-iH_{\mathrm{free}}}$

- **Why** Gaussian states/unitaries:
  - Model free-fermion physics (many metals, semi- and superconductors)
  - Classically easy to simulate



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Majorana operators

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(They are just some Pauli strings)

Fermionic Gaussian states

- Gaussian states ρ are **fully characterized** by their "correlation matrix" $\Gamma(\rho) \in \mathbb{R}^{2n\times 2n}$,

How to learn fermionic Gaussian states?

- Gaussian states ρ are **fully identified** by their correlation matrix $\Gamma(\rho)$.

- So it is enough to estimate $\Gamma(\rho)$, but to which accuracy?

Problem (Learning states/Tomography)

Let $\varepsilon > 0$. Given N copies of the (unknown) state $\rho \in \mathcal{S}$, the goal is to output $\tilde{\rho}$ such that (with high probability)

$$\|\rho - \tilde{\rho}\|_1 \leq \varepsilon$$

- We need norm bounds between Gaussian states and their correlation matrices!

(Our first main) Theorem

Let $\rho, \tilde{\rho}$ be Gaussian states, then:

$$\|\rho - \tilde{\rho}\|_1 \leq \frac{1}{2}\|\Gamma(\rho) - \Gamma(\tilde{\rho})\|_1$$

Norm bounds between Gaussian states

Theorem
Let $\rho, \tilde{\rho}$ be Gaussian states, then:

$$\|\Gamma(\rho) - \Gamma(\tilde{\rho})\|_\infty \leq \|\rho - \tilde{\rho}\|_1 \leq \frac{1}{2}\|\Gamma(\rho) - \Gamma(\tilde{\rho})\|_1$$

- "If we know $\Gamma(\rho)$ with accuracy ε, we know the Gaussian state itself with **trace distance** error ε."

Theorem
Let $\rho, \tilde{\rho}$ be pure Gaussian states, then:

$$\|\rho - \tilde{\rho}\|_1 \leq \frac{1}{2}\|\Gamma(\rho) - \Gamma(\tilde{\rho})\|_2$$

- These bounds are **"optimal"** !

How to learn fermionic Gaussian states?

Theorem (Efficient learning of Gaussian states)

$N = O\left(n^{\alpha}/\varepsilon^{2}\right)$ copies of the unknown Gaussian state ρ suffice to learn $\tilde{\rho}$ such that $\|\tilde{\rho} - \rho\|_1 \leq \varepsilon$.

$\alpha = 4$ if ρ is possibly mixed,

$\alpha = 3$ if ρ is pure.

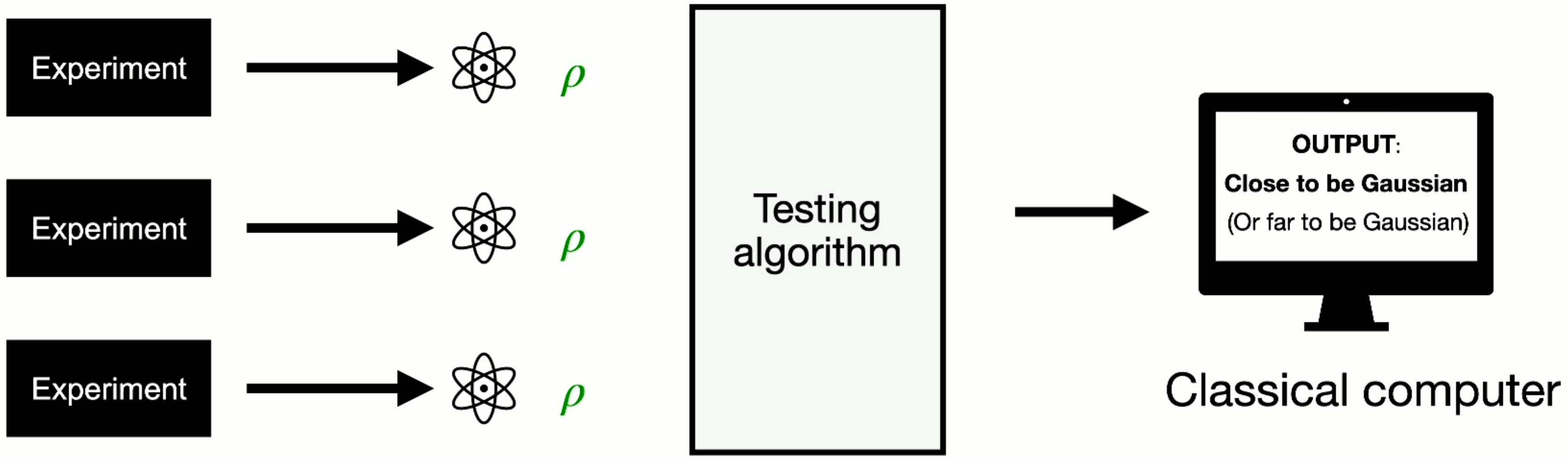
- Previous state-of-art bound (known only for pure-states) was $O(n^5/\varepsilon^4)$, while our is $O(n^3/\varepsilon^2)$.

[11] Aaronson et al, Efficient tomography of non-interacting fermion states (2023)
[12] O'Gorman. Fermionic tomography and learning, (2022)

- The algorithm is just: estimate the correlation matrix and "regularize it".

- **Experimentally feasible protocol:** 'simple' measurements, time-efficient and "noise robust".

Testing whether an unknown state is Gaussian



Problem (Property testing)

Given N copies of the (unknown) state ρ, decide (for $\varepsilon_B > \varepsilon_A \geq 0$) if:

- Case A (ρ is **close** to be Gaussian): There exists a Gaussian state σ such that $\|\rho - \sigma\|_1 \leq \varepsilon_A$, or
- Case B (ρ is **far** from being Gaussian): $\|\rho - \sigma\|_1 > \varepsilon_B$, for all σ Gaussian states.

Testing whether an unknown state is Gaussian

Theorem (Testing Gaussian states is Hard!)

To solve the testing problem, $N \geq \Omega(2^n)$ copies of the unknown state are necessary.

$\downarrow$

There is no measure of **'fermionic magic (non-Gaussianity)'** which can be efficiently estimated.

- What if the unknown state—or the states in the Gaussian set—have **rank** $\leq R$?

  $N \geq \Omega(R)$ copies necessary.

- Is there an efficient algorithm for $R = \mathrm{poly}(n)$?

Theorem (Efficient testing for **bounded rank** states)

The Gaussian testing problem can be solved with sample&time complexity $\mathrm{poly}(n, R)$.
(under appropriate conditions on $\varepsilon_A, \varepsilon_B$).

Outline

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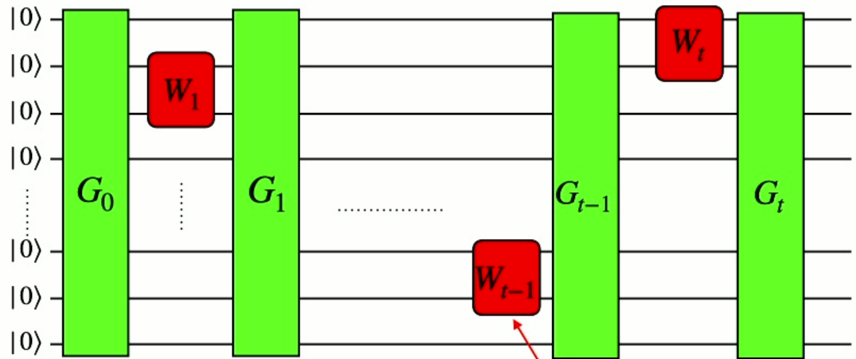
- Learning t-doped fermionic Gaussian states

t-doped fermionic Gaussian states

- Gaussian states are efficient to <u>classically simulate</u> and to <u>learn</u>, unlike general quantum states.
- How to interpolate between the two?
- ***t*-doped Gaussian state** = state prepared by Gaussian (1D-matchgates) unitaries + at most t 'magic' gates.

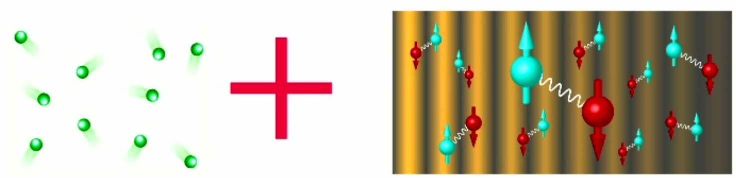


E.g.:
- $\exp(i\theta\,\gamma_3\gamma_4\gamma_5\gamma_6)$,
- SWAP gate …

Gaussian unitary

κ-mode (non-Gaussian) unitary

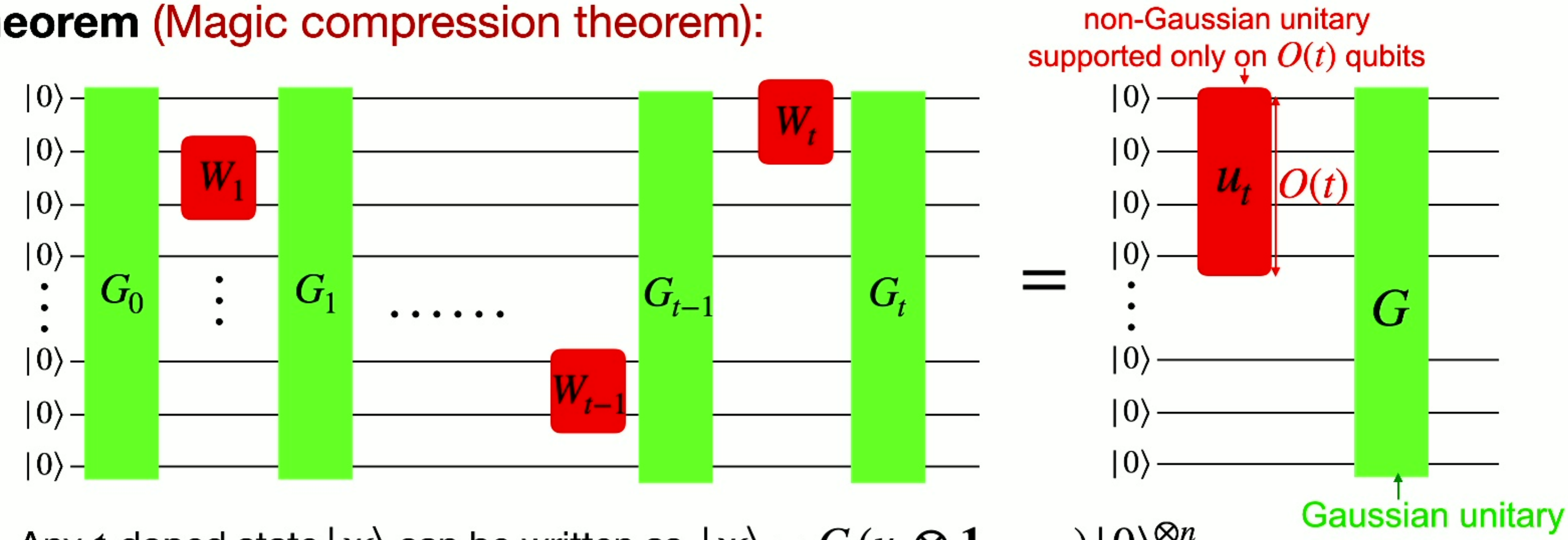
- **Why** non-Gaussian circuits/states:
  - They model interacting physics
  - Universal for Quantum Computation



- Classically simulable if $t = O(\log(n))$, no longer for $t \geq \omega(\log(n))$.

What about their **learnability**?
Spoiler: The same!

Theorem (Magic compression theorem):



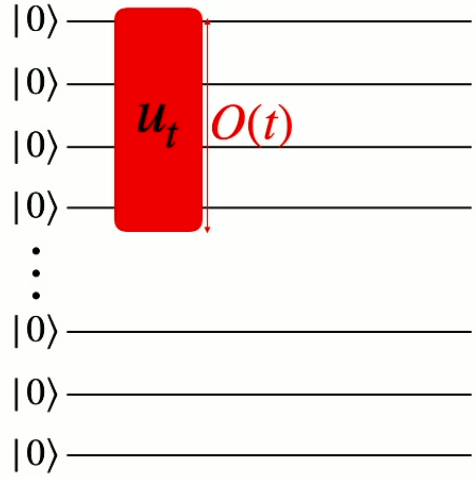
Any t-doped state $|\psi\rangle$ can be written as $|\psi\rangle = G\,(u_t \otimes \mathbf{1}_{n-O(t)})\,|0\rangle^{\otimes n}$.

Implications: • More efficient compilation of non-Gaussian circuits ("avoid redundancy").

Analogous theorem holds for "Clifford + T":

[1] Oliviero, Leone, Lloyd, and Hamma, Unscrambling Quantum Information with Clifford Decoders ,Phys. Rev. Lett. 132, 080402 (2024).
[2] Grewal, Iyer, Kretschmer, Liang, Efficient learning of quantum states prepared with few non-clifford gates (2023)

Idea for Learning t-doped Gaussian states



Crucial idea for tomography algorithm:

1) Imagine that we can learn G (….Yes, we can!)

2) Apply G^{-1} to $|\psi\rangle$

3) Do full state tomography on the first $O(t)$ qubits.

By estimating and processing the correlation matrix of $|\psi\rangle$.

Theorem (Efficient learning of t-doped Gaussian states)

For $t = O(\log(n))$, t-doped Gaussian states can be learnt in $\mathrm{poly}(n)$-time & sample.

- What if t is larger than $\log(n)$?

Theorem (Hardness learning of $\omega(\log(n))$-doped Gaussian states)

If $t \geq \omega(\log(n))$, there is no $\mathrm{poly}(n)$-time algorithm to learn t-doped Gaussian states, up to common crypto-assumptions (i.e., "RING-LWE cannot be solved by quantum computer in sub-exp-time").

- The runtime of our algorithm $\mathrm{poly}(n, 2^t)$ is "optimal".

Further remarks

- **Experimentally feasible protocol:** single copy, "simple" measurements, "noise robust".
  ("approximate t-doped"/mixed state learning).
- Our algorithm extends to all "t-compressible states". (e.g., ground states of impurity models [1])

$$|\psi\rangle = G\,(u_t \otimes \mathbf{1}_{n-O(t)})\,|0\rangle^{\otimes n}$$

- We provide an efficient testing algorithm for t-compressible states.

[1] S. Bravyi and D. Gosset, Complexity of quantum impurity problems, Commun. Math. Phys. 356, 451–500 (2017)

Summary

- Optimal trace distance bounds for Gaussian states, and efficient learning.
- Hardness for testing general Gaussian states, but efficient for low-rank states.
- Magic-compression theorem for t-doped states, and efficient learning/testing of t-compressible.
- Critical threshold for efficient 'Learnability' $= \log(n)$ magic gates.

  ↓

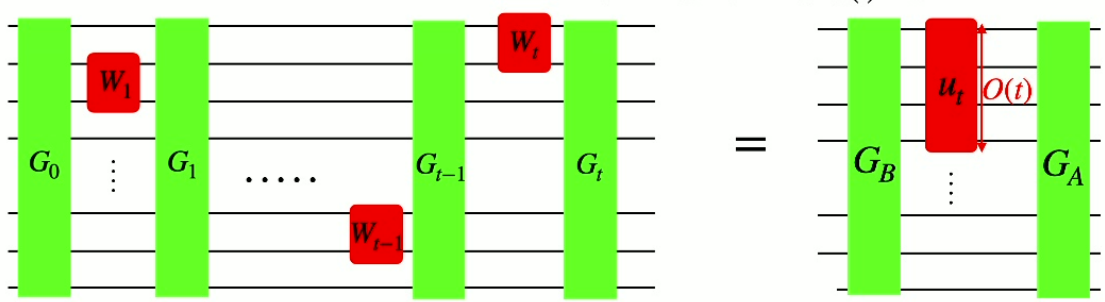
  "A new form of state-complexity coming into play".

- We showed analogous results for Bosons. [1,2]

Open questions

- Learning t-doped Gaussian unitaries.
  (Very recently solved for fermions! [3].)
- Testing Gaussian unitaries.
- Optimal learning and testing of Gaussian states.
- Agnostic tomography.

(They can be 'compressed' as well, i.e., $U_t = G_A(u_t \otimes I_{n-O(t)})G_B$)



[1] F. A. Mele, A. A. Mele, L. Bittel, J. Eisert, V. Giovannetti, L. Lami, L. Leone, and S. F. E. Oliviero, *Learning quantum states of continuous variable systems*, arXiv 2024.
[2] L. Bittel, F. A. Mele, A. A. Mele, S. Tirone, and L. Lami, *Optimal estimates of trace distance between bosonic Gaussian states and applications to learning*, arXiv 2024.
[3] Vishnu Iyer, *Mildly-Interacting Fermionic Unitaries are Efficiently Learnable*, ArXiv, 2025.