Title: Quantum Gravity and Effective Topology

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Collection/Series: Quantum Gravity

Subject: Quantum Gravity

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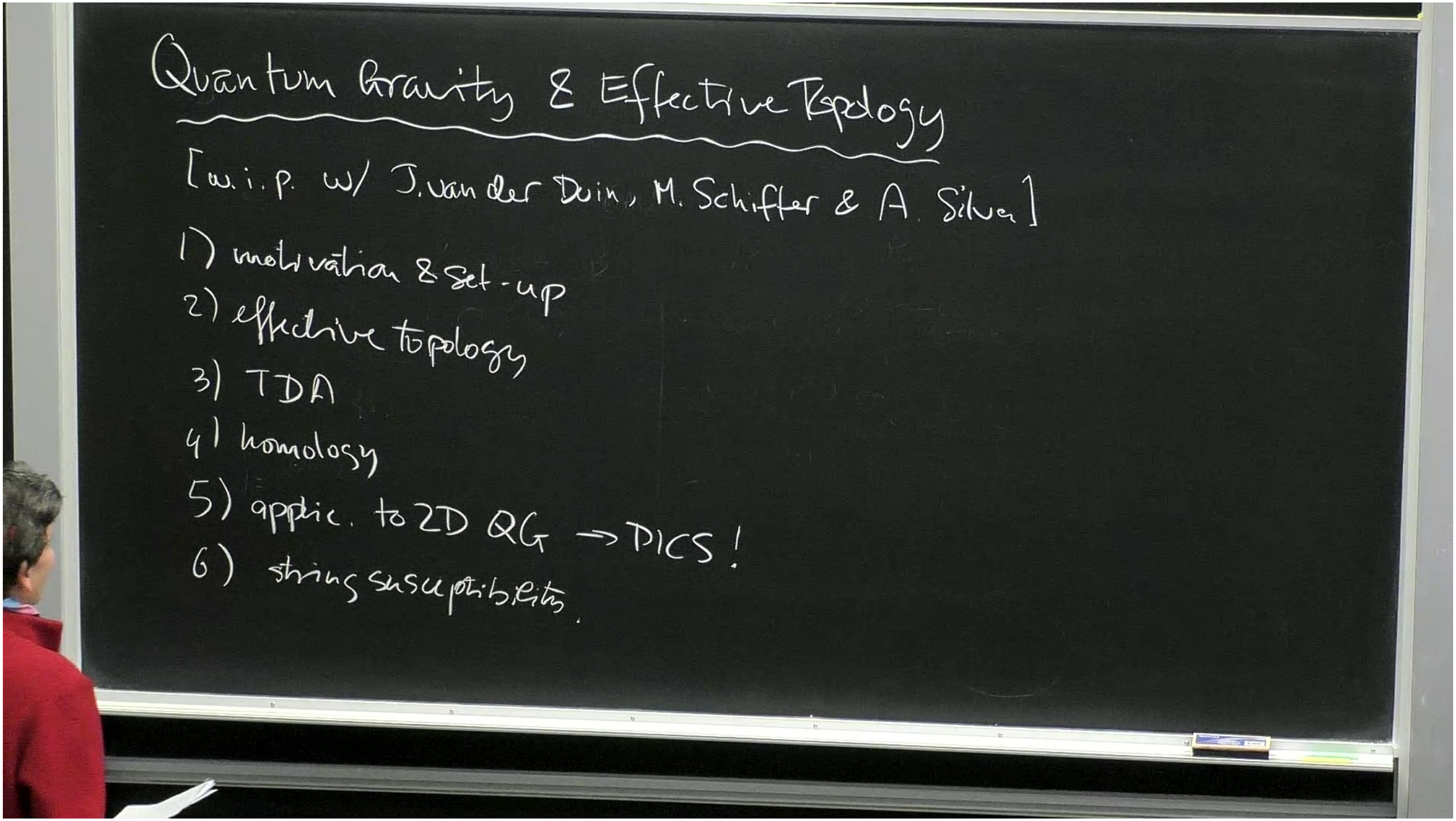
Abstract:

My presentation will introduce a new methodology to characterize properties of quantum spacetime in a strongly quantum-fluctuating regime, using tools from topological data analysis. Starting from a microscopic quantum geometry, generated nonperturbatively in terms of dynamical triangulations (DT), we compute the homology of a sequence of coarse-grained versions of the geometry as a function of the coarse-graining scale. This gives rise to a characteristic "topological finger print" of the quantum geometry. I discuss the results for Lorentzian and Euclidean 2D quantum gravity, defined via lattice quantum gravity based on causal and Euclidean DT. For the latter, our numerical analysis reproduces the well-known string susceptibility exponent governing the scaling behaviour of the partition function.

[Joint work with Jesse van der Duin, Marc Schiffer and Agustin Silva, to appear.]

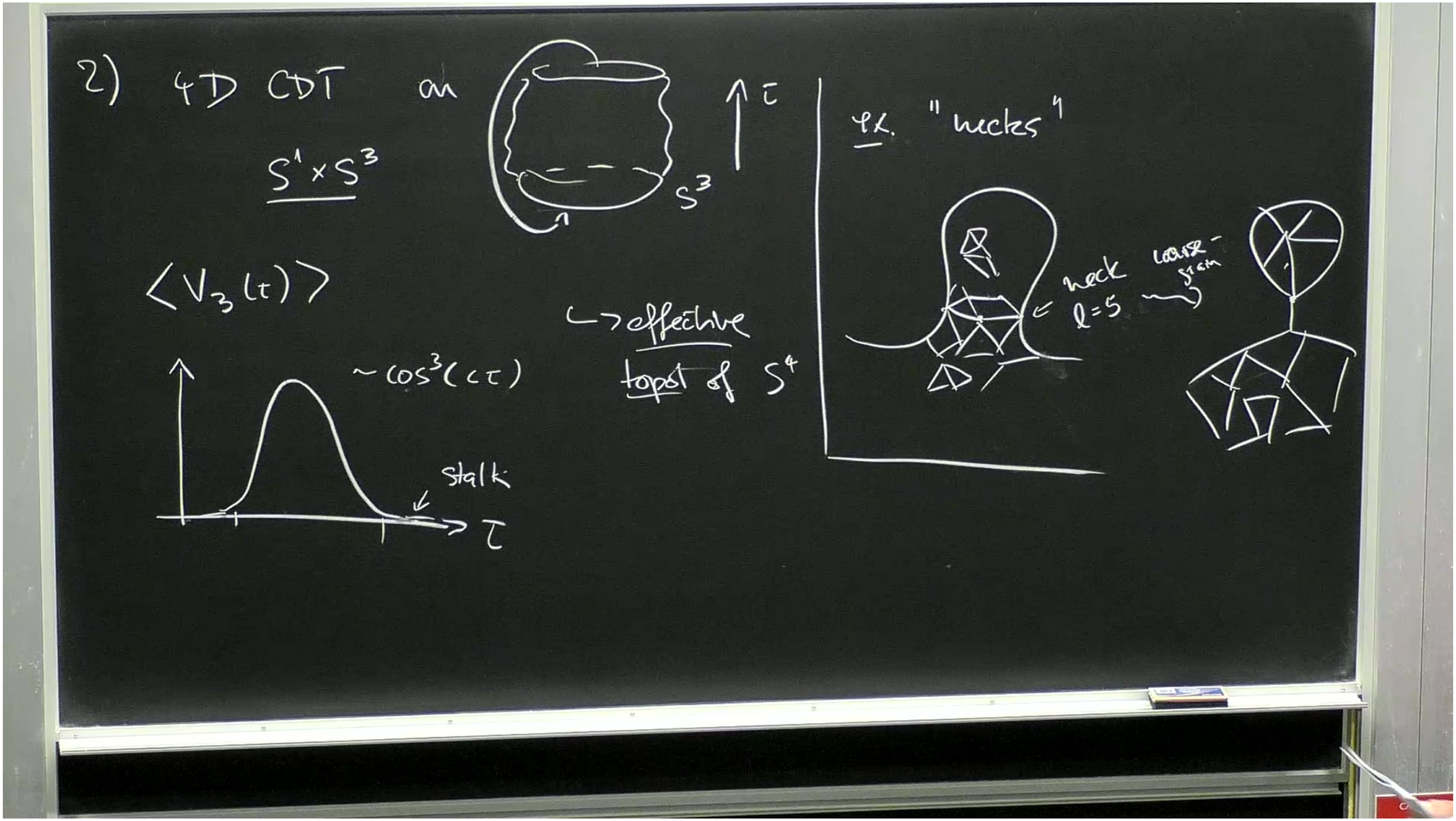

Quantum Gravity & Effective Topology
[w.i.p. w/ J. van der Duin, M. Schiffer & A. Silva]
1) motivation & set-up
2) effective topology
3) TDA
4) homology
5) applic. to 2D QG → PICS!
6) string susceptibility.

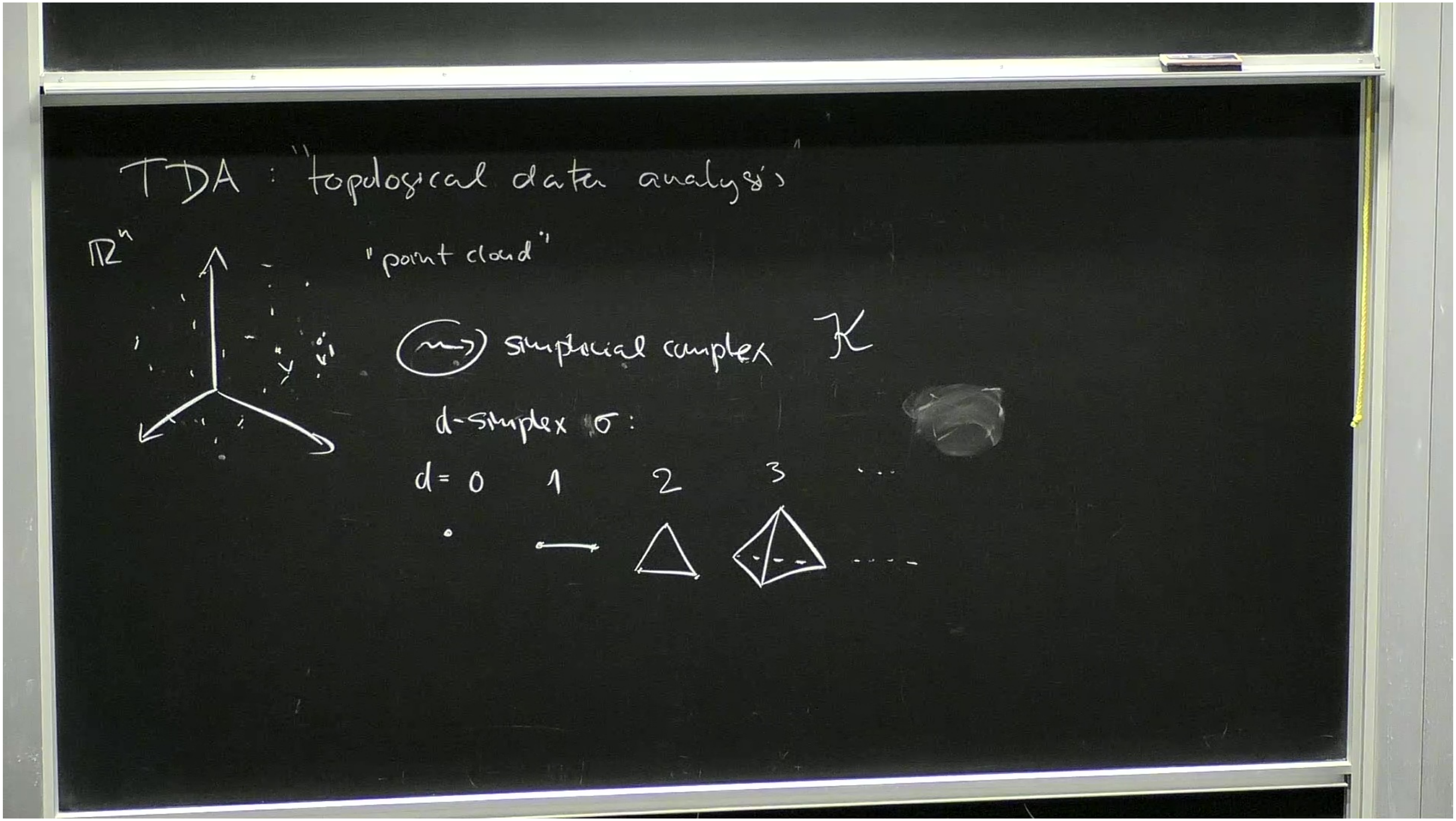
lattice QG, based on CDT, $Z = \int \mathcal{D}[g]\, e^{iS^{EH}[g]}$

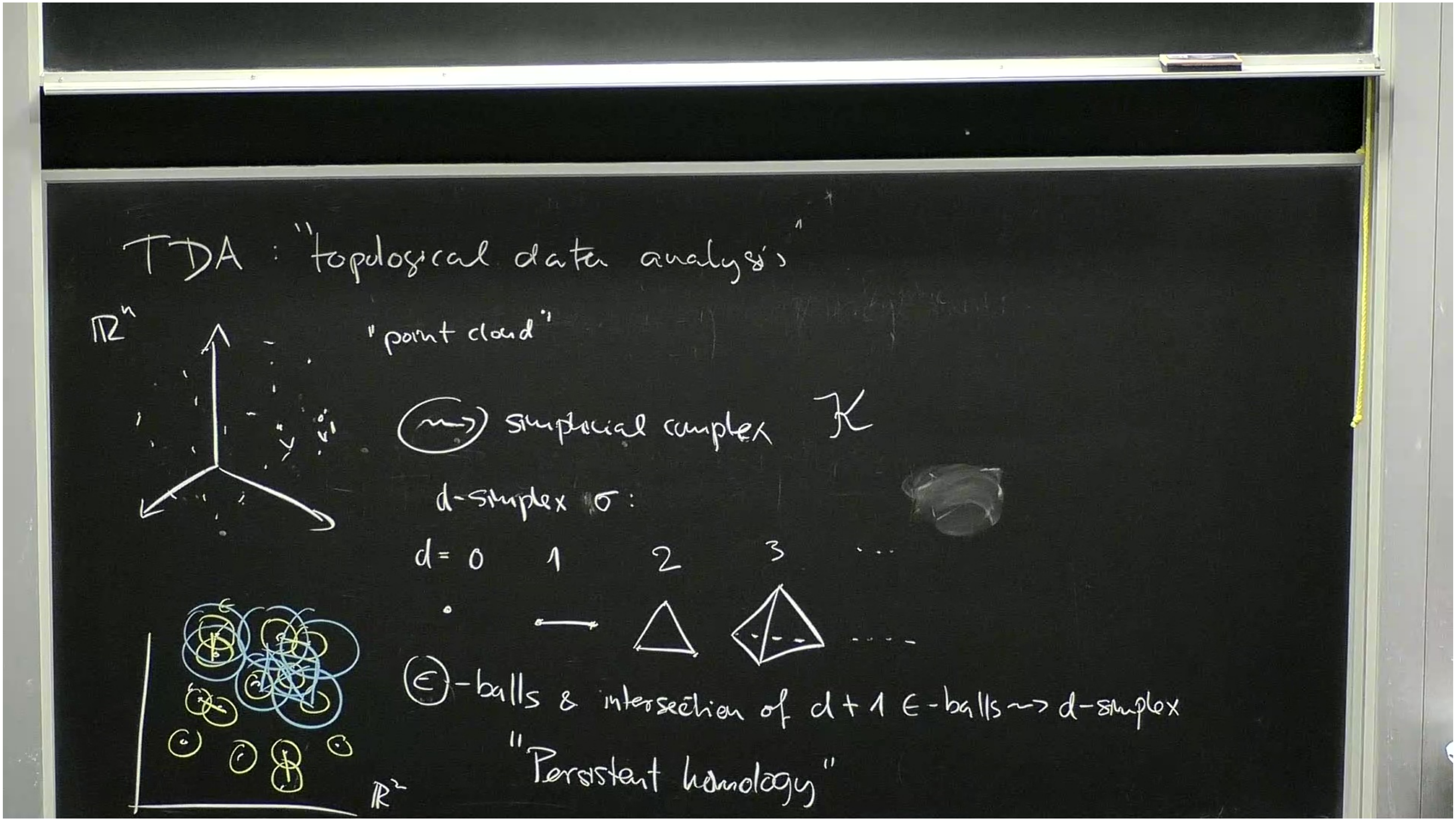
$$\hookrightarrow Z^{CDT} = \lim_{a \to 0} \sum_{T} \frac{1}{C(T)} e^{-S^{EH}_{eu}[T]} \qquad (NP)$$

(T ↑ causal)

- dynamical lattices
- exact "lattice diffeos" ~ exact relabelling
- well-defined Wick rotation $\rightsquigarrow$ MCMC


2) 4D CDT on
S^1 × S^3
S^3
τ
ex. "necks"
⟨V_3(t)⟩
~cos^3(cτ)
↪ effective
topol of S^4
neck
l=5
coarse-grain
stalk
τ


TDA : "topological data analysis"
$\mathbb{R}^n$
"point cloud"
$\rightsquigarrow$ simplicial complex K
d-simplex σ :
d = 0 1 2 3 ...


TDA : "topological data analysis"
$\mathbb{R}^n$
"point cloud"
$\rightsquigarrow$ simplicial complex $\mathcal{K}$
d-simplex σ :
d = 0 1 2 3 ...
ϵ
ϵ-balls & intersection of d + 1 ϵ-balls $\rightsquigarrow$ d-simplex
"Persistent homology"
$\mathbb{R}^2$

Homology p-chain: sum of p-simplices σ_i, $c = \sum_i a_i \sigma_i$ $\quad a_i \in \{0, 1\}$

$\hookrightarrow$ group $C_p(\mathcal{K})$:

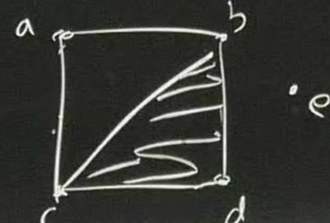
"boundary map" $\partial_p : C_p \to C_{p-1}$: $\sigma^{(p)} = \{v_0, \dots, v_p\}$. $\partial_p \sigma^{(p)} = \sum_{i=0}^{p} \{v_0, \dots, \hat{v}_i, \dots, v_p\}$

p-cycle c : $\partial_p c = 0 \Rightarrow Z_p(\mathcal{K})$

p-boundary c : $c = \partial_{p+1} d \Rightarrow B_p(\mathcal{K})$

} homology group is $H_p := \dfrac{Z_p}{B_p}$

since $\partial_p \partial_{p+1} d = 0$



Betti number β_i =

= rank H_i =

= rank Z_i − rank B_i

β_0 = # components [2]

β_1 = # loops [1]

β_2 = # 2dim holes [0]

β_2 = # 2dim holes [O]

2D QG: $Z = \sum_{(\text{causal})\,T} \frac{1}{C(T)} e^{-\lambda N_2(T)}$, $\langle O \rangle = \frac{1}{Z} \sum_{T} \frac{1}{C(T)} O(T) e^{-\lambda N_2(T)}$

DT on S^2  $\beta_0 = 1, \beta_1 = 0, \beta_2 = 1$

CDT on T^2  $\beta_0 = 1, \beta_1 = 2, \beta_2 = 1$

$\mathcal{S} \subset V(T)$ "evenly spread" $\sim \delta$

$\delta = 2, 3, 4, \ldots$

Voronoi cells

dual Delaunay triangul

(coarse grained)

Quantum Gravity and Effective Topology

Plots & Figures

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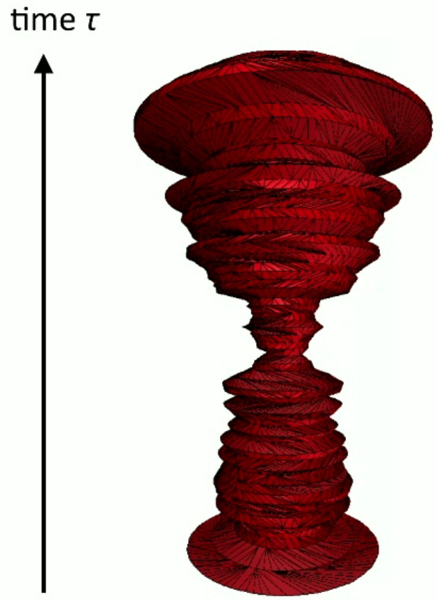
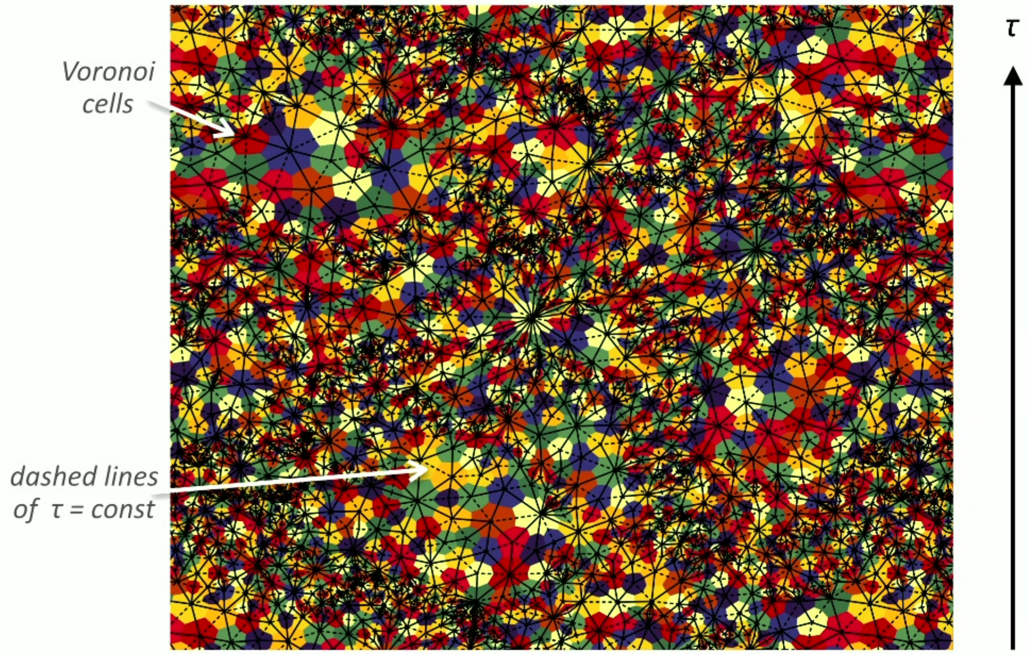
Radboud University (NL) & Perimeter Institute

PI Quantum Gravity Seminar **Waterloo, 22 May 2025**

Illustrating the Voronoi cell covering, CDT lattice

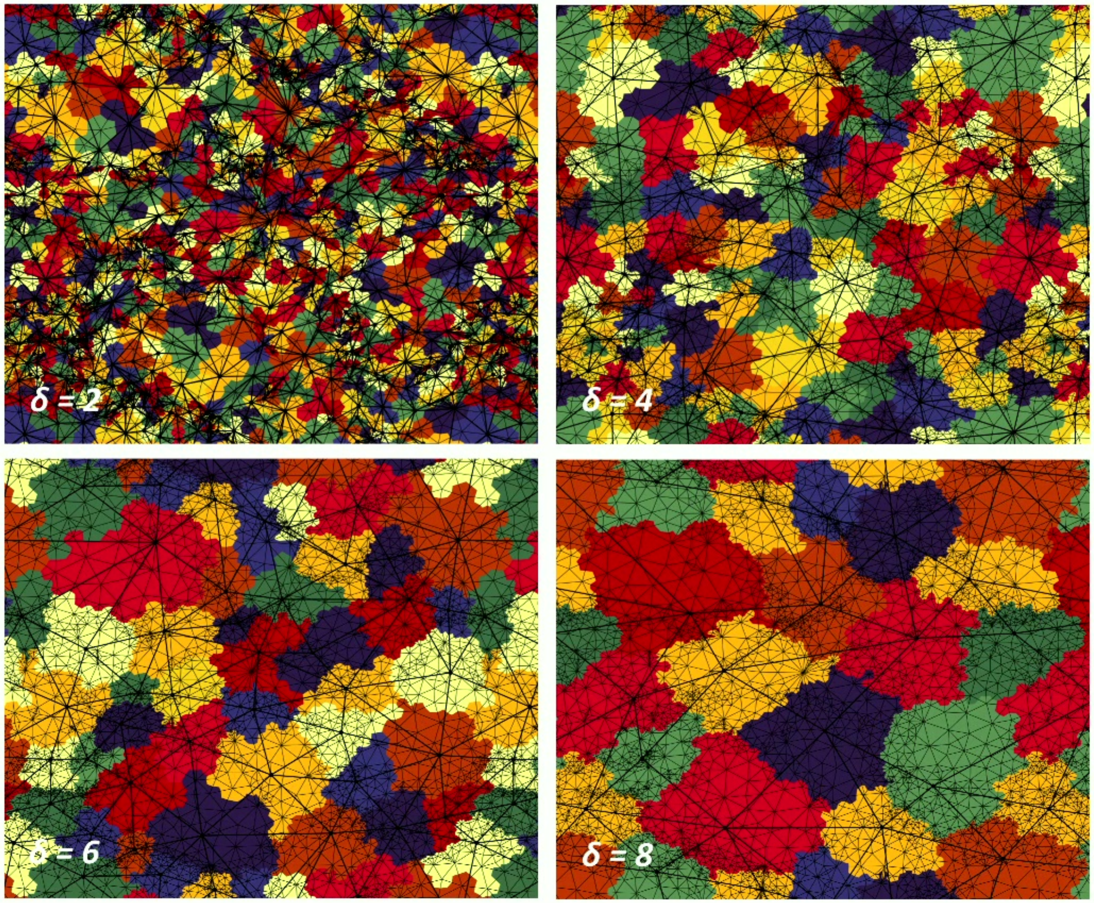


typical CDT configuration of a cut-open torus — spatial universe of fluctuating size



no coarse-graining: initial triangulation at resolution $\delta = 1$, for volume $N_2 = 5040$ and time extension $T = 35$ [harmonic embedding on periodic rectangle]

Voronoi cell covering, CDT lattice: coarse-graining



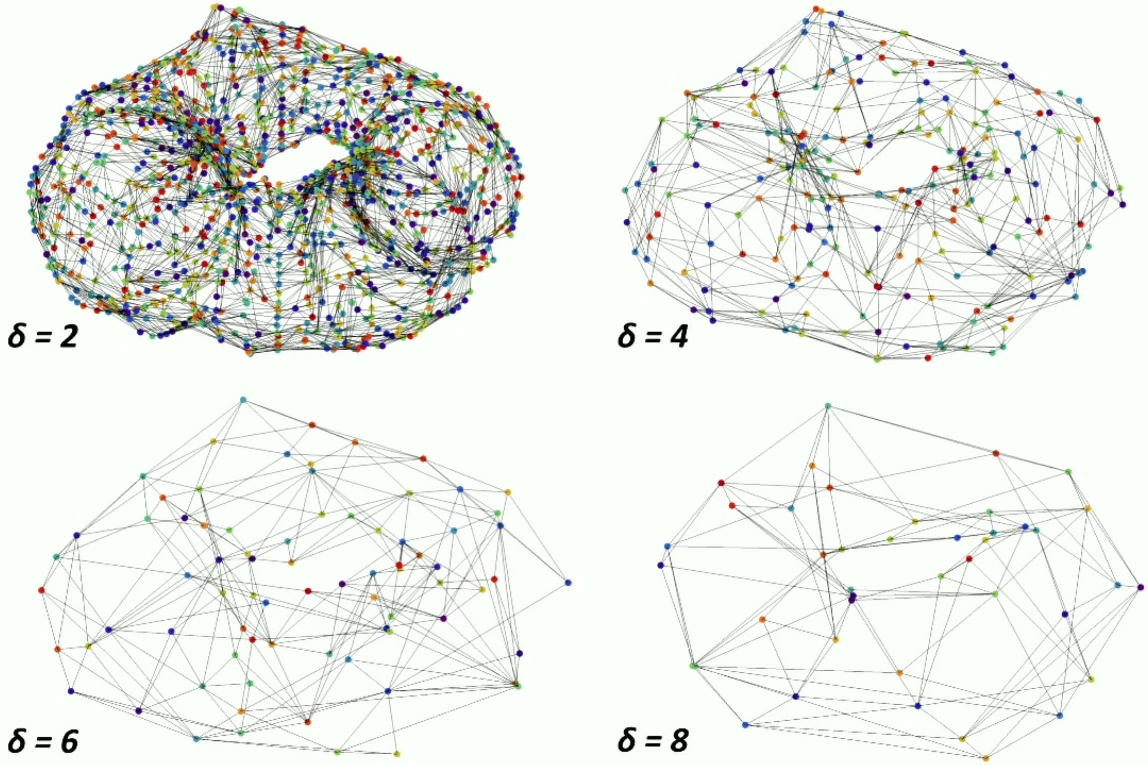
coarse-graining at resolution *δ = 2, 4, 6 and 8*:

fat dots = evenly spread sample points,

thin edges = original triangulation

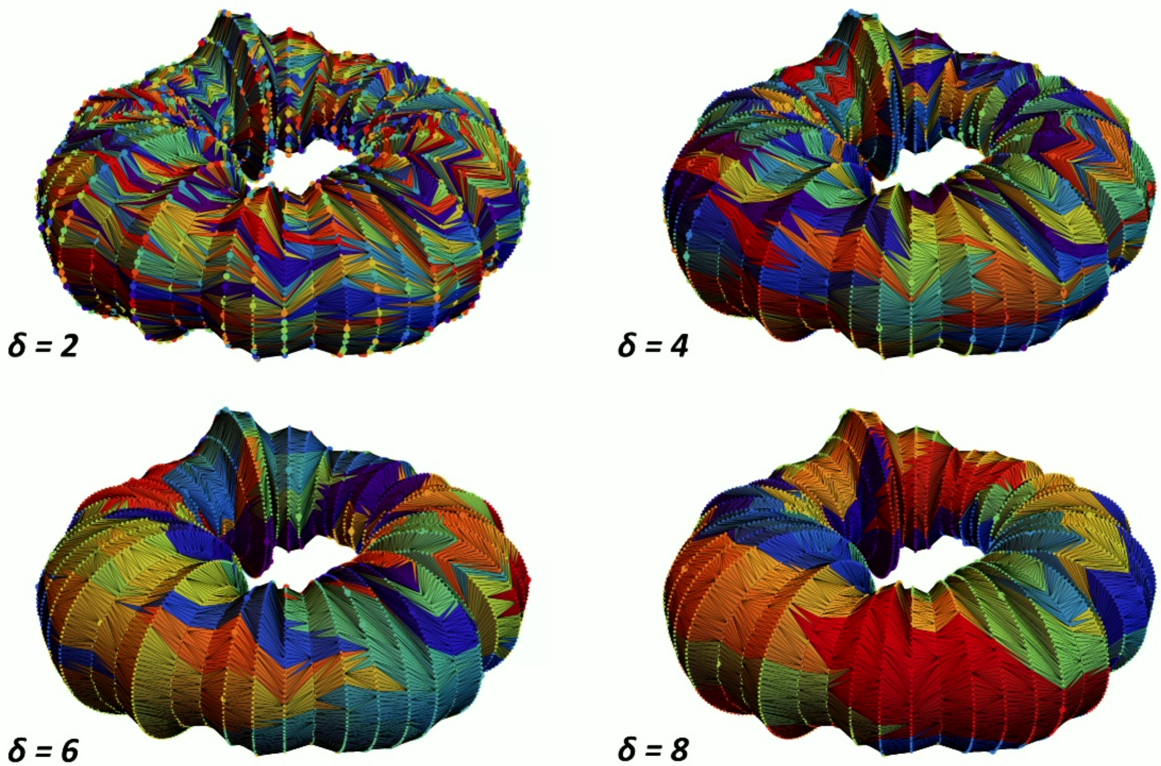
thick edges = dual Delaunay triangulation

CDT lattice: coarse-grained triangulations I



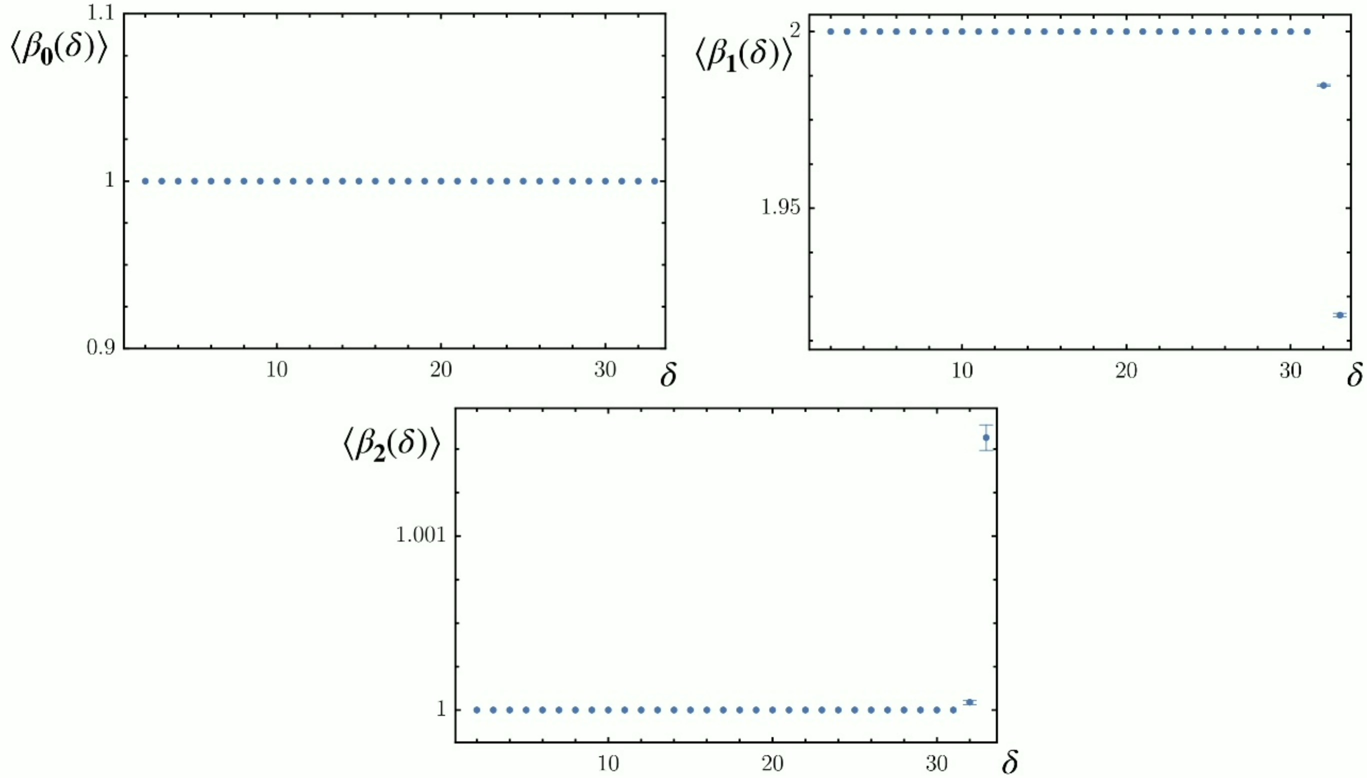
coarse-grained torus embedded in 3D, at resolution *δ = 2, 4, 6 and 8* *[N_2 = 10k, T = 52]*
showing only evenly spread sample points and edges of the coarse-grained Delaunay triangulation

CDT lattice: coarse-grained triangulations II



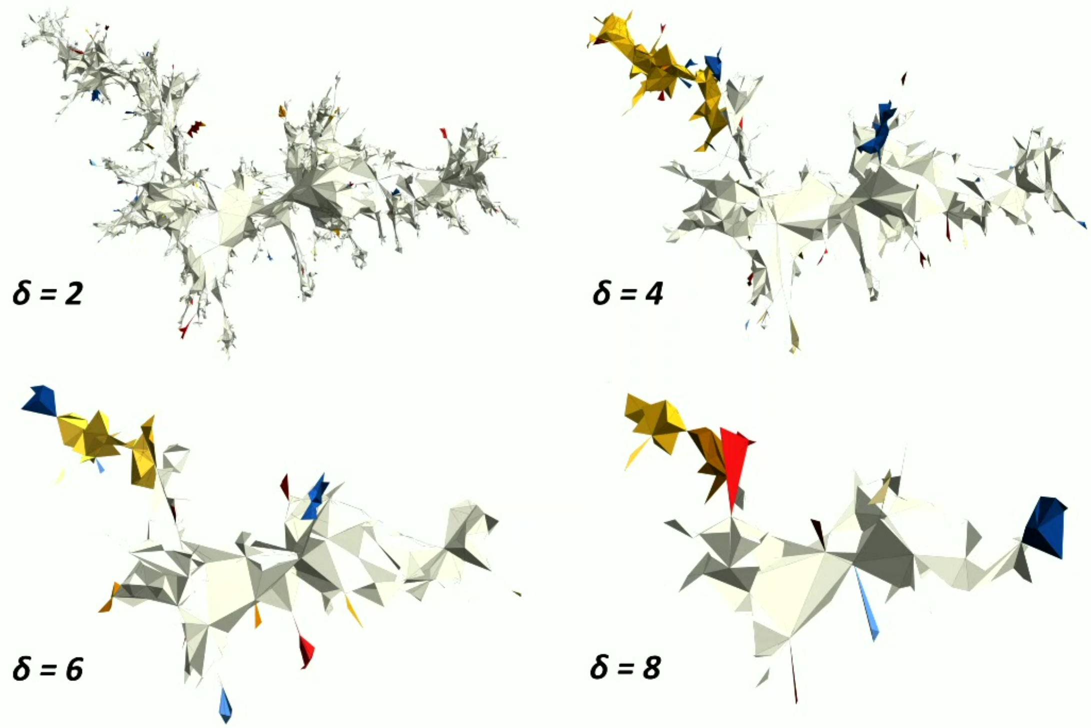
coarse-grained torus embedded in 3D, at resolution *δ = 2, 4, 6 and 8*
showing the evenly spread sample points and associated Voronoi cells

Effective homology:
Betti numbers for 2D Lorentzian QG



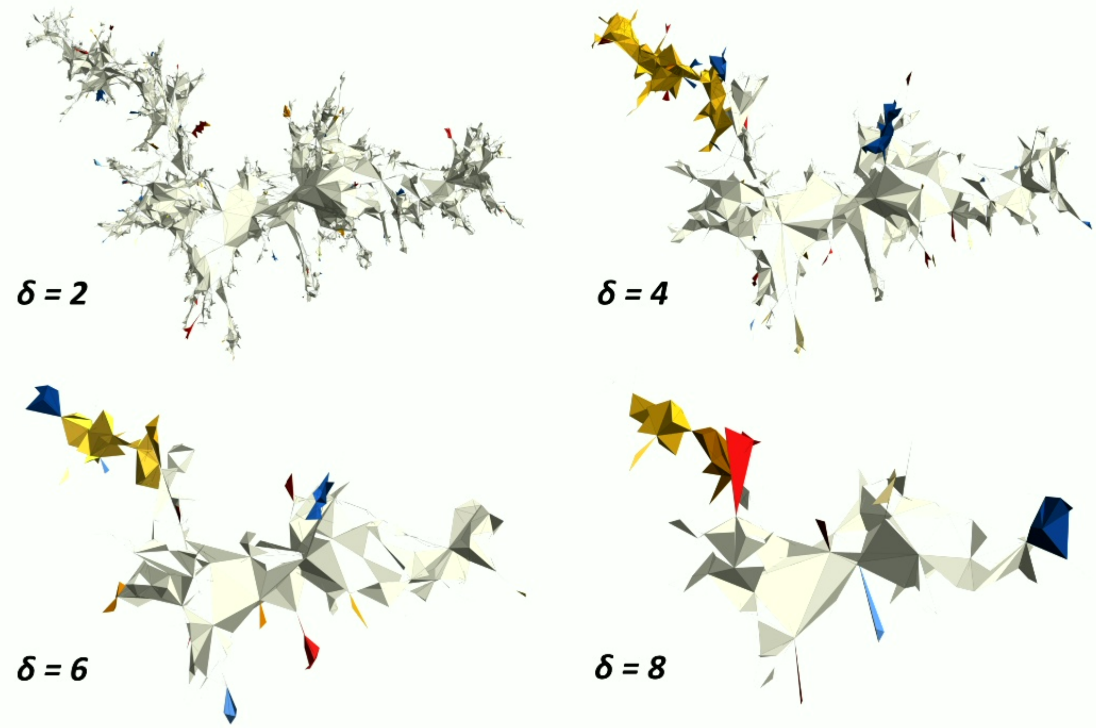
Betti numbers $\langle\beta_i(\delta)\rangle$ as a function of the resolution $\delta \in$ *[2, 34]* *[N_2 = 50k, T = 63]*

DT lattice: coarse-grained triangulations



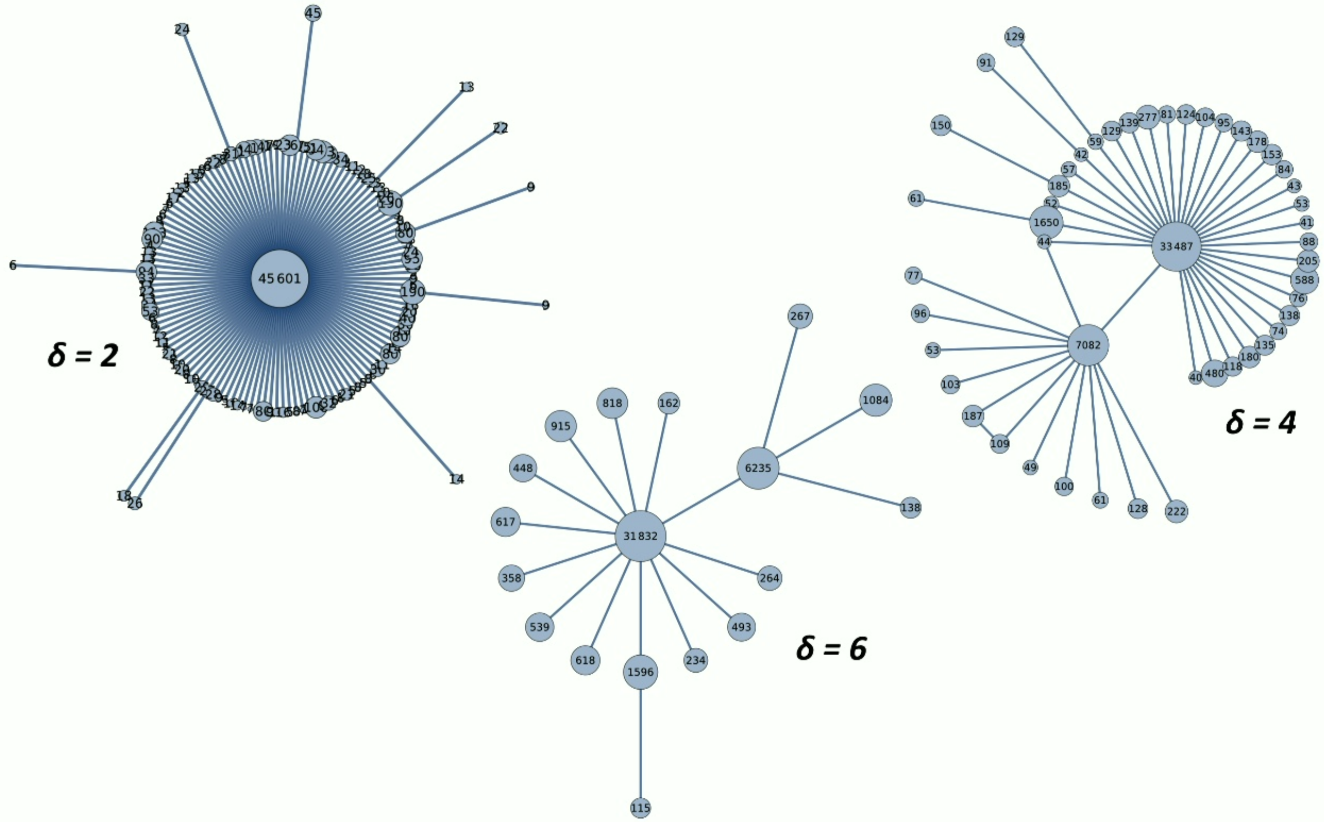
coarse-grained sphere embedded in 3D, at resolution *δ = 2, 4, 6 and 8* *[$N_2 = 50k$]*
showing the "mother universe" (grey) and pinched "bubbles" (coloured)

DT lattice: coarse-grained triangulations



coarse-grained sphere embedded in 3D, at resolution *δ = 2, 4, 6 and 8* *[N_2 = 50k]*
showing the “mother universe” (grey) and pinched “bubbles” (coloured)

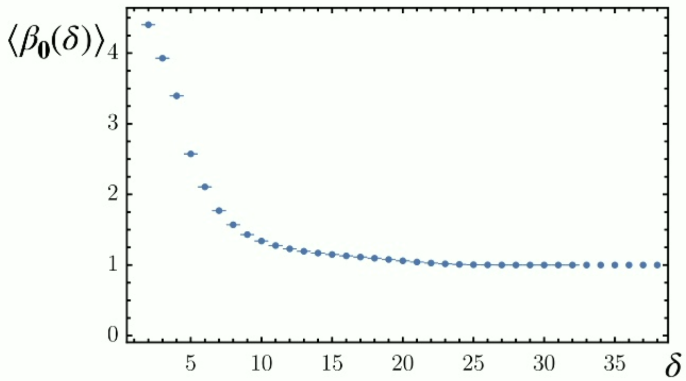
DT lattice: illustrating the bubble structure

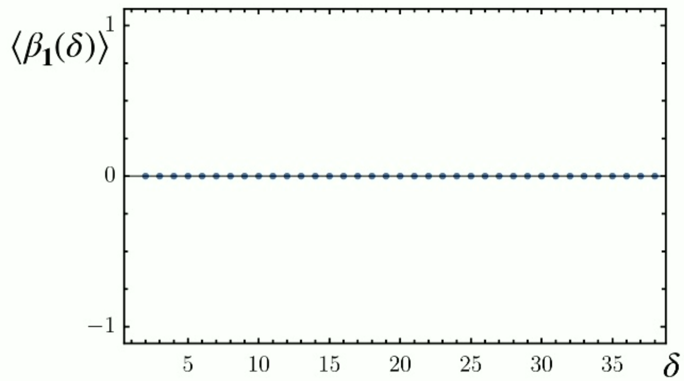


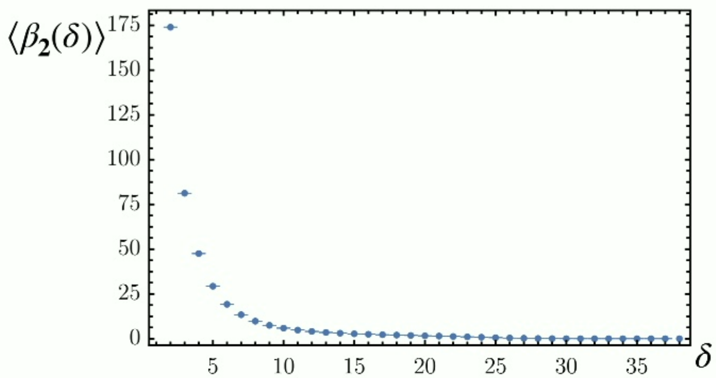
"Bubble graphs" at resolution *δ = 2, 4 and 6* *[N_2 = 50k]*

connectivity + volumes of the "mother universe" and of pinched bubbles (in original lattice units)

Effective homology: Betti numbers for 2D Euclidean QG

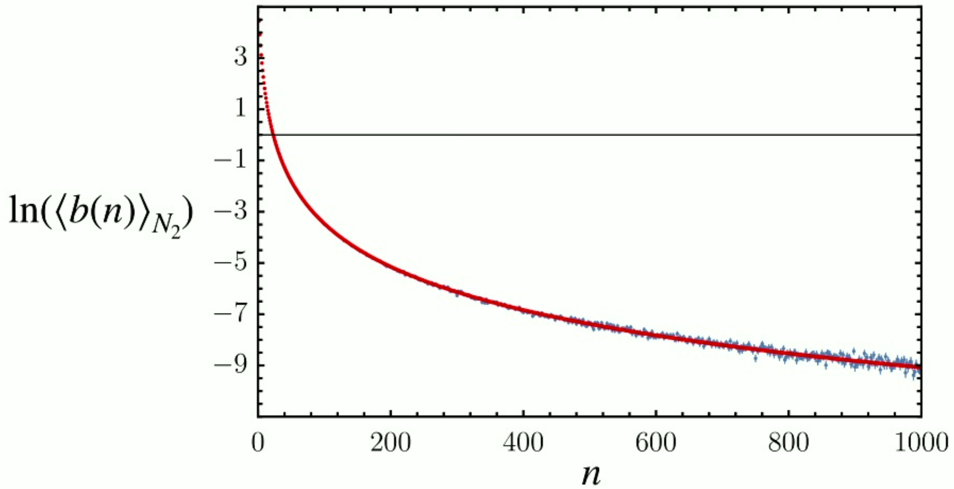






Betti numbers $\langle\beta_i(\delta)\rangle$ as a function of the resolution $\delta \in$ *[2, 40]* *[N_2 = 50k]*

Measuring γ_{str} from bubble volumes at $\delta = 2$



fitting to $\ln(\langle b(n)\rangle_{N_2}) = \alpha + (\gamma_{\rm str} - 2)\ln\Big(n(1 - \frac{n}{N_2})\Big)$

for coarse-grained volume $N_2 = 15362$ yields $\gamma_{\rm str} = -0.4550 \pm 0.0042$

in good agreement with the known value $\gamma_{\rm str} = -1/2$!