Title: Extra Lecture - Quantum Matter, PHYS 777 2/2

Speakers: Chong Wang

Collection/Series: Quantum Matter (Elective), PHYS 777, March 31 - May 2, 2025

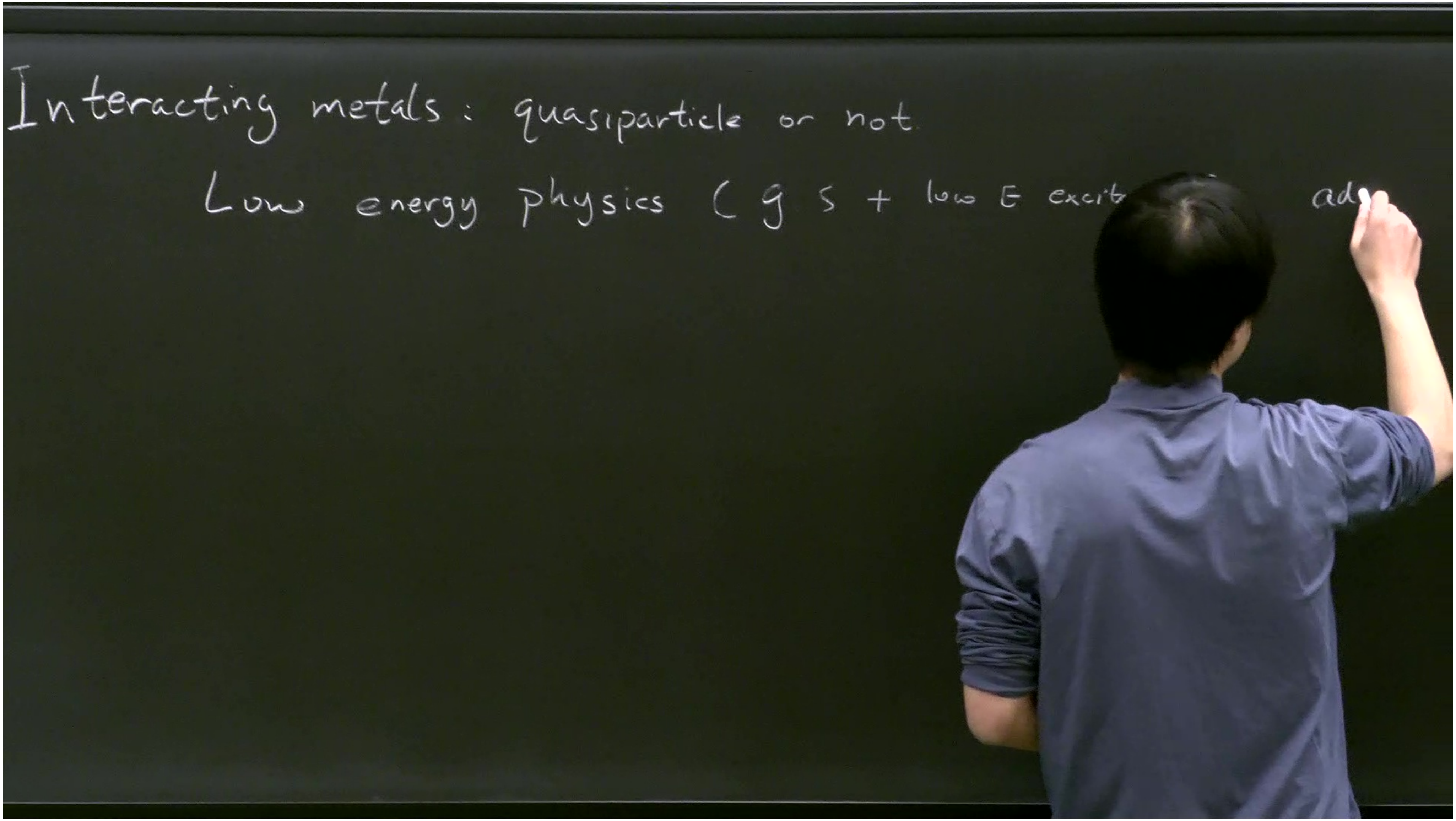
Subject: Condensed Matter

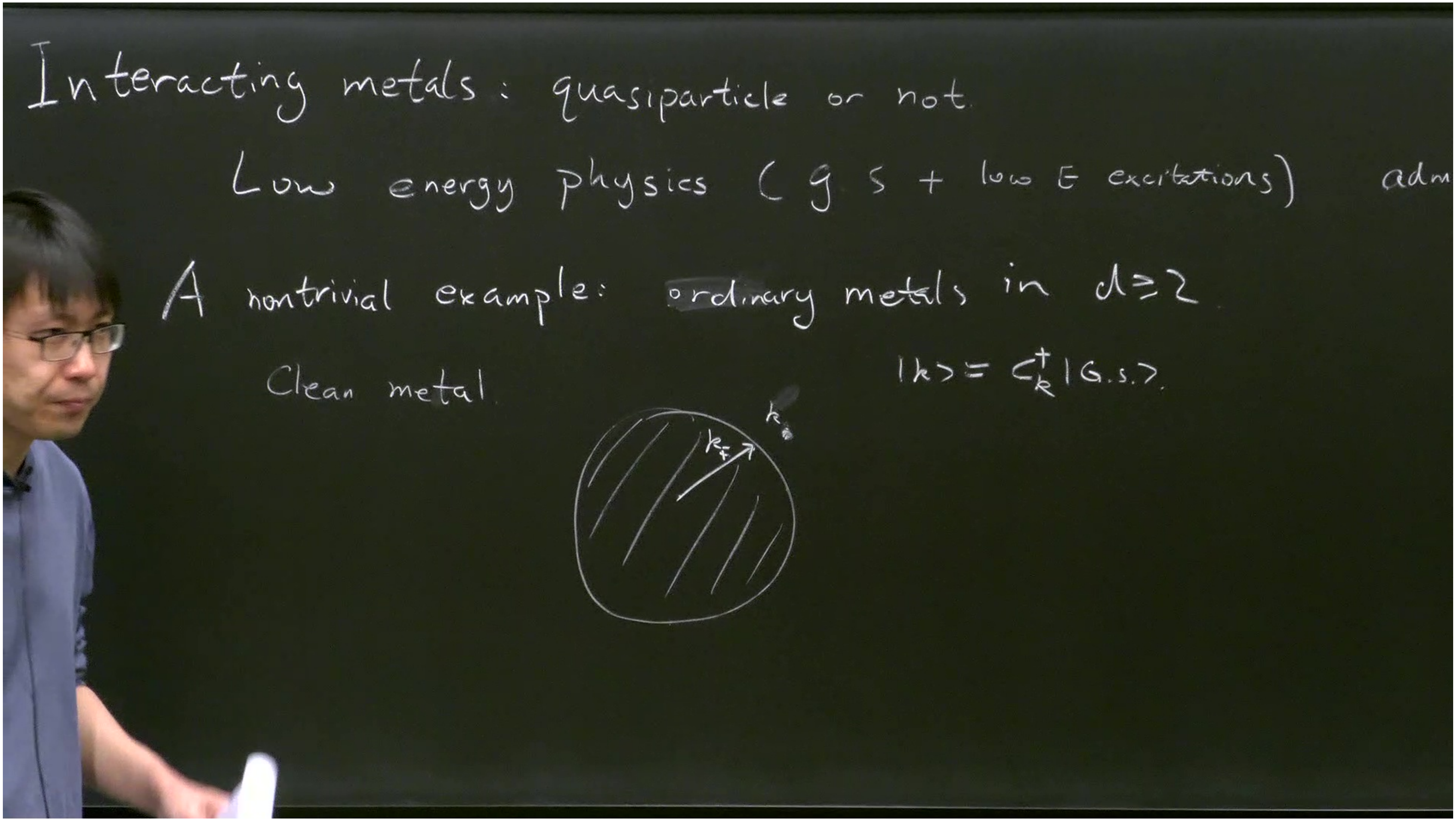
Date: May 09, 2025 - 2:00 PM

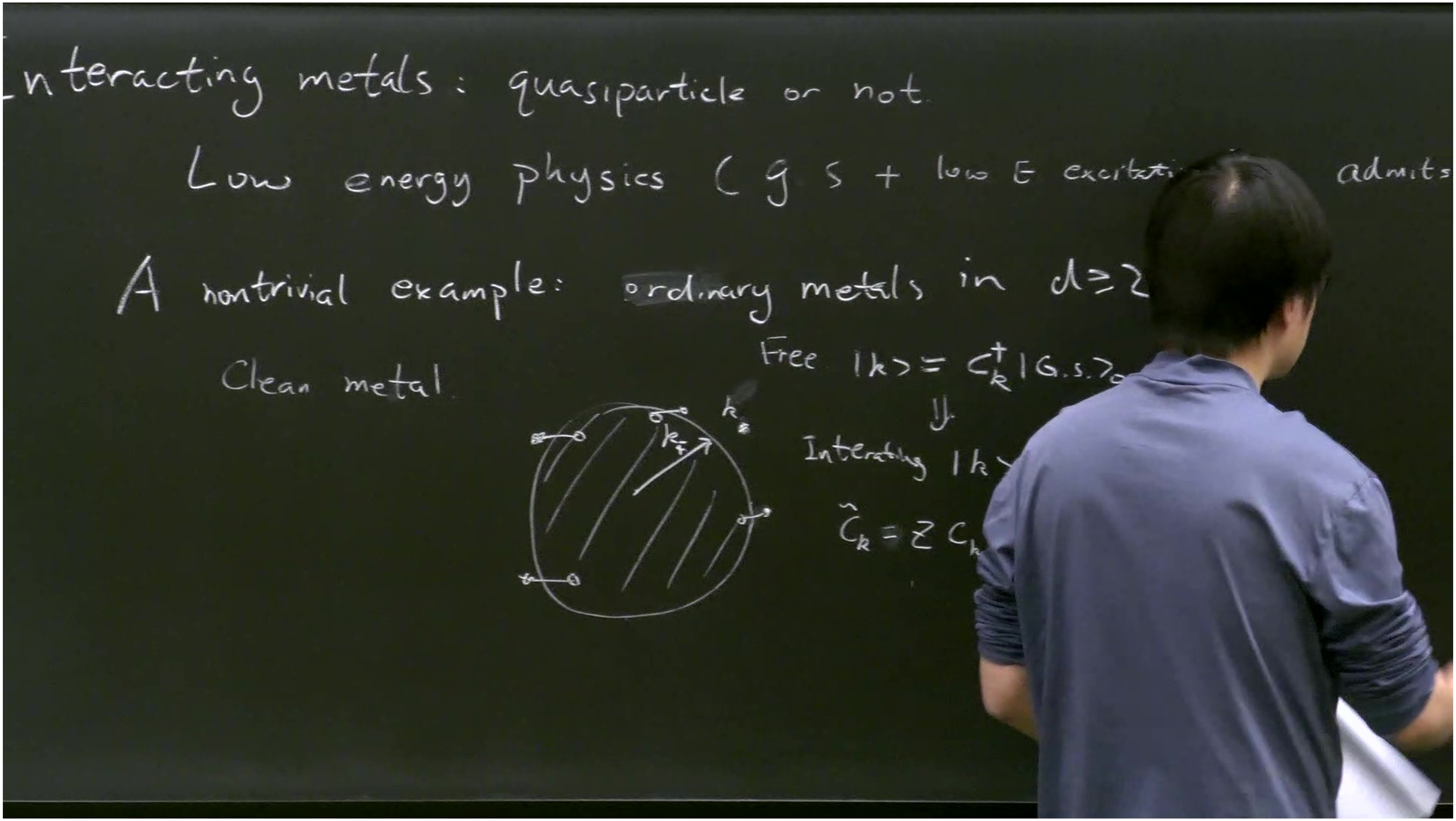
URL: https://pirsa.org/25050030

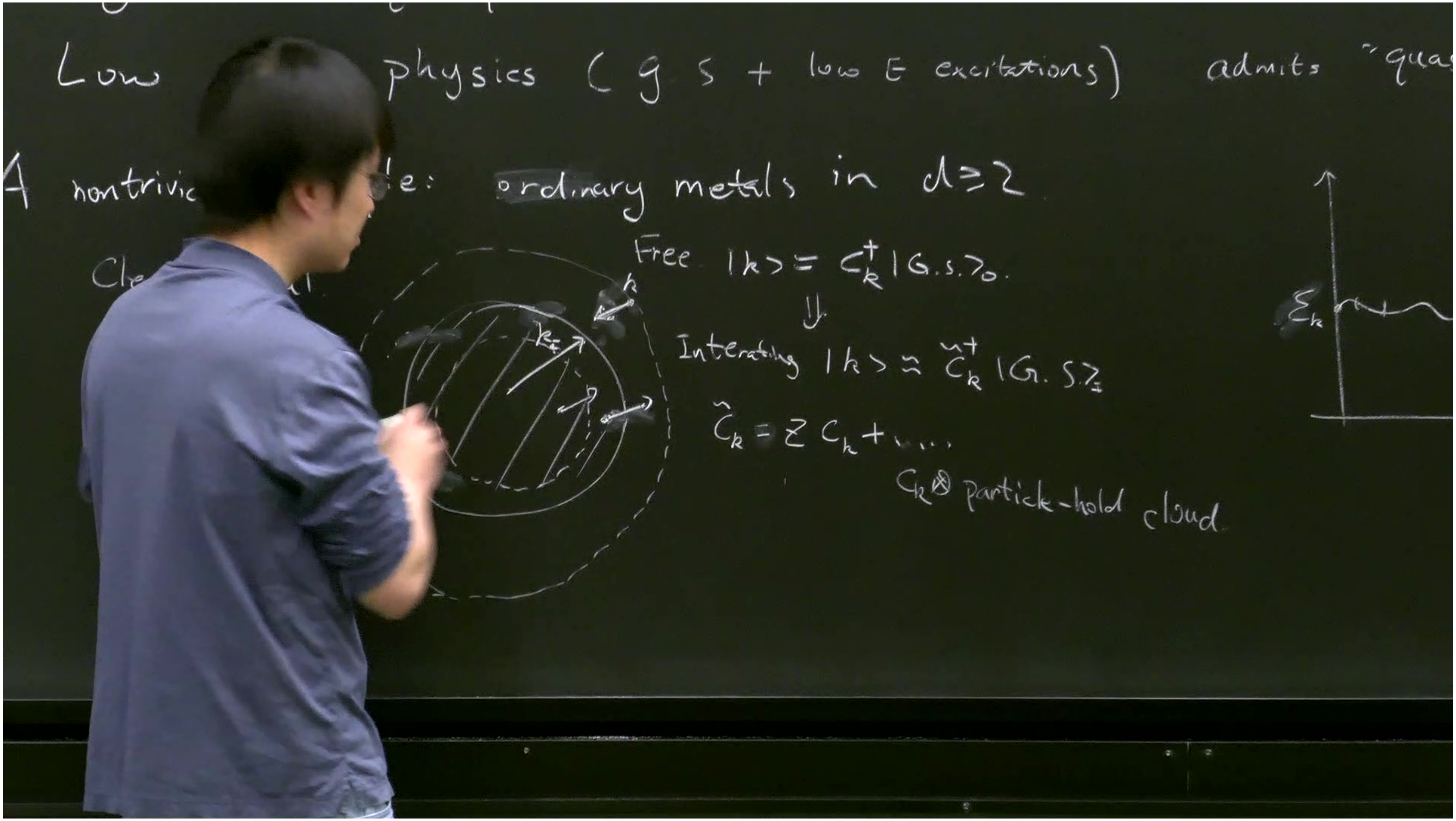
Abstract:

Optional


Interacting metals : quasiparticle or not
Low energy physics (g s + low E excit
ad


Interacting metals: quasiparticle or not.
Low energy physics (g s + low E excitations) adm
A nontrivial example: ordinary metals in $d \geq 2$
Clean metal
$|k\rangle = c_k^\dagger |G.s.\rangle$
k_F
k

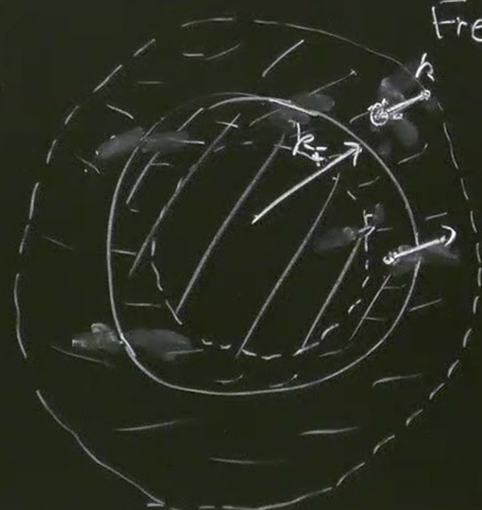

nteracting metals: quasiparticle or not
Low energy physics (g s + low E excit
admits
A nontrivial example: ordinary metals in d≥2
Free |k> = c†k |G.s.>
⇓
Interacting |k
c̃k = Z c
Clean metal
kF
k


Low physics (g s + low E excitations) admits
nontrivi
e: ordinary metals in d≥2
Free |k> = $c_k^\dagger$ |G.s.>$_0$.
Interacting |k> ≃ $\tilde{c}_k^\dagger$ |G. S.>$_I$
$\tilde{c}_k = Z c_k + \ldots$
c_k ⊗ particle-hole cloud
Σ_k

Low energy physics (g.s + low E excitations) admits

A nontrivial example: ordinary metals in $d \geq 2$

Clean metal



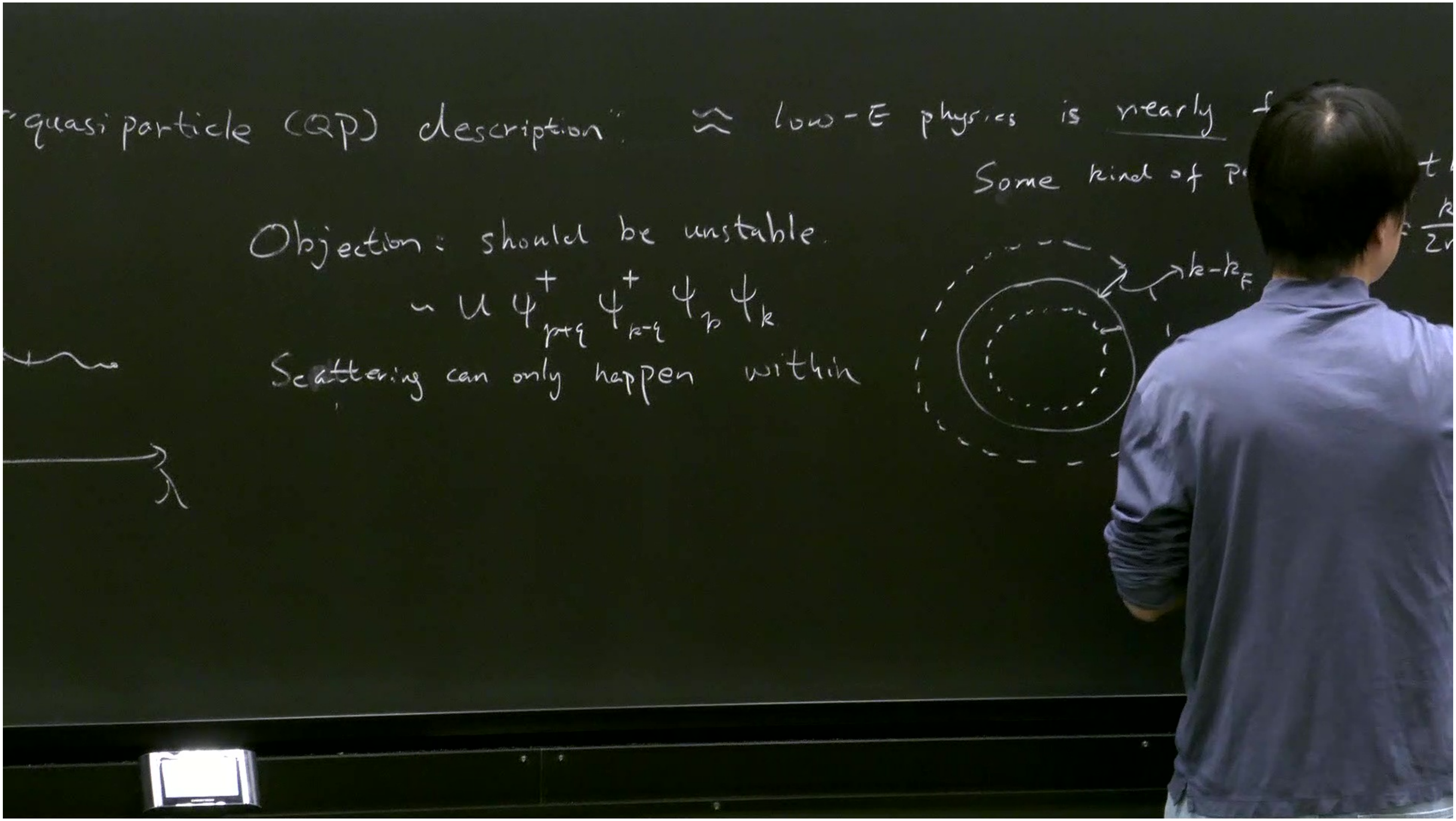
Free $|k\rangle = c_k^\dagger |G.s.\rangle_0$.

$\Downarrow$

Interacting $|k\rangle \simeq \tilde{c}_k^\dagger |G.S.\rangle_I$

$\tilde{c}_k = Z c_k + \dots$

$c_k \otimes$ particle-hole cloud.

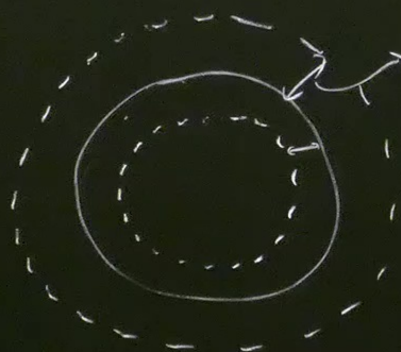

quasiparticle (QP) description: low-E physics is nearly f
Some kind of p
Objection: should be unstable.
$\sim U \psi^{+}_{p+q} \psi^{+}_{k-q} \psi_{p} \psi_{k}$
Scattering can only happen within
$k-k_F$
λ

ation": $\approx$ low-E physics is nearly free.

Some kind of perturbation th. makes sense.

uld be unstable.

$$\varepsilon_k = \frac{k^2}{2m} - \frac{k_F^2}{2m} \approx v_F \cdot (k - k_F)$$



$^+ \psi^+_{p+q} \psi^+_{k-q} \psi_p \psi_k$

ly happen within

Scattering rate $\sim \dfrac{1}{\tau_k} \propto$ # of states available for scattering

$$\propto (k-k_F)^2 \sim \epsilon_k^2$$

quasiparticle lifetime

Crucial: $\epsilon_k^2 << \epsilon_k$ if ϵ_k is small.

$\Rightarrow$ QP asymptotically stable at low E.

In terms correlation function (Green function)

$$G(k,\omega) = \langle c^\dagger_{k,\omega} c_{k,\omega} \rangle \simeq \frac{Z \le 1}{\omega - \epsilon_k + \frac{i}{\tau_k}}$$

For a more systematic justification. see e.g. Shankar. "Renormalization approach to interacting fermions".

What's important:

QP: long lived & can be used to label (almost) all low

$|k_1, k_2 \ldots k_3\rangle$ labled through n_k.

$$\Delta E = \sum_k \varepsilon_k \delta n_k + \underbrace{\sum_{k,k'} f_{k,k'} \delta n_k \delta n_{k'}}_{\text{Landau interaction}}$$

One consequence: $\frac{1}{\tau_k} \sim \varepsilon_k^2$: low-T resistivity. $\rho = \underset{\text{disorder}}{\rho(T=0)} + \underset{\text{e-e scat}}{\rho}$

"Strange metal": $\rho = \rho_0 + \alpha T$. , $\frac{1}{\tau} \sim \varepsilon$

lifetime

Crucial: $\varepsilon_k^2 << \varepsilon_k$ if ε_k is small.

$\Rightarrow$ QP asymptotically stable at low E.

In terms correlation function (Green function)

$$G(k,\omega) = \langle c^\dagger_{k\omega} c_{k\omega} \rangle \simeq \frac{Z \le 1}{\omega - \varepsilon_k + \frac{i}{\tau_k}}$$

What's important:

QP: long lived. & can be used to label (almost)

$|k_1, k_2 \dots k_3\rangle$ labled through n_k.

$$\Delta E = \sum_k \varepsilon_k \delta n_k + \underbrace{\sum_{k,k'} f_{k,k'} \delta n_k \delta n_{k'}}_{\text{Landau interaction}}$$

"Landau Fe

One consequence: $\frac{1}{\tau_k} \sim \varepsilon_k^2$. low-T resistivity $\rho = \underset{\substack{\uparrow \\ \text{disorder}}}{\rho(T=0)} + \underset{\substack{\uparrow \\ \text{e-e scattering}}}{\rho} = \rho_0 + \alpha T^2$

"Strange metal". $\rho = \rho_0 + \alpha T$. , $\frac{1}{\tau} \sim \varepsilon$. Definitely no QP. not a Fermi liquid.

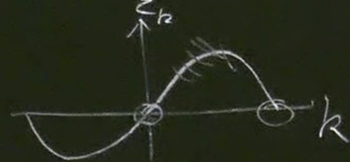
Two ways to organize quantum phases of matter. (g.s. + lo

In terms of # of d.o.f. at low energy.

① "Nothing" - Gapped.

② "Something" - Critical. e.g. CFT.

low-E d.o.f around finite # of momenta

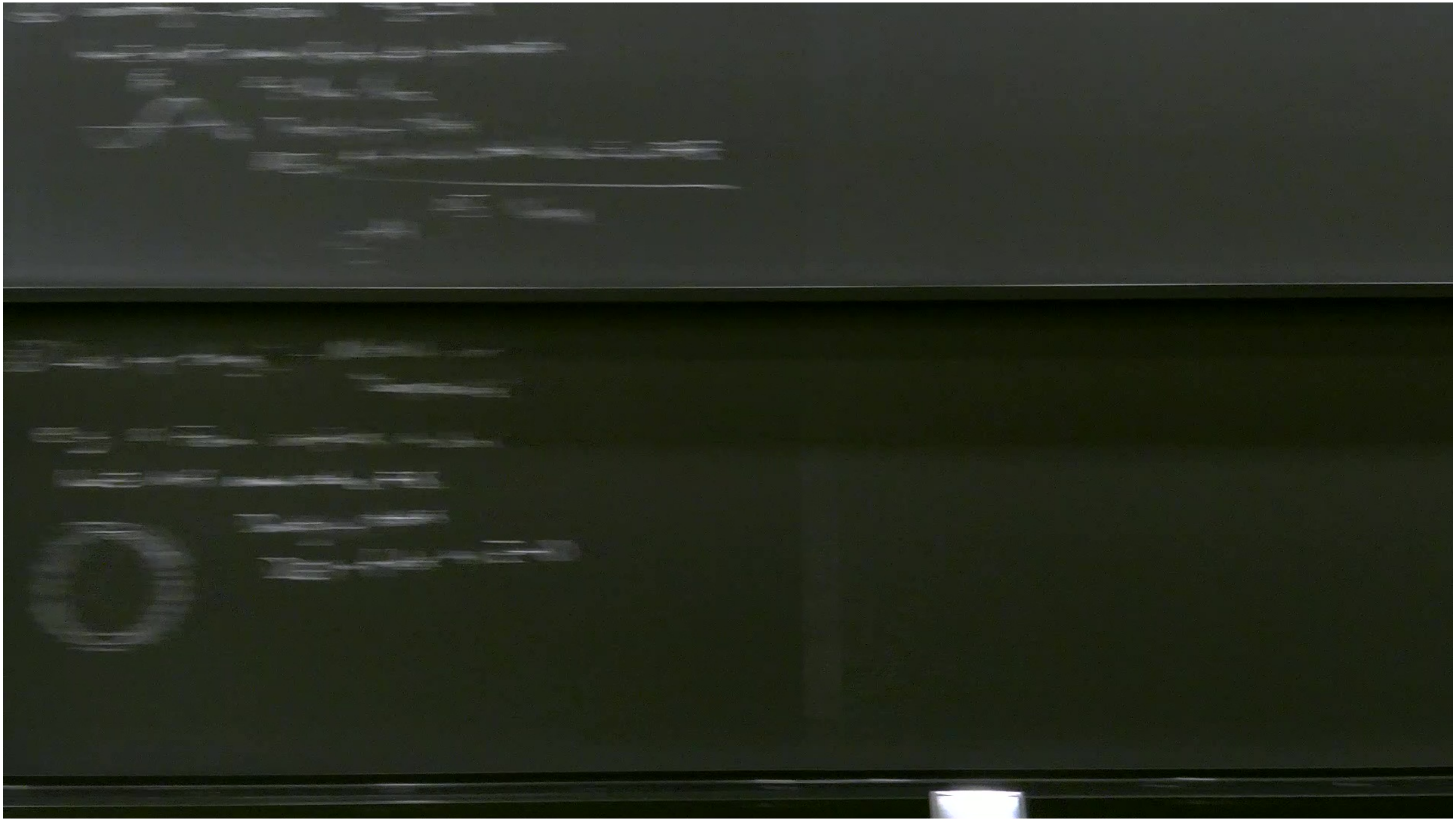


e.g. $\gamma_{k\sim 0}$, $\gamma_{k\sim\pi}$

Density of states

$$D(E) \sim \frac{\#\text{ of Quantum states between } E \text{ and } E+\Delta E}{\Delta E \cdot \text{Volume}}$$

$$\sim E^{d-1}$$



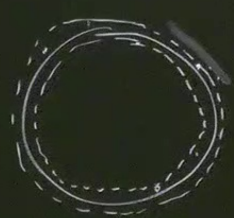
$$D(E) \sim \frac{\#\text{ of Quantum states between } E \text{ and } E+\Delta E}{\Delta E \cdot \text{Volume}}$$

$$\sim E^{d-1}$$

③ "Lots of things": – Metallic or "Compressible"

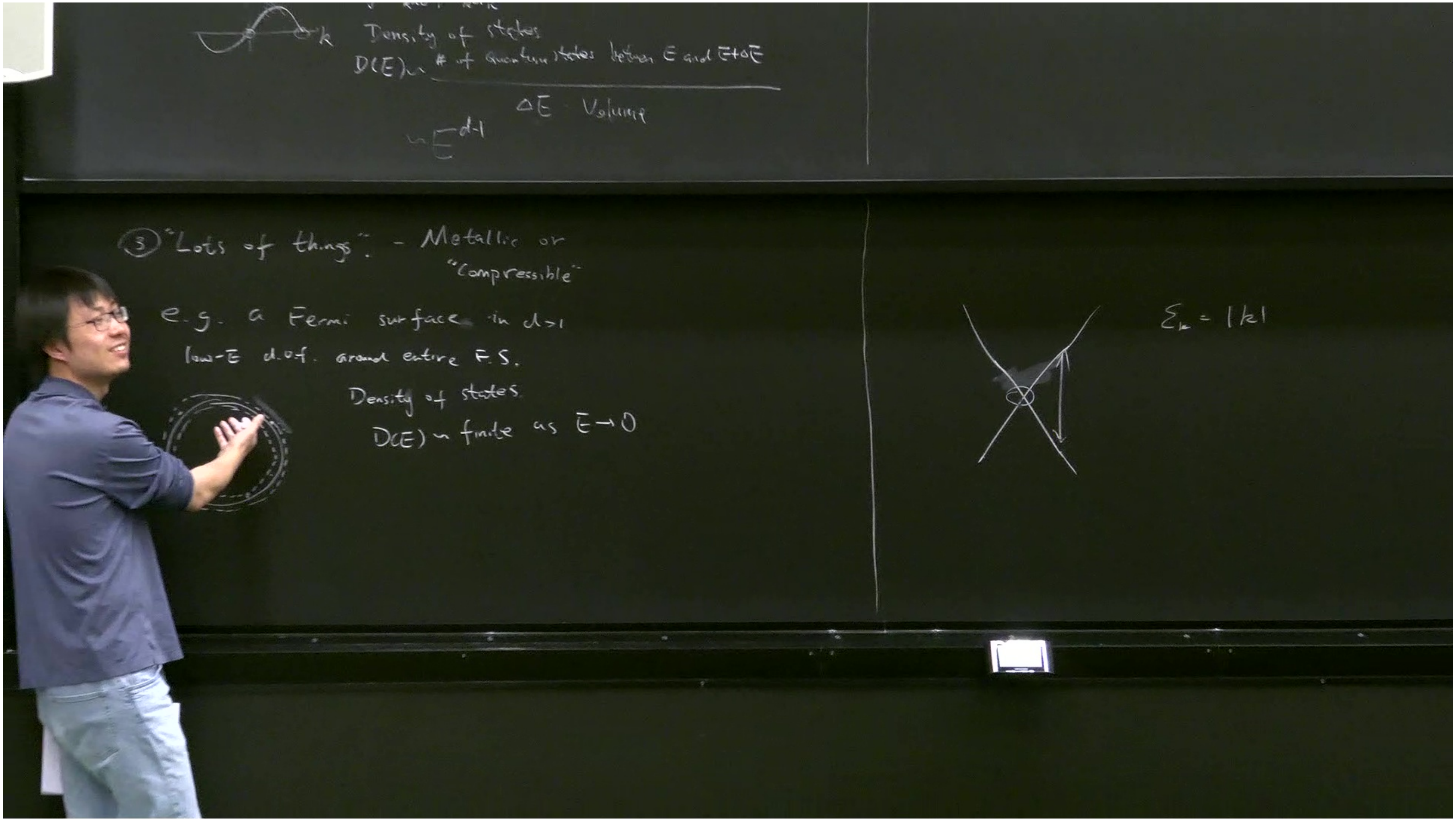
e.g. a Fermi surface in $d>1$

low-E d.o.f. around entire F.S.



Density of states

$D(E) \sim$ finite as $E \to 0$


k
Density of states
D(E) ~ # of quantum states between E and E+ΔE / ΔE · Volume
~ E^{d-1}
③ "Lots of things": - Metallic or "Compressible"
e.g. a Fermi surface in d>1
low-E d.o.f. around entire F.S.
Density of states
D(E) ~ finite as E→0
ε_k = |k|

low E excitations)

In terms of QP description (or not)

(a) QP ✓, in terms of microscopic d.o.f. e.g. Fermi liquid

QP. $\tilde{c}_k \sim Z c_k + \cdots$

(b) QP ✓, but in terms of "collective" or "emergent" d.o.f.

e.g. FQHE (Q.P. ~ anyons)

1d metal, a.k.a. "Luttinger liquid" (Free boson CFT)

(c) No QP at all. e.g. interacting CFT $\langle O(x)\, O(0)\rangle \sim \frac{1}{|x|^{2\Delta}}$

$\Rightarrow \langle O(k)\, O(-k)\rangle \sim \int d^{d+1}x \, \frac{e^{ikx}}{|x|^{2\Delta}} \sim \frac{1}{k^{d+1-2\Delta}}$. if 2Δ not integer, no pole.

$\Rightarrow \langle O(k)\, O(-k)\rangle \sim \int d^{d+1}x \frac{e^{ikx}}{|x|^{2\Delta}} \sim \frac{1}{k^{d+1-2\Delta}}$. if 2Δ not integer, no po

Things we've seen so far:

Short-ranged entangled (invertible) : 1a

SPT. IQHE

Topological orders (Toric code. FQHE). 1b

Quantum critical point (Ising) : 2c

Fermi liquid metal: 3a

"Non-Fermi liquid". : 3c

Two popular modes of NFL.

(1) 2d Fermi surface coupled to $U(1)$ gauge field (or other critical boson

realized in Landau level at $\nu = 1/2$.

$\frac{1}{\tau} \sim \varepsilon$ or $\varepsilon^{2/3}$. See. e.g. Sung-Sik Lee, "Recent Developments in

(2) Sachdev-Ye-Kitaev model (SYK)

Ref. e.g. Sachdev's lecture vedio (Youtube) on "Quantum Phases of Matter"

Fermi surface coupled to $U(1)$ gauge field

realized in Landau level at $\nu = 1/2$.

$\frac{1}{\tau} \sim \varepsilon$ or $\varepsilon^{2/3}$. See. e.g. Sung-Sik Lee, "Recent Develop

dev-Ye-Kitaev model (SYK)

Ref. e.g. Sachdev's lecture vedio (Youtube) on "Quantum Phases of Mat

$$H = \frac{1}{(2N)^{3/2}} \sum_{ijkl=1}^{N} U_{ijkl}\, c_i^{\dagger} c_j^{\dagger} c_k c_l .$$

(0d, all-to-all coupling)

Large-N solvable

Key property: at low E:

$$D(E) \sim \frac{1}{N} \underbrace{\boxed{e^{S_0 N}}}_{\text{exp. large in } N.} \sinh \sqrt{2N\gamma E}$$

If $\exists$ QP description,

$E \sim f(\delta n)$

$O(N)$ Q.P.

$\Rightarrow D(E) \sim \mathrm{Poly}(N) \ll e^{S_0 N}$.