Title: Extra Lecture - Quantum Matter, PHYS 777 1/2

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Collection/Series: Quantum Matter (Elective), PHYS 777, March 31 - May 2, 2025

Subject: Condensed Matter

Date: May 06, 2025 - 2:00 PM

URL: https://pirsa.org/25050029

Abstract:

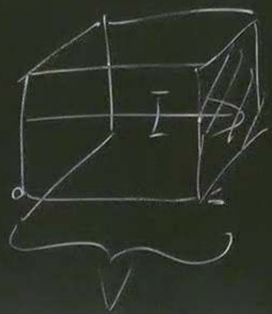
Optional

Disordered Metal I. - Diffusion & Localization in free fermions.

Ref. "Disordered electronic systems", Patrick A. Lee & T.V. Ramakrishnan

Why do metals exist? (at $T=0$)

Electric conductance $\underline{G} := \frac{I}{V}$. Crucial observation: $G = \frac{e^2}{h} \times \underbrace{\text{dimensionless}}_{g}$



Conductivity: $\sigma := \frac{j}{E}$

Ohm's law: $G = \frac{\text{Area}}{L} \cdot \sigma = L^{d-2} \cdot \sigma$

$\sigma = \underset{0<}{\ } \underset{<\infty}{\text{Const}}$ as $L \to \infty$

ee fermions.

Ramakrishnan

$\frac{e^2}{h} \times \underbrace{\text{dimensionless}}_{g}$

Not an insulator. $\sigma \neq 0$.

Not a superconductor. $\sigma \to \infty$

Not a <u>clean</u> metal:

$$\dot{p} = E \quad . \quad j = \rho \cdot v = \frac{\rho}{m} \cdot p = \frac{\rho}{i m \omega} E$$

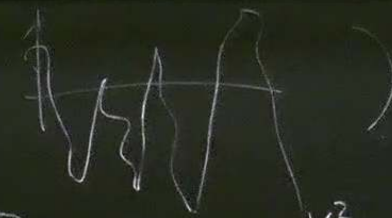
$$\sigma(\omega) = \frac{\rho}{i m \omega} \to \infty \text{ as } \omega \to 0$$

More generally, translation symmetry $\to$ Conserved P.

If $P \neq 0$. g.s. will (generically) have $j \neq 0$.

(Unless forbidden by other symmetry. Lorentz, CRT)

Finite $\sigma(\omega \to 0)$ requires breaking translation. (disorders $\Rightarrow$ $V(x)$



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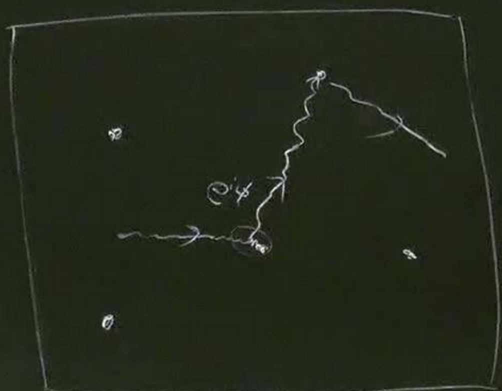
Crucial length scale :

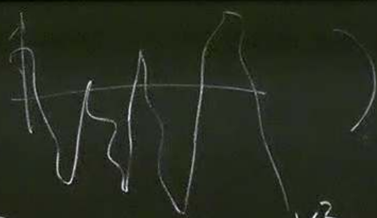
mean free path l e.g. $V(x)$, $P[V(x)] = e^{-V^2/2\sigma^2}$

$L \gg l$, electrons go through many scattering, each with some random phase factor $e^{i\phi}$



Assumption: $e^{i\phi}$ so random, no interference effect

$\Rightarrow$ particle behaves classically (random walk) $\rightarrow$ Boltzmann eq.

$\Rightarrow$ $V(x)$ )

$V(x)$, $P[V(x)] = e^{-V^2/2\sigma^2}$

$\rightarrow$ Boltzmann eq.

...e effect

...m walk)

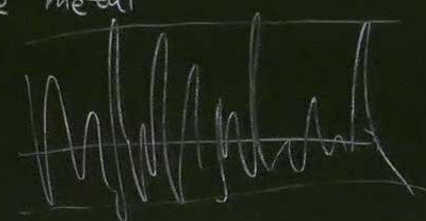
$\sigma(\omega) = \frac{e}{im\omega}$ $\xrightarrow{\text{Keep accerating for time } \tau = \frac{l}{v_F}}$ $\frac{\rho}{m}\tau . \propto \frac{1}{a^{d-1}} l$

$\rho \propto \frac{1}{a^d}$ $\quad \uparrow$ some UV scale.

$\frac{1}{i\omega} \rightarrow \tau.$

$$\boxed{\sigma \propto \frac{1}{a^{d-1}} l}$$

"Diffusive metal"

Wavefunction  

"Extended".

Is it ok. to treat e^- classically?

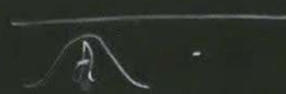
Anderson (1958): not if disorder is strong

e.g. $H = -\sum_i V_i c_i^\dagger c_i - \sum_{\langle ij \rangle} t_{ij} c_i^\dagger c_j$

V_i random, independent. range $(-W, W)$, $\boxed{W \gg t}$

Single particle eigenstate $c_a = \sum_i \psi_a(i) c_i$ $\quad a = 1, \ldots, L$, $\psi_a(i)$: wavefunction

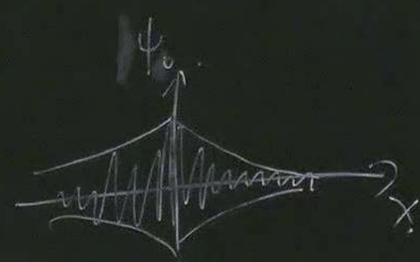
0'th order in $\frac{t}{W}$. $\psi_0(i) = \delta_{i,0}$



1st order in $\frac{t}{W}$. $|V_0 - V_{\pm 1}| \sim W \gg t$, non-degenerate perturbation justified.

$\psi_a(i)$: wavefunction.

Higher order $\psi_0(i=x) \sim e^{-|x|/\xi}$ ξ : localization length



perturbation justified. $\psi_0(i=\pm 1) \sim \dfrac{t_{0i}}{V_0-V_i} \sim \dfrac{t}{W}$.

One might worry. some rare site j s.t. $|V_j - V_0| < \varepsilon$

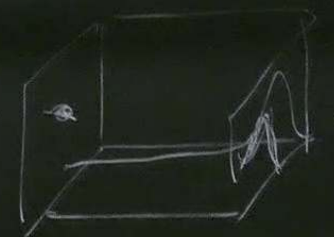
If allow tunneling between $j, 0.$ $\Rightarrow$ delocalize

Typical distance $|j-0| \sim O\left(\frac{W}{\varepsilon}\right)$

Tunneling $\sim \left(\frac{t}{W}\right)^{|j-0|} \sim e^{-\# \cdot \frac{W}{\varepsilon}} << \varepsilon$ No need to worry

$\Rightarrow$ Wave function exponentially localized.

some rare site j s.t. $|V_j - V_0| < \varepsilon$



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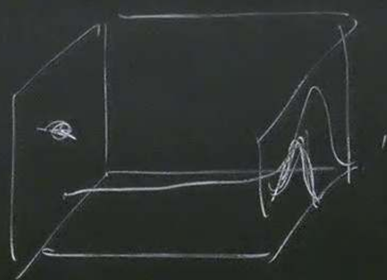
Typical distance $|j-0| \sim O\left(\frac{W}{\varepsilon}\right)$

Tunneling $\sim \left(\frac{t}{W}\right)^{|j-0|} \sim e^{-\#\cdot\frac{W}{\varepsilon}} \ll \varepsilon$ No need to worry.

$\Rightarrow$ Wave function exponentially localized. Conductance $G \sim e^{-L/\xi} \cancel{\propto} L^{d-2}$

"Anderson insulator".

ize

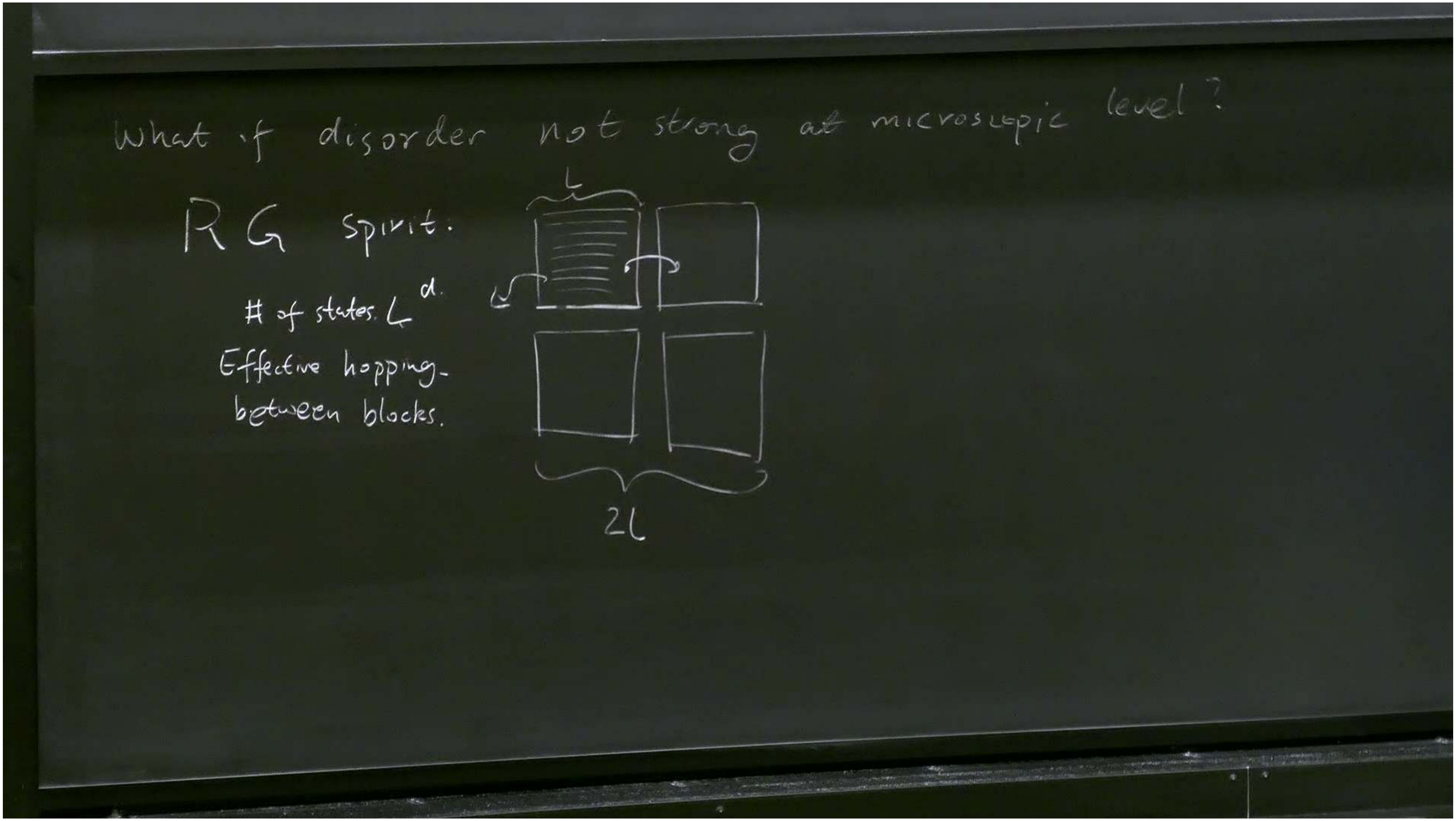


o need to worry.

ized. Conductance $G \sim e^{-L/\xi} \cancel{\propto} L^{d-2}$ not Ohmic

"Anderson insulator" ~ A random product state.

$$|G.S.\rangle = \prod_{\varepsilon_a < 0} c_a^\dagger |0\rangle \sim \text{product state}$$


What if disorder not strong at microscopic level?
RG spirit:
# of states. L^d.
Effective hopping-
between blocks.
L
2L

microscopic level ?

Thoules: one-pameter scaling hypothesis.

At lage scale $L \gg l$, disorder pattern captureed by a single dimensionless para

"like "t/W"", any other one will be a function of "t/W" : $g = G/(e^2/h)$

In RG language: assuming at most 1 relevant parameter.

$\Rightarrow$ $g(bL)$ determined by $g(L)$

$$\Rightarrow \frac{dg}{d(\ln L)} = \underbrace{\beta(g)}_{\beta\text{-function only depends on } g.} \cdot g \quad (\text{"Gang of Four", 1979})$$

Can do a more serious calculation to get $\beta(g)$

But we already have some info about β.

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① if $g \ll 1$, strong disorder. Anderson localization.

$$g \sim g_0 e^{-L/\xi}$$

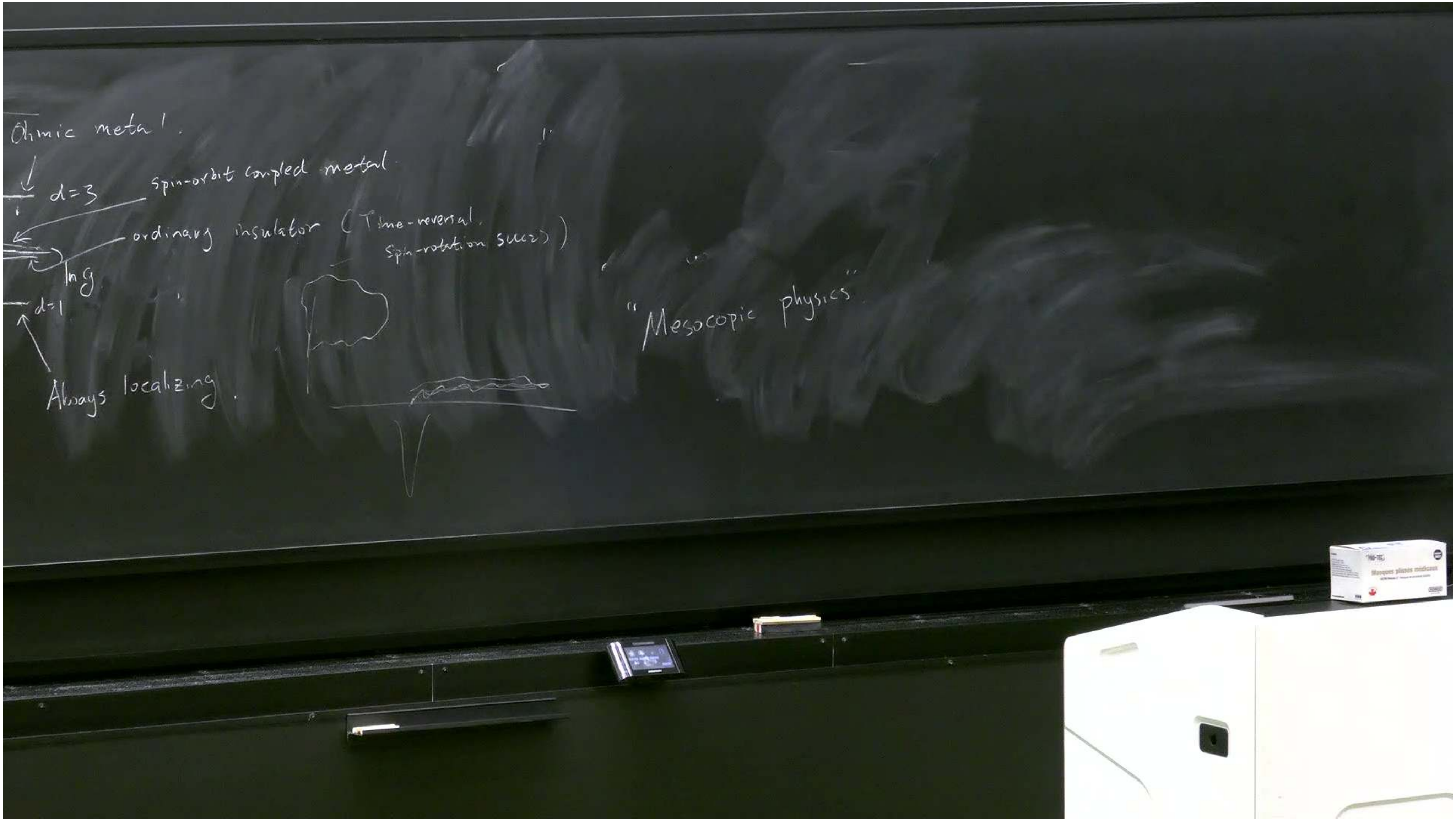
$$\Rightarrow \beta(g) = \ln(g/g_0)$$

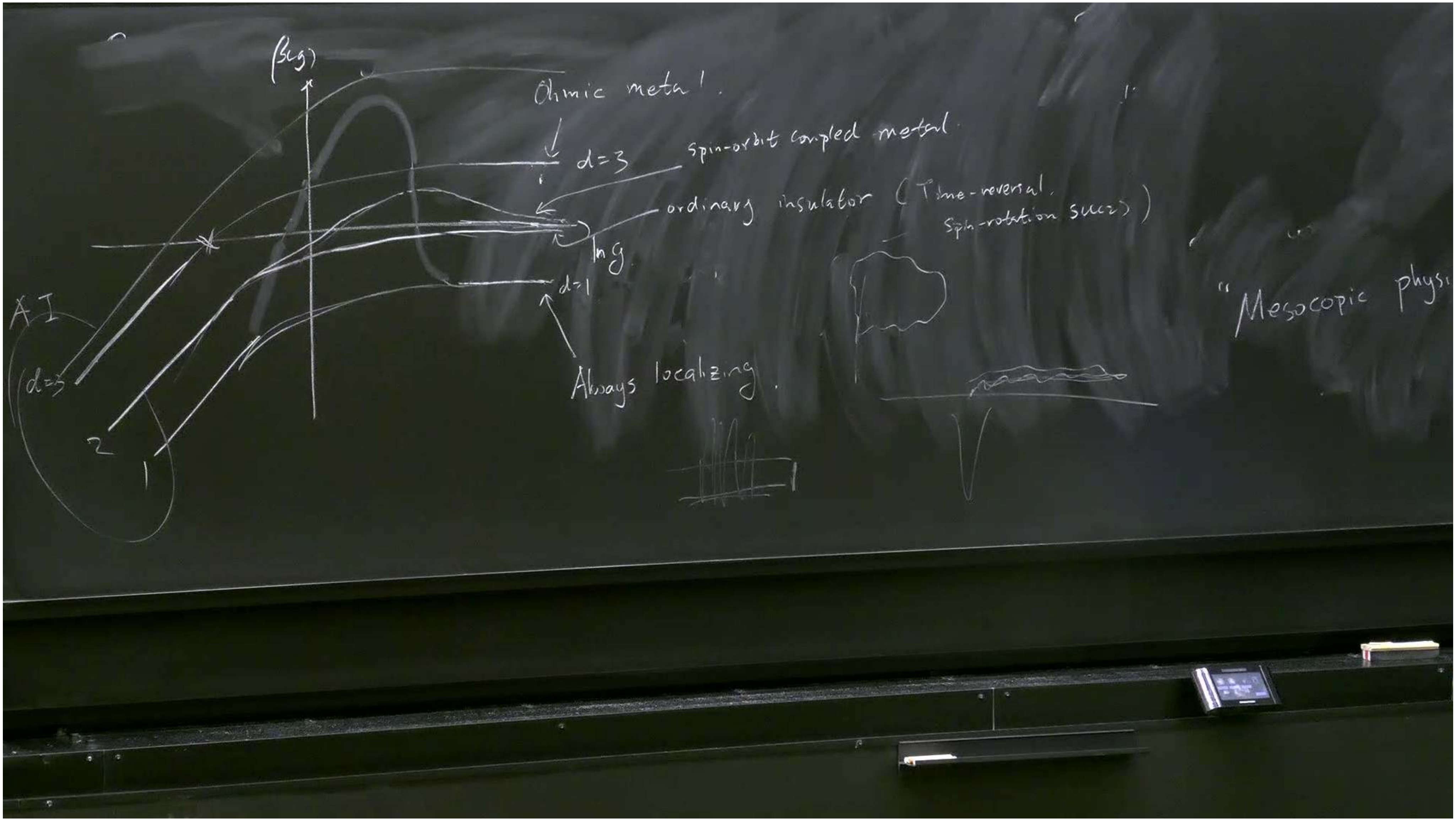
② if $g \gg 1$, weak disorder. diffusive. Ohm's law. $g \sim g_0 L$

$$\Rightarrow \beta(g) = d-2 + O(\tfrac{1}{g})$$

$\beta > 0$. $g \uparrow$ as $L \uparrow$ metal.

$\beta < 0$. $g \downarrow$ as $L \uparrow$. insulator.


Ohmic metal.
d=3
Spin-orbit coupled metal
ordinary insulator (Time-reversal,
Spin-rotation
ln g
d=1
Always localizing
"Mesocopic physics"



β(g)
Ohmic metal.
d=3
Spin-orbit coupled metal.
ordinary insulator (Time-reversal,
ln g
d=1
AI
d=3
2
1
Always localizing.
"Mesocopic