**Title:** Long-term stable non-linear evolutions of ultracompact black hole mimickers

**Speakers:** Seppe Staelens

**Collection/Series:** Strong Gravity

**Subject:** Strong Gravity

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**Abstract:**

Ultracompact black hole mimickers formed through gravitational collapse under reasonable assumptions obtain light rings in pairs, where one is unstable and the other one is not. Stable light rings are believed to be a potential source for dynamical instability due to the trapping of massless perturbations, as their decay is relatively slow.

We study the stability of ultracompact boson stars admitting light rings combining a perturbative analysis with 3+1 numerical-relativity simulations with and without symmetry assumptions. We observe excellent agreement between all perturbative and numerical results which uniformly support the hypothesis that this family of black-hole mimickers is separated into stable and unstable branches by extremal-mass configurations. This separation includes, in particular, thin-shell boson stars with light rings located on the stable branch which we conclude to represent long-term stable black-hole mimickers. Our simulations suggest that the proposed mechanism may not be efficient after all to effectively destroy ultracompact black hole mimickers.

# Stable evolutions of ultracompact black hole mimickers

(2504.17775)

Seppe J. Staelens

- with G. Marks, T. Evstafyeva, U. Sperhake -

*Strong Gravity Seminar, Perimeter Institute for Theoretical Physics*

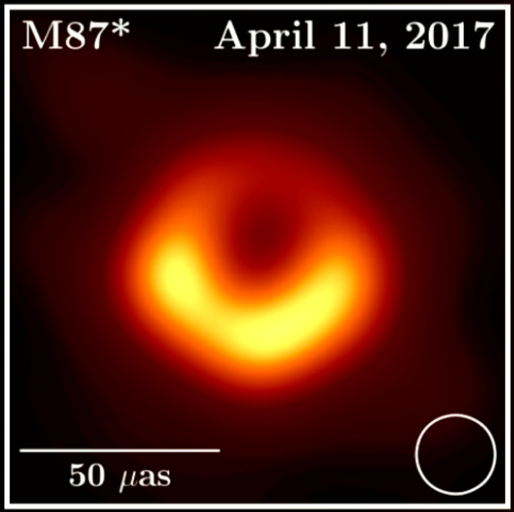
UNIVERSITY OF CAMBRIDGE

St. Edmund's College

LEUVEN GRAVITY INSTITUTE

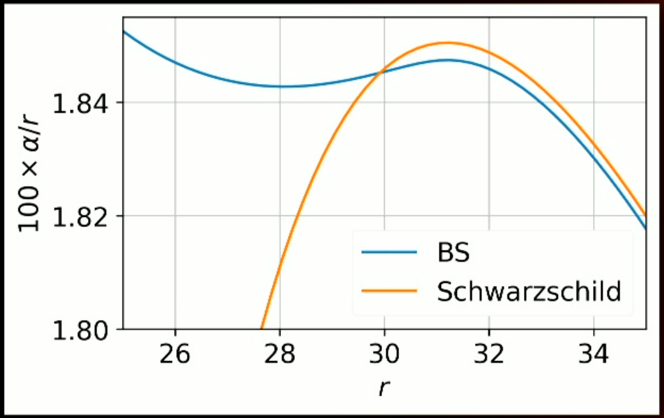
# Introduction

- Black hole mimickers
  - Horizonless / regular
  - Dark matter ?
- Light ring vs. Event horizon
  - Ringdown
  - VLBI images
- Ultracompact objects



# Introduction

- Light rings (LRs) come in pairs (Cunha+, '17)
  - Stable + unstable
- Stable LR
  - Slow decay of linear waves (Keir, '16)
  - Trapping well
- LR instability is still debated (Cardoso+, '14) (Cunha+, '23) (Guo+, '24) (Benomio+, '24) (Siemonsen, '24) (Redondo-Yuste+, '25)



# Contents

- Boson stars
- Radial stability
- Numerical simulations
- Effective potential & light rings
- Teaser: VLBI images

# Boson stars

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# Boson stars

- GR + complex scalar field

$$S = \int \left( \frac{R}{16\pi G} + \mathscr{L}_\varphi \right) \sqrt{-g}\ \mathrm{d}^4 x\,, \quad \mathscr{L}_\varphi = -\frac{1}{2} \left( \nabla_\alpha \overline{\varphi} \nabla^\alpha \varphi + V(\varphi) \right)$$

- Einstein-Klein-Gordon equations

$$G_{\alpha\beta} = 8\pi G T_{\alpha\beta}\,, \quad \nabla_\mu \nabla^\mu \varphi = \frac{\mathrm{d}\, V}{\mathrm{d}\, \overline{\varphi}}$$

# Boson stars

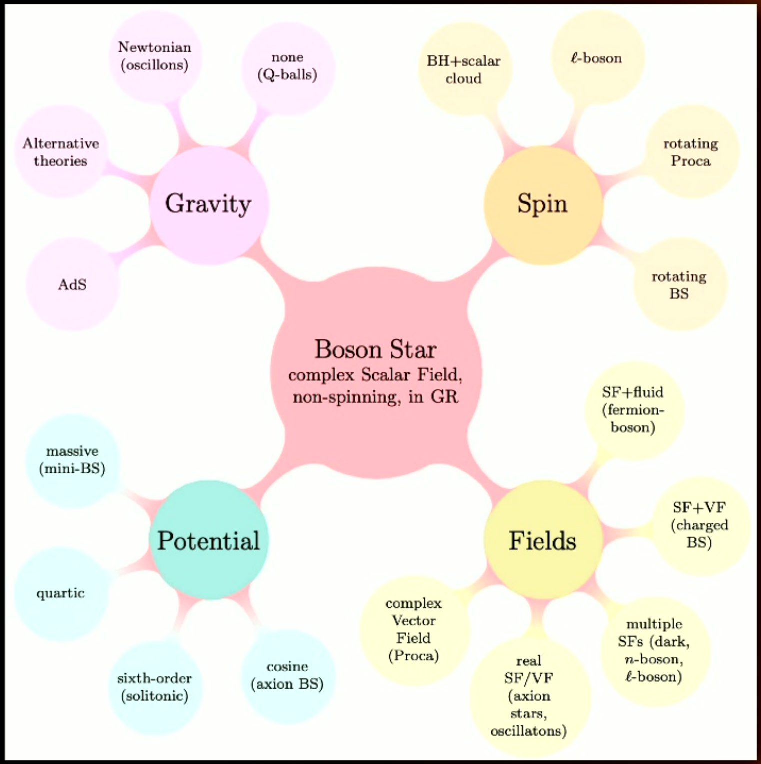
(Liebling+, '23)

- GR + complex scalar field

$$S = \int \left( \frac{R}{16\pi G} + \mathscr{L}_\varphi \right) \sqrt{-g}\ \mathrm{d}^4 x, \quad \mathscr{L}_\varphi = -\frac{1}{2}\left( \nabla_\alpha \bar{\varphi} \nabla^\alpha \varphi + V(\varphi) \right)$$

- Einstein-Klein-Gordon equations

- Scalar potential
  - Mini $V(\varphi) = \mu^2 |\varphi|^2$
  - Solitonic $V(\varphi) = \mu^2 |\varphi|^2 \left( 1 - 2\frac{|\varphi|^2}{\sigma^2} \right)^2$
  - Axion (real scalar) $V(\varphi) = \mu^2 f^2 \left| 1 - \cos \frac{\varphi}{f} \right|$

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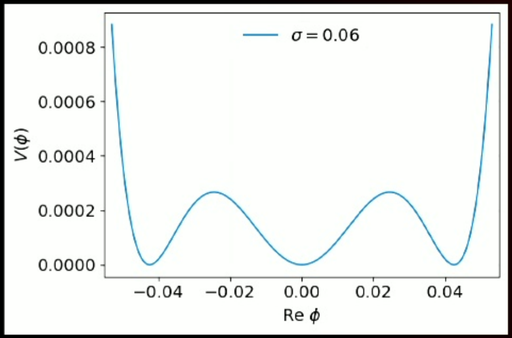
# Why boson stars?

- BH mimickers
  - Black if no coupling to EM
  - Regular and no horizon
  - Varying compactness
- Only one additional complex scalar
- ‘Easy’ to simulate
- Good first-order approximation of non-BH?

# Solitonic boson stars

- Solitonic potential
  - Parameter $\sigma$

$$V(\varphi) = \mu^2 |\varphi|^2 \left(1 - 2\frac{|\varphi|^2}{\sigma^2}\right)^2$$




- Spherically symmetric, harmonic Ansatz

$$\varphi = A(r)\, e^{i\omega t}$$

# Solitonic boson stars

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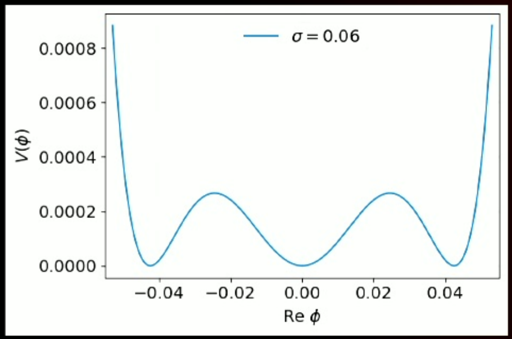
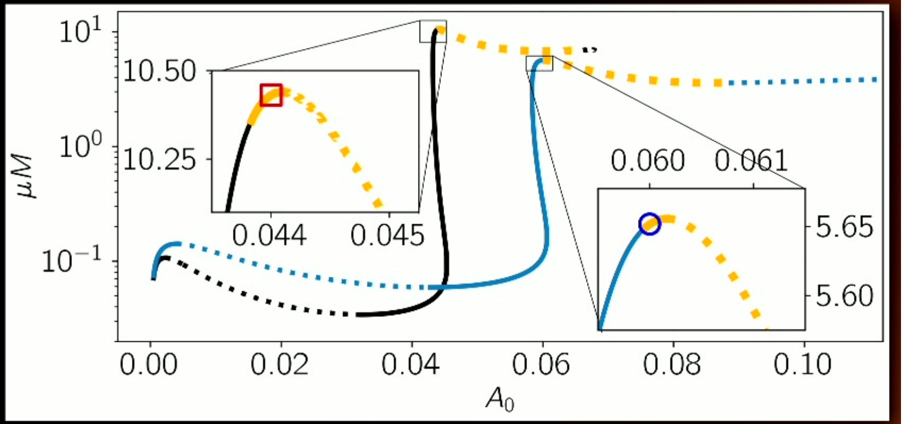



- Spherically symmetric, harmonic Ansatz

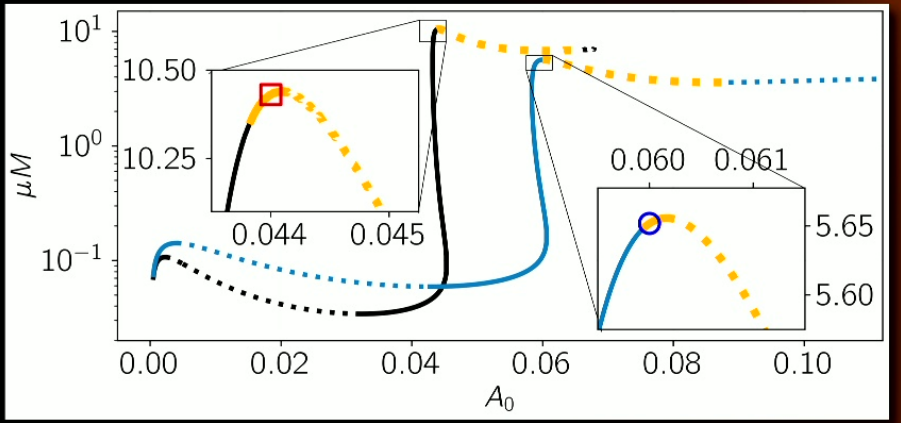
$$\varphi = A(r)\, e^{i\omega t}$$

- Shooting method
  - Fix central amplitude $A_0$
  - Guess $\omega$
  - Integrate EKG equations outwards
  - Repeat for different $\omega$



# Thin shell models

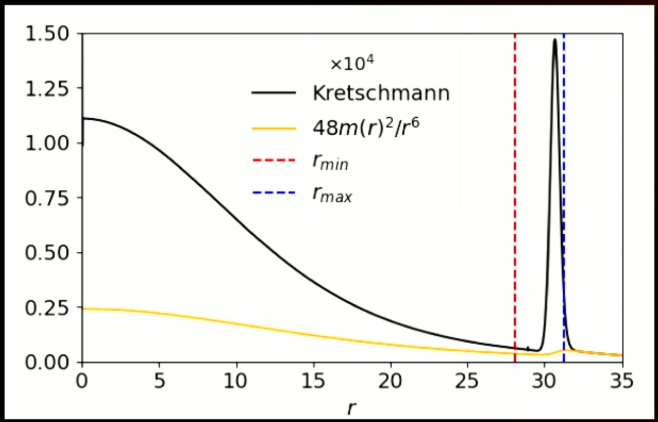
- For small $\sigma$
  - Multivalued
  - Ultracompact models (orange)

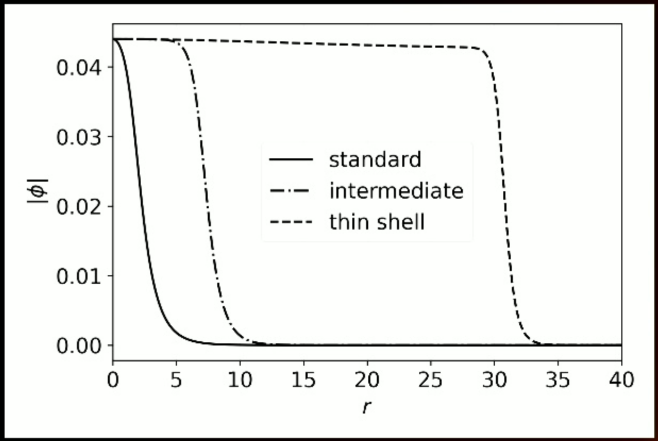


# Thin shell models

- For small $\sigma$
  - Multivalued
  - Ultracompact models (orange)

- Thin shell models
  - Center of BS close to non-zero vacuum
  - Energy concentrated in a shell
  - $\approx$ Schwarzschild outside of the shell

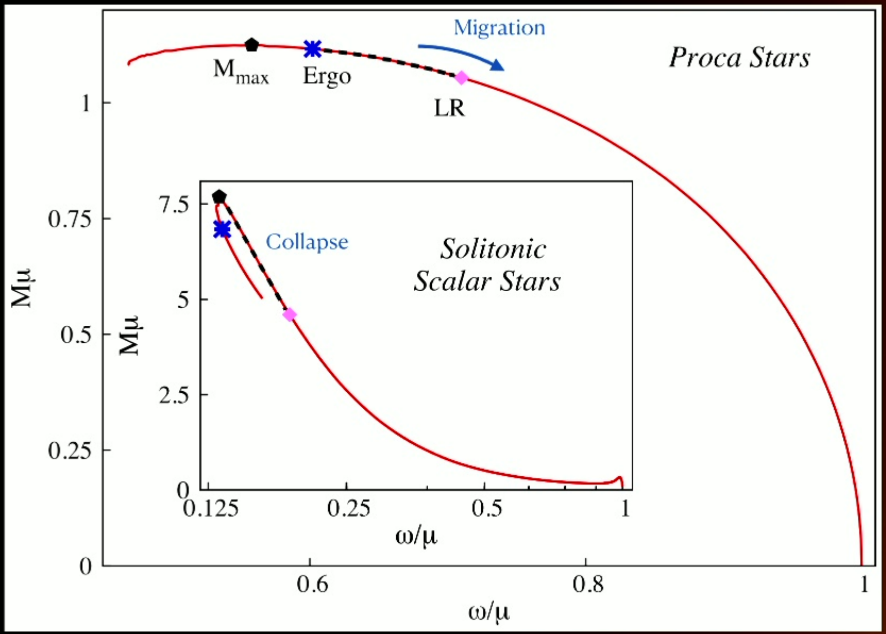


# Instability: past work

- (Cunha+, '23)

- BS and Proca stars
- Find instability onset at LR regime
- Difference: rotating models

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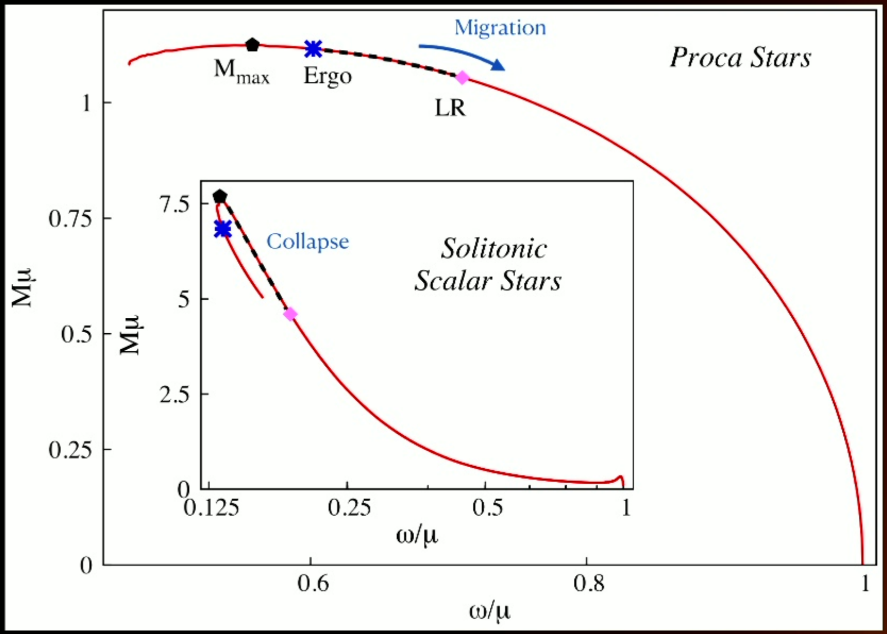
# Instability: past work

- (Cunha+, ’23)

- BS and Proca stars
- Find instability onset at LR regime
- Difference: rotating models

- Our motivation
  - Simplify by neglecting rotation
  - If unstable: non-spherical instability?
  - Analytical arguments for / against

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# Radial Perturbative Stability (Gareth's work)

# Theory

- Linear stability against radial perturbations
- Expand EKG equations around background BS

$$\mathrm{d}s^2 = -\alpha^2(r,t)\ \mathrm{d}t^2 + X^2(r,t)\ \mathrm{d}r^2 + r^2\ \mathrm{d}\Omega^2$$

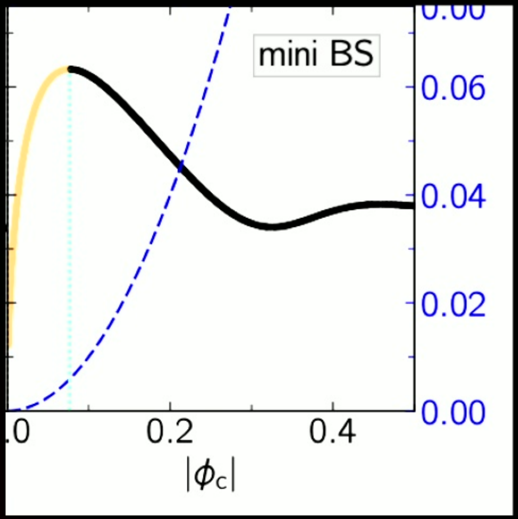
$$\varphi = e^{i\omega t}\left(\psi_1 + i\psi_2\right)\ , \quad \psi_1 = A\left(1 + \delta\psi_1\right)\ , \quad \psi_2 = A\delta\psi_2\,,$$
$$\alpha = \alpha_0\left(1 + \frac{1}{2}\delta\nu\right)\ , \quad X = X_0\left(1 + \frac{1}{2}\delta\lambda\right)$$

# Theory

- Assume perturbations of the form $\delta f(r,t) = e^{i\chi t} g(r)$
- Result: system of two coupled ODEs
- Positive (negative) $\chi^2$ corresponds to linear radial (in)stability

# Theory

- (Un)stable branches are separated by extrema of $M(A_0)$




(Ge+, 2410.23839)

# Theory

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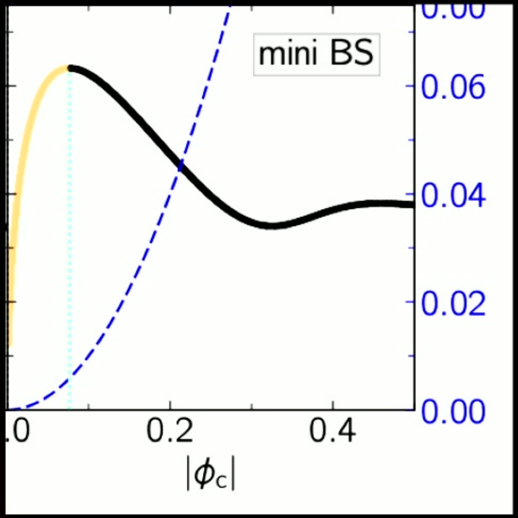


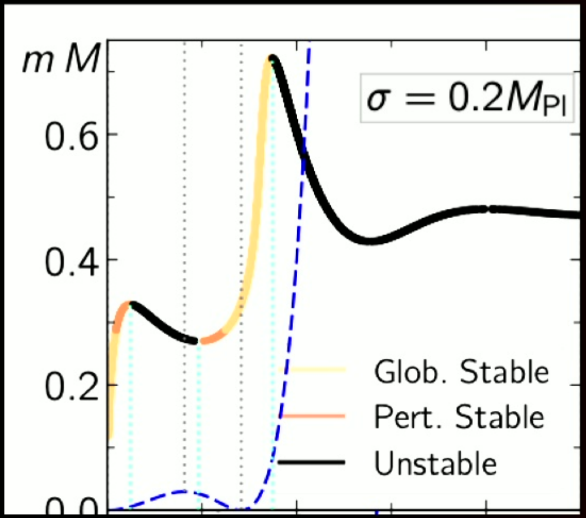


(Ge+, 2410.23839)

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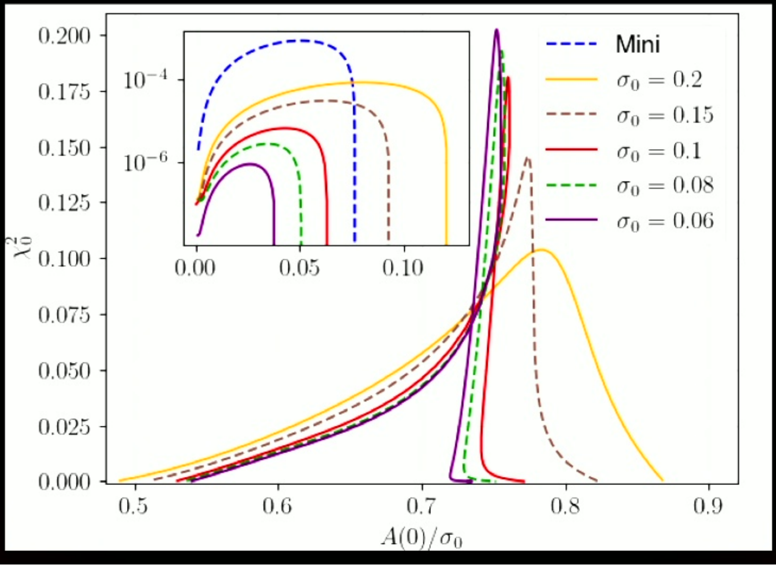
# Gareth's results

- Calculated perturbation frequencies for solitonic families that include thin-shell models.
- Multi-stage shooting code with quadruple precision



# Numerical Evolutions

# Numerical Relativity

- 3+1 metric

$$\mathrm{d}\,s^2 = -\,\alpha^2\;\mathrm{d}\,t^2 + \gamma_{ij}\left(\mathrm{d}\,x^i + \beta^i\,\mathrm{d}\,t\right)\left(\mathrm{d}\,x^j + \beta^j\,\mathrm{d}\,t\right)$$

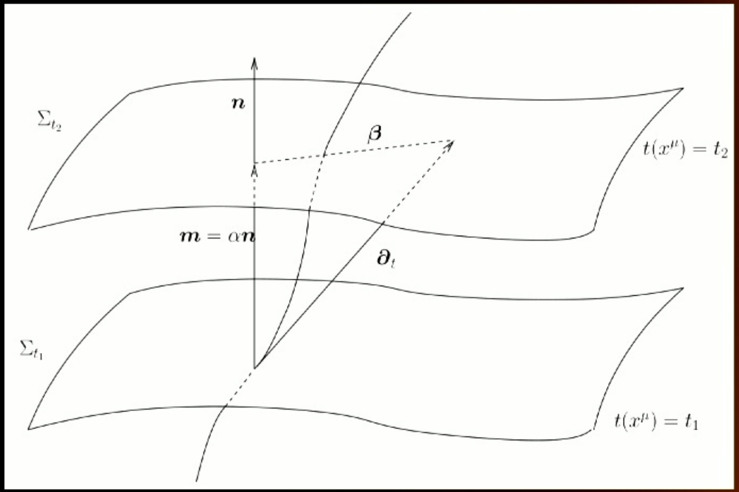
- 6 spatial evolution equations

$$\partial_t \gamma_{ij} = -\,2\alpha K_{ij} + \ldots\,, \quad \partial_t K_{ij} = \ldots$$

- 4 constraint equations

$$\mathscr{H} = 0\,, \quad \mathscr{M}_i = 0$$

- 4 evolution equations for freely chosen $\alpha, \beta^i$
- In practice: BSSN / CCZ4 formulation
  - Latter: Constraint damping $\kappa_i$



# Numerical Relativity

- Initial data: FORTRAN for quadruple precision
- 4 codes
  - Cartoon method
    - Spherical symmetry (GAM)
    - Axial symmetry: ExoZvezda (TE)
  - Full NR*
    - LEAN (US)
    - ExoZvezda (SJS)

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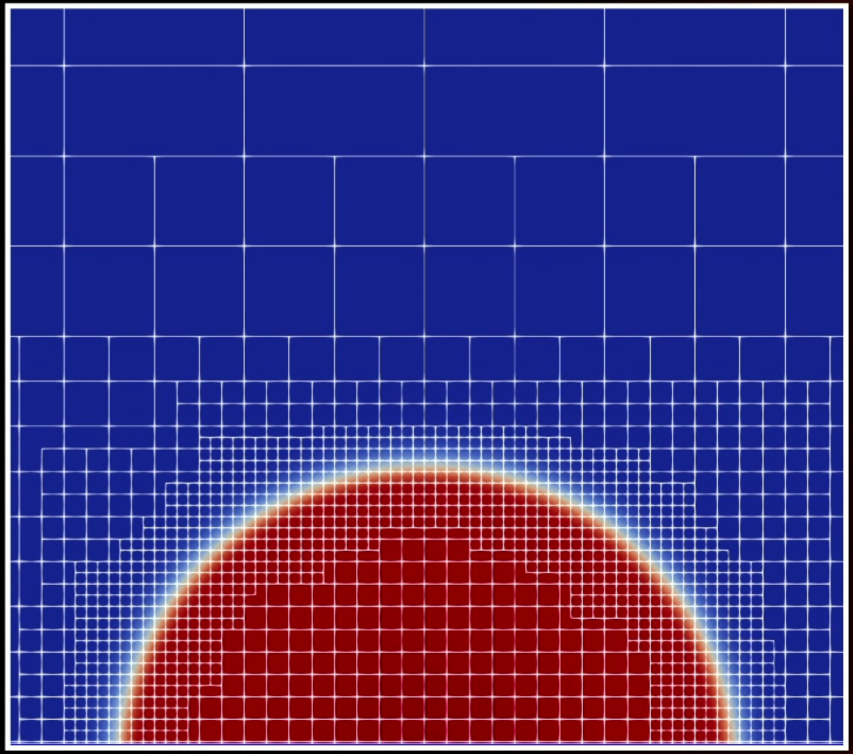
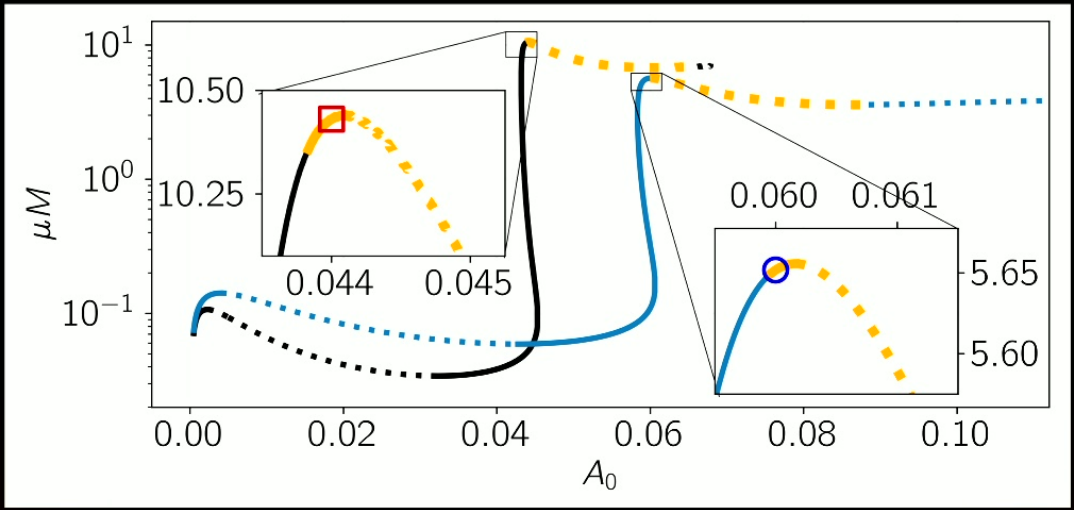
# 3+1 simulations

- **ExoZvezda**: Numerical Relativity with Adaptive Mesh Refinement (AMR)
- Challenging simulations
  - Initial data (quadruple precision)
  - Sensitive and slow
  - Separation of scales



# Reminder

- Two ultracompact thin-shell boson stars

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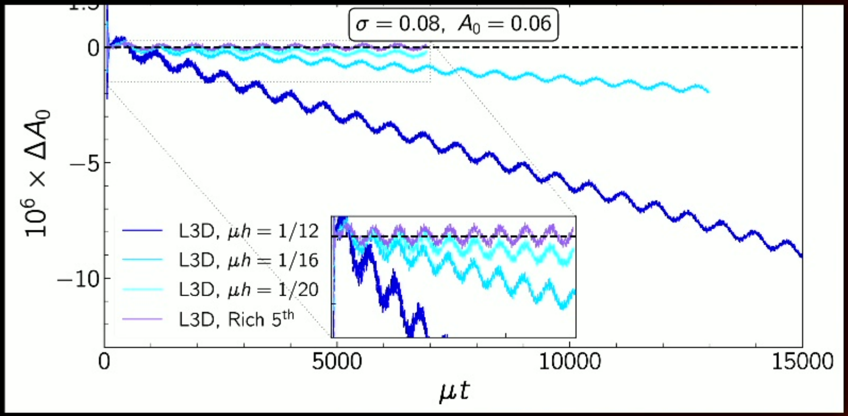
# Central amplitude

- $\sigma = 0.08$: Evolution dominated
  - Observed drift converges away



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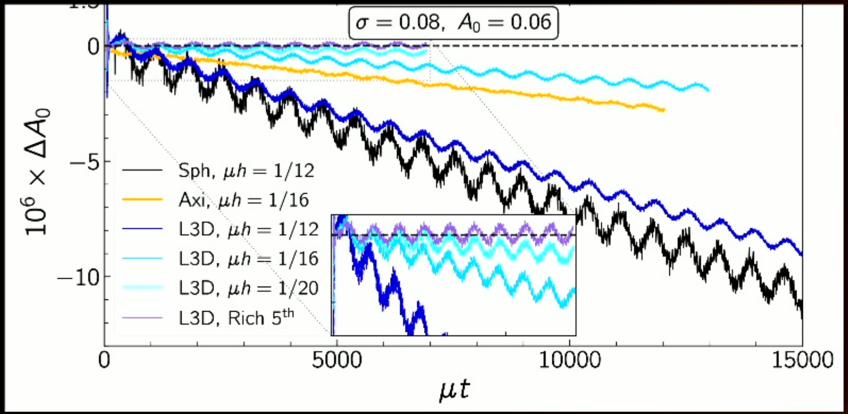


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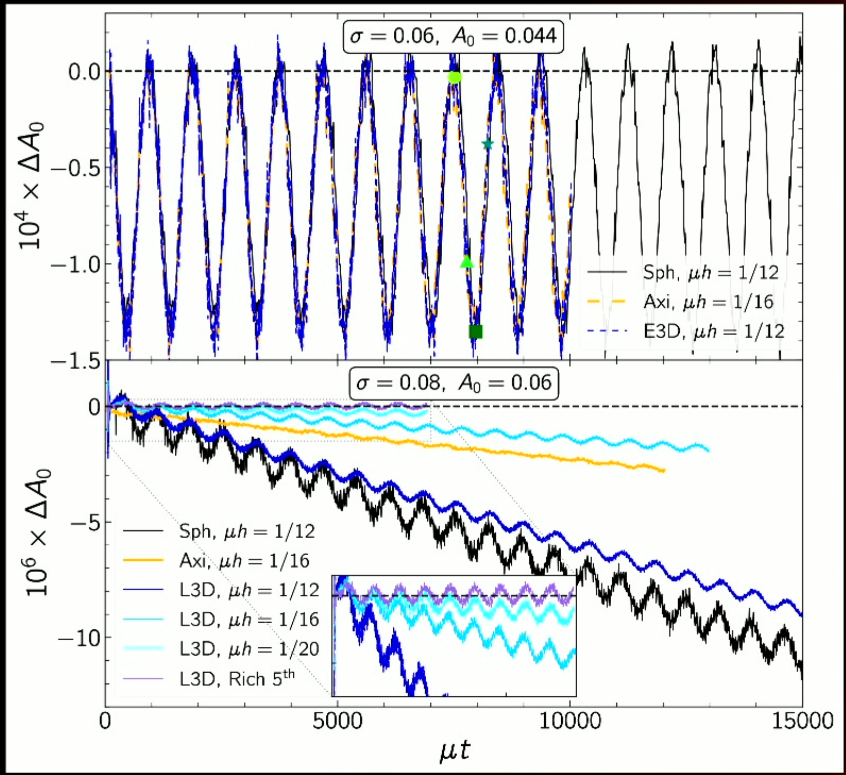
# Central amplitude

- $\sigma = 0.06$ **:** Initial data dominated

**BOTTOM LINE**: Stability

- $\sigma = 0.08$**:** Evolution dominated
  - Observed drift converges away

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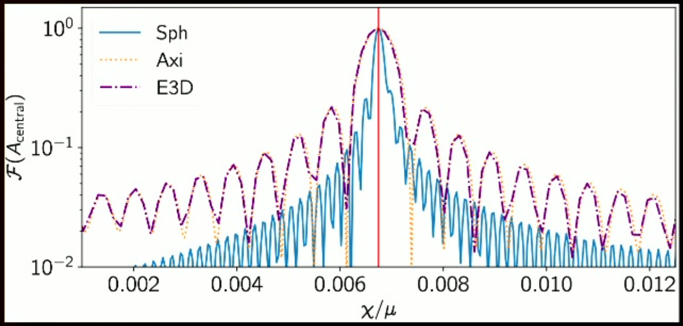
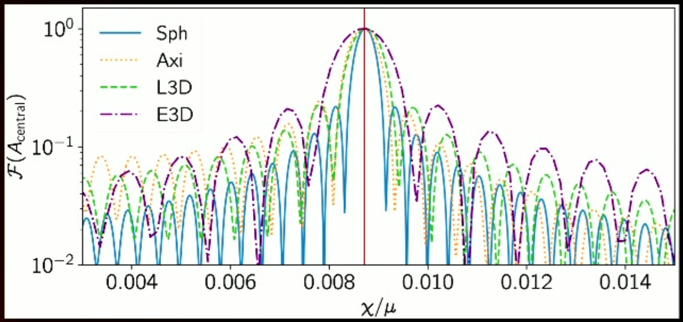
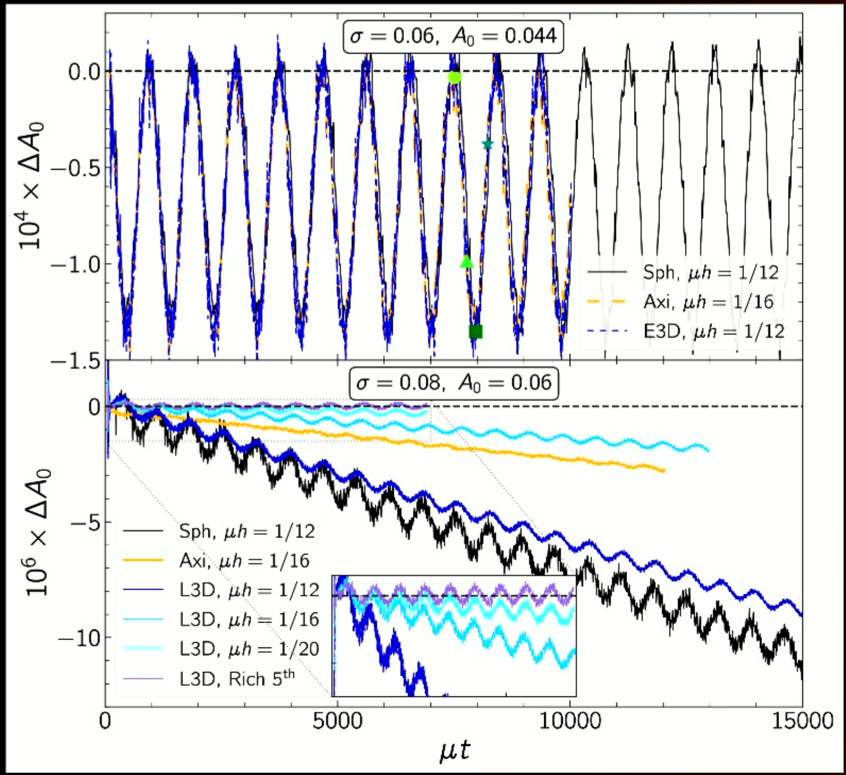
# Oscillation Frequencies




Consistent with perturbation theory



# **<u>But</u>: is the BS still ultracompact?**

# Effective potential

- Stationary and spherical symmetric spacetime

$$\mathrm{d}s^2 = -\alpha^2(r)\ \mathrm{d}t^2 + X^2(r)\ \mathrm{d}r^2 + r^2\ \mathrm{d}\Omega^2$$

- **Light rings**: circular null geodesics $\dot{r} = \ddot{r} = 0$

# Effective potential

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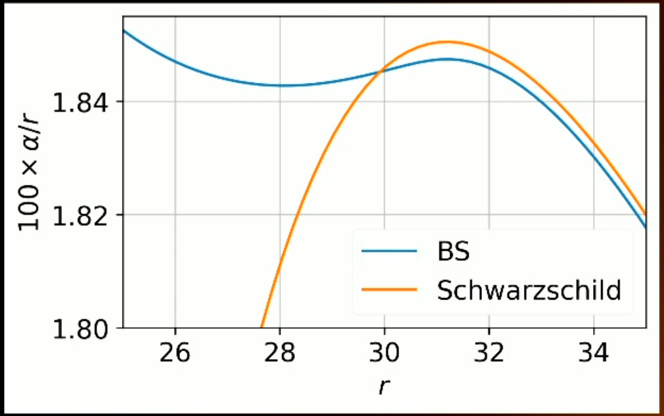
- **Light rings**: circular null geodesics $\dot{r} = \ddot{r} = 0$

- Effective potential

$$V_{\text{eff}}(r) = \sqrt{\frac{-g_{tt}}{g_{\phi\phi}}} = \frac{\alpha}{r}, \quad \partial_\mu V_{\text{eff}} = 0$$

- (in)stability: second derivative

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# Adiabatic effective potential

- Numerical evolutions: departure from assumed symmetries
  - Assume slow evolution and small departures
- **Adiabatic Effective Potential (AEP)** (Cunha+, '23)

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- Numerical evolutions: departure from assumed symmetries
  - Assume slow evolution and small departures
- **Adiabatic Effective Potential (AEP)** (Cunha+, '23)
  - Average over coordinate spheres

$$V_{\text{eff}}(r) = \frac{\alpha}{r} \longrightarrow H_{\text{eff}}(r) = \sqrt{\frac{\langle \alpha^2 \rangle - \langle \gamma_{ij} \beta^i \beta^j \rangle}{\left( \oint \sqrt{\det q} \right) / 4\pi}}$$

Lapse

Spatial metric

Shift

Induced metric on sphere

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# Adiabatic effective potential

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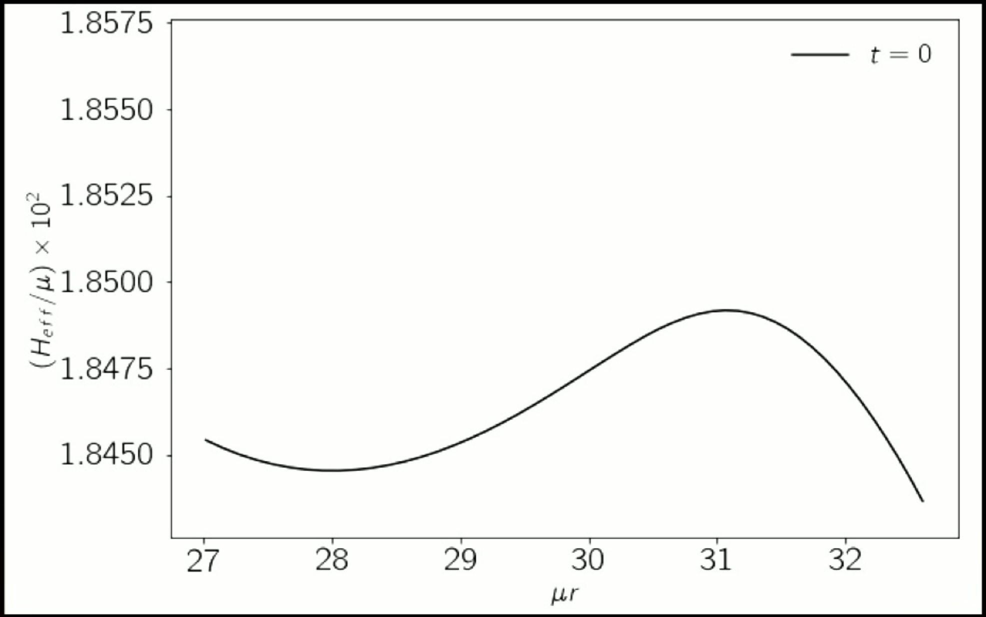
$$H_{\text{eff}}(r) = \sqrt{\frac{\langle \alpha^2 \rangle - \langle \gamma_{ij}\beta^i\beta^j \rangle}{\left(\oint \sqrt{\det q}\right)/4\pi}}$$



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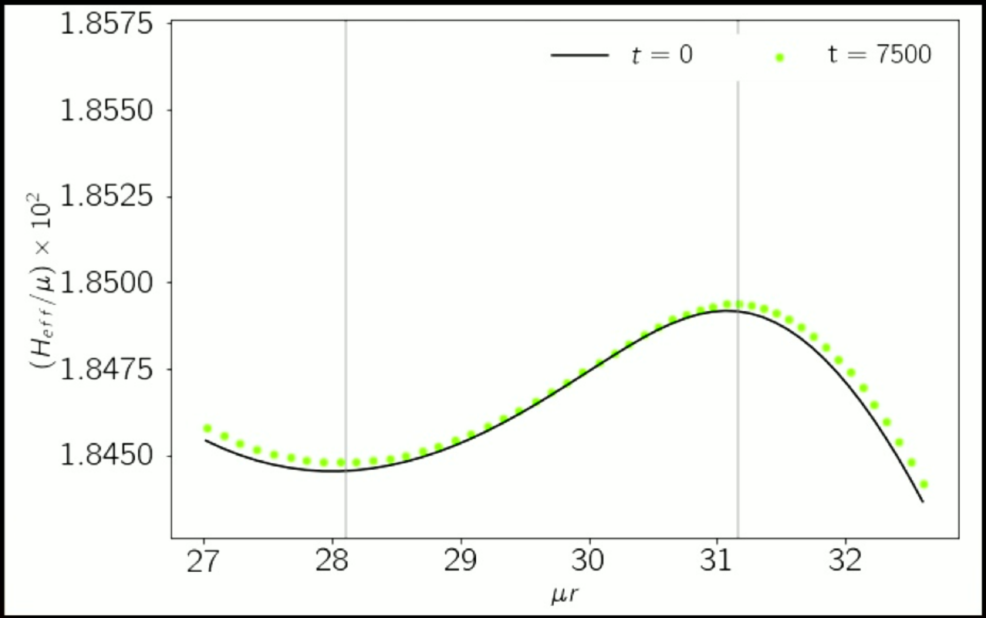


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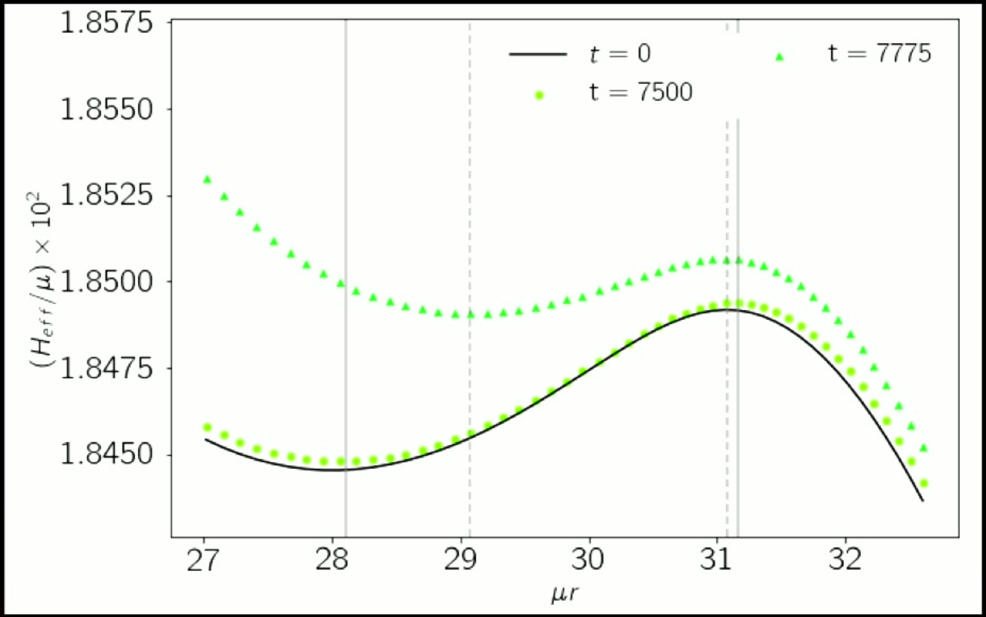

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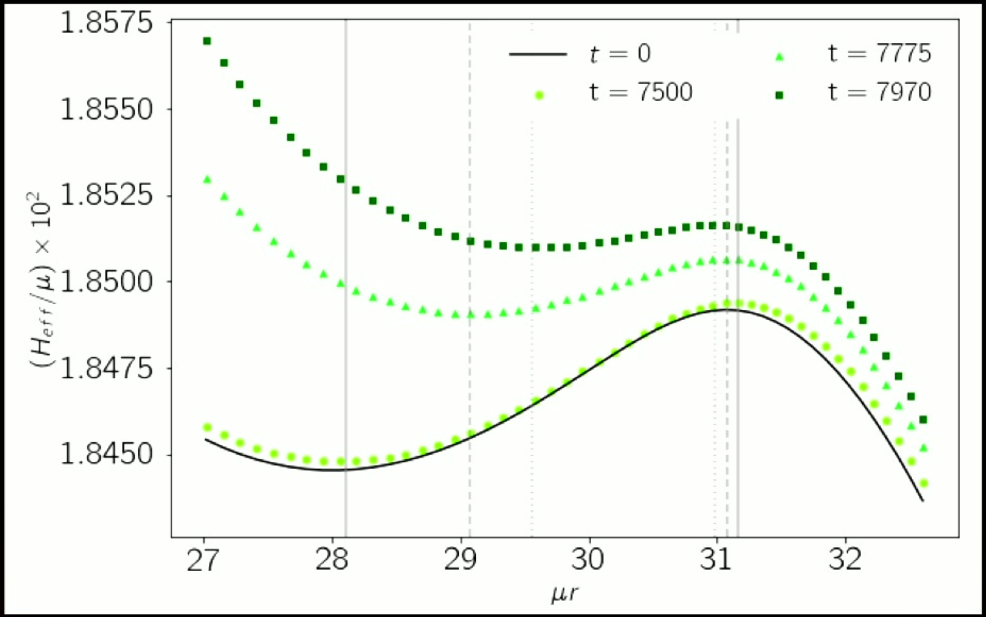


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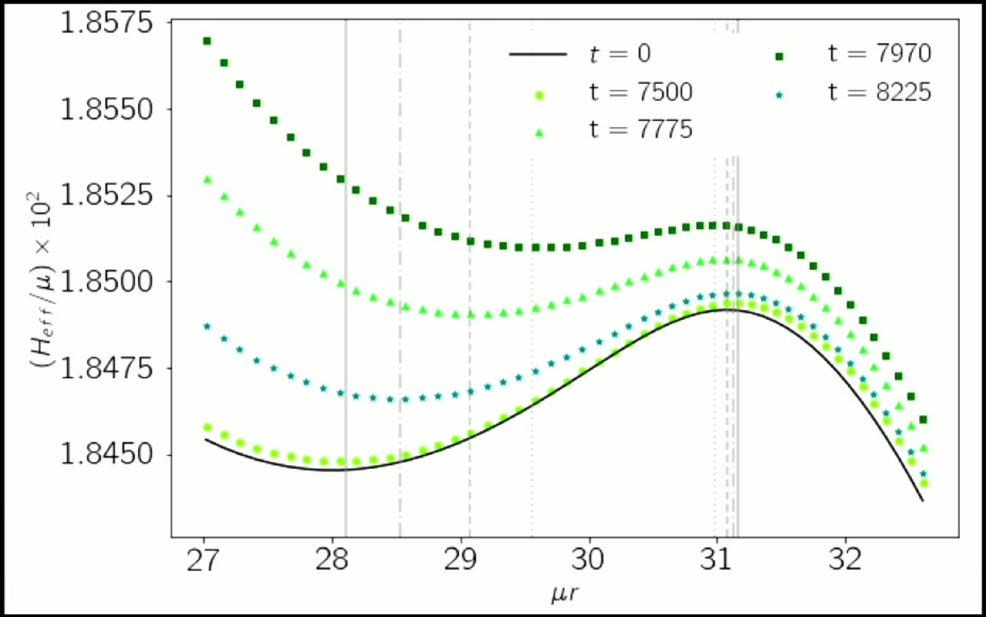
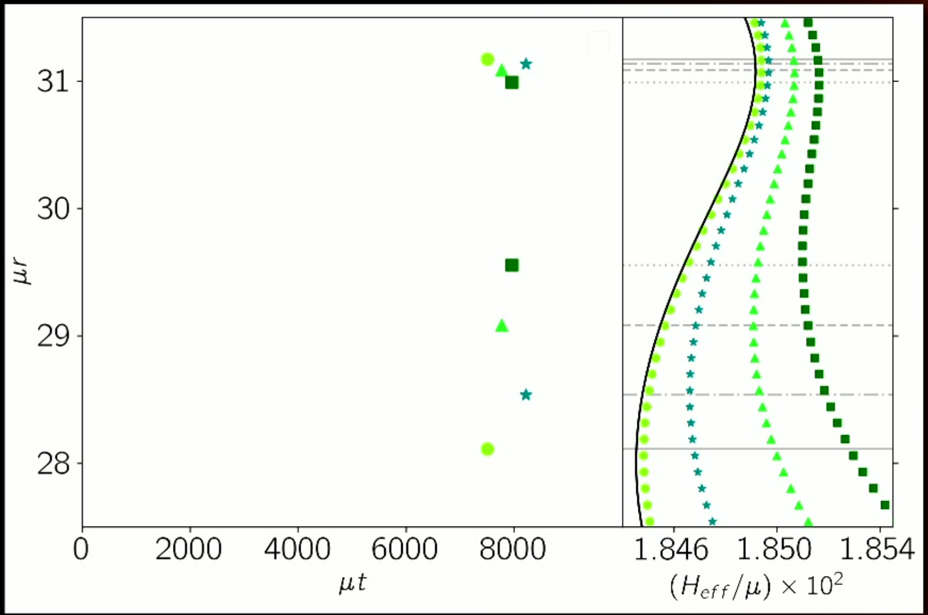


# Light ring evolution

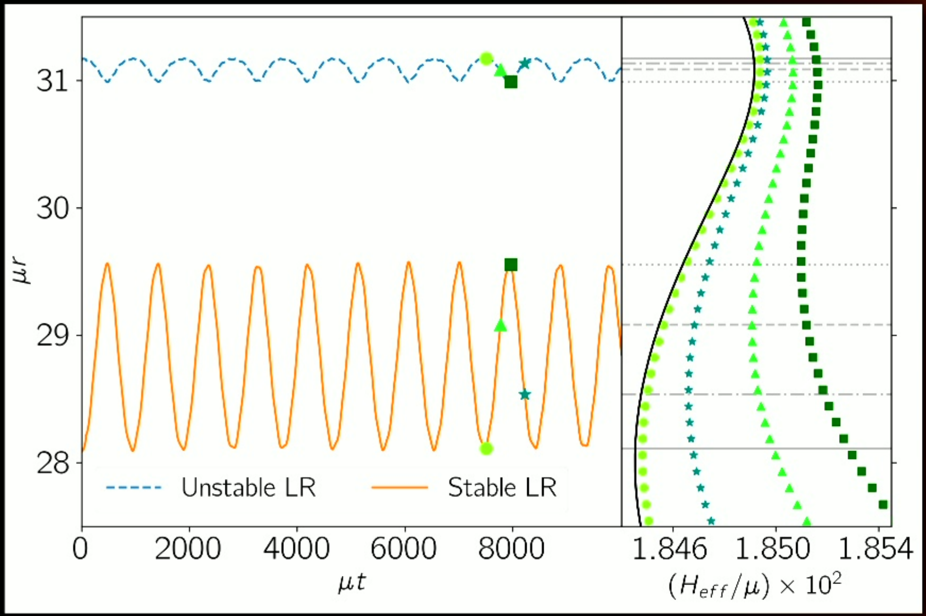
- Extract light rings at extrema




$\sigma = 0.06$

# Light ring evolution

- Extract light rings at extrema
- Light ring structure preserved

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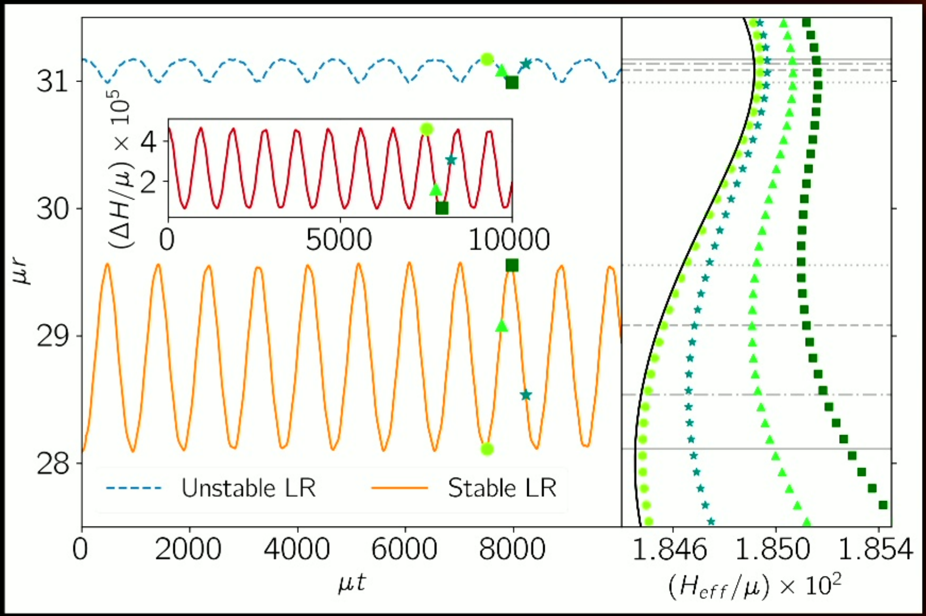


$\sigma = 0.06$

# Robustness of AEP

- Variances over the extraction sphere

$$H_{\text{eff}}(r) = \sqrt{\frac{\langle \alpha^2 \rangle - \langle \gamma_{ij} \beta^i \beta^j \rangle}{\left( \oint \sqrt{\det q} \right) / 4\pi}}$$

$$\sigma(X)\,,\ \langle X \rangle$$

$$\delta(X) = \frac{\sigma(X)}{\langle X \rangle}$$

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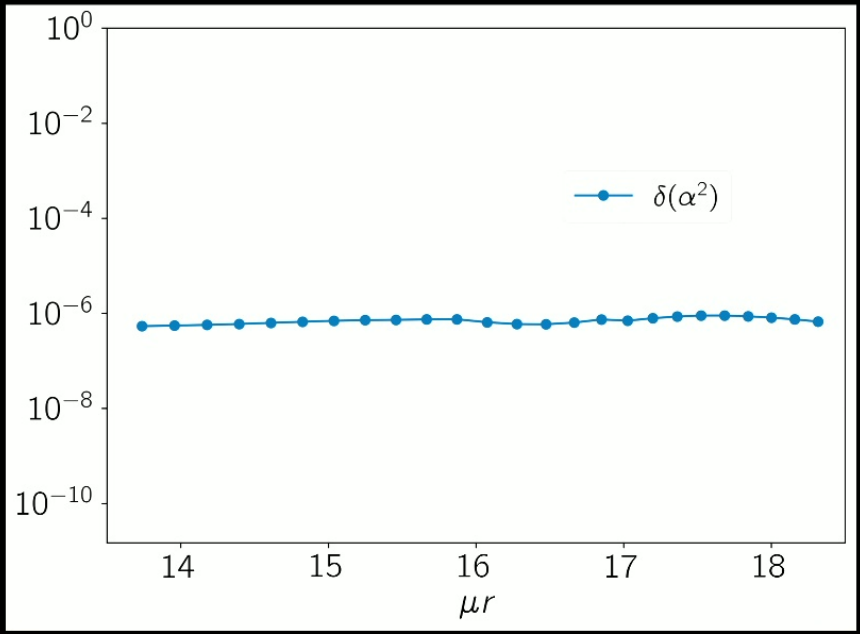


$\sigma = 0.08$

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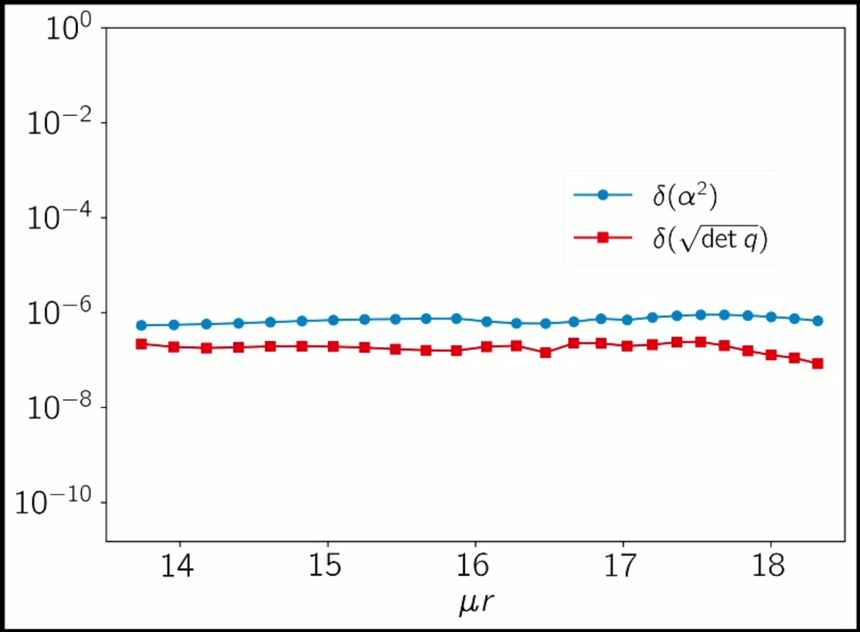


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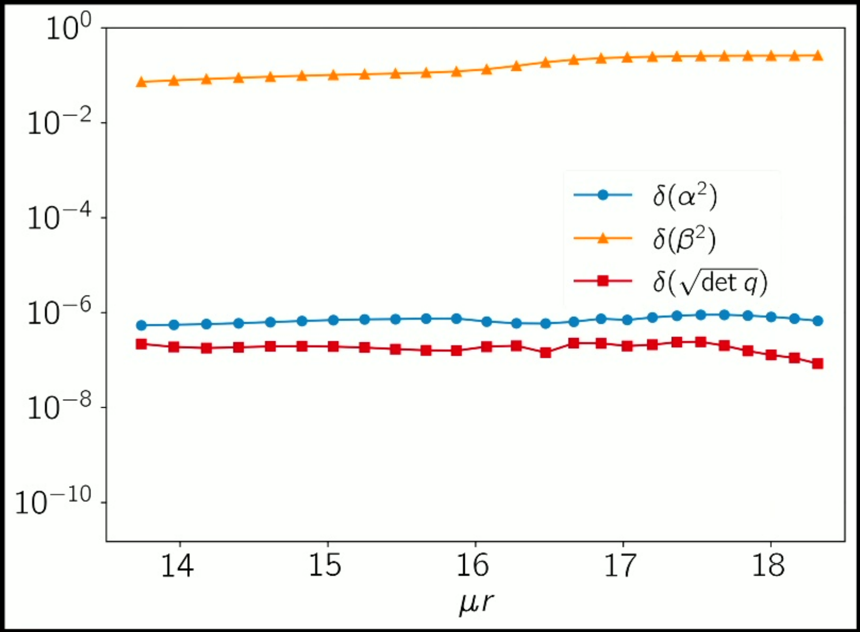


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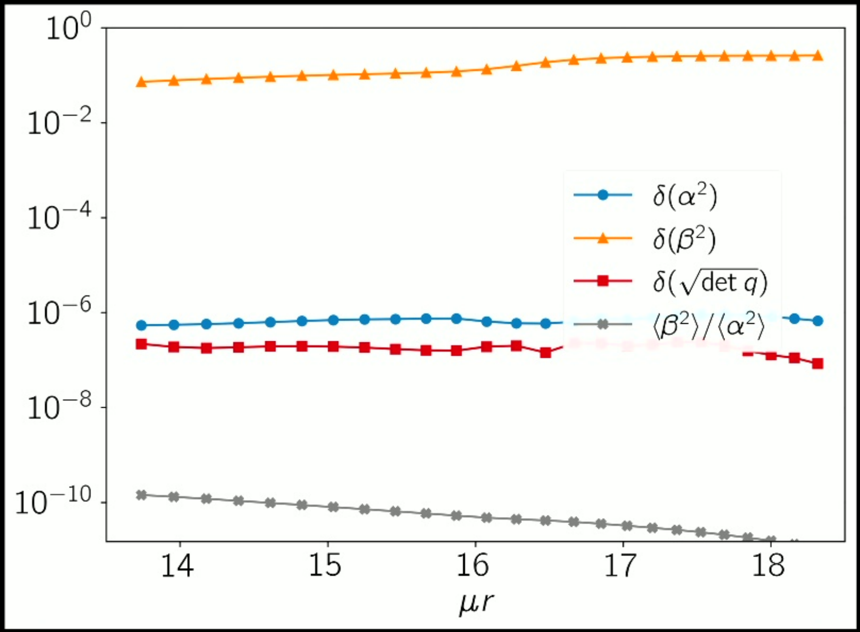


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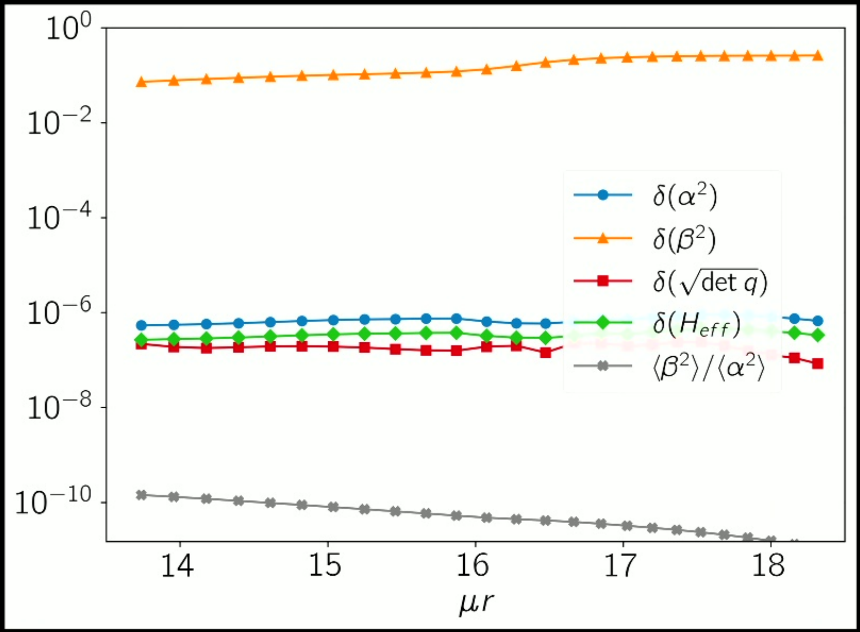


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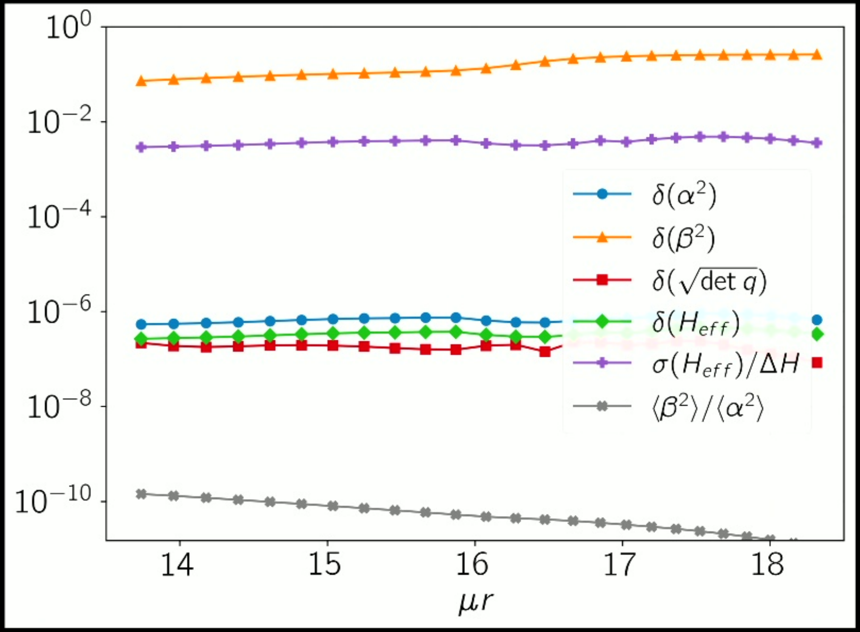
$$\delta(X) = \frac{\sigma(X)}{\langle X \rangle}$$

$$H_{\text{eff}}(r) = \sqrt{\frac{\langle \alpha^2 \rangle - \langle \gamma_{ij} \beta^i \beta^j \rangle}{\left(\oint \sqrt{\det q}\right)/4\pi}}$$

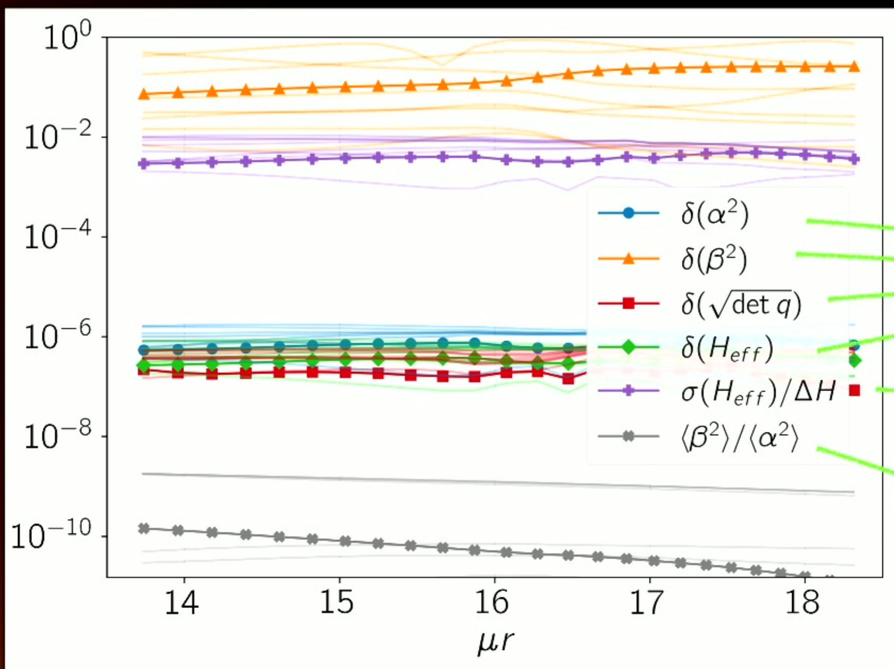



$\sigma = 0.08$

# Robustness of AEP

- Variances over the extraction sphere

$$\delta(X) = \frac{\sigma(X)}{\langle X \rangle}$$




Spherical symmetry

Robustness of light ring presence

Stationarity

$\sigma = 0.08$

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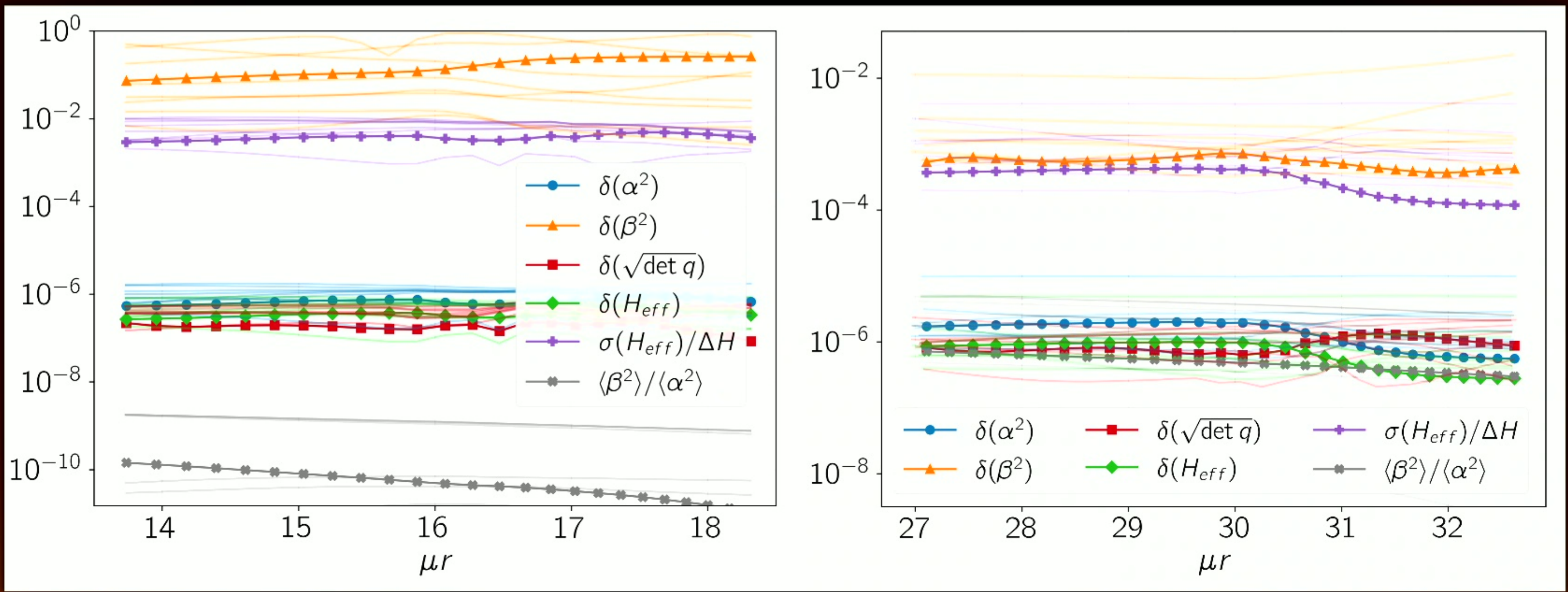
# Robustness of AEP

- Variances over the extraction sphere $\delta(X) = \frac{\sigma(X)}{\langle X \rangle}$




$\sigma = 0.08$

$\sigma = 0.06$

# Teaser: VLBI images

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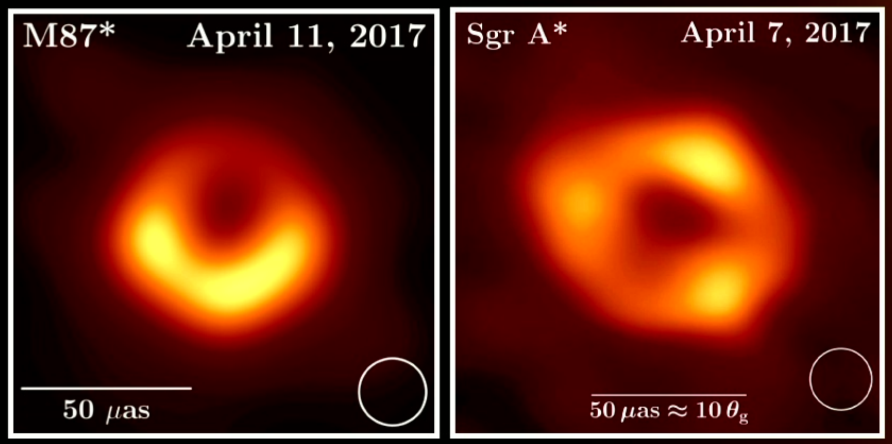
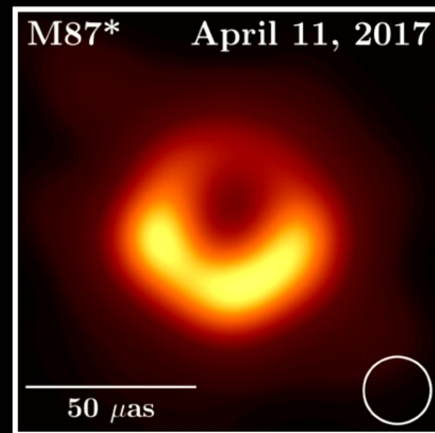
# VLBI images

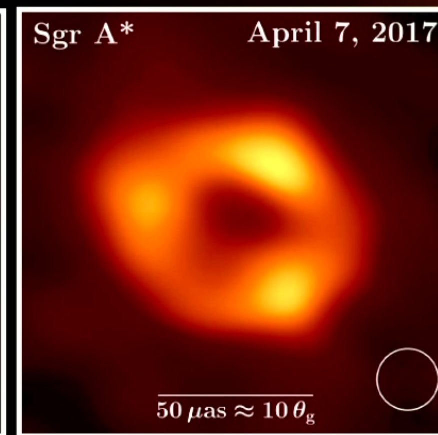
- Event Horizon Telescope Collaboration
- Test the BH paradigm



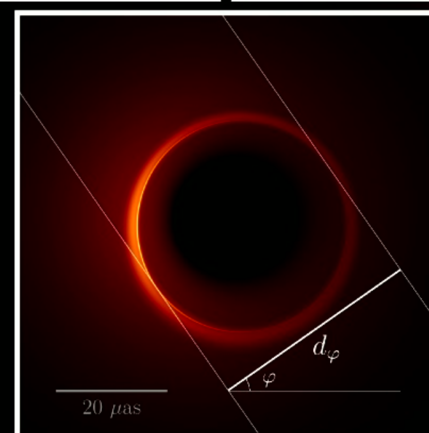
# VLBI images

- Event Horizon Telescope Collaboration
- Test the BH paradigm




- Photon rings of special interest (Johnson+, '20) (Gralla+, '20)

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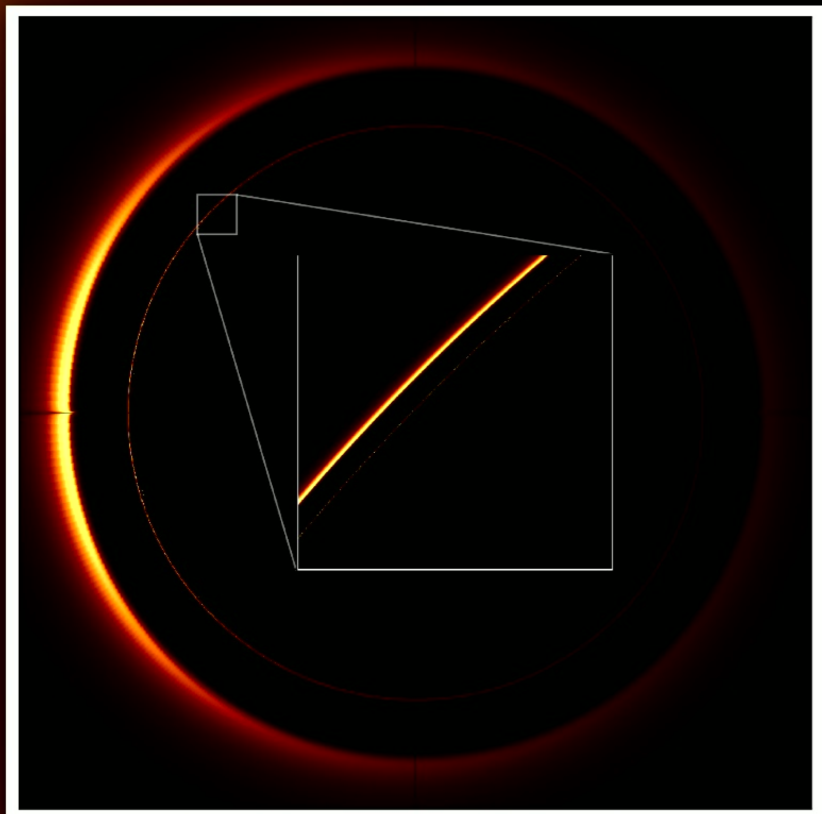
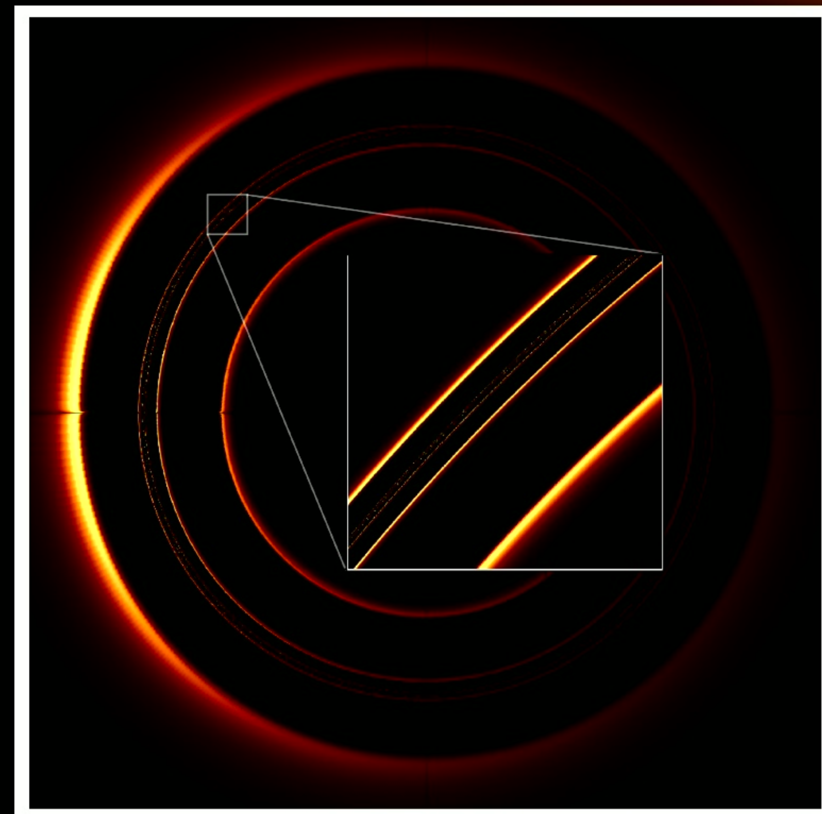


Schwarzschild



Boson Star

(Images produced with FOORT (Mayerson+ (incl. SS), in prep.) )

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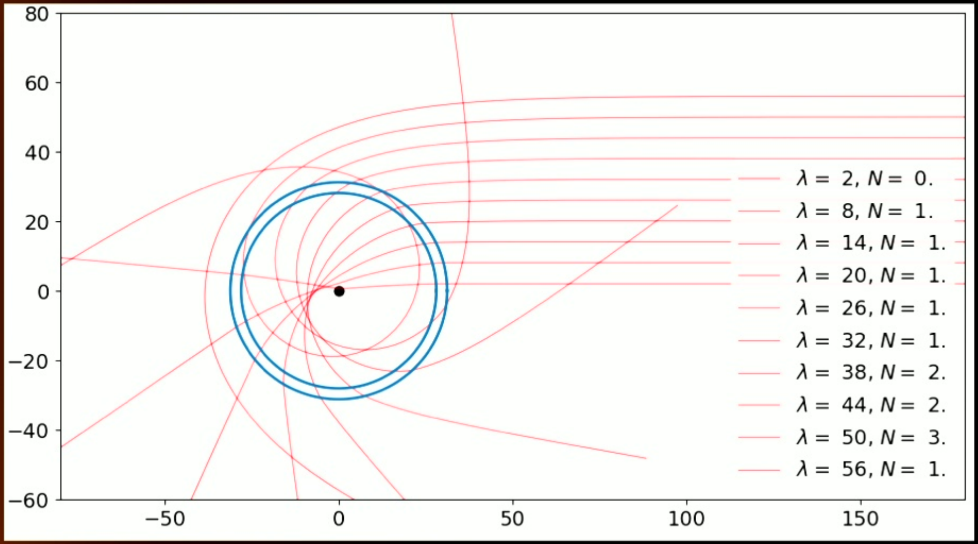
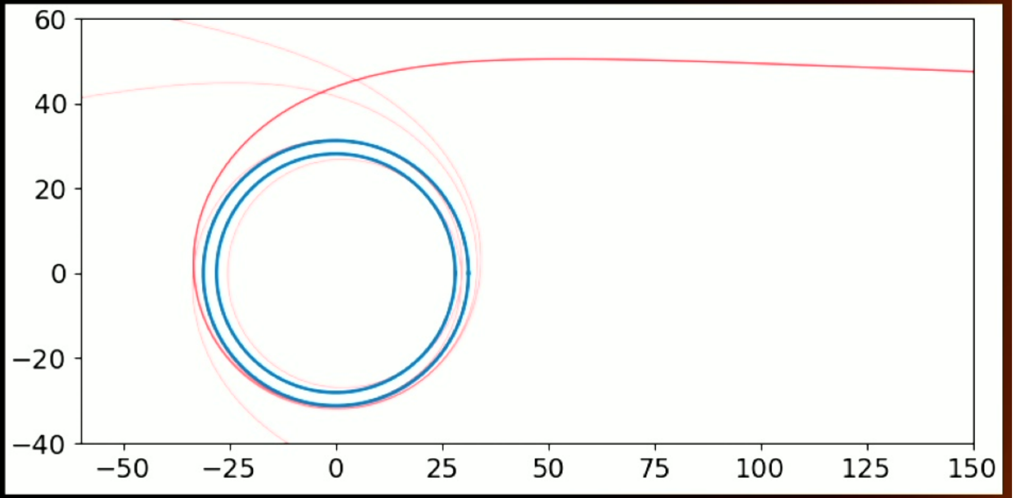
# Ray tracing



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# Conclusions & Outlook

- Stable evolutions of ultracompact solitonic BSs
- Light ring structure is preserved throughout
- Excellent agreement with radial perturbation analysis

# Conclusions & Outlook

- Stable evolutions of ultracompact solitonic BSs
- Light ring structure is preserved throughout
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- Extend to rotating BSs
- Simulations of mergers, formation, non-spherical perturbations