Title: Superconductivity, with Anyons

Speakers: Hart Goldman

Collection/Series: Quantum Matter

Subject: Condensed Matter

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Abstract:

I will discuss the phenomenology of superconductors hosting both order parameter vortices and fractionally charged anyon excitations. I will demonstrate that in such systems superconductivity and topological order are intertwined under applied magnetic fields, leading to surprising observable consequences departing from traditional superconductivity from electronic pairing. In particular, I will show that vortices nucleated by perpendicular magnetic fields must trap anyons in their cores. However, because only some vortices can trap an integer number of anyons, this places a constraint on the vortex phase winding. In general, rather than the expected hc/2e quantization of superconducting vortices, we find instead the enhanced flux quantum of hc/e, which I will argue should affect a wide range of observables. I will further develop a general Landau-Ginzburg theory describing vortex fluctuations and discuss the phase diagram as perpendicular magnetic field is increased, showing that condensation of the intertwined vortices leads to exotic insulating phases hosting neutral anyons and a nonvanishing thermal Hall effect.

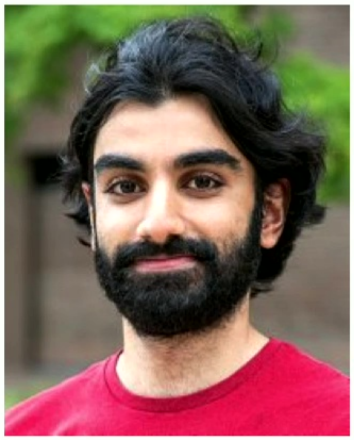
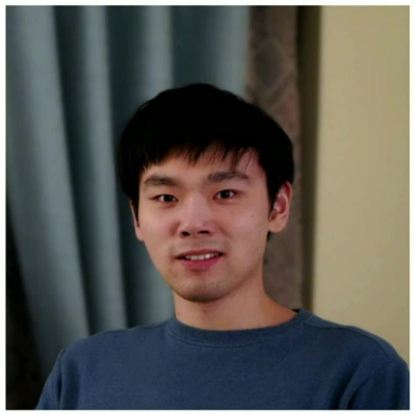
Superconductivity, *with anyons*

Hart Goldman

University of Minnesota

This talk is based on

arXiv:2505.XXXXX

Raman Sohal
UChicago

Xiao-Chuan Wu
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Adarsh Patri
UBC

Alex Thomson
UC Davis

The 2d materials revolution

- Rapid development of atomically thin 2d materials like *graphene* and *transition metal dichalcogenides* (TMDs).

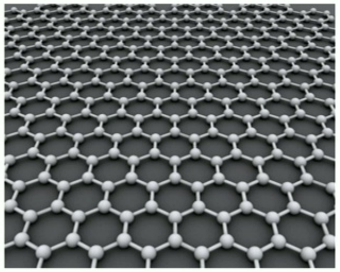
- **Extraordinary tunability** (density, substrate, layers) and high purity, access new physical regimes

- **Prominent example:** twisted bilayers, $\theta \sim \mathcal{O}(1^\circ)$.

  - *Moiré superlattice*: $a_{\mathrm{M}} \sim \dfrac{a}{\theta} \sim 10\,\mathrm{nm}$, *reconstructs electronic bands*, enhances correlations.

- **Playground for correlated physics:** engineer crystals, superconductivity, strange metals, topological phases, ...

  - ***Window into interplay of topology and strong correlations. New quantum phenomena!***



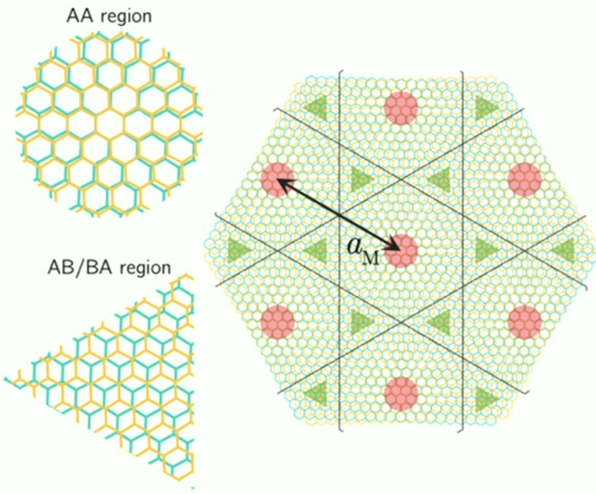
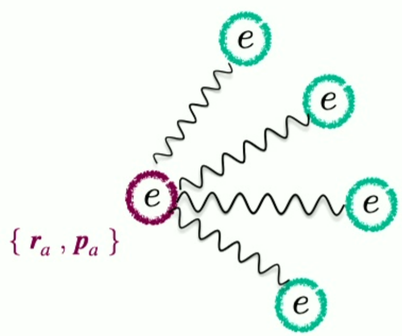


Figure credit: Alex Thomson (UC Davis)

Tuning through the quantum many-body problem

- **Microscopic foundation of quantum condensed matter physics:**



Kinetic energy: K

Potential energy: $V \sim 1/r$

(Interactions)

$$\hat{H} = \sum_{a=1}^{N} \left[\frac{|\boldsymbol{p}_a|^2}{2m_e} + U(\boldsymbol{r}_a) \right] + \sum_{a<b} V(\boldsymbol{r}_a - \boldsymbol{r}_b)$$

- Impossible to solve outright, but can make some basic limiting statements.

Tuning through the quantum many-body problem

Nearly free

electrons are plane waves

$E(\boldsymbol{p}) \sim |\boldsymbol{p}|^2$

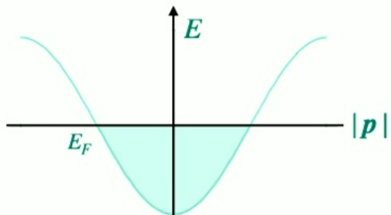
Electrons localized

$E(\boldsymbol{p}) \sim$ constant ("flat bands")

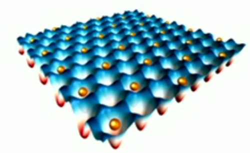
Possibility of non-trivial Hilbert space topology

New exotic quantum phenomena!

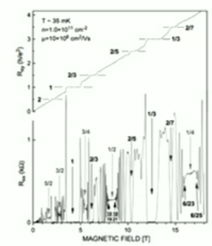
$V/K:\quad 0 \qquad\qquad \infty$



Ordinary metals live here



Wigner crystals



The quantum Hall effect

(2d electrons in large $\boldsymbol{B}$ - field)

Tuning through the quantum many-body problem

Nearly free

electrons are plane waves

$E(\boldsymbol{p}) \sim |\boldsymbol{p}|^2$

moiré materials:
tune through
uncharted territory!

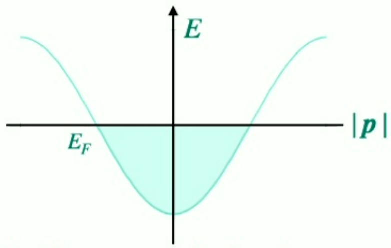
Electrons localized

$E(\boldsymbol{p}) \sim$ constant ("flat bands")

Possibility of non-trivial Hilbert space topology

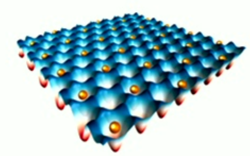
New exotic quantum phenomena!

$V/K:\ 0$ ———— ∞

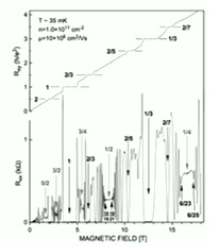


Ordinary metals live here

???



Wigner crystals



The quantum Hall effect

(2d electrons in large $\boldsymbol{B}$ - field)

- **2d materials:** can tune K/V through twisting, displacement field.
  Promising for finding new phenomena

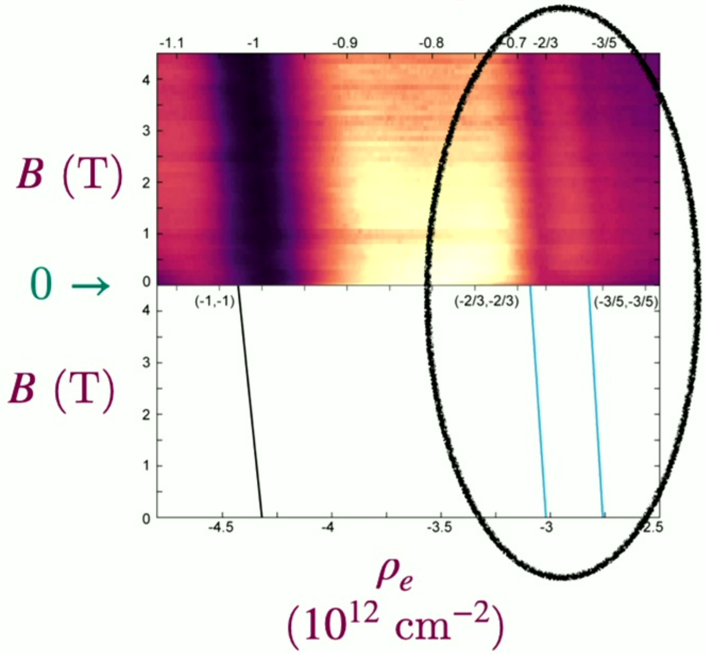
New discovery:
The fractional quantum anomalous Hall (FQAH) effect

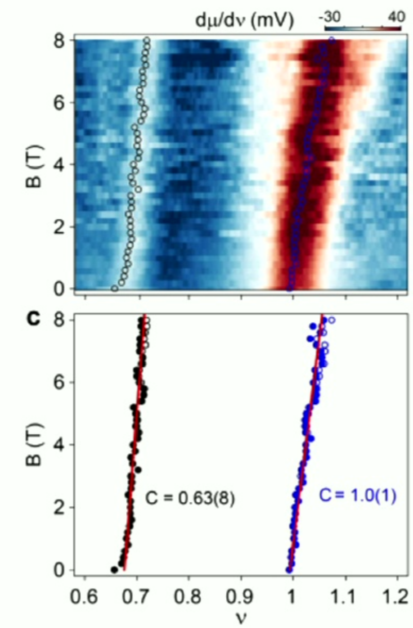
- **April 2023**: Fractional quantum *anomalous* ($B = 0$) Hall effect in twisted $MoTe_2$ from Xiaodong Xu's group [Cai *et al.*, Nature (2023)]
- **Optics measurement of compressibility:** dark regions are incompressible states with quantized Hall conductivity (σ_{xy}).
- ***Requires narrow $|C| = 1$ bands + spontaneous T breaking:*** *previously such phenomena only belonged to 2DEGs in large magnetic field!*

Středa formula

$$\frac{d\rho_e}{dB} = \sigma_{xy}$$

twisted $MoTe_2$, $\theta \approx 3.7°$

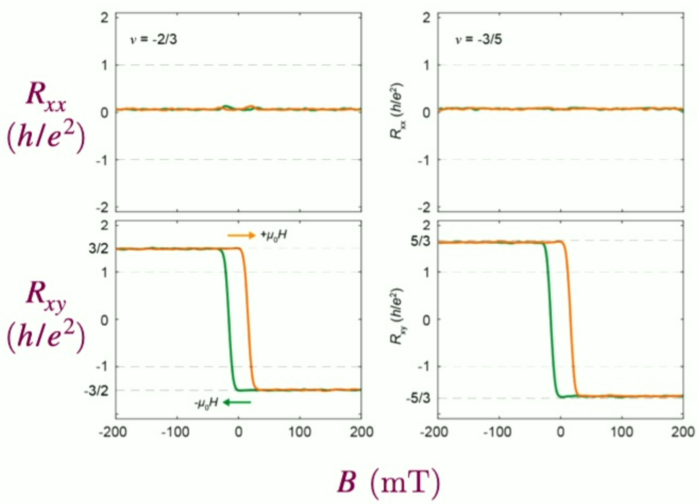




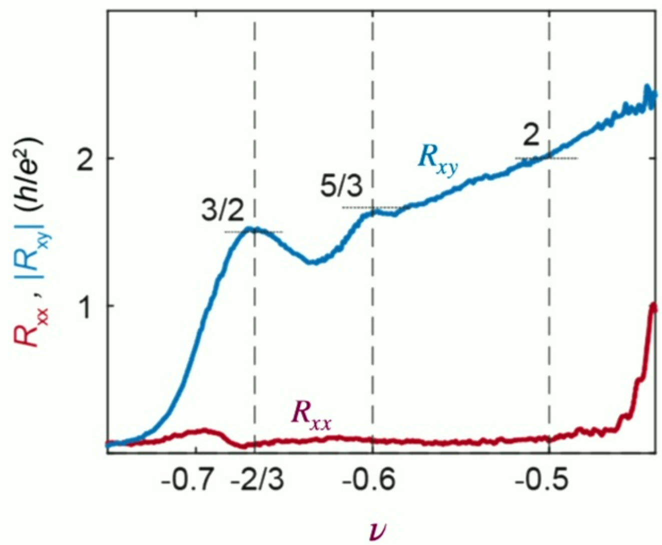
Also Jie Shan and Kin Fai Mak's group:
[Zeng *et al.*, Nature (2023)]

Rapid experimental progress

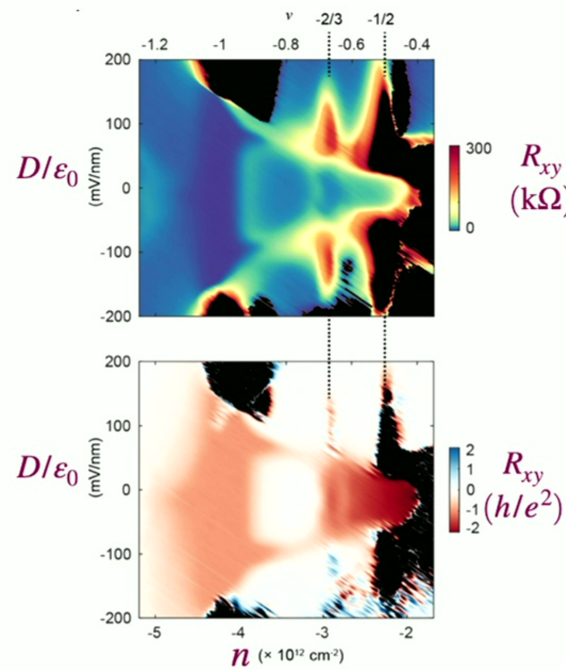
- Subsequently, transport experiments were reported in tMoTe$_2$ further confirming FQAH and fleshing out the phase diagram



Good quantization!



Plateaus sweeping filling at $B = 0$

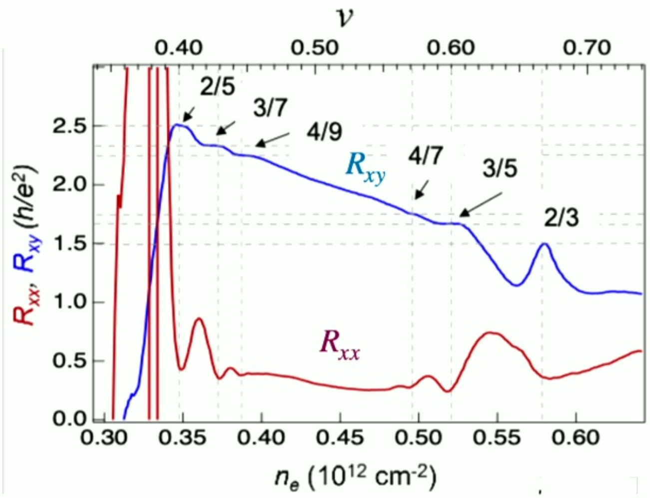


Phase diagram tuning density and displacement field

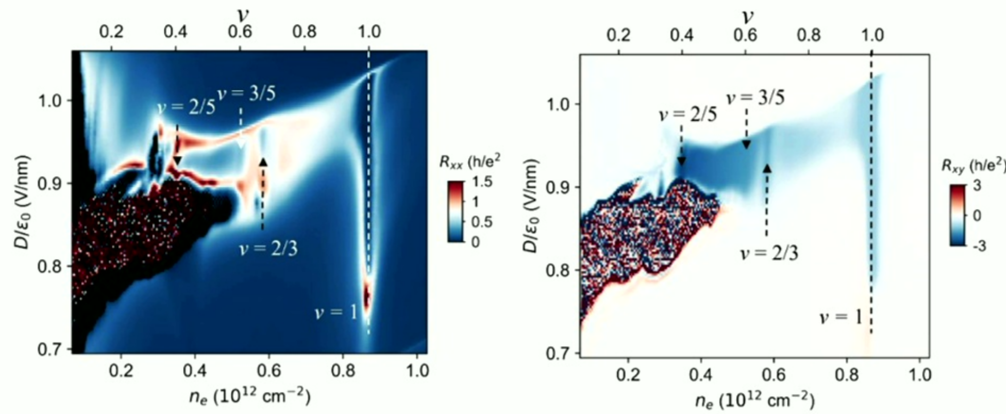
[Park *et al.*, Nature (2023)],
see also: [Xu *et al.*, PRX (2023)]

FQAH in rhombohedral graphene

- Amazingly, similar physics was found in rhombohedral pentalayer graphene/hBN [Lu *et al.*, Nature (2024)]



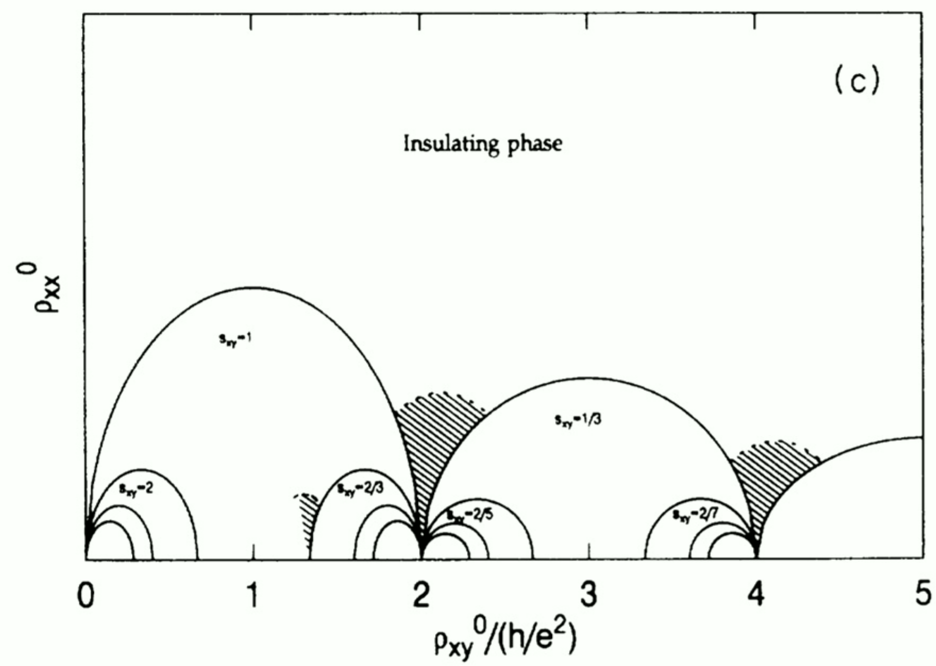
Plateaus sweeping filling at $B \approx 0$



Phase diagram tuning density and displacement field

Opening up the FQAH phase diagram

- *The FQH diagram is vastly enriched at zero magnetic field.*
- Ordinary FQH systems: phase transitions are between QH plateaus and/or ordinary insulating phases, **tuned by filling** (ν, " R_{xy} ") **and disorder** (random impurities, " R_{xx} ") [Kivelson, Lee, Zhang, PRB (1992)].



Opening up the FQAH phase diagram

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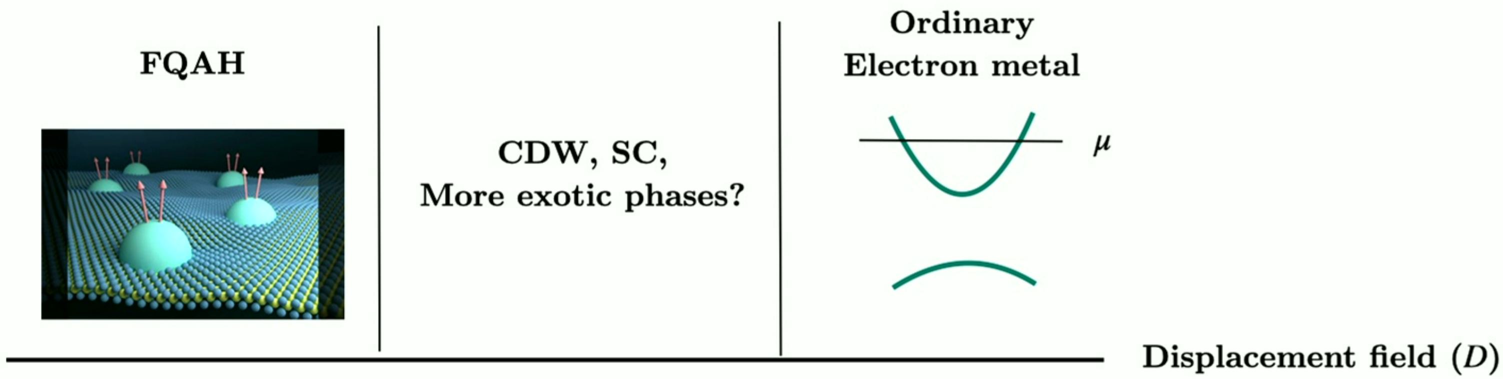
- FQAH: ***new competing phases*** possible at zero field — metals, CDWs, superconductivity, ... — and ***new tuning knobs*** such as displacement field.
  - Possible "*intertwinement*" of topological and Landau orders [Sohal and Fradkin, PRB (2020)], [Song and Senthil, 2311.16216], [Sohal, Wu, Thomson, Patri, and **HG**, 2505.XXXXX]
- *Exotic new kinds of phase transitions! Some well understood, some less so.*
  [Barkeshli and McGreevy, PRB (2012)], [Song, **HG**, and Fu, PRB (2023)], [Song, Zhang, and Senthil, arXiv:2308.10903], ...

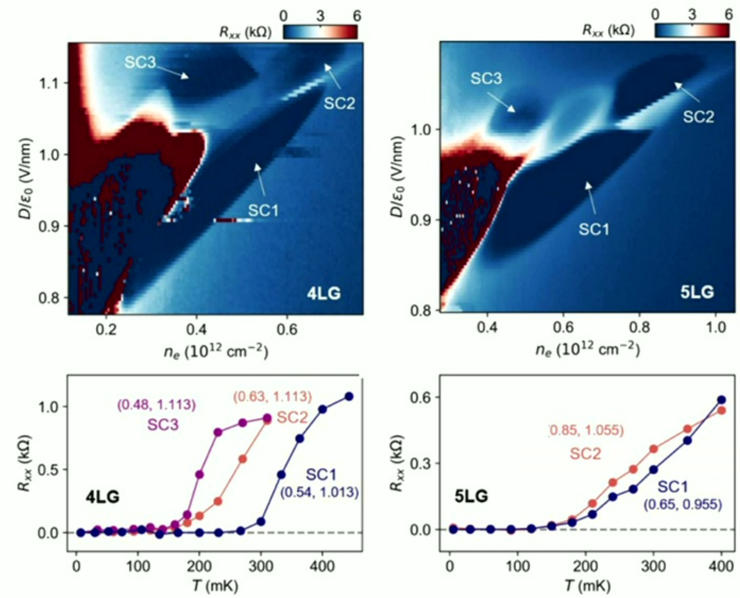


Enter superconductivity

[Han *et al.*, arXiv:2408.15233]

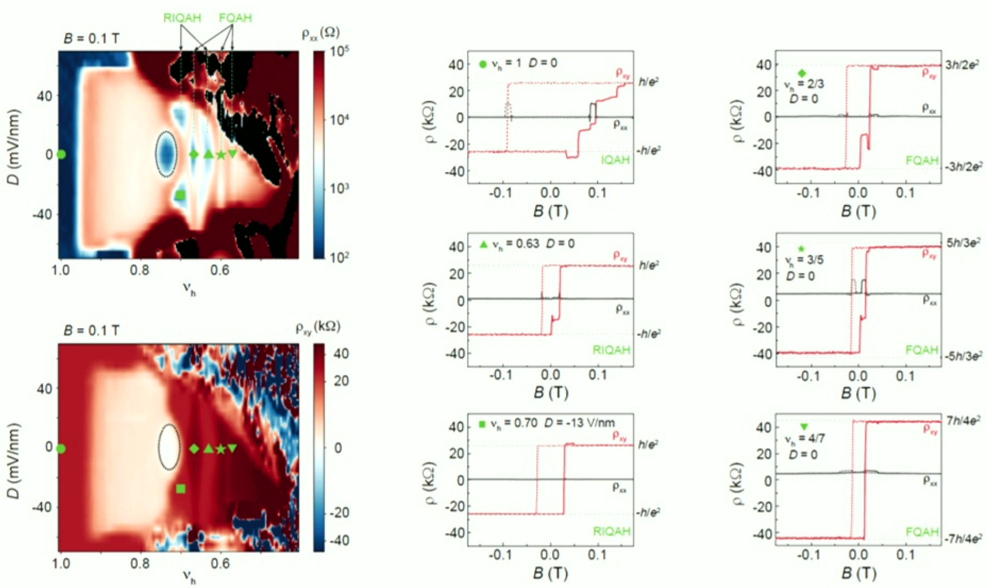
- Recently, signatures of chiral superconductivity seen in rhombohedral graphene *without moiré*, but in a similar region of gate voltage and displacement field to the FQAH phases.

- These SC states arise in the *absence* of a moiré potential, meaning that no FQAH phases are present...

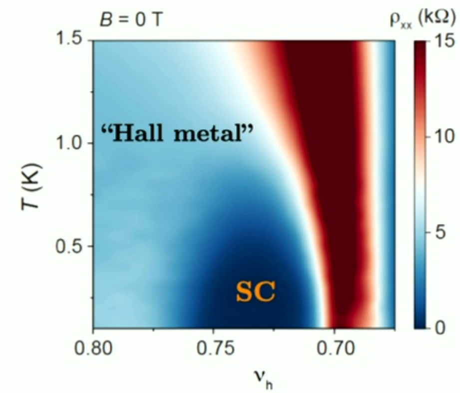


Enter superconductivity

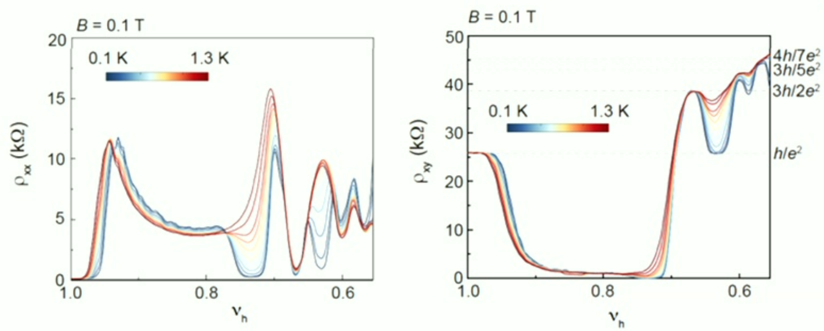
- But now even more recently in $tMoTe_2$, [Xu *et al.*, arXiv:2504.06972] observed signatures of SC near quantum Hall phases!
  - ***Tantalizing possibility: SC born from a FQAH normal state?***



Several FQAH phases, followed by a "re-entrant" integer QAH state

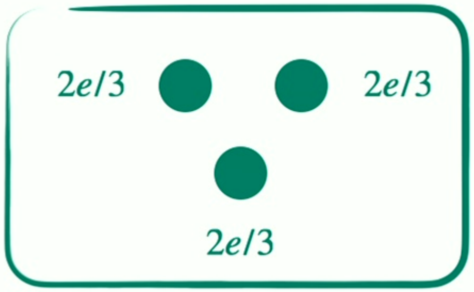


$D = 0$ cuts



Anyon superconductivity

- The possibility of superconductivity in proximity to a FQAH normal state has led to the revival of an old idea, *anyon superconductivity.*
- Motivated by fact that in a FQH fluid (for reasons reviewed later on), low energy excitations are ***anyons*** with fractional charge and quantum statistics.
- **Rough idea:** instead of SC from pairing electrons, obtain it as an instability of an anyon gas where charge-$2e$ objects condense:



[Laughlin, PRL (1988)], [Chen, Wilczek, Witten, and Halperin, Int. J. Mod. Phys. (1989)], [Lee and Fisher, PRL (1989)], ...

Old proposals: "Spontaneous violation of a fact"

- **Basic proposal:** Dope a topological order to make an anyon gas with density ρ, which can be modeled as of fermions attached to $-1/m$ flux quanta.

$$\mathscr{L} \supset j_f^\mu a_\mu + \frac{m}{4\pi}(a+A)d(a+A)$$

CS imparts statistics

$$\theta = \pi\left(1 - \frac{1}{m}\right)$$

- **Mean field theory:** Each fermion sees a *fictitious* magnetic field b, fills $-m$ Landau levels. *Integrating out fermion LLs cancels the CS term...*

$$\rho_f = -m\, b_{\text{eff}}/2\pi \Rightarrow \nu_{\text{anyons}} = 2\pi\frac{\rho_f}{b_{\text{eff}}} = -m \Rightarrow \mathscr{L}_{\text{eff}} = \frac{m}{2\pi}adA + \frac{m}{4\pi}AdA$$

- **Transmutation of ordinary translations into magnetic translations** was associated with a charged "Goldstone mode," claimed to cause superconductivity.

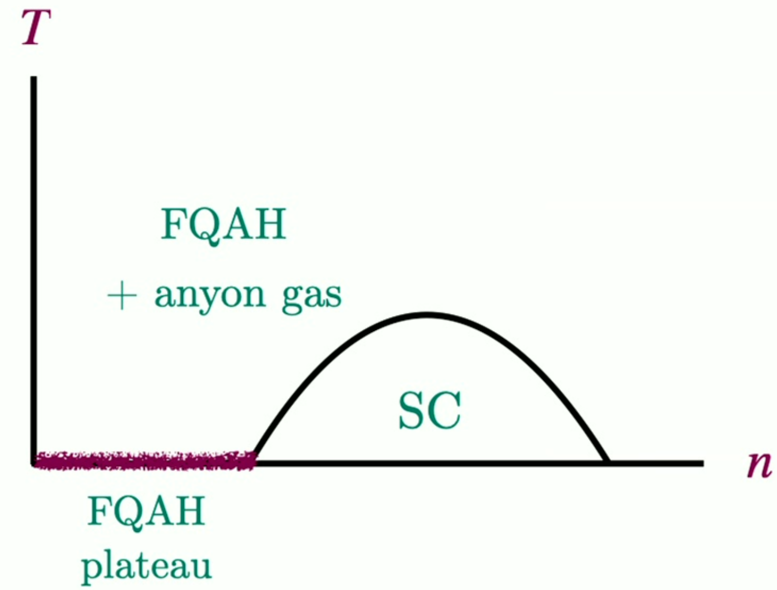
[Laughlin, PRL (1988)], [Chen, Wilczek, Witten, and Halperin, Int. J. Mod. Phys. (1989)], [Lee and Fisher, PRL (1989)], ...

Anyon superconductivity

- **Possible phase diagram:** Imagine doping FQAH to obtain an anyon gas. The anyon gas then experiences an instability to SC at low temperatures.

- **One question:** How to obtain SC from a fractionalized normal state microscopically?
  - *Spontaneous fact violation* avoided by new parton mean field approaches [Shi and Senthil, arXiv:2409.20567], [Divic *et al.*, arXiv:2410.18175]

- **Could there be anything unique about the superconducting phase itself?**



[Laughlin, PRL (1988)], [Chen, Wilczek, Witten, and Halperin, Int. J. Mod. Phys. (1989)], [Lee and Fisher, PRL (1989)], ...

Guiding question

If you meet an anyon SC, how would you be able to tell?

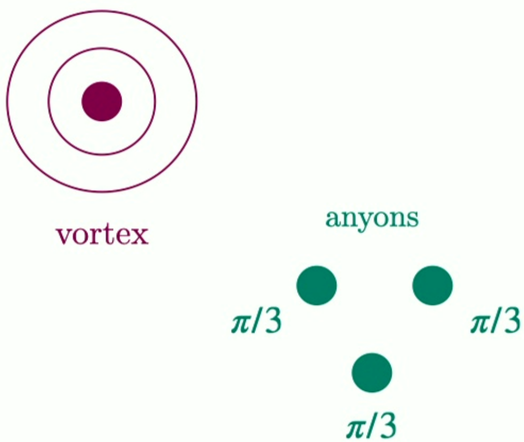
How could you tell it apart from an ordinary SC, without referring to the normal state?

This talk:

Magnetic fields "intertwine" SC and topological order!

Central dogmas

- **Our strategy:** Start with basic expectations for the superconducting phase, work out from there. These furnish a set of *central dogmas*,



1. The system is very clean, but we can tolerate long-wavelength impurities.
2. The microscopic particles are charged fermions.
3. **3.** The gapped excitations of the superconductor include ***both SC vortices*** **and** ***anyons.***

- *Encompass every theory on the market, but not necessarily every anyon SC possible...*

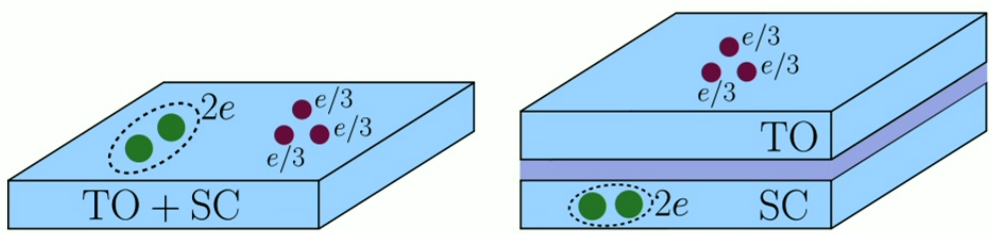
Superconductivity, with anyons

- **To keep in mind:** The anyons in question exist apart from the superconductivity (although charge-$2e$ combinations of them may be allowed to tunnel into the SC condensate). *They are fractionalized degrees of freedom left over after the formation of the SC.*
- Our low energy models (and many of our results) are valid both for:

  (1) an *intrinsically coexisting* SC + TO

  (2) a bilayer of e.g. a FQAH system and a SC, separated by an insulating layer.

  What could possibly be interesting about two morally decoupled systems?
- **Key fact: both sectors couple to the same physical EM field...**



Superconductivity, with anyons

- **K-matrix model:** Need a concrete model of both SC vortices + anyons. Can study the interplay of SC with topological order by writing down a gauge theory at long wavelengths,

K_{mn} : matrix of integers

A_μ : EM field

t_m : anyon charge vector

Differential Notation:
$bdc = \varepsilon_{\mu\nu\lambda} b_\mu \partial_\nu c_\lambda$

$$\mathscr{L} = \frac{K_{mn}}{4\pi} a_m d a_n + \frac{t_m}{2\pi} a_m dA + \frac{Q}{2\pi} \alpha dA - \frac{1}{2g^2} (\varepsilon_{\mu\nu\lambda} \partial_\nu \alpha_\lambda)^2 + \ldots$$

Topological sector:

$m = 1, \ldots, N$ gauge fields, $a_{m,\mu}$,

Labeling different anyon species

Ground state degeneracy: $\det K$

Charge-Qe SC sector:

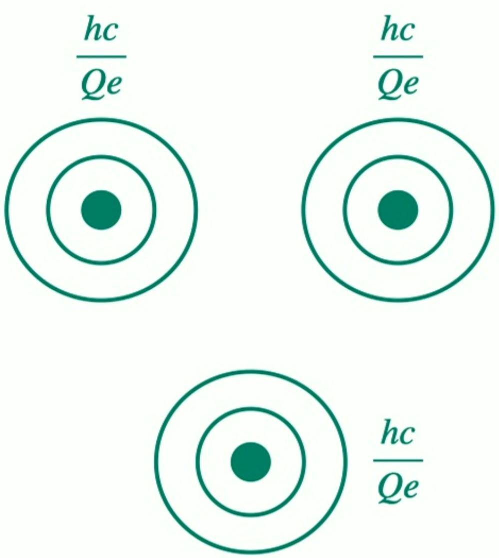
α_μ : "dual" to order parameter phase, ϑ

$\partial_\mu \vartheta = \varepsilon_{\mu\nu\lambda} \partial_\nu \alpha_\lambda / 2\pi$

[Wen and Zee, PRB (1991)], [Shi and Senthil, arXiv:2409.20567], [Kim, Timmel, Ju, and Wen, PRB (2025)], [Sohal, Wu, Patri, Thomson, and **HG**, 2505.XXXXX]

Threading flux: SC sector

- Usual expectation: Meissner effect means that for a charge-$2e$ SC, magnetic flux nucleates half-vortices carrying π flux ($hc/2e$). Lets see how this works in the gauge theory.



- Effective gauge theory for charge-Qe SC:

$$\mathscr{L}_{\mathrm{SC}} = \frac{Q}{2\pi}\alpha dA$$

(Neglecting Maxwell term, which provides stiffness)

- α acts as a Lagrange multiplier, imposing a *Meissner effect* :

$$\Phi_{\mathrm{ext}} = Q\int_{\Sigma} dA = 0 \bmod 2\pi$$

Σ : closed $2d$ surface

Threading flux: SC sector

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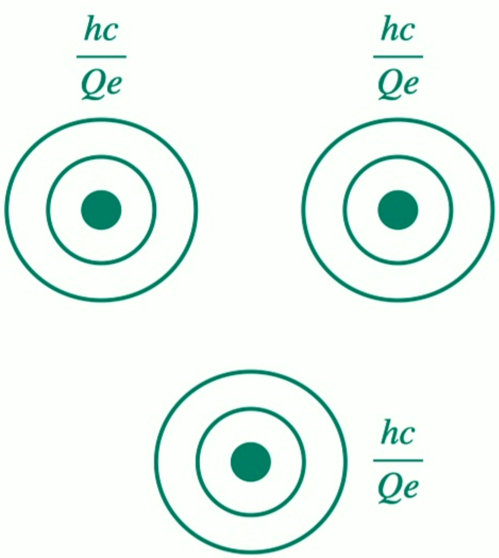


- Effective gauge theory for charge-Qe SC:

$$\mathscr{L}_{\text{SC}} = \frac{Q}{2\pi}\alpha dA$$

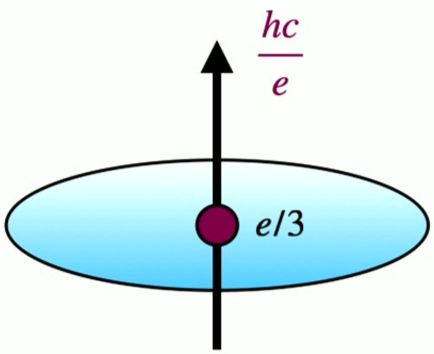
(Neglecting Maxwell term, which provides stiffness)

- Means only fluxes in units of hc/Qe can penetrate the sample,

$$\Phi_Q = \frac{2\pi}{Q}$$

- ***Key consequence:*** *Magnetic fields nucleate vortex lattices, with each vortex carrying* hc/Qe *flux.*

Threading flux: anyon sector



- Now consider the anyon sector alone:

$$\mathscr{L}_{\text{top}} = \frac{K_{mn}}{4\pi} a_m da_n + \frac{t_m}{2\pi} a_m dA$$

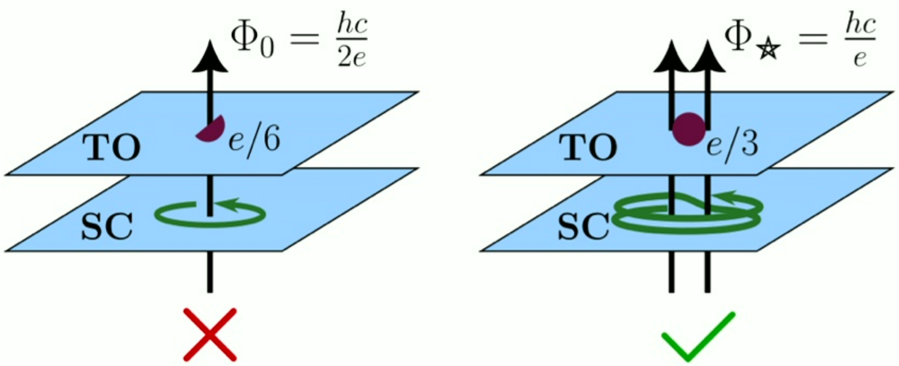
- Here the famous Laughlin argument applies. Thread 2π (hc/e) flux. *Then t_m anyons of species m will be nucleated.* For any flux configuration Φ_{ext} penetrating the sample,

$$t_m \frac{\Phi_{\text{ext}}}{2\pi} = t_m \int_\Sigma \frac{dA}{2\pi} \in \mathbb{Z}$$

Number of m - type anyons must be an integer

- ***Example:*** *For a $\nu = 1/3$ Laughlin state, a solenoid possessing a single flux quantum nucleates a single $e/3$ quasiparticle.*

Synthesis: Which vortices are allowed?

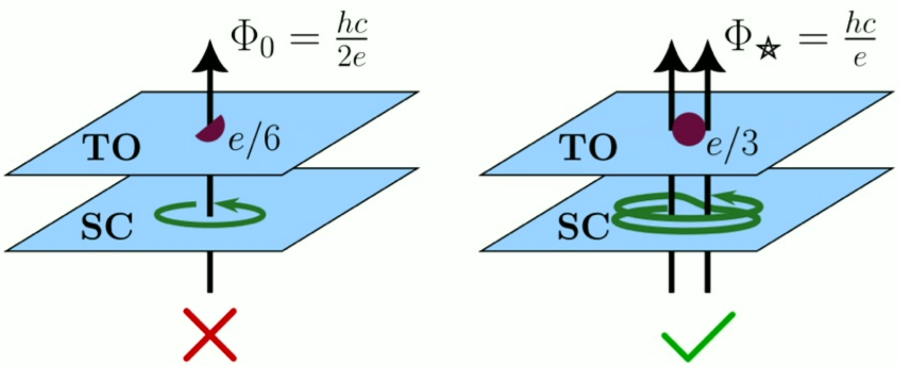


- **Tension:** magnetic fields nucleate partial vortices in superconductors, but these vortices would trap partial anyons...
- **Resolution**: Only fluxes which trap an integer number of anyons will be nucleated by applied field!
- **Magnetic fields intertwine SC and TO!**
  - *Emergent intertwinement*: individual anyons and partial vortices still exist, they just aren't nucleated by a magnetic field.

[Sohal, Wu, Patri, Thomson, and **HG**, 2505.XXXXX]

Analogous proposal for detecting TO in cuprates: [Senthil and Fisher, PRL (2001)]

Commensurability condition



- What is the smallest number of SC flux quanta $(2\pi/Q)$ that traps an integer number of anyons?
- Can obtain a minimum unit of flux for the intertwined vortex:

$$\Phi_\star = \frac{2\pi}{\mathfrak{q}} \qquad \mathfrak{q} = \gcd(Q, t_1, t_2, \ldots, t_N)$$

t_m : anyon charge vector , Q : charge of SC

- **On turning on a magnetic field, the *intertwined* vortices will be nucleated!** Alternatives cost energy growing with system size.

[Sohal, Wu, Patri, Thomson, and **HG**, 2505.XXXXX]

Interpretation

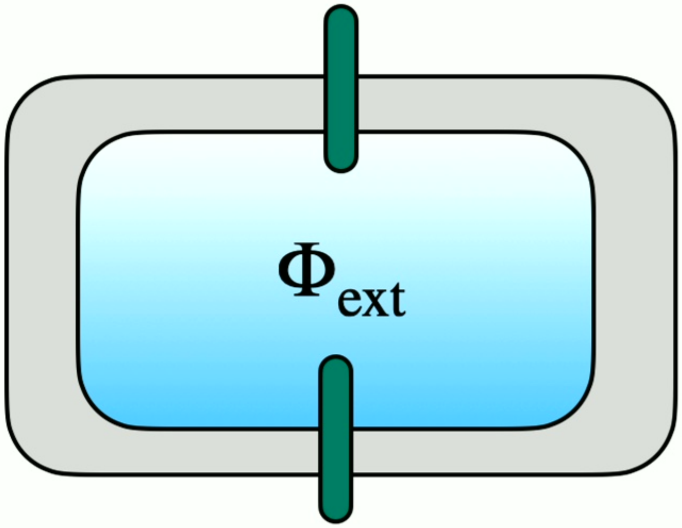
$$\Phi_\star = \frac{2\pi}{\mathfrak{q}} \qquad \mathfrak{q} = \gcd(Q, t_1, t_2, \ldots, t_N)$$

- $\Phi_\star$ **determines the periodicity of the full system under magnetic flux.**
- $\mathfrak{q} = 1$ **generically** (at least one $t_m = 1$). Means hc/e vortices are the general outcome.
- **TO not strictly essential:** If the topological sector is integer quantum Hall, the commensurability condition still holds.
  A vortex cannot trap half an electron!

[Sohal, Wu, Patri, Thomson, and **HG**, 2504.XXXXX]

Observing the doubling

- In principle, the periodicity of the system under threading $\mathbf{\Phi}_\star$ flux should be observable. **One possibility:** Little-Parks experiment



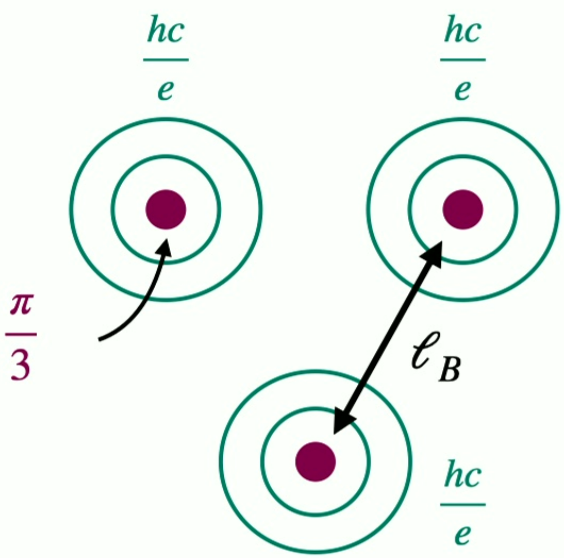
Measure resistance or critical current as a function of flux:

$$R \sim R_0 \sin\left(2\pi \frac{\Phi_{\text{ext}}}{\Phi_\star}\right)$$

- Expect almost any measurement sensitive to flux periodicity to see signatures of $\mathbf{\Phi}_\star$

"Intertwined" Landau-Ginzburg mean field theory

- With an understanding of the elementary vortices, can understand the behavior of the system under perpendicular magnetic field, $B = \nabla \times A$



- **Expect *intertwined vortices* are nucleated, form a vortex lattice.** Write down a Landau-Ginzburg theory for the vortices:

$$\mathscr{L} = \mathscr{L}_v - \frac{1}{2\pi}(t_m a_m + \alpha)\, dA + \frac{K_{mn}}{4\pi} a_m da_n + \ldots$$

$$\mathscr{L}_v = |D_\mu \phi_v|^2 - V[\phi_v] \qquad D_\mu = \partial_\mu - i\frac{\Phi_\star}{2\pi}(t_m a_{m,\mu} + Q\alpha_\mu)$$

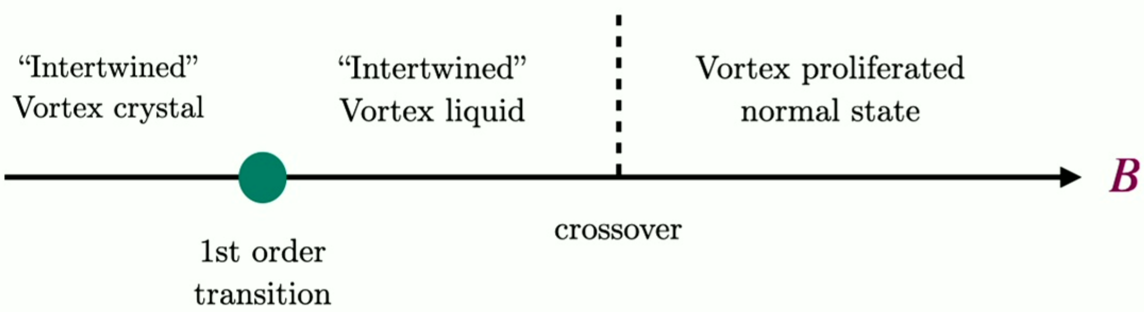
ϕ_v : intertwined vortex field, $\langle \phi_v \rangle = 0$ gapped in SC phase

[Zhang, Hansson, Kivelson, PRL (1989)], [Wen and Zee, PRB (1991)], ... , [Sohal, Wu, Patri, Thomson, and **HG**, 2505.XXXXX]

Evolution under magnetic field

- As B is increased, vortex crystal eventually melts. What happens next will depend on detailed energetics of the system — require (1) intertwined vortices to remain lightest objects, (2) anyon tunneling between vortices is weak.

- If this is the case, eventually the vortices will proliferate and $\langle \phi_v \rangle \neq 0$...



What happens when the vortices condense?
Thermal Hall crystal

- **Consider what happens when the vortices condense, $\langle \phi_v \rangle = 0$.** Then the gauge fields are Higgesed such that $t_m a_m = -Q\alpha$. If $\Phi_\star = 2\pi$, obtain a final theory *independent of the background EM field*

$$\mathscr{L} = \frac{1}{2\pi}\beta\, d(t_m a_m + Q\alpha) + \frac{K_{mn}}{4\pi}\, a_m d a_n$$

β_μ : gauge field dual to the vortex current

Enhanced ground state degeneracy: $\mathfrak{D} = Q^2 |\det K|$

- **Thermal Hall crystal:** Insulating state with neutral anyons, no EM Hall effect, but $\kappa_{xy} \neq 0$!
- No charged anyons: if state exists in a lattice system, must be accompanied by coexisting translation breaking.
- Direct transition from SC to this state also possible, perhaps with disorder.

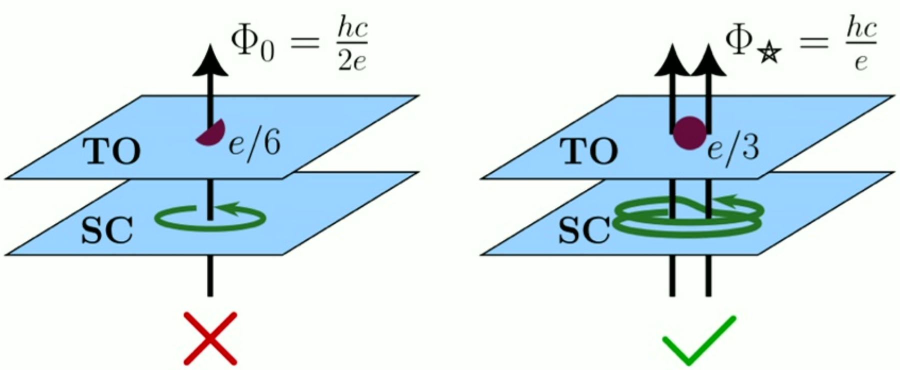
[Sohal, Wu, Patri, Thomson, and **HG**, 2504.XXXXX],

similar states found in [Song, Zhang, and Senthil, PRB (2024)], [Song and Senthil, arXiv:2311.16216]

Summary

- Considered general effective theories of superconductivity, with anyons.

- Because magnetic vortices trap an integer number of anyons, **the effective flux quantum is generically enhanced to hc/e** , *could be sought after experimentally as a means of determining the presence of gapped anyons in the spectrum of a SC.*



- Developed a LG theory for studying vortex fluctuations, allowed access to an exotic phase with "dark" topological order!

- **Detailed understanding of anyon dynamics still very much needed!** *Major direction for future work...*

[Sohal, Wu, Patri, Thomson, and **HG**, 2504.XXXXX]

The future is anyon dynamics

- **Microscopic foundations:** Dynamics of anyon gases at finite T giving rise to SC, beyond simple parton mean field approaches? How to map onto traditional frameworks for SC?
- **Constraints on transport and thermodynamics in anyon gases.**
  Ongoing work with Yuto Nakajima and Umang Mehta.
- **Collective modes** of the intertwined vortex crystal.
- **Continuous "anyonic" quantum phase transitions beyond partons?**
- **Tunnelling of anyons between vortices?** *Basis for an "anyon Hubbard model"*?
- **Dispersion of "itinerant" anyons?**

Thanks!

[Sohal, Wu, Patri, Thomson, and **HG**, 2504.XXXXX]