Title: Temperature-Resistant Order in 2+1 Dimensions

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Subject: Other, Quantum Fields and Strings

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Abstract:

High temperatures are typically thought to increase disorder. Here we examine this idea in Quantum Field Theory in 2+1 dimensions. For this sake we explore a novel class of tractable models, consisting of nearly-mean-field scalars interacting with critical scalars. We identify UV-complete, local, unitary models in this class and show that symmetry breaking $\mathbb{Z}_2 \to \emptyset$ occurs at any temperature in some regions of the phase diagram. This phenomenon, previously observed in models with fractional dimensions, or in the strict planar limits, or with non-local interactions, is now exhibited in a local, unitary 2+1 dimensional model with a finite number of fields.

Entropic Order

2503.22789, 2412.09459

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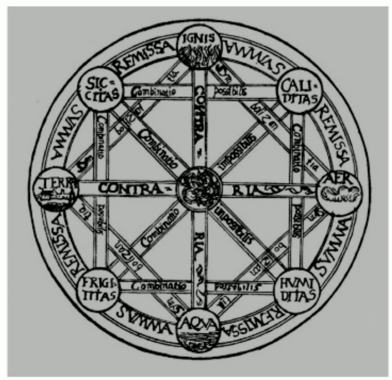
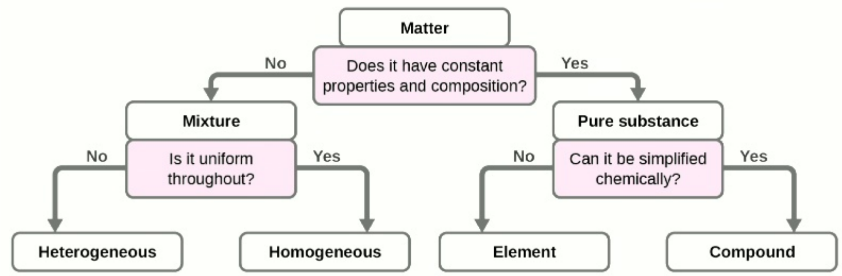
13 May 2025

The Nature of Matter

Bertrand Russell, *The History of Western Philosophy*, Allen & Unwin, 1945.

- The classification of states of matter and the understanding of their nature are among the most fundamental problems, dating back to antiquity.





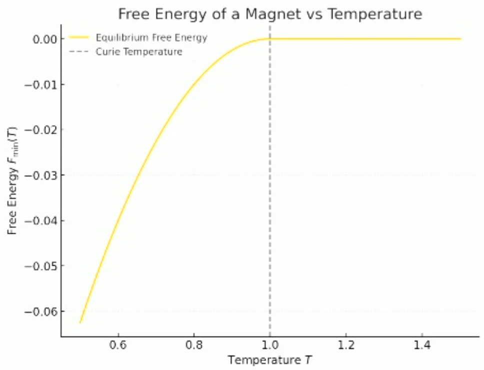
Spontaneous Magnetization

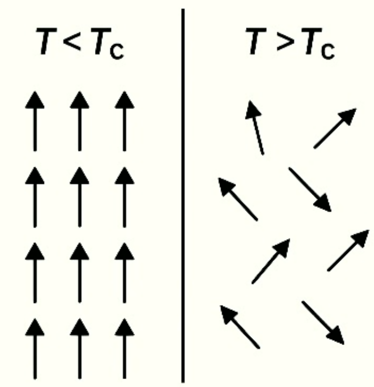
P. Curie, *Sur la symétrie dans les phénomènes physiques*, J. Phys., 1894.

- Pierre Curie observed that magnets can become magnetized without the presence of an external magnetic field, indicating spontaneous symmetry breaking.

- “When certain effects show a certain dissymmetry, this dissymmetry must be found in the causes that gave rise to them.”

- Pierre Weiss proposed that spins interact via an internal field, causing spontaneous magnetization.





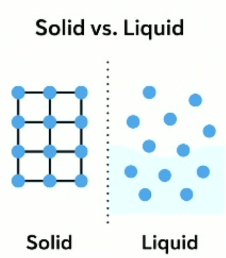
Landau Theory of Phase Transitions

L. D. Landau, *On the Theory of Phase Transitions*, Zh. Eksp. Teor. Fiz. **7**, 19–32 (1937).

- Landau introduced the idea of symmetry and symmetry breaking as a universal classificatory.
- Based not this Landau proposed a phenomenological theory of second-order phase transitions.

$$F(M,T) = a(T)M^2 + b(T)M^4$$

$$H = -\sum_i \frac{\hbar^2 \nabla^2}{2m} + \sum_{ij} U(x_i - x_j)$$



Ferromagnet	Paramagnet → Ferromagnet	Spin rotation symmetry	SO(3) → SO(2) (or Z_2 for Ising)
Superconductor (BCS)	Normal metal → Superconductor	U(1) gauge symmetry (global in BCS)	U(1) → 1
Liquid → Solid (Crystal)	Translation / Rotation symmetry	Continuous translational and rotational symmetry	Euclidean group E(3) → space group
Superfluid (Helium-4)	Normal fluid → Superfluid	Global U(1) phase symmetry	U(1) → 1
Electroweak transition	High-T → Higgs phase	SU(2)×U(1) gauge symmetry	SU(2)×U(1)_Y → U(1)_EM
Quark-gluon plasma → Hadron	Deconfinement → Confinement	Center symmetry (approximate)	Z_3 (in SU(3)) → 1
Nematic liquid crystal	Isotropic liquid → Nematic phase	Rotational symmetry	SO(3) → SO(2)
Bose-Einstein Condensate	Thermal gas → Condensate	Global U(1) phase symmetry	U(1) → 1
Time crystal	Continuous time → Periodic	Time translation symmetry	$\mathbb{R} \to \mathbb{Z}$ (discrete subgroup)
Chiral symmetry breaking (QCD)	Quark-gluon → Hadronic matter	Chiral SU(N)_L × SU(N)_R symmetry	SU(N)_L × SU(N)*R → SU(N)* {L+R}

Symmetries and Temperature

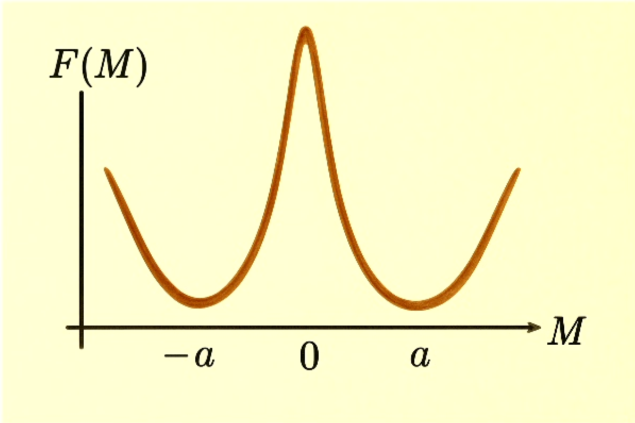
L. D. Landau, *On the Theory of Phase Transitions*, Zh. Eksp. Teor. Fiz. **7**, 19–32 (1937).

- Landau argued that as temperature decreases, entropy becomes less important, and the system tends to minimize its internal energy by breaking some symmetries.

$$\langle O \rangle_T = \frac{\int dE\, O(E) e^{S(E)-E/T}}{\int dE\, e^{S(E)-E/T}}$$

$$F = E - TS$$

$$T \to 0, E \text{ is more important}$$
$$T \to \infty, S \text{ is more important}$$

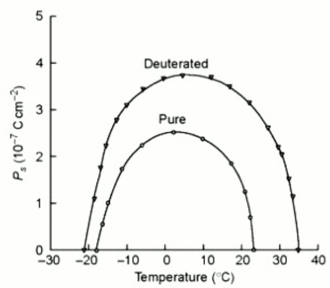


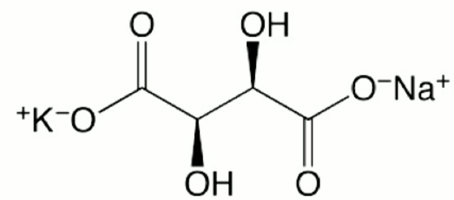
Free Energy as a function of order parameter

Counterexamples to Simple Symmetry Arguments

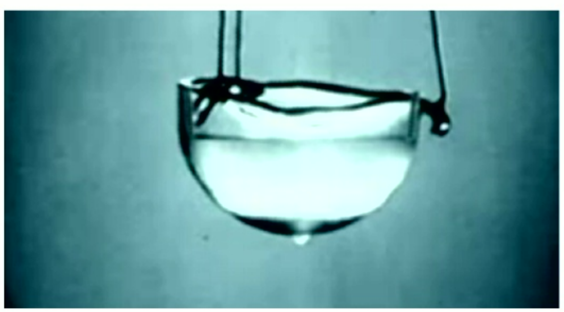
G. Busch, *Ferroelectricity*, Springer, 1949.**I. Pomeranchuk**, *On the possibility of cooling liquid He3 by solidification*, Zh. Eksp. Teor. Fiz. **20**, 919 (1950).

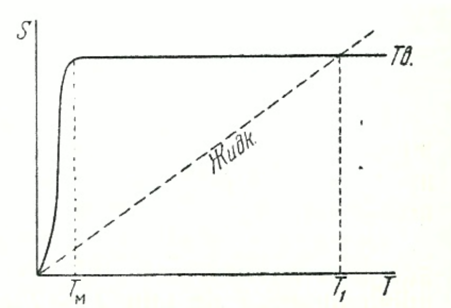
- Rochelle salt has two Curie points





- Pomeranchuk effect: He-3 remains solid at very low temperatures due to higher entropy of the liquid phase.





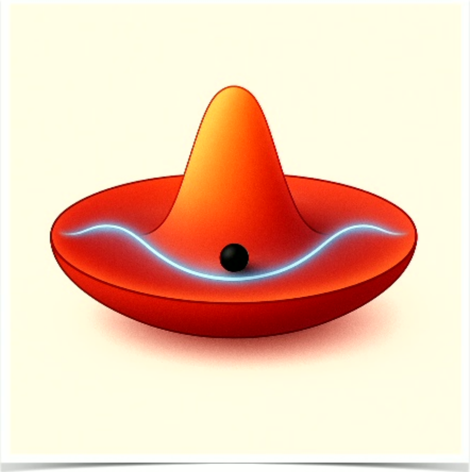
SSB in Classical and Quantum Mechanics

S. Weinberg, *The Quantum Theory of Fields, Volume II: Modern Applications*, Cambridge University Press, 1996.

- In classical mechanics, the ground state can break symmetry at zero temperature.

- At finite temperature, thermal fluctuations allow the system to explore all states, restoring the symmetry on average.

- In quantum mechanics, multiple degenerate vacuum states can exist, and we can form symmetric linear combinations. Symmetry is still restored at finite temperature due to statistical mixing:



$$U\rho U^{-1} = \rho$$

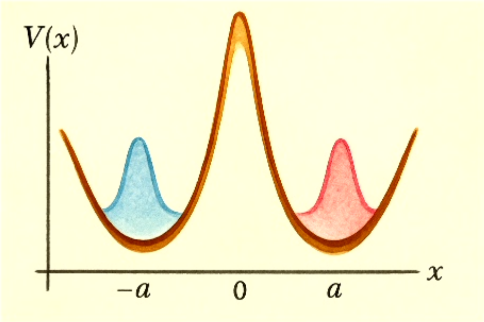
SSB in Quantum Mechanics

S. Coleman, *Aspects of Symmetry*, Cambridge University Press, 1985.

- Consider the double well potential

$$H = -\Delta + \lambda \left(x^2 - a^2\right)^2$$

- In the large coupling limit, the spectrum develops near-degenerate states localized in the left and right wells.



- Tunneling between the two vacua is exponentially suppressed.
- Superpositions of vacua are unstable and effectively decohere under small perturbations

$$E^i_+ - E^i_- \approx \mathcal{O}\left(e^{-\lambda a^3}\right)$$

$$\rho \approx \frac{1}{2}\rho_+ + \frac{1}{2}\rho_- \xrightarrow{\text{pert}} \rho_+$$

$$\langle \psi^i_+ | A | \psi^i_- \rangle \approx \mathcal{O}\left(e^{-\lambda a^3}\right)$$

SSB in Quantum Field Theory

***F. Strocchi**, Symmetry Breaking, 2nd Edition, Springer, 2008.*

$$S = \frac{1}{2}\left(\partial_\mu \phi\right)^2 + \frac{\lambda}{4}\left(\phi^2 - v^2\right)^2$$

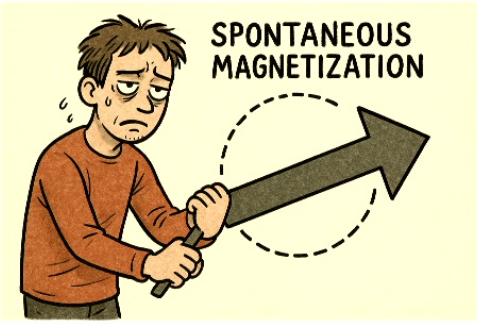
- In quantum field theory, infinite volume leads to superselection sectors, corresponding to distinct vacua.

$$\langle + | \, \mathcal{O} \, | - \rangle \propto e^{-CV}$$

- Averaging over vacua would lead to a violation of the cluster decomposition property:

$$|\pm\rangle\,, \quad \langle \pm | \, \phi \, | \pm \rangle = \pm v, \quad |s\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}}$$

$$\langle s|A(x)B(y)|s\rangle \neq \langle s|A(x)|s\rangle \langle s|B(y)|s\rangle \,, |x-y| \to \infty$$



- The ground state must belong to one sector; superpositions are forbidden.

$$\rho = e^{-H/T} = \frac{1}{2}\rho_+ + \frac{1}{2}\rho_-,$$

- Sometimes these super selection sectors do not exist **S. Coleman, "There are no Goldstone bosons in two dimensions"**,

$$\textbf{Bogoliubov:} \quad \rho_p = \lim_{\epsilon \to 0} e^{-\frac{H + \epsilon V}{T}} = \rho_+$$

$$\textbf{Ruelle:} \quad \rho_p = \lim_{A \to \infty} e^{-H_{\partial A}/T} = \rho_+$$

Dobrushin Theorem and Gibbs Uniqueness at High Temperature

R. L. Dobrushin, *Prescribing a system of random variables by conditional distributions*, Theory of Probability and Its Applications, **15**, 458–486 (1970).

- At infinite temperature, the system behaves as:

$$\lim_{T\to\infty} e^{-\frac{H+\epsilon V}{T}} = 1 \qquad \langle\phi\rangle_T = \sum_E \langle E|\phi|E\rangle\, e^{-E/T} = \sum_E \langle E|\phi|E\rangle = \mathrm{tr}[\phi] = 0$$

- Assume that locally the system has a maximally mixed (identity) state, and impose boundary conditions at infinity(Ruelle's approach)

- The expansion is analytic with finite radius of convergence. The influence of boundary conditions appears only at high orders:

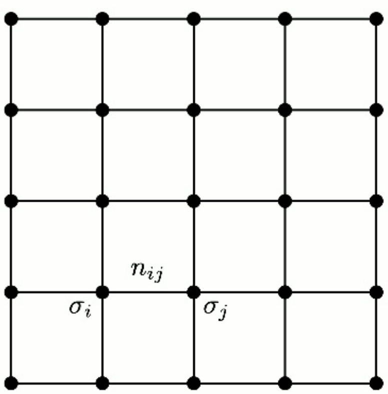
$$\rho = e^{-H/T} = \sum_{n=0}^{\infty} \frac{1}{n!}\frac{H^n}{T^n}, \quad T > T_*$$

Loopholes in Dobrushin's theorem

- The Dobrushin theorem assumes existence of local maximally mixed (identity) state.
- Many systems, such as spin models, satisfy this assumption.
- However, introducing bosonic degrees of freedom violates this assumption, as it leads to an infinite number of states.

$$H = \sum_{<i,j>} (a - bn_{ij})(\sigma_i \sigma_j + 1) \qquad H_{\text{eff}} = \frac{1}{T_{\text{eff}}(a,b,T)} \sum_{<i,j>} \sigma_i \sigma_j$$

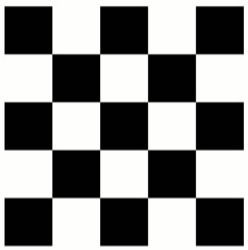
$$\sum_{n>0} e^{(a-bn)\sigma_i\sigma_j} = C e^{-\frac{1}{T_{\text{eff}}}\sigma_i\sigma_j} \qquad T_{\text{eff}} < \frac{2}{\log \frac{a+b}{a-b}}$$



Lattice Boson Gas

T.D. Lee and C.N. Yang, "Statistical Theory of Equations of State and Phase Transitions. II. Lattice Gas and Ising Model,"

- Lattice Gas Model (antiferromagnetic Ising Model)



$$H = \mu \sum_i n_i - U \sum_{<i,j>} n_i n_j, \quad n_i = 0, 1$$

$$P(n) = 2^{-L^2}$$

- Allow unbounded local occupation numbers

$$n_i = 0, 1, \ldots, \infty$$

- To ensure stability, we introduce positive couplings

$$H = \mu \sum_i n_i + U \sum_{<i,j>} n_i^a n_j^a$$

Mean-Field Approximation and Monte Carlo Simulations

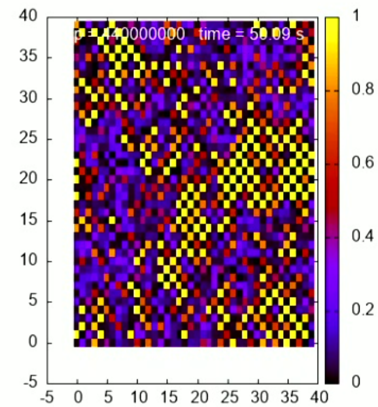
- Checkerboard phase $\qquad n \lesssim T$

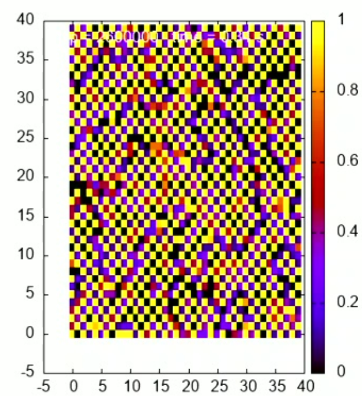
$$T \gg 1, \quad Z_1 = \sum e^{-\beta n} = \frac{1}{e^{\beta} - 1} \sim T, \quad Z \sim T^{\frac{L^2}{2}}$$

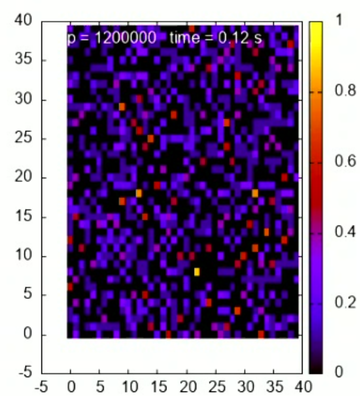
- Gas phase $\qquad Un^{2a} \lesssim T$

$$T \gg 1, \quad Z_1 = \sum_{n=0}^{T^{\frac{1}{2a}}} e^{-\beta n} \sim T^{\frac{1}{2a}}, \quad Z \sim T^{\frac{L^2}{2a}}$$

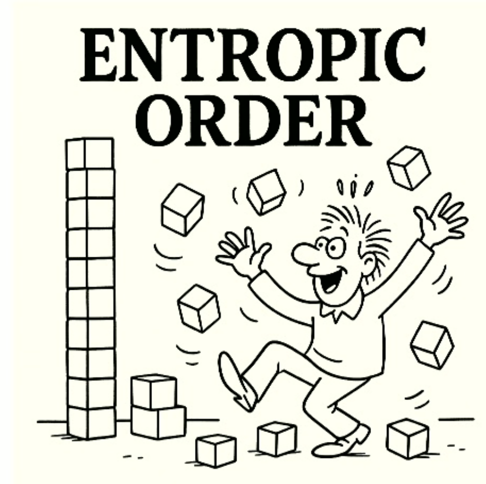
- For a = 1, a more refined analysis is necessary.







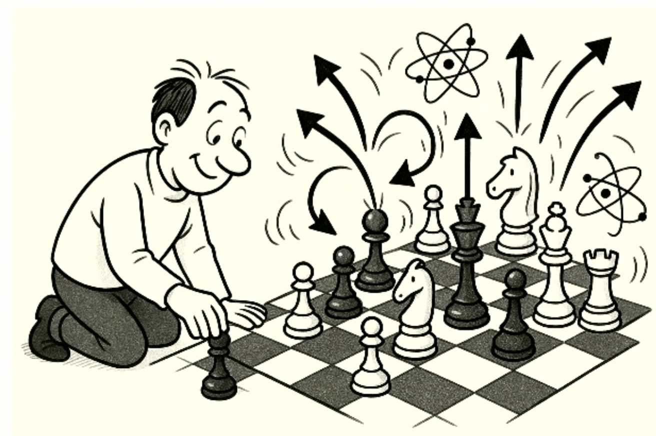
Entropic Order



- When the system forms a checkerboard-ordered state, it allows for the addition of arbitrarily many particles on top of the ordered background.

- As a result, the system favors the ordered configuration, despite the high energy cost.

- Again we have splitting of the Hilbert spaces, and the tunneling between two worlds is suppressed in the thermodynamic limit



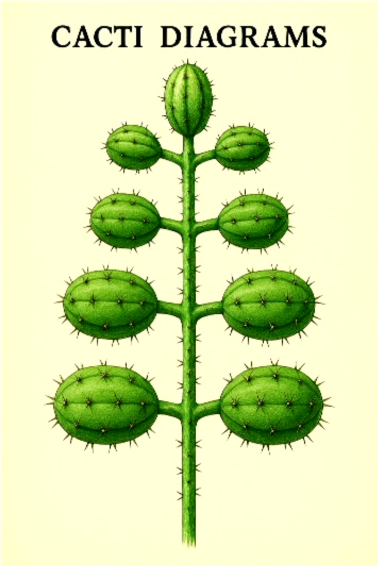
Persistent Symmetry Breaking in QFT

Noam Chai, Soumyadeep Chaudhuri, Changha Choi, Zohar Komargodski, Eliezer Rabinovici, Michael Smolkin

- In QFT, standard arguments for symmetry restoration at high temperatures do not apply.
- In d=3 only discrete global symmetries can be spontaneously broken at finite temperatures.
- Unitarity must be preserved.
- Ideally, the model should possess some integrability or solvability to validate the phase structure.

Large N vector models at finite temperatures

S. Coleman, *1/N*, in *Aspects of Symmetry*, Cambridge University Press, 1985.



- The Large N expansion provides a controlled approximation for theories with many degrees of freedom.
- Quantum fluctuations are effectively averaged, making the theory tractable and exactly solvable in certain limits.
- These models are ideal toy laboratories

$$S = \int d^3x \left[\frac{1}{2} \left(\partial_\mu \phi_i \right)^2 + \frac{1}{2} m_0 \phi_i^2 + \frac{1}{4} \lambda \left(\phi_i^2 \right)^2 \right],$$

$$\sigma = \phi_i^2$$

$$S = \int d^3x \left[\frac{1}{2} \left(\partial_\mu \phi_i \right)^2 + \frac{1}{2} \sigma \phi_i^2 + N \left(\frac{\sigma^2}{2\lambda} + r_0 \sigma \right) \right],$$

$$\frac{1}{-\Delta + \sigma} = 0 \Rightarrow \sigma = T^2 \log^2 \left(\frac{\sqrt{5}+1}{2} \right)^2$$

$$S = \frac{N}{2} \operatorname{tr} \log \left[-\Delta + \sigma \right] + \int d^3x r_0 \sigma$$

Scalar field in an O(N) spa

- Let us consider an additional scalar field on top of the N interacting fields

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi_i\right)^2 + \frac{1}{2}\sigma\phi_i^2 + \frac{1}{2}(\partial_\mu\psi)^2 + t\sigma\psi^2$$

- At leading order in N, t is marginal. But 1/N corrections drive the flow of t to -1,0,1

- The solution t = -1 forbids the condensation of the sigma field.

$$\frac{1}{2\beta}\sum_n \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{1}{\omega_n^2 + k^2 + \sigma} + \frac{1}{2}\left(\phi^2 + t\psi^2\right) = 0$$

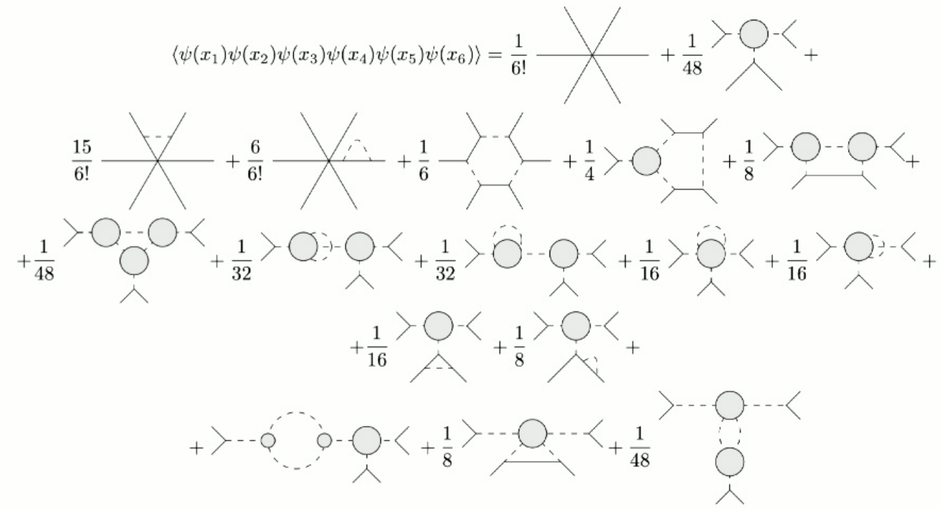
$$-t\psi^2 = \frac{\Gamma\left(\frac{d-2}{2}\right)\zeta(d-2)}{4\pi^{\frac{d}{2}}\beta^{d-2}} > 0$$

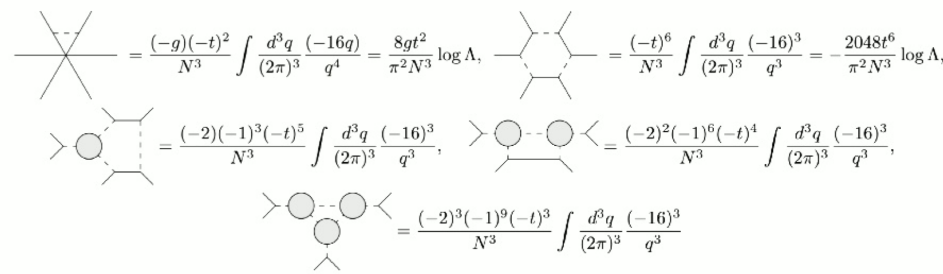
Beta function of the scalar coupling for O(N) model

$$\beta(t) = -\frac{32}{3\pi^2}(t - t^3)$$

$$\beta(g) = \frac{128gt^2}{\pi^2 N} - \frac{245760}{\pi^2 N}t^3(t-1)^3 + \frac{32768}{3\pi^2 N}t^3(t^2-1),$$

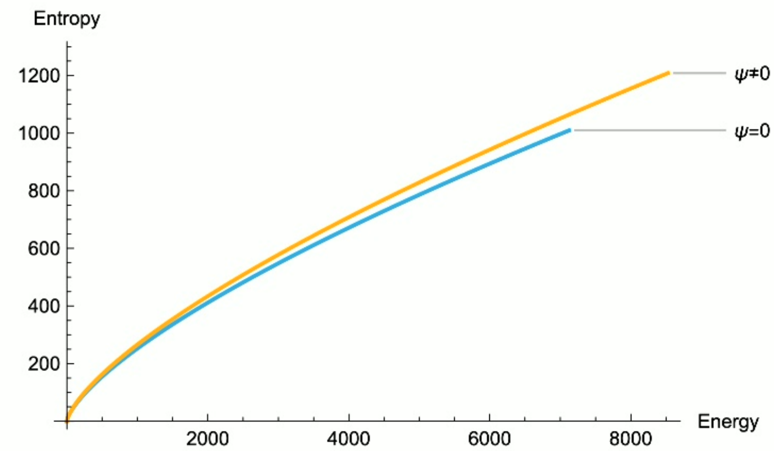
$$\langle\sigma(x_1)\sigma(x_2)\sigma(x_3)\rangle = \frac{512}{N^2}\delta^{(3)}(x_1 - x_2)\delta^{(3)}(x_2 - x_3)$$





Entropic Order in QFT

- By condensing psi field we modify sigma and release additional entropy
- The mechanism is analogous to that in Rochelle salt and Pomeranchuk effect.
- Thus, we obtain entropic order.



Casimir Energy

$$\mathcal{F}[\sigma,\psi,T] = T\int \frac{\mathrm{d}^d p}{(2\pi)^d}\log\left[1-\mathrm{e}^{-\frac{\sqrt{p^2+\sigma}}{T}}\right] + \frac{1}{2}\int \frac{\mathrm{d}^d p}{(2\pi)^d}\left(\sqrt{p^2+\sigma}-p\right) - \frac{1}{2}\sigma\psi^2,$$

Thermal free energy

Interaction with

condensate

Spontaneous Breaking of U(1)

- In 4d, we can break continuous symmetries:

$$\mathcal{L} = -\frac{1}{4e_0^2} F_{\mu\nu}^2 + |\partial_\mu \psi - i A_\mu \psi|^2 + \frac{1}{2}(\partial \phi_i)^2 + \frac{\lambda}{4N}\left(|\psi|^2 - \phi_i^2\right)^2$$

- The mechanism is analogous to the one proposed earlier:

$$\frac{1}{N}\mathcal{F}[\psi, \phi, \sigma] = \frac{1}{2}\left[\log(-\Delta + \sigma)\right]_{xx} + \frac{\sigma^2}{4\lambda_0} + \frac{1}{2}\sigma\left(\phi^2 - |\psi|^2\right) + A_0^2 |\psi|^2$$

- A non-trivial condensate forms:

$$\frac{1}{12\beta^2} = |\psi|^2 - \phi^2,$$

- The theory is not UV-complete, so the results apply at very high, but finite, temperatures.

Conclusion

- Symmetry breaking is possible even at high temperatures.
- Entropic order is a genuine phenomenon.
- Standard no-go theorems can be bypassed
- Quantum Field Theories can also support persistent order.
- Could be potentially interesting to realize these models in real life