Title: String Theory Course Q&A

Speakers:

Collection/Series: String Theory Course

Date: May 08, 2025 - 9:00 AM

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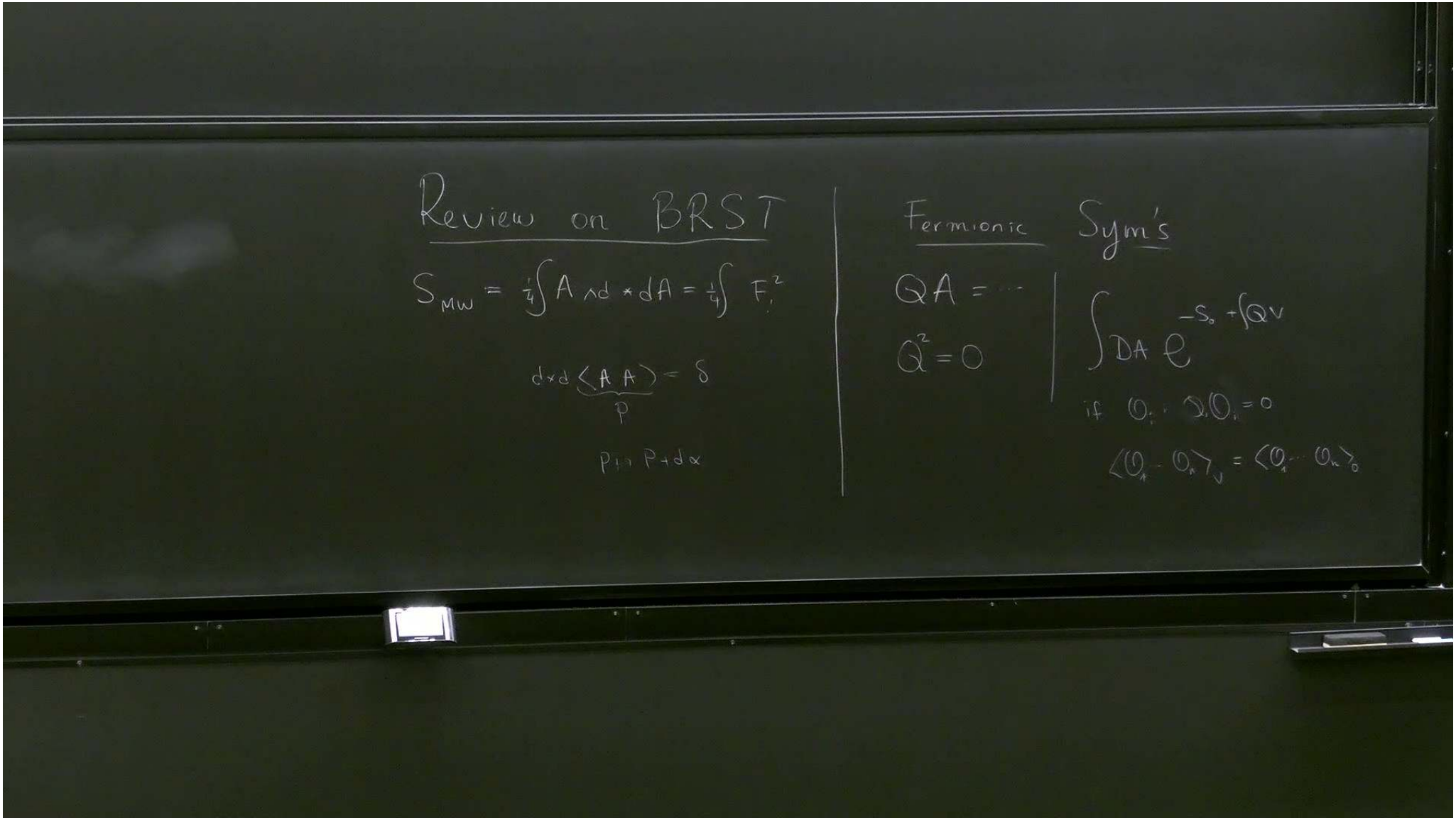
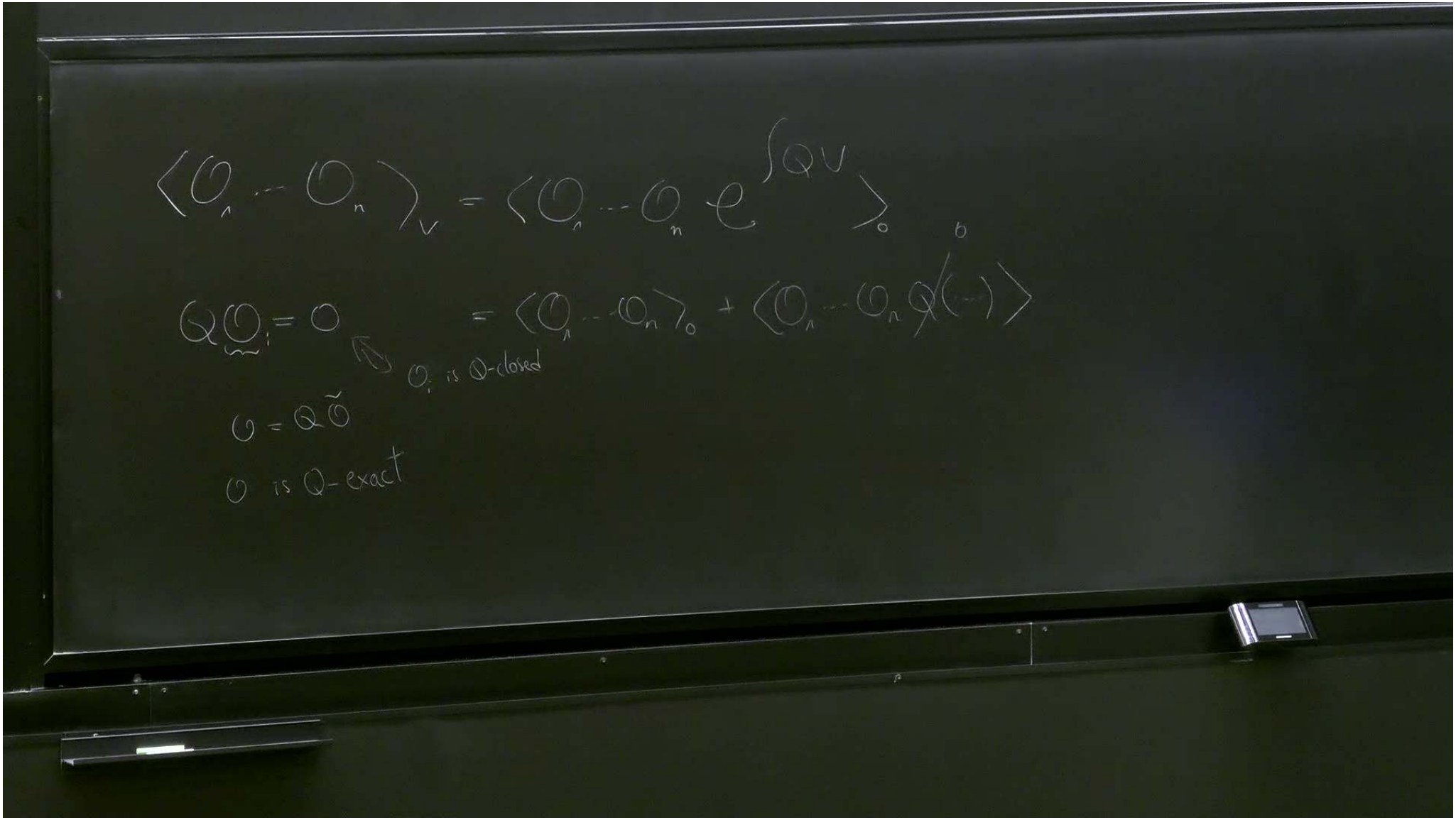
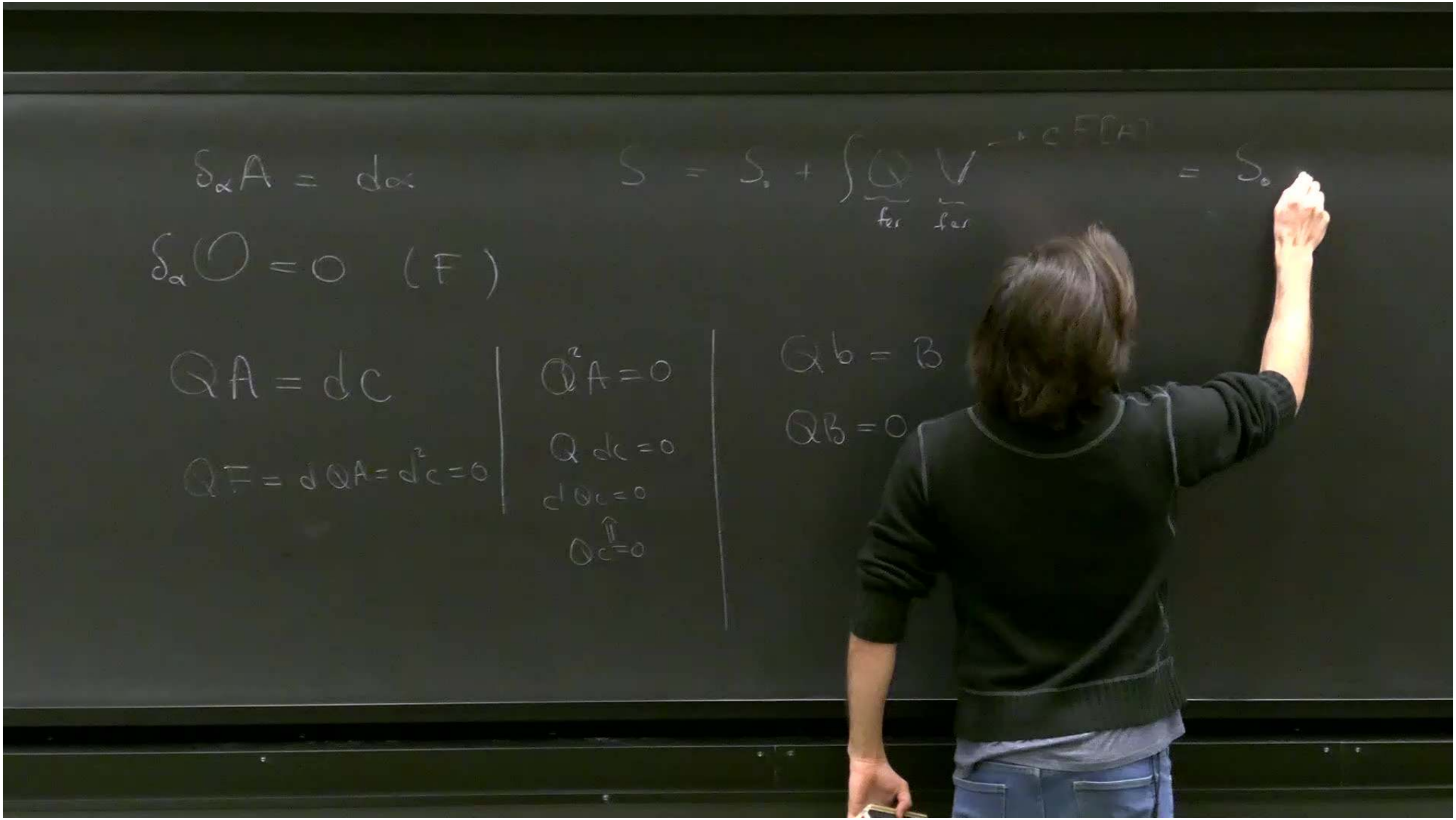
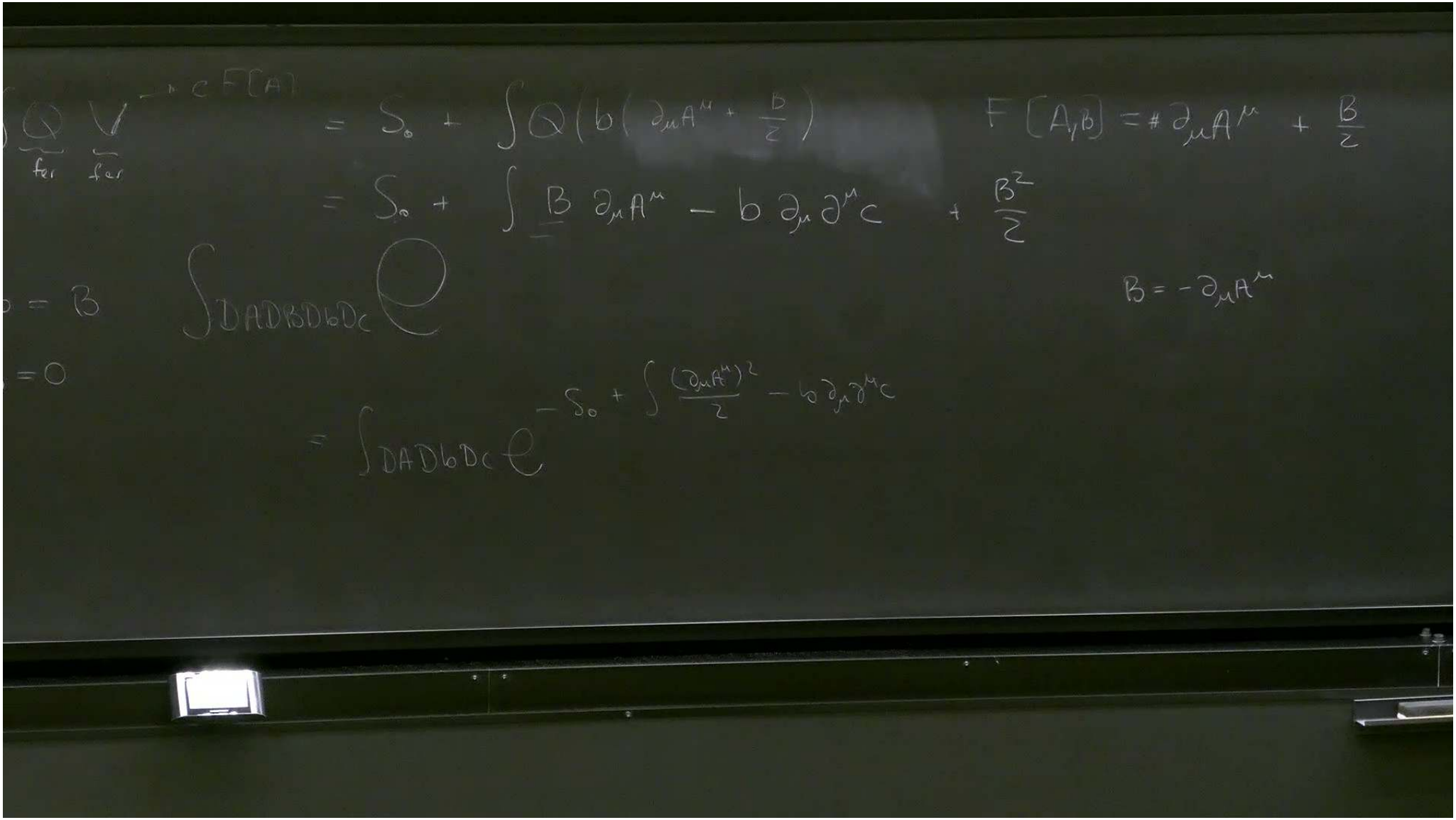
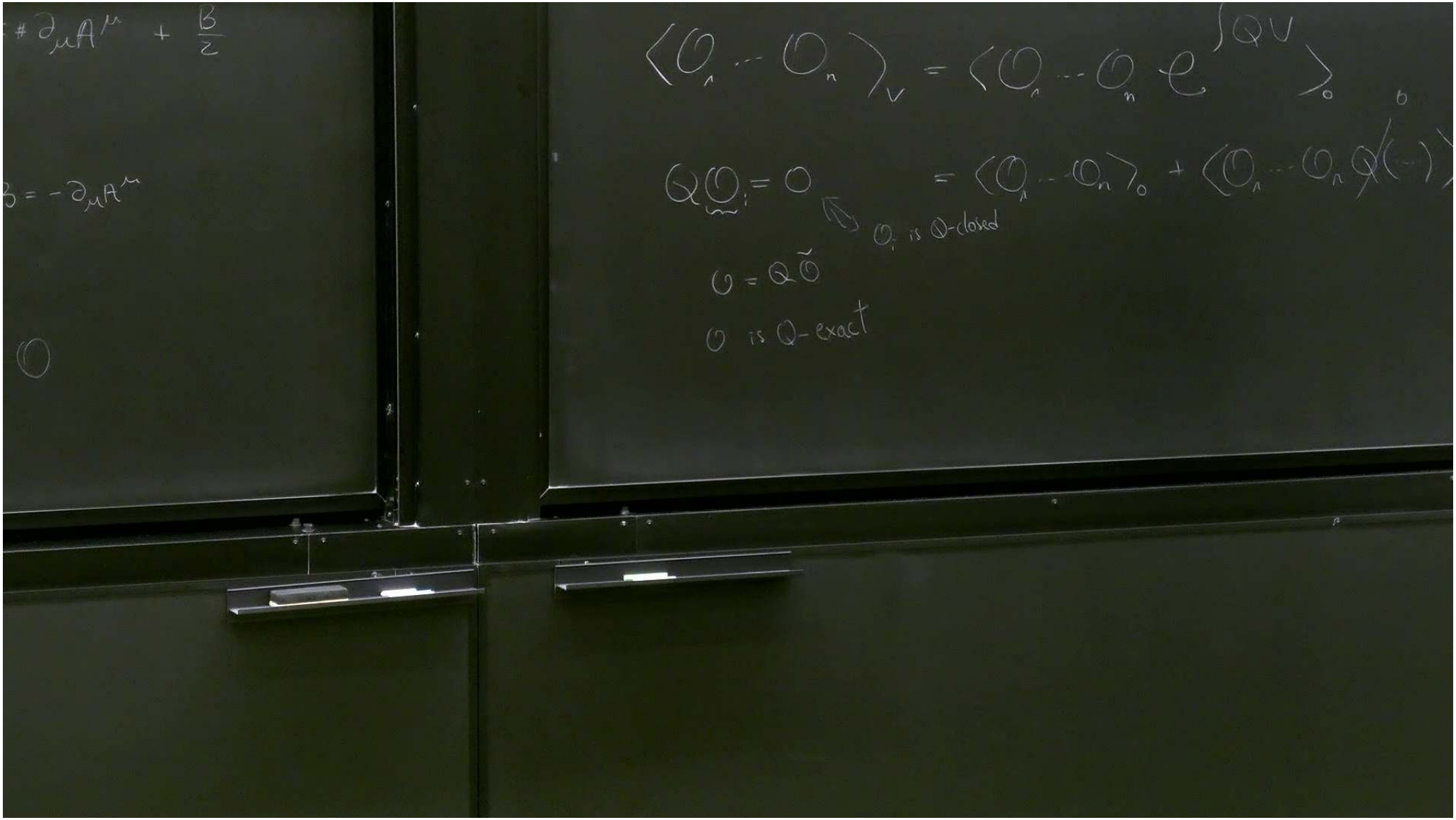
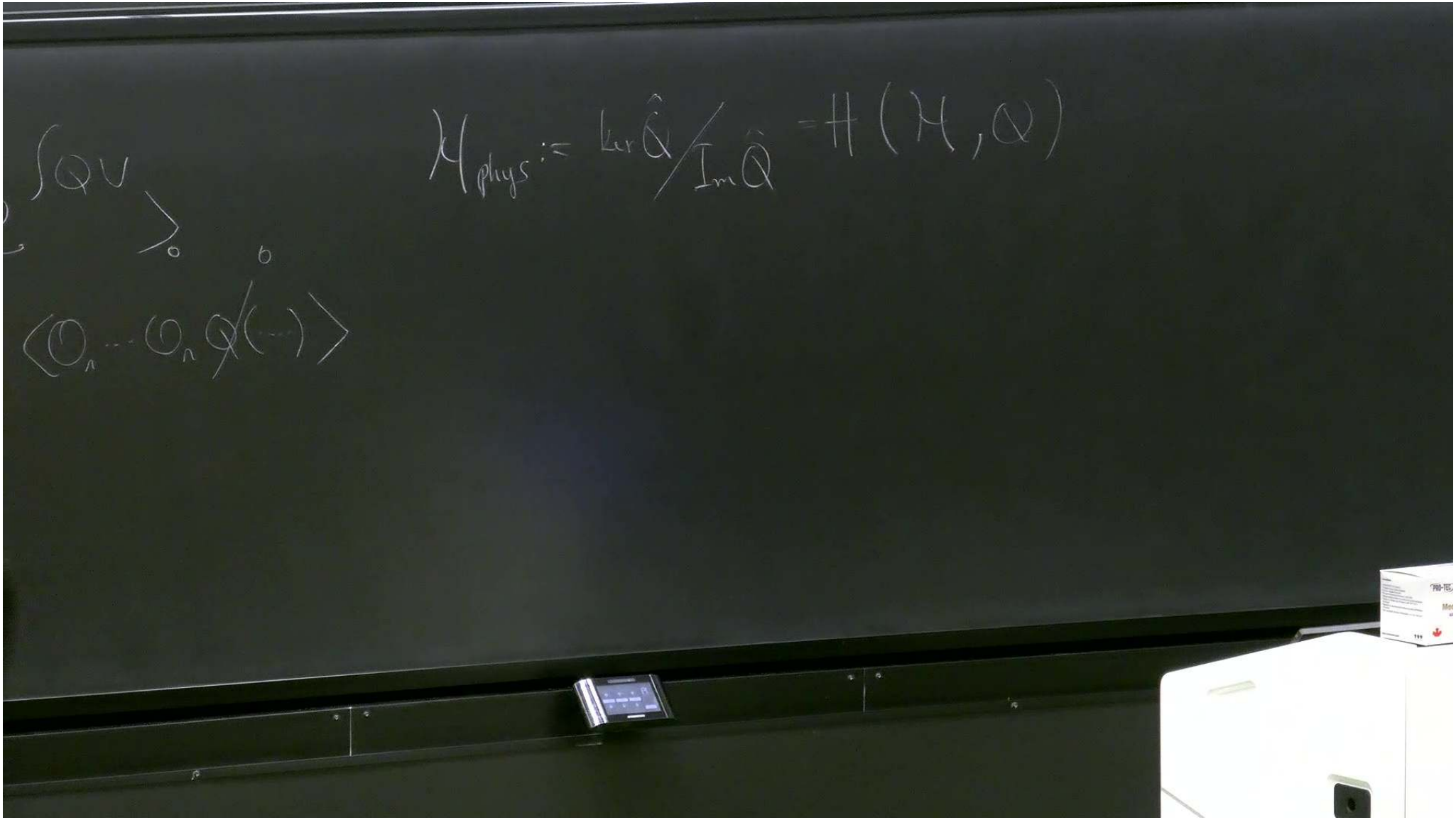

Review on BRST
$S_{MW} = \frac{1}{4}\int A \wedge d * dA = \frac{1}{4}\int F_i^2$
$d*d\underbrace{\langle A\,A\rangle}_{P} = \delta$
$P \mapsto P + d\alpha$
Fermionic Sym's
$QA = \cdots$
$Q^2 = 0$
$\int_{DA} e^{-S_0 + tQV}$
if $\mathcal{O}_i \cdot Q\mathcal{O}_i = 0$
$\langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle_V = \langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle_0$

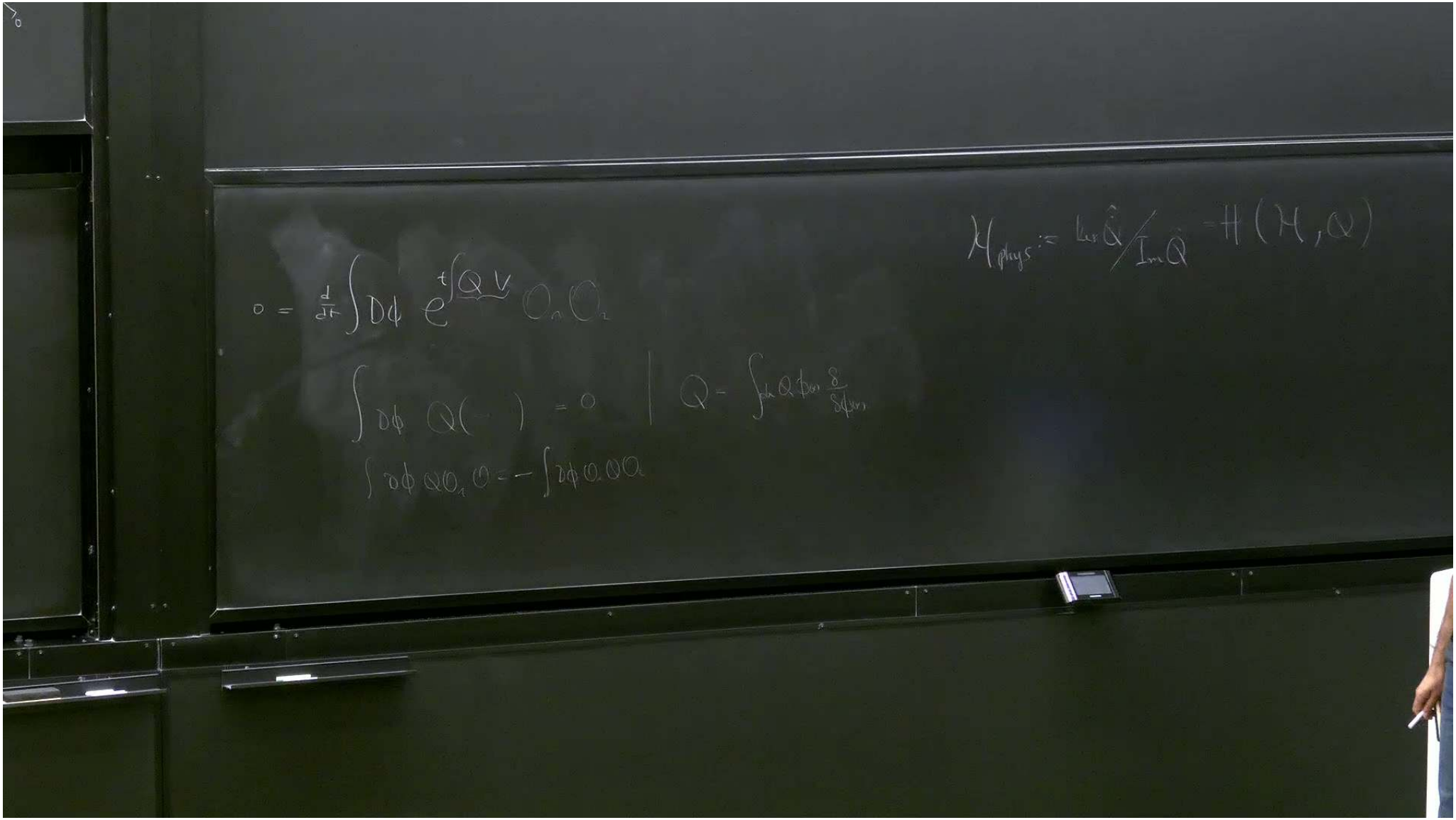

$\langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle_V = \langle \mathcal{O}_1 \cdots \mathcal{O}_n e^{\int Q V} \rangle_0$
$Q\mathcal{O}_i = 0$
$= \langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle_0 + \langle \mathcal{O}_1 \cdots \mathcal{O}_n Q(\cdots) \rangle_0$
$\mathcal{O}_i$ is Q-closed
$\mathcal{O} = Q\tilde{\mathcal{O}}$
$\mathcal{O}$ is Q-exact


$\delta_\alpha A = d\alpha$
$S = S_0 + \int Q\,V$
fer fer
$= S_0$
$\delta_\alpha O = 0 \quad (F)$
$QA = dC$
$Q^2 A = 0$
$Qb = B$
$QB = 0$
$QF = dQA = d^2C = 0$
$Q\,dC = 0$
$d\,QC = 0$
$QC = 0$


Q V
for for
$= S_0 + \int Q\left(b\left(\partial_\mu A^\mu + \frac{B}{2}\right)\right.$
$= S_0 + \int B\,\partial_\mu A^\mu - b\,\partial_\mu\partial^\mu c + \frac{B^2}{2}$
$F[A,B] = \#\,\partial_\mu A^\mu + \frac{B}{2}$
$B = -\partial_\mu A^\mu$
$= B$
$= 0$
$\int DA\,DB\,Db\,Dc\,e$
$= \int DA\,DB\,Db\,Dc\,e^{-S_0 + \int \frac{(\partial_\mu A^\mu)^2}{2} - b\partial_\mu\partial^\mu c}$


$\langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle_V = \langle \mathcal{O}_1 \cdots \mathcal{O}_n e^{\int QV} \rangle_0$
$Q\mathcal{O}_i = 0$
$= \langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle_0 +$
$\mathcal{O}_i$ is Q-closed
$\mathcal{O} = Q\tilde{\mathcal{O}}$
$\mathcal{O}$ is Q-exact

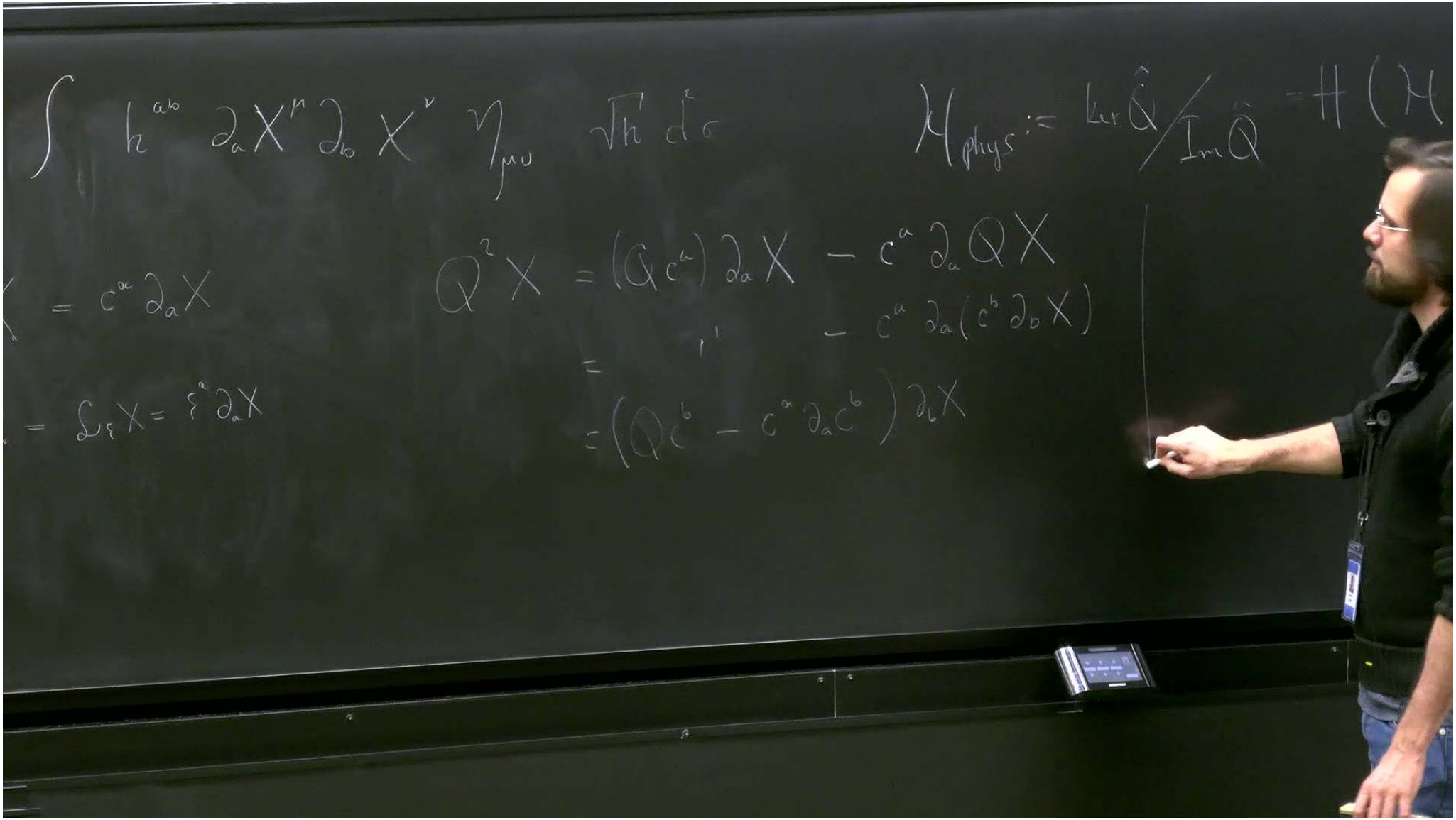

$\mathcal{H}_{phys} := \ker \hat{Q} / \mathrm{Im}\, \hat{Q} = H(\mathcal{H}, Q)$
$\int QV$
$\langle O_1 \cdots O_n Q(\cdots) \rangle$

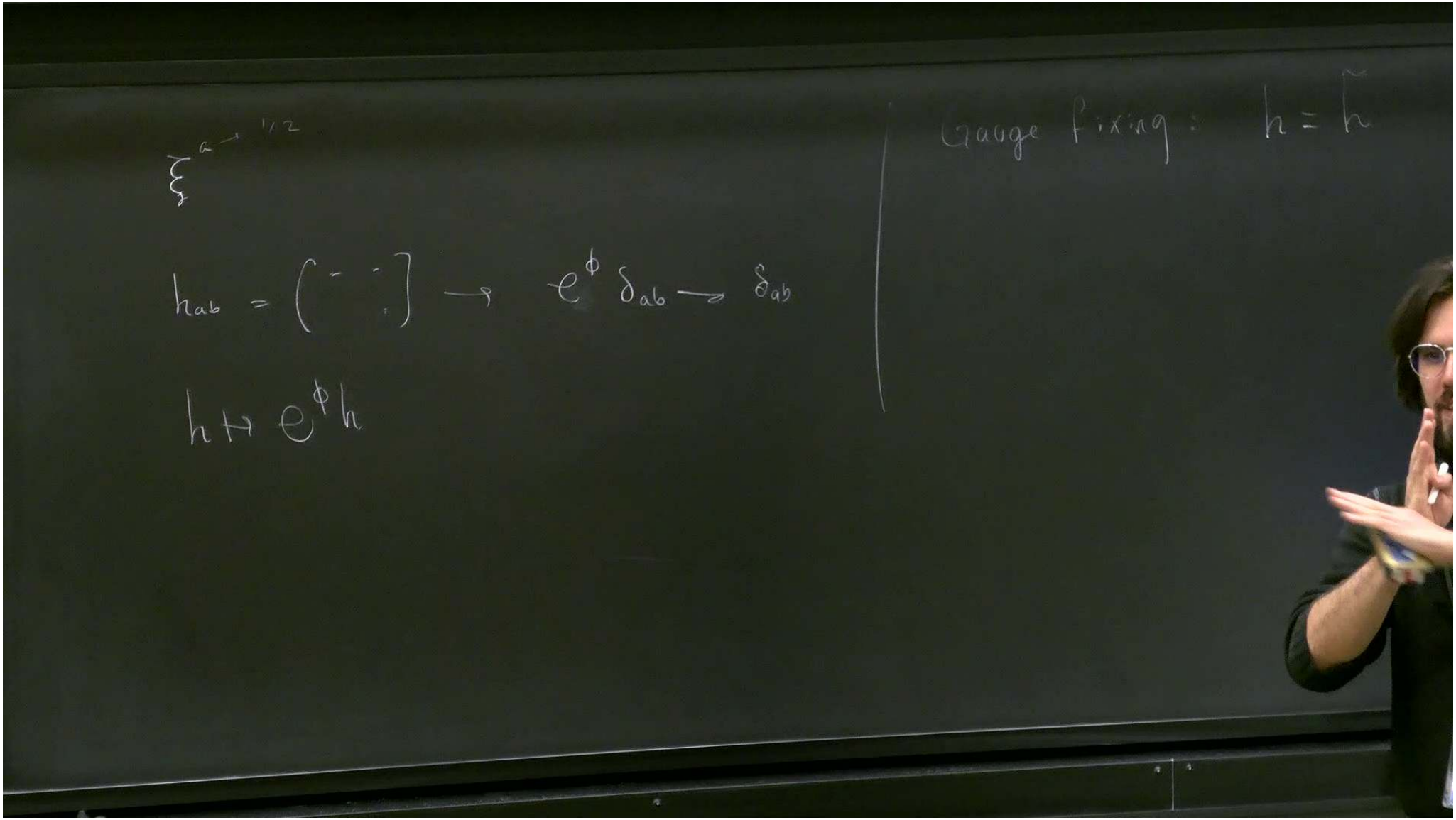

$0 = \frac{d}{dt}\int D\phi\, e^{t\int Q V}\, O_1 O_2$
$\int D\phi\, Q(\ \ ) = 0$
$Q = \int dx\, Q\phi(x) \frac{\delta}{\delta\phi(x)}$
$\int D\phi\, Q O_1 O = -\int D\phi\, O_1 Q O_2$
$\mathcal{H}_{phys} := \ker \hat{Q} / \mathrm{Im}\, \hat{Q} = H(\mathcal{H}, Q)$

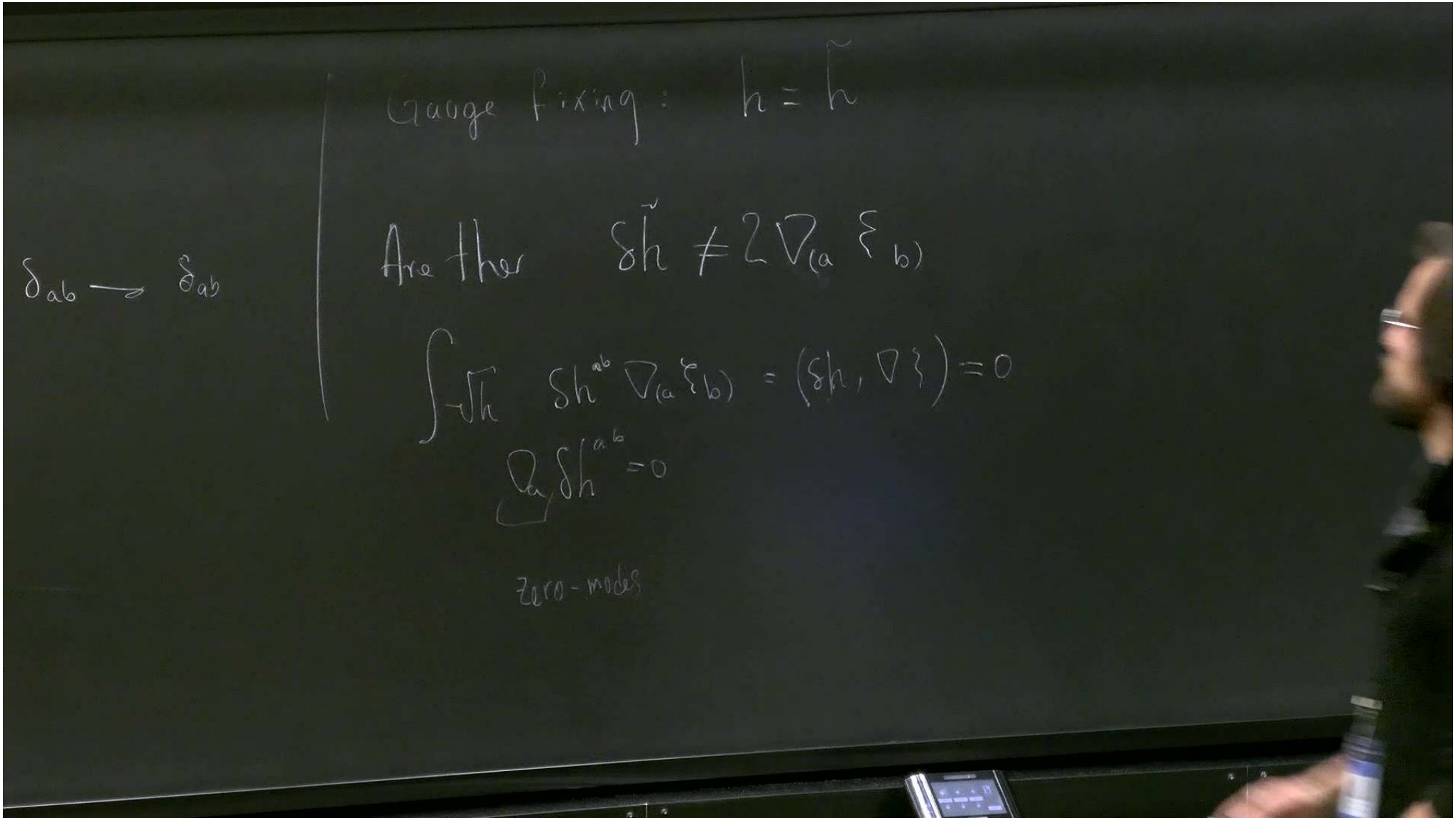
$$S_{poly} = \int h^{ab}\, \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu} \sqrt{h}\, d^2\sigma \qquad \mathcal{H}_{ph}$$

$$Q X = c^\alpha \partial_\alpha X$$

$$\delta_\xi X = \mathcal{L}_\xi X = \xi^\alpha \partial_\alpha X$$


$\int h^{ab}\,\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}\,\sqrt{h}\,d^2\sigma$
$\mathcal{H}_{phys} := \ker \hat{Q} / \mathrm{Im}\,\hat{Q}$
$= c^a \partial_a X$
$= \mathcal{L}_\xi X = \xi^a \partial_a X$
$Q^2 X = (Q c^a)\partial_a X - c^a \partial_a Q X$
$- c^a \partial_a (c^b \partial_b X)$
$= (Q c^b - c^a \partial_a c^b)\partial_b X$


$\xi^a \to 1/2$
$h_{ab} = \begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix} \to e^{\phi}\delta_{ab} \to \delta_{ab}$
$h \mapsto e^{\phi} h$
Gauge fixing: $h = \tilde{h}$

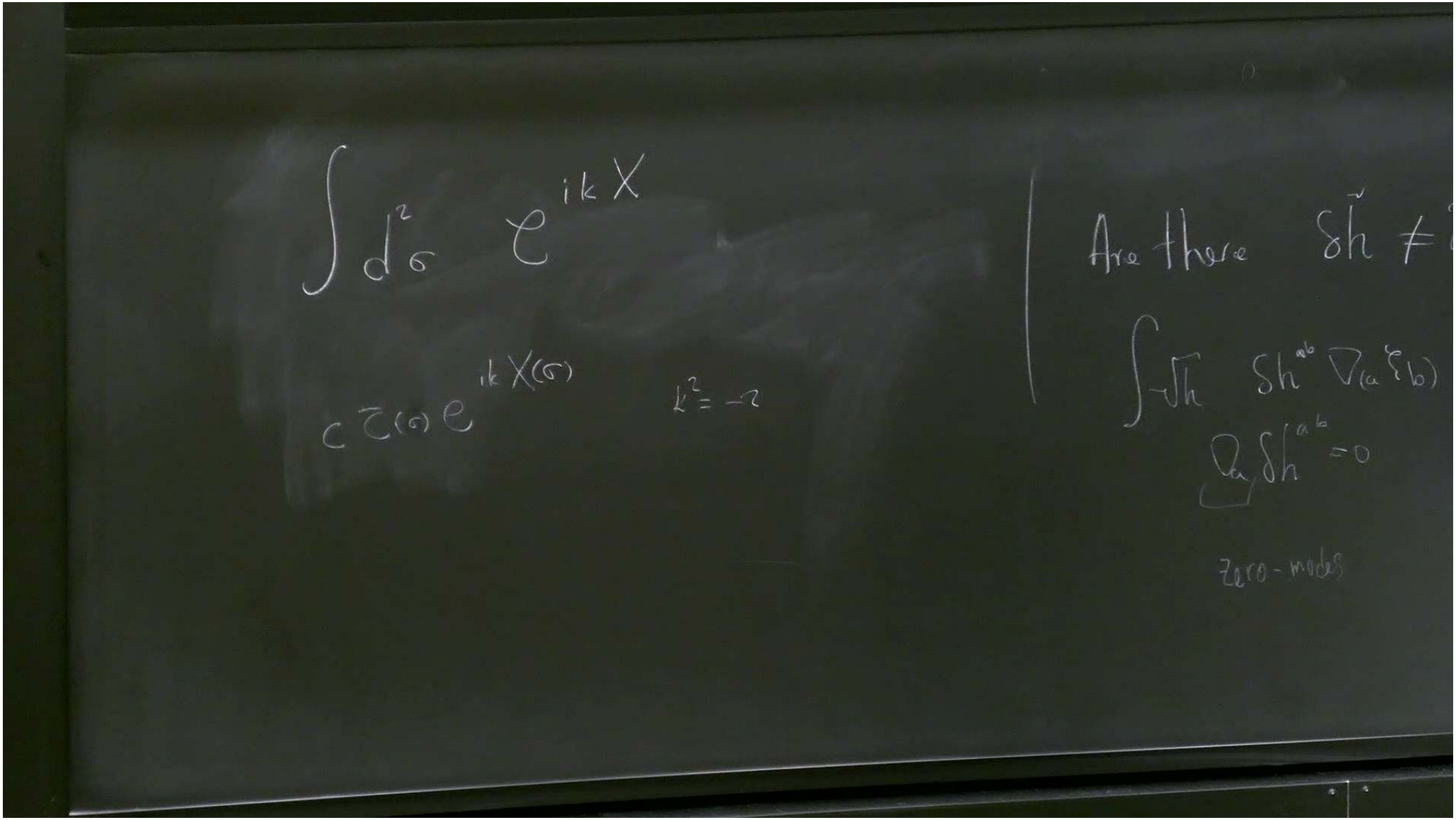

$\delta_{ab} \longrightarrow \delta_{ab}$
Gauge fixing: $h = \tilde{h}$
Another $\delta\tilde{h} \neq 2\nabla_{(a}\xi_{b)}$
$\int \sqrt{h}\, \delta h^{ab} \nabla_{(a}\xi_{b)} = (\delta h, \nabla\xi) = 0$
$\nabla_a \delta h^{ab} = 0$
zero-modes

$$S_0 + \int Q\left(b_{ab} \underbrace{\left(h^{ab} - \tilde{h}^{ab} \right)}_{F[h]} \right)$$

$$Q\, h_{ab} := \mathcal{L}_c h_{ab} = 2 D_{(a} c_{b)}$$

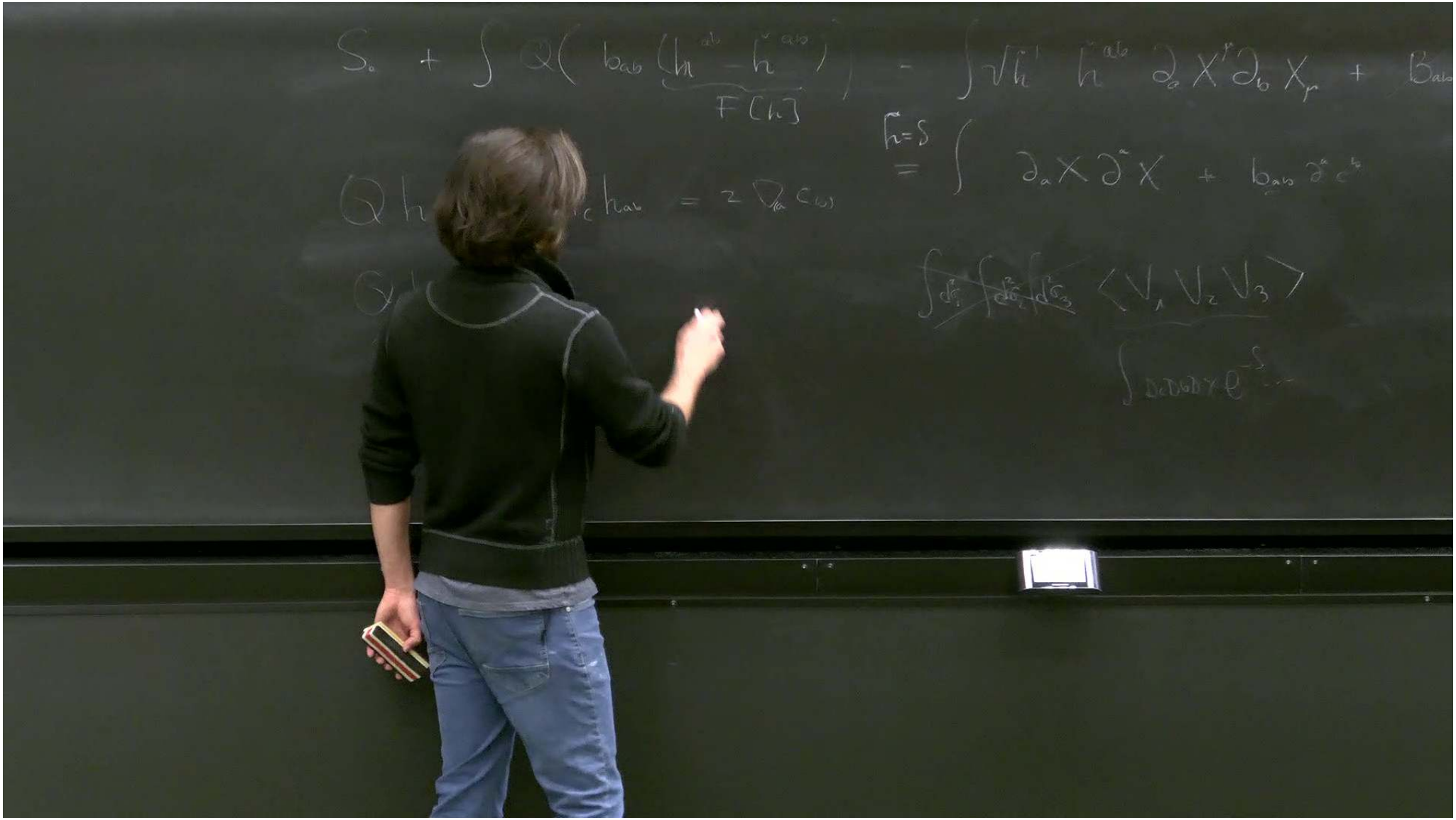
$$Q\, b_{ab} = B_{ab}$$

$$Q\, B_{ab} = 0$$


$\int d^2\sigma \; e^{ikX}$
$c\,\tau(\sigma)\, e^{ikX(\sigma)}$
$k^2 = -2$
Are there $\delta\tilde{h} \neq$
$\int \sqrt{h}\, \delta h^{ab} \nabla_{(a}\xi_{b)}$
$\nabla_a \delta h^{ab} = 0$
zero-modes

$$= \int \sqrt{\tilde{h}}\, \tilde{h}^{ab}\, \partial_a X^\mu \partial_b X_\mu + B_{ab}\left(h^{ab} - \tilde{h}^{ab}\right) - b_{ab}\, \nabla^a_{\tilde{h}}\, c^b$$

$$\overset{\tilde{h}=\delta}{=} \int \partial_a X\, \partial^a X + b_{ab}\, \partial^a c^b$$

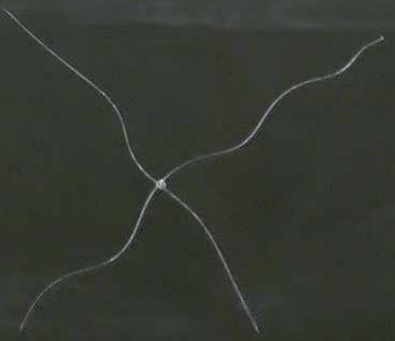


$$= \int \sqrt{h}\, h^{ab}\, \partial_a X^\mu \partial_b X_\mu + B_{ab}\,(h^{ab} - \tilde{h}^{ab}) - b_{ab}\, \nabla^a_{\tilde{h}}\, c^b)$$

$\tilde{h}=\delta$

$$= \int \partial_a X\, \partial^a X + b_{ab}\, \partial^a c^b$$

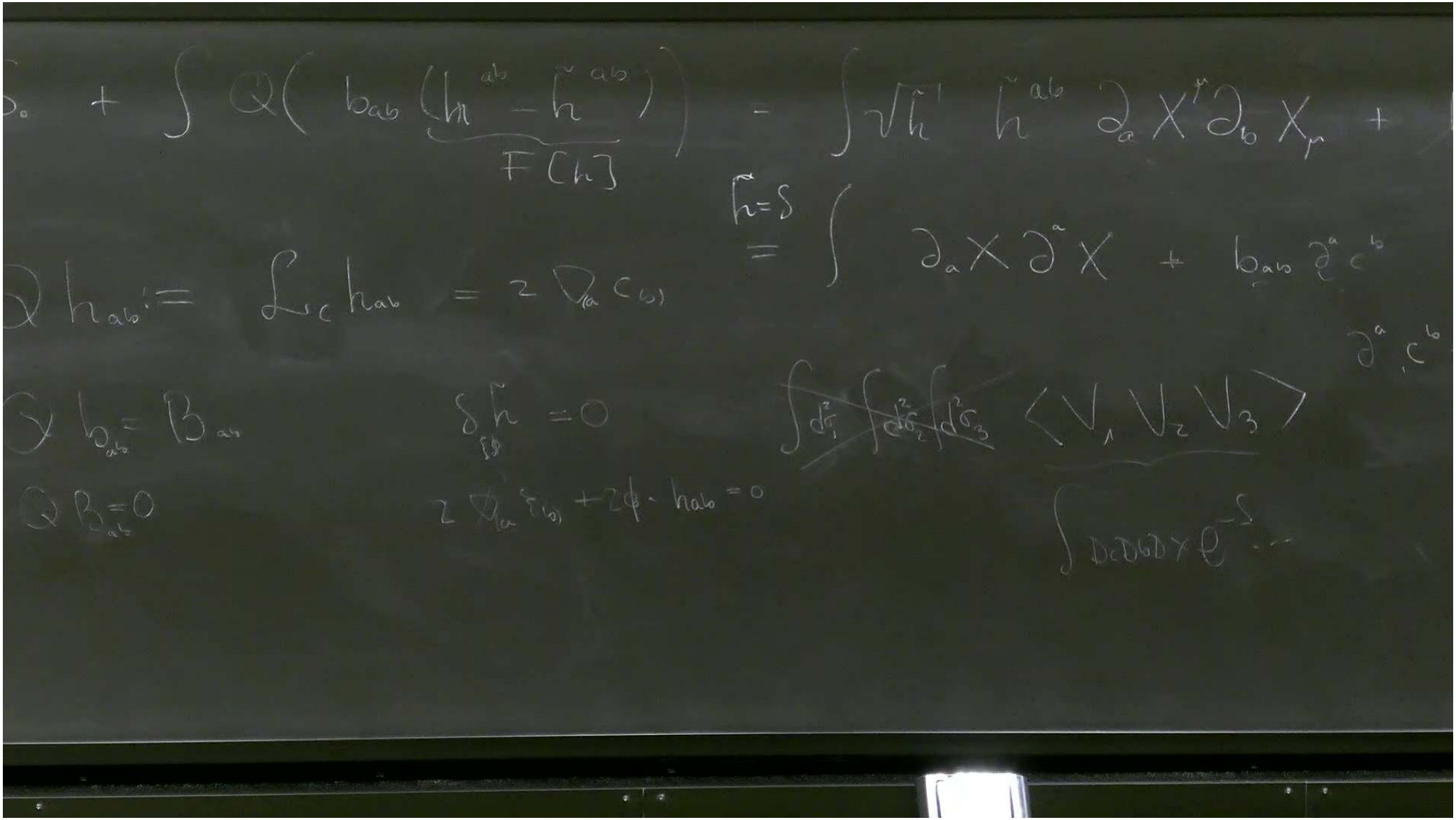
$$\partial^a c^b = 0$$

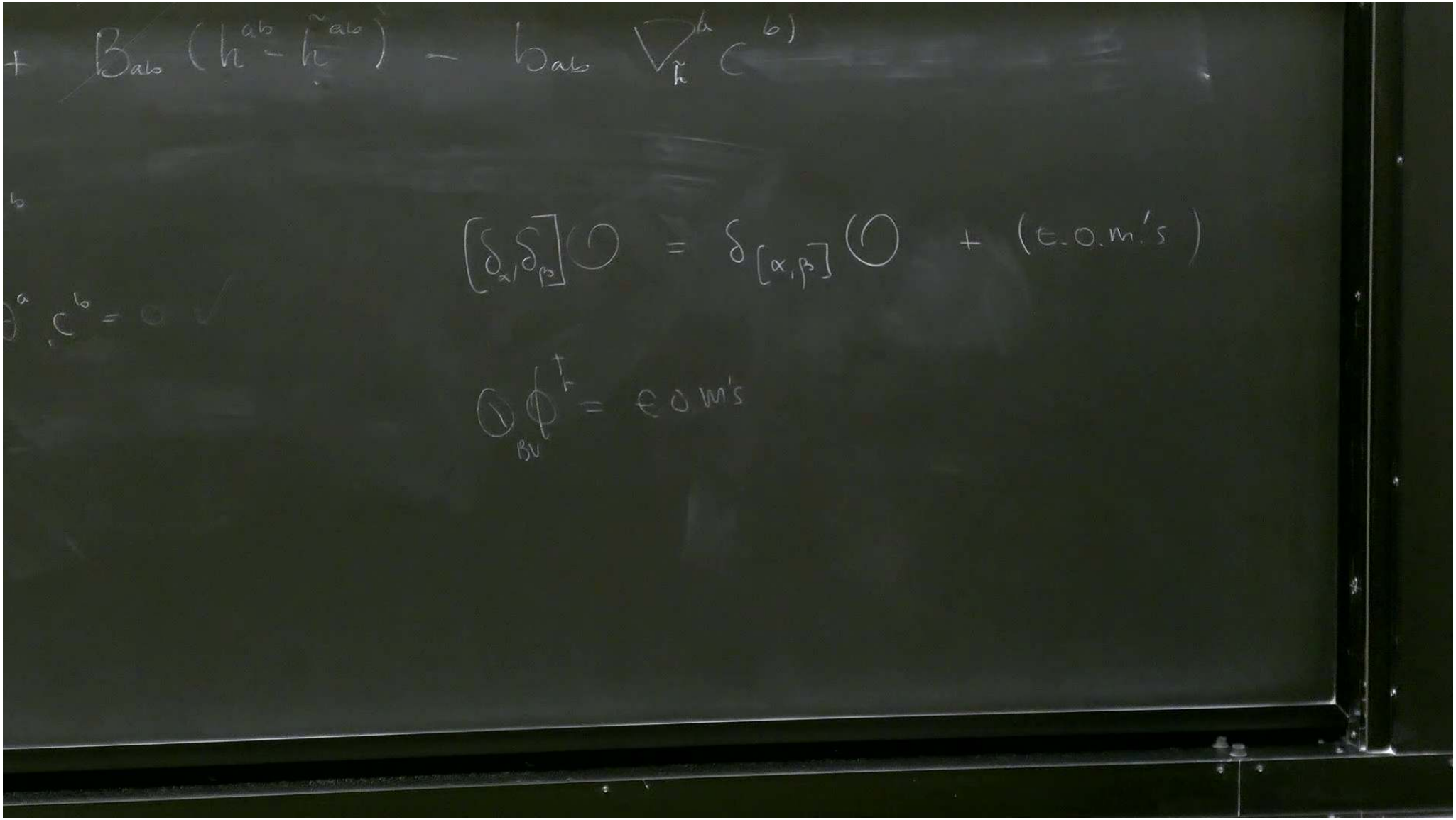


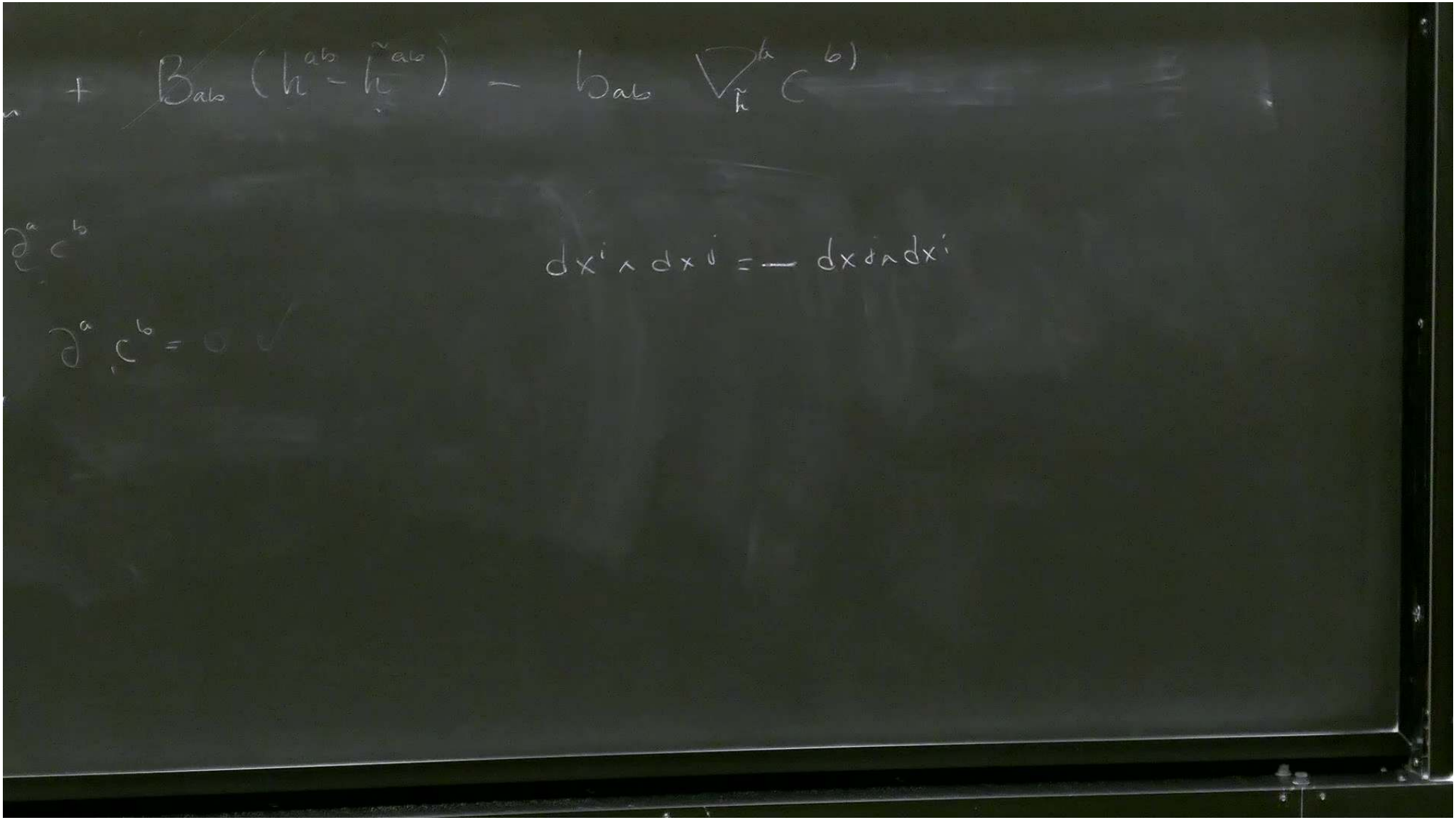
$$\int d^2\sigma_1 \int d^2\sigma_2 \int d^2\sigma_3 \langle V_1 V_2 V_3 \rangle$$

$= 0$

$$\int \mathcal{D}c\mathcal{D}b\mathcal{D}X\, e^{-S} \cdots$$


$+ \int Q\left(b_{ab} \underbrace{(h^{ab} - \tilde{h}^{ab})}_{F[h]} \right) = \int \sqrt{h'}\, h'^{ab}\, \partial_a X^\mu \partial_b X_\mu +$
$\tilde{h}=\delta$
$= \int \partial_a X \partial^a X + b_{ab} \partial^a c^b$
$Q\, h_{ab}' = \mathcal{L}_c h_{ab} = 2 \nabla_{(a} c_{b)}$
$\partial^a c^b$
$Q\, b_{ab} = B_{ab}$
$\delta_{\tilde{h}\phi} \tilde{h} = 0$
$\int d^2\sigma_1 \int d^2\sigma_2 \int d^2\sigma_3 \langle V_1 V_2 V_3 \rangle$
$Q\, B_{ab} = 0$
$2 \nabla_{(a} \xi_{b)} + 2\phi \cdot h_{ab} = 0$
$\int \mathcal{D}c\mathcal{D}b\mathcal{D}X\, e^{-S} \cdots$


+ $B_{ab}(h^{ab} - \tilde{h}^{ab}) - b_{ab} \nabla^a_{\tilde{h}} c^b$
$[\delta_\alpha, \delta_\beta] \mathcal{O} = \delta_{[\alpha,\beta]} \mathcal{O} + (\text{e.o.m's})$
$\mathcal{O} \phi^\dagger_L = \text{e.o.m's}$


+ $B_{ab}(h^{ab} - \tilde{h}^{ab}) - b_{ab} \nabla^{h}_{\tilde{h}} C^{b)}$
$\partial^a C^b$
$\partial^a C^b = 0$
$dx^i \wedge dx^j = - dx^j \wedge dx^i$