Title: String Theory Course Q&A

Speakers:

Collection/Series: String Theory Course

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Review on BRST

$$S_{MW} = \frac{1}{4}\int A \wedge d * dA = \frac{1}{4}\int F^2$$

$$d * d \underbrace{\langle A\, A \rangle}_{P} = \delta$$

$$P \mapsto P + d\alpha$$

Fermionic Sym's

$$QA = \cdots$$

$$Q^2 = 0$$

$$\int DA\, e^{-S_0 + \int QV}$$

if $\mathcal{O}_i : Q\mathcal{O}_i = 0$

$$\langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle_V = \langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle_0$$

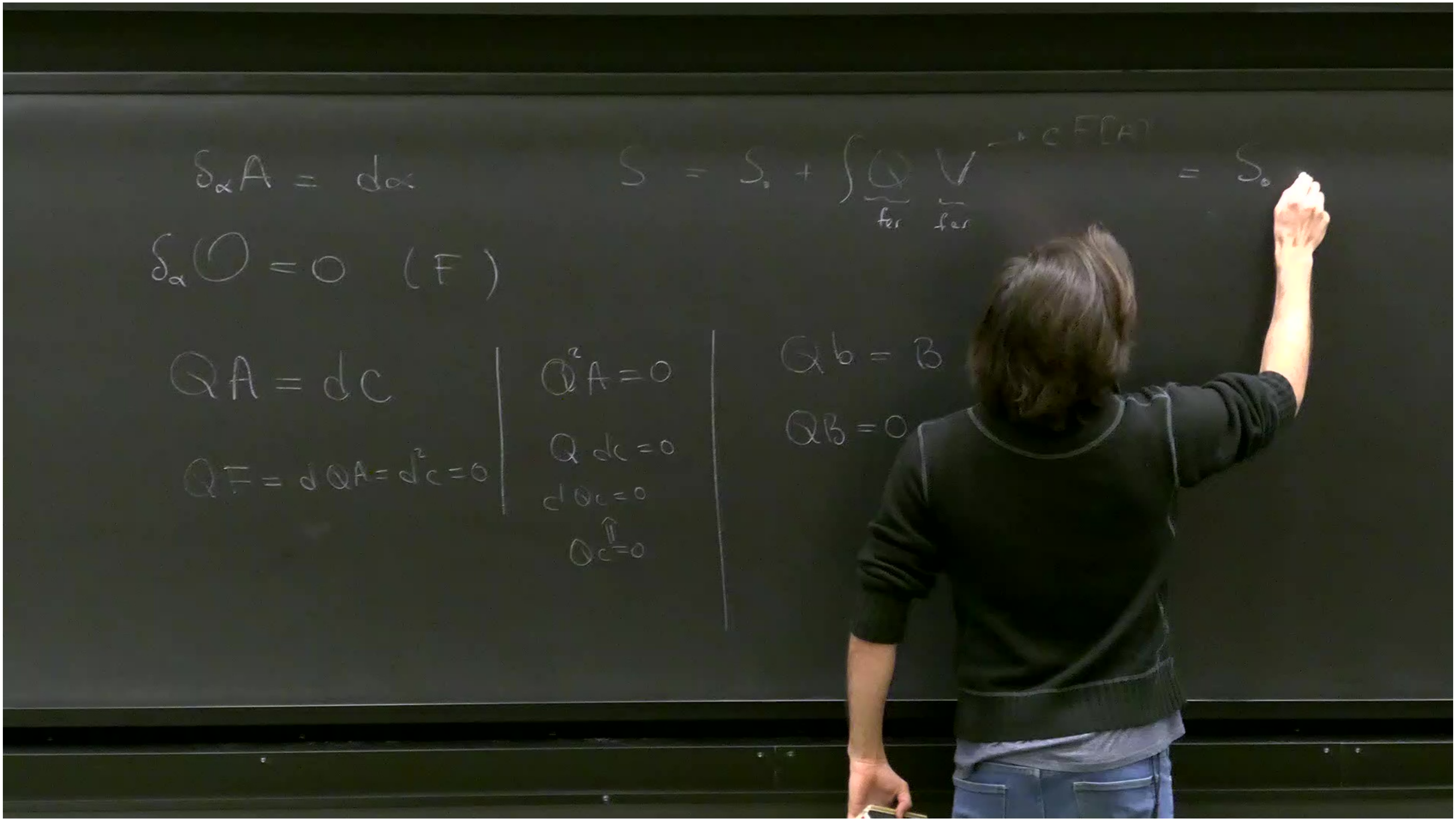
$$\langle O_1 \cdots O_n \rangle_V = \langle O_1 \cdots O_n \, e^{\int QV} \rangle_0$$

$$Q O_i = 0 \qquad = \langle O_1 \cdots O_n \rangle_0 + \langle O_1 \cdots O_n \, \cancel{Q}(\cdots) \rangle_0$$

$\Leftrightarrow$ O_i is Q-closed

$$O = Q\tilde{O}$$

O is Q-exact


δ_α A = dα
S = S_0 + ∫ Q V = S_0
fer fer
δ_α O = 0 (F)
QA = dc
QF = d QA = d²c = 0
Q²A = 0
Q dc = 0
d Qc = 0
Qc = 0
Qb = B
QB = 0

$\int Q \underbrace{V}_{fer} \xrightarrow{} c F[A]$

$\underbrace{Q}_{fer}$

$$= S_0 + \int Q\left(b\left(\partial_\mu A^\mu + \frac{B}{2}\right)\right) \qquad F[A, B] = \# \partial_\mu A^\mu + \frac{B}{2}$$

$$= S_0 + \int \underline{B}\, \partial_\mu A^\mu - b\, \partial_\mu \partial^\mu c + \frac{B^2}{2}$$

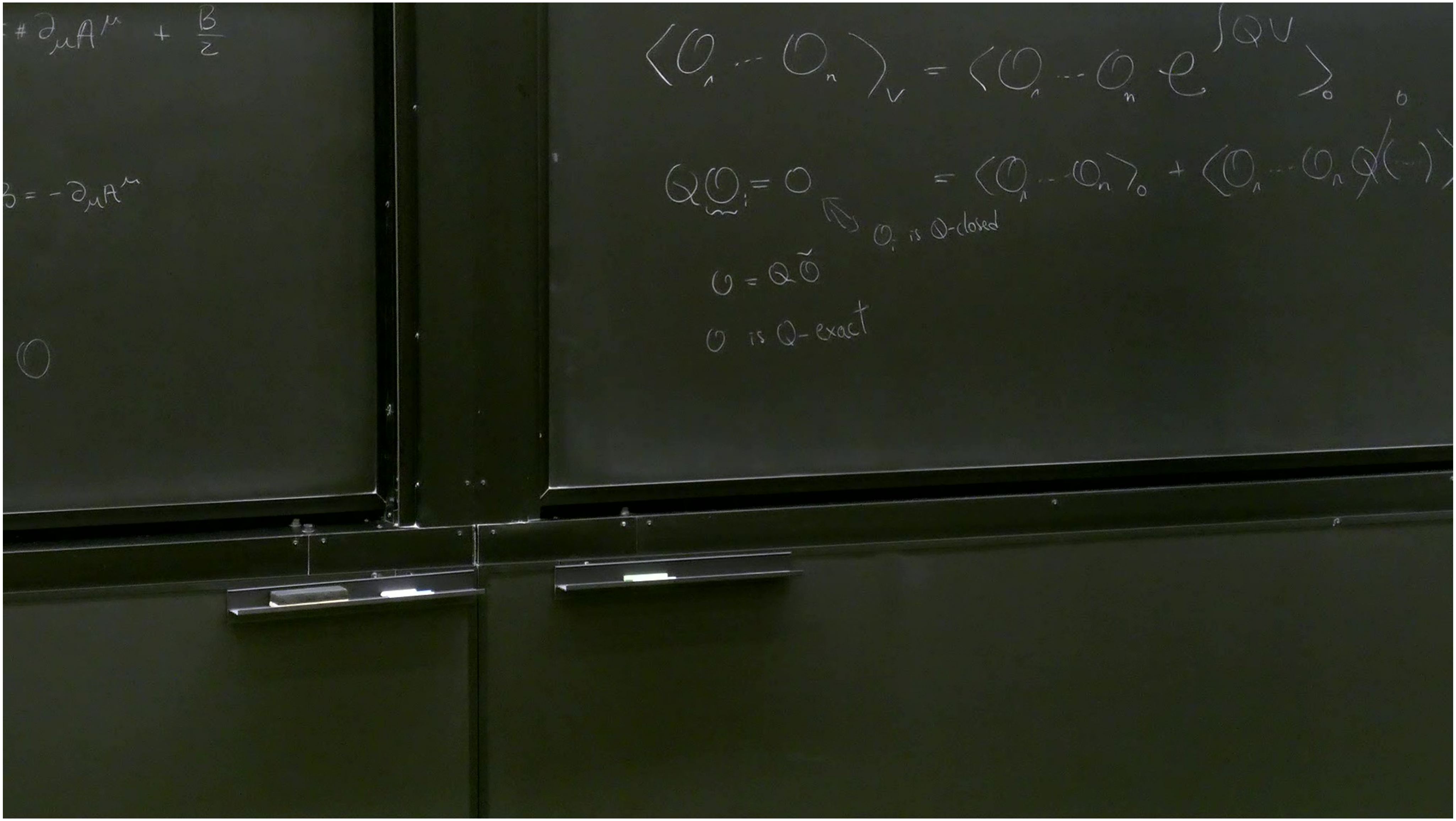
$= B$

$$\int DA DB Db Dc\, e$$

$$B = -\partial_\mu A^\mu$$

$= 0$

$$= \int DA Db Dc\, e^{-S_0 + \int \frac{(\partial_\mu A^\mu)^2}{2} - b \partial_\mu \partial^\mu c}$$

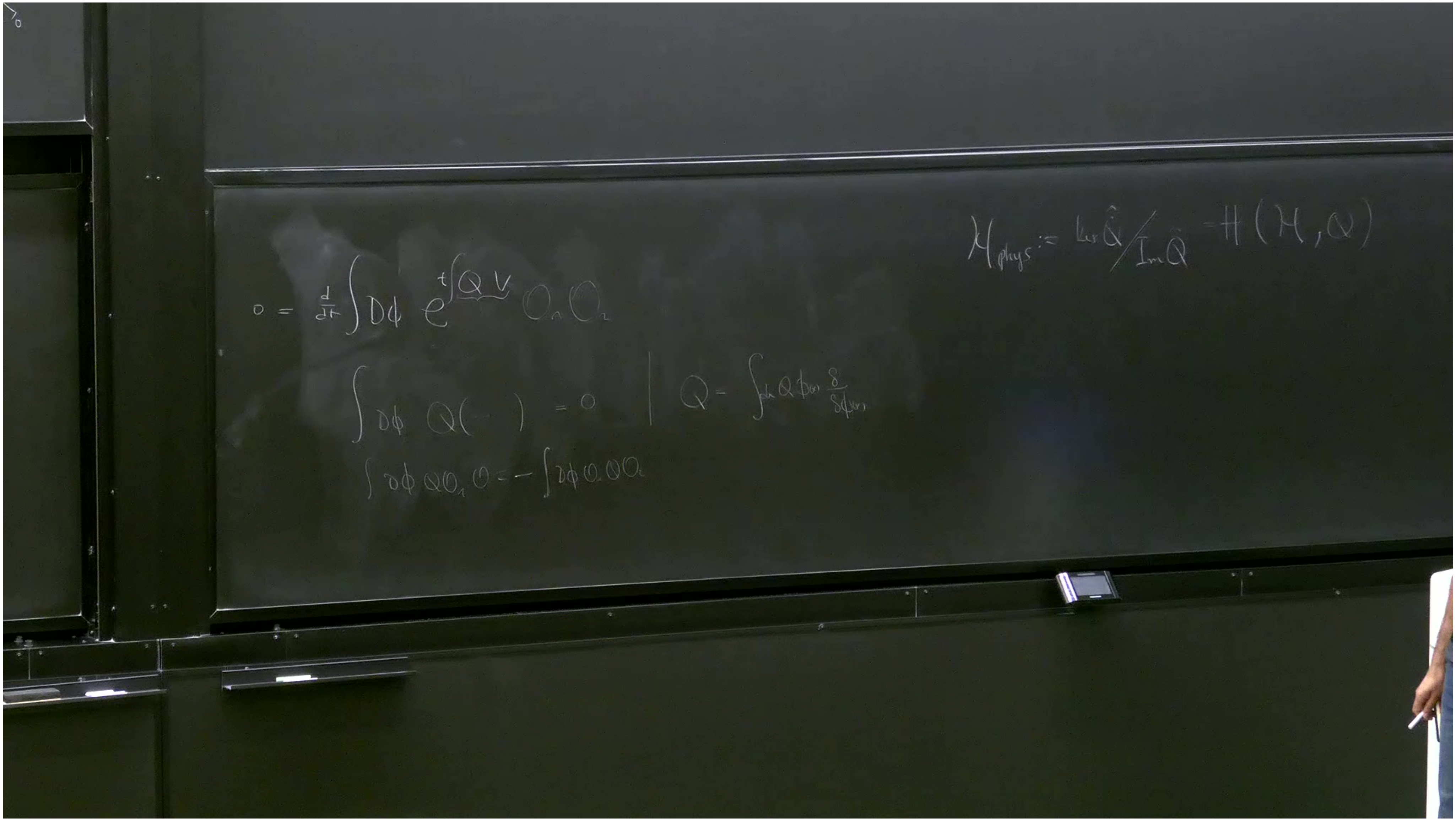

$\# \partial_\mu A^\mu + \frac{B}{2}$
$\langle O_1 \cdots O_n \rangle_V = \langle O_1 \cdots O_n e^{\int QV} \rangle_0$
$B = -\partial_\mu A^\mu$
$Q O_i = 0$
$= \langle O_1 \cdots O_n \rangle_0 + \langle O_1 \cdots O_n \cancel{Q}(\cdots) \rangle$
O_i is Q-closed
$O = Q\tilde{O}$
O is Q-exact
O

$\int Q V$

$\rangle_0$ $\qquad 0$

$\langle \mathcal{O}_1 \cdots \mathcal{O}_n \cancel{Q}(\cdots) \rangle$

$$\mathcal{H}_{phys} := \ker \hat{Q} / \mathrm{Im}\,\hat{Q} = H(\mathcal{H}, Q)$$

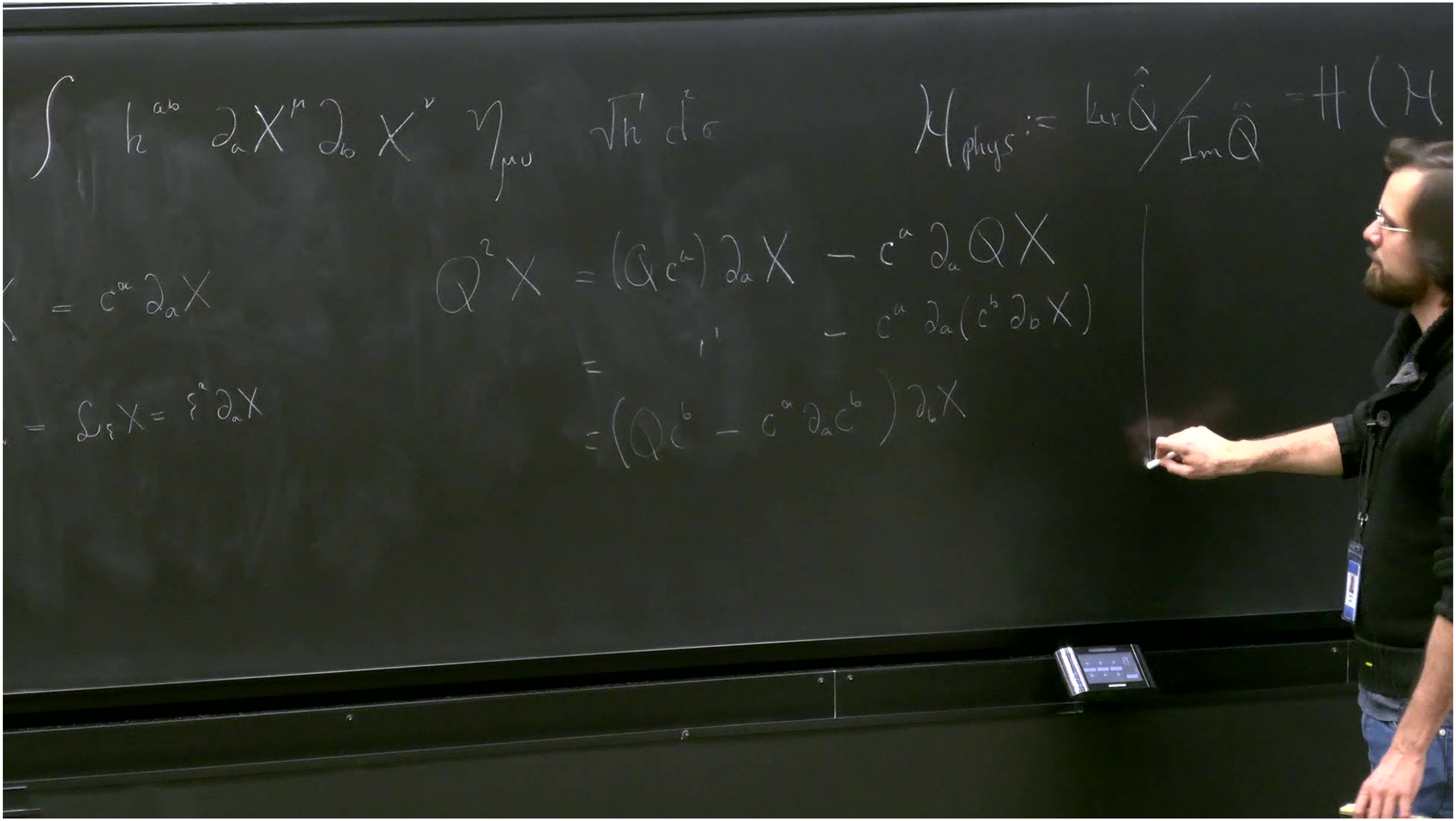


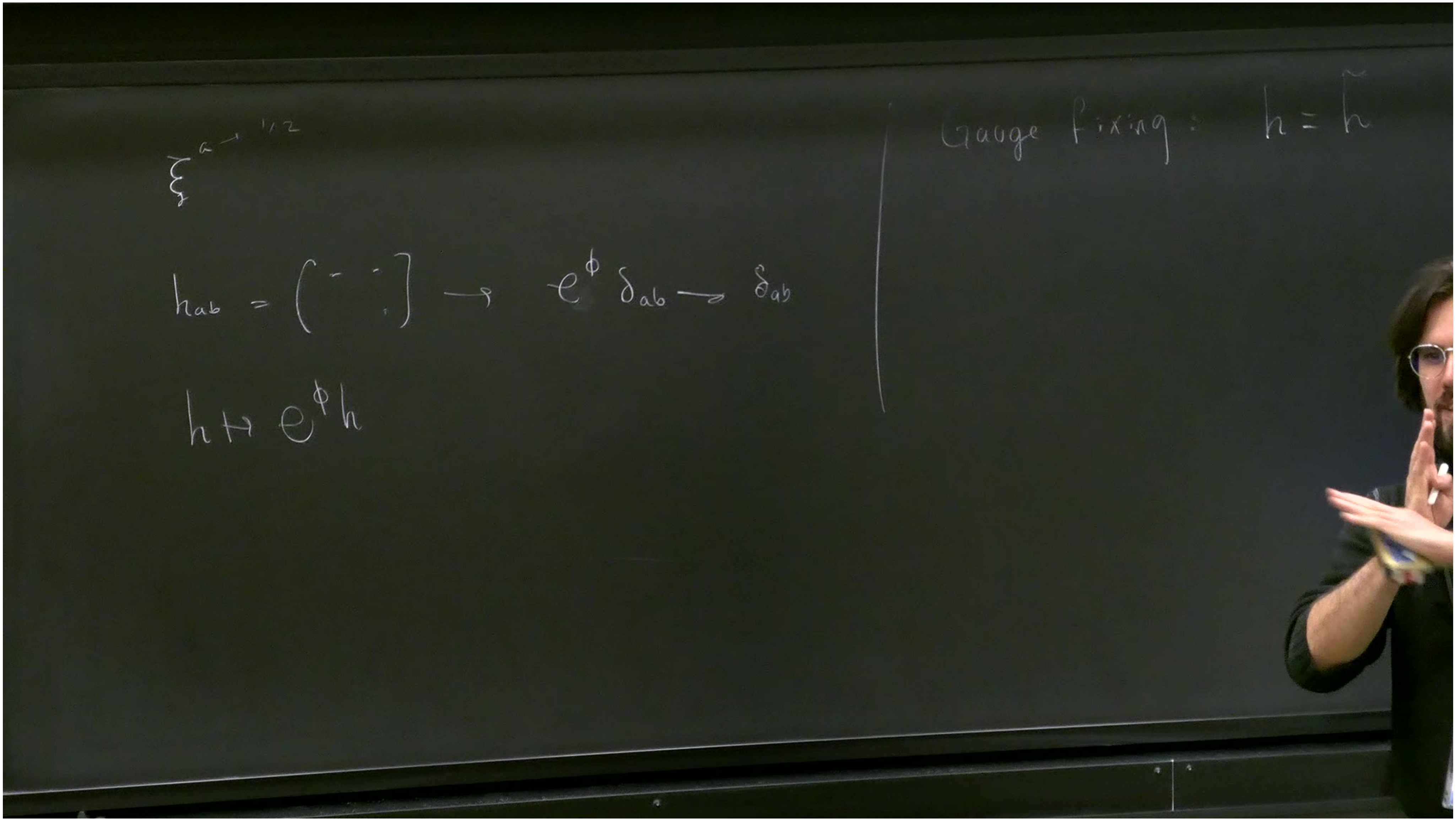
$\mathcal{H}_{phys} := \ker \hat{Q} / \operatorname{Im} \hat{Q} = H(\mathcal{H}, Q)$
$0 = \frac{d}{dt} \int D\phi \, e^{t \int \underbrace{QV}} \mathcal{O}_1 \mathcal{O}_2$
$\int D\phi \, Q(\cdots) = 0$
$Q = \int dx \, Q\phi(x) \frac{\delta}{\delta \phi(x)}$
$\int D\phi \, Q\mathcal{O}_1 \mathcal{O} = - \int D\phi \, \mathcal{O}_1 Q \mathcal{O}_2$

$$S_{poly} = \int h^{ab}\, \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu} \sqrt{h}\, d^2\sigma \qquad \mathcal{H}_{ph}$$

$$Q X = c^a \partial_a X$$

$$\delta_\xi X = \mathcal{L}_\xi X = \xi^a \partial_a X$$


$\int h^{ab} \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu} \sqrt{h}\, d^2\sigma$
$\mathcal{H}_{phys} := \mathrm{Ker}\,\hat{Q} / \mathrm{Im}\,\hat{Q} = H(\mathcal{H}$
$= c^a \partial_a X$
$= \mathcal{L}_\xi X = \xi^a \partial_a X$
$Q^2 X = (Q c^a) \partial_a X - c^a \partial_a Q X$
$- c^a \partial_a (c^b \partial_b X)$
$= (Q c^b - c^a \partial_a c^b) \partial_b X$


ξ^a → 1/2
h_ab = ( ) → e^ϕ δ_ab → δ_ab
h ↦ e^ϕ h
Gauge fixing: h = h̃

$\delta_{ab} \to \tilde{\delta}_{ab}$

Gauge fixing: $h = \tilde{h}$

Are ther $\delta\tilde{h} \neq 2\nabla_{(a}\xi_{b)}$

$$\int\sqrt{h}\;\delta h^{ab}\,\nabla_{(a}\xi_{b)} = (\delta h, \nabla\xi) = 0$$

$$\nabla_a \delta h^{ab} = 0$$

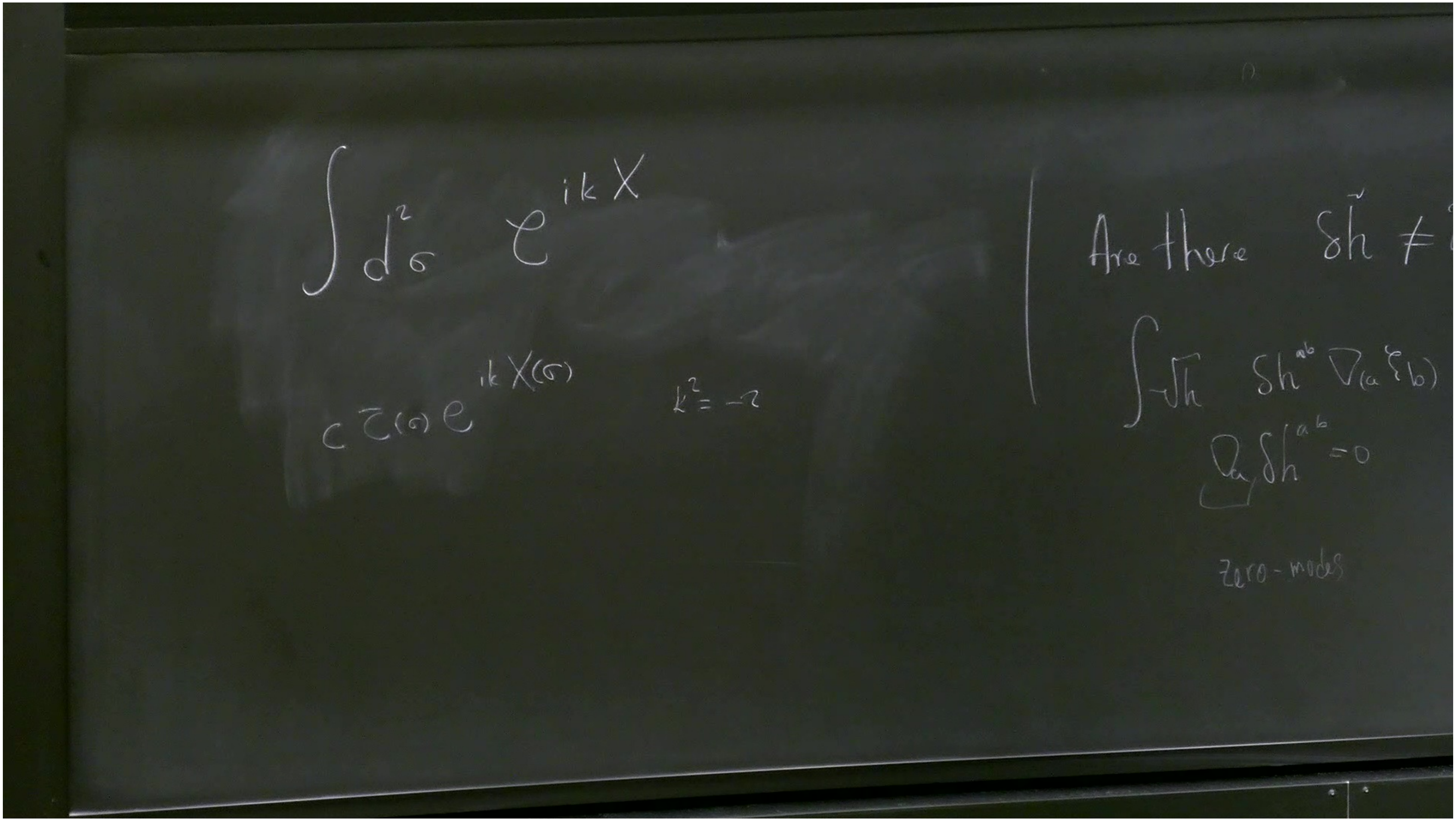
zero-modes

$$S_0 + \int Q\Big(b_{ab} \underbrace{(h^{ab} - \tilde{h}^{ab})}_{F[h]} \Big)$$

$$Q h_{ab} := \mathcal{L}_c h_{ab} = 2 \nabla_{(a} c_{b)}$$

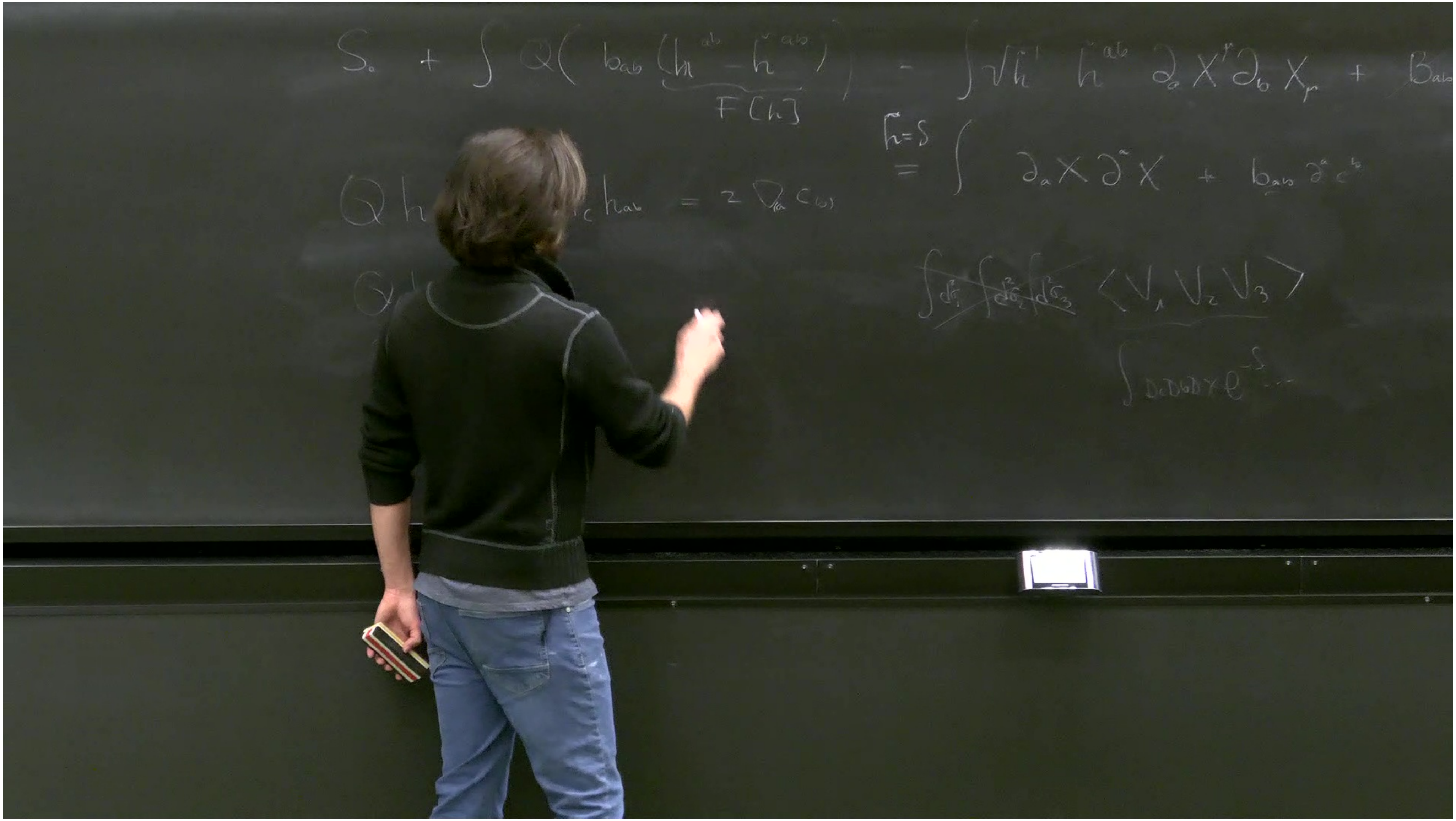
$$Q b_{ab} = B_{ab}$$

$$Q B_{ab} = 0$$


∫ d²σ e^{ikX}
c τ(σ) e^{ikX(σ)}
k² = -2
Are there δh̃ ≠
∫ √h δh^{ab} ∇_(a ξ_b)
∇_a δh^{ab} = 0
zero-modes

$$- \int \sqrt{\tilde{h}}\ \tilde{h}^{ab} \partial_a X^\mu \partial_b X_\mu + B_{ab} (h^{ab} - \tilde{h}^{ab}) - b_{ab} \nabla_{\tilde{h}}^{a} c^{b)}$$

$$\overset{\tilde{h}=\delta}{=} \int \partial_a X \partial^a X + b_{ab} \partial^a c^b$$

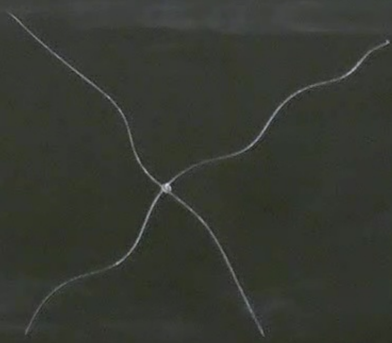

$S_0 + \int Q(b_{ab} \underbrace{(h^{ab} - \check{h}^{ab})}_{F[h]}) - \int \sqrt{h}\, \check{h}^{ab} \partial_a X^\mu \partial_b X_\mu + B_{ab}$
$\check{h} = \delta$
$= \int \partial_a X \partial^a X + b_{ab} \partial^a c^b$
$Q h_{ab} = \mathcal{L}_c h_{ab} = 2 \nabla_{(a} c_{b)}$
$\int d^2\sigma_1 \int d^2\sigma_2 \int d^2\sigma_3 \langle V_1 V_2 V_3 \rangle$
$\int DcDbDX\, e^{-S} \dots$

$$= \int \sqrt{\tilde{h}}\ \tilde{h}^{ab}\, \partial_a X^\mu \partial_b X_\mu + B_{ab}\left(h^{ab} - \tilde{h}^{ab}\right) - b_{ab} \nabla_{\tilde{h}}^{a} c^{b})$$

$\tilde{h} = \delta$

$$= \int \partial_a X \partial^a X + b_{ab} \partial^a c^b$$

$$\partial^a c^b = 0$$



$$\cancel{\int d^2\sigma_1 \int d^2\sigma_2 \int d^2\sigma_3}\ \underbrace{\langle V_1 V_2 V_3 \rangle}_{\int DcDbDX\, e^{-S} \cdots}$$

$= 0$

$$S_0 + \int Q\Big(b_{ab} \underbrace{(h^{ab} - \tilde{h}^{ab})}_{F[h]} \Big) = \int \sqrt{\tilde{h}}\, \tilde{h}^{ab} \partial_a X^\mu \partial_b X_\mu +$$

$$\overset{\tilde{h}=\delta}{=} \int \partial_a X \partial^a X + b_{ab} \partial^a c^b$$

$$Q h_{ab} := \mathcal{L}_c h_{ab} = 2 \nabla_{(a} c_{b)}$$

$$\partial^a c^b$$

$$Q b_{ab} = B_{ab}$$

$$\delta_{\xi \phi} \tilde{h} = 0$$

$$\cancel{\int d^2\sigma_1 \int d^2\sigma_2 \int d^2\sigma_3} \underbrace{\langle V_1 V_2 V_3 \rangle}_{\int DcDbDX e^{-S} \cdots}$$

$$Q B_{ab} = 0$$

$$2 \nabla_{(a} \xi_{b)} + 2\phi \cdot h_{ab} = 0$$

$$+ \; B_{ab}\,(h^{ab} - \tilde{h}^{ab}) \; - \; b_{ab}\, \nabla_{\tilde{h}}^{(a} c^{b)}$$

b

$\nabla^a{}_{,} c^b = 0$ ✓

$$[\delta_\alpha, \delta_\beta]\,\mathcal{O} = \delta_{[\alpha,\beta]}\,\mathcal{O} + (\text{e.o.m.'s})$$

$$\mathcal{O}_{BV}\,\phi^{\dagger} = \text{e.o.m's}$$

$$+ \; B_{ab}\left(h^{ab} - \tilde{h}^{ab}\right) - b_{ab} \nabla^{(a}_{\tilde{h}} c^{b)}$$

$$\partial^a c^b$$

$$\partial^a{}_{,} c^b = 0 \checkmark$$

$$dx^i \wedge dx^j = - dx^j \wedge dx^i$$