Title: Lecture - AdS/CFT, PHYS 777

Speakers: David Kubiznak

Collection/Series: AdS/CFT (Elective), PHYS 777, March 31 - May 2, 2025

Subject: Quantum Fields and Strings, Quantum Gravity

Date: May 01, 2025 - 9:00 AM

URL: https://pirsa.org/25050002

L13: HOLOGRAPHIC ENTANGLEMENT ENTROPY

QM: VON NEUMANN ENTROPY:

$$S_{VN} = -\mathrm{Tr}\, \rho \log \rho$$

- 0 ... PURE STATE
- MAXIMAL (EQUAL PROB)

- IF $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

REDUCED DENSITY MATRIX FOR A

$$\rho_A = \mathrm{Tr}_B \rho$$

- ENTANGLEMENT ENTROPY

$$S_A = -\mathrm{Tr}(\rho_A \log \rho_A)$$

$$S_A = \#(R\Lambda)^{d-2} + \#(R\Lambda)^{d-3} + \dots$$

R .. LINEAR SIZE OF A

Λ ... UV CUTOFF

- PURE STATE IN $\mathcal{H}$, $B = \bar{A}$

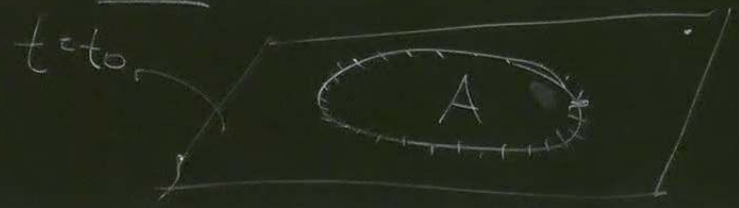
$$\Rightarrow S_A = S_B$$

QFT: (FIX $t = t_0$) d-DIMS

NOTE:

$$S_A \sim \text{AREA}(\partial A) \sim R^{d-2}$$

AREA LAW



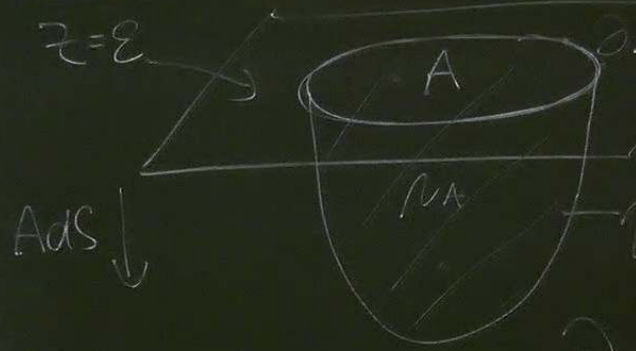
SUBADDITIVITY:

$$S_A + S_B \geq S_{A\cup B} + S_{A\cap B}$$

$\rho_A = T_{rB}\rho$

A

AdS/CFT PRESCRIPTION. RYU & TAKAYANAGI



ENTANGLING SURFACE

γ_A .. MINIMAL SURFACE IN AdS_{d+1}

$$S_A = \min_{\gamma_A} \frac{\text{AREA}(\gamma_A)}{4G_{d+1}}$$

$\partial\gamma_A = A \cup \gamma$

γ ANCHORED ON ∂A

$S_A + S_B \geq S_{A\cup B} + S_{A\cap B}$

HG SURFACE

$$S_A = \min_{\gamma \sim A} \frac{\text{AREA}(\gamma)}{4G_{d+1}} \sim \left(\frac{\ell^{d-1}}{G_{d+1}}\right) \frac{\text{AREA}(\partial A)}{\varepsilon^{d-2}} + \dots$$

RFACE

CENTRAL CHARGE

∂A

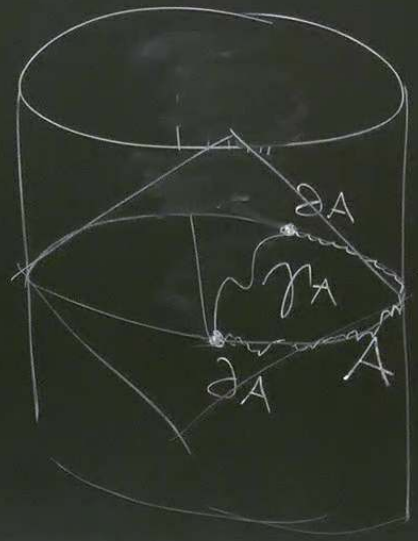
- NOTE SUB ADITIVITY:

$$= \frac{A\cup B}{\gamma_{A\cup B}} + \frac{A\cap B}{\gamma_{A\cap B}}$$

$$S_A + S_B \geq S_{A\cup B} + S_{A\cap B}$$

S_{A+B}

- MORE GENERALLY: IF BULK M & STATES ON ∂M ARE TIME VARYING (GEOMETRY ON ∂M FIXED)



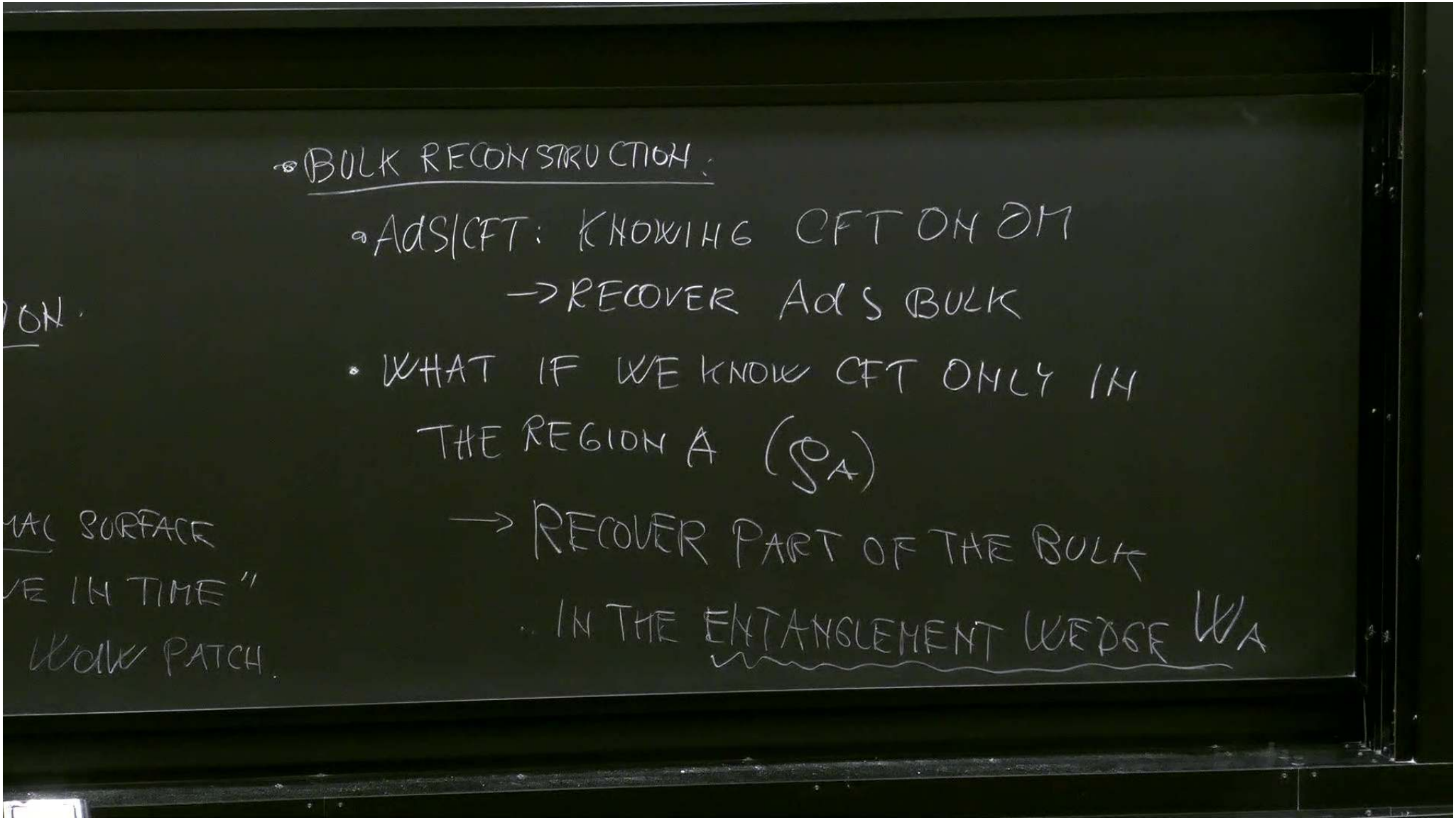
HRT COVARIANT PRESCRIPTION.

γ MINIMAL SURFACE AT FIXED t $\longrightarrow$ γ_A ... EXTREMAL SURFACE "CAN MOVE IN TIME"

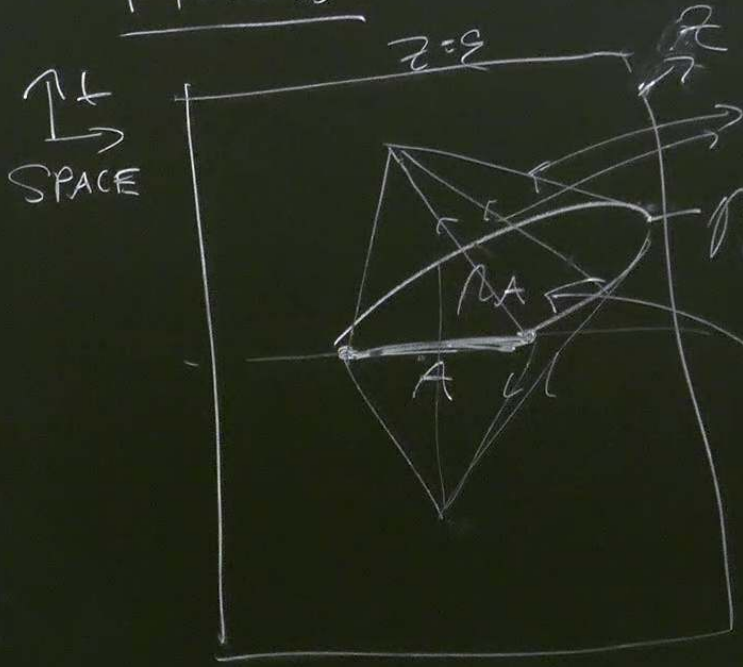
- BULK RECONSTRUCTION:
  - AdS/CFT: KNOWING CFT ON ∂M
    → RECOVER AdS BULK
  - WHAT IF WE KNOW CFT ONLY IN THE REGION A (ρ_A)
    → RECOVER PART OF THE BULK

ON.

AL SURFACE
E IN TIME"
WICK PATCH.


• BULK RECONSTRUCTION:
• AdS/CFT: KNOWING CFT ON ∂M
→ RECOVER AdS BULK
• WHAT IF WE KNOW CFT ONLY IN
THE REGION A (ρ_A)
→ RECOVER PART OF THE BULK
IN THE ENTANGLEMENT WEDGE W_A

PICTURE:



NULL GEODESICS FROM $\mathcal{I}A$

$\mathcal{I}A$.. EXTREMAL SURFACE

WA.. INSIDE OF THIS;

"DUAL OF $\mathcal{I}A$"

ENTANGLEMENT = SPACETIME

(QUANTUM ENTANGLEMENT OF MICROSCOPIC DOF GIVES RISE TO SMOOTH SPACETIME GEOMETRY OF BULK GRAV. THEORY)

$$S_A = -\mathrm{Tr}(\rho_A \log \sigma_A)$$

σ: REFERENCE STATE

- PERT: $\rho_A \to \tilde{\rho}_A = \rho_A + \delta\rho$ $\quad (\mathrm{Tr}\,\delta\rho = 0)$

$$\delta S_A = -\mathrm{Tr}(\delta\rho \log \sigma_A) - \mathrm{Tr}\left(\rho_A \frac{1}{\rho_A}\delta\rho\right) = \emptyset =$$

DEFINE: MODULAR HAMILTONIAN $\rho_A \equiv \frac{1}{Z} \exp(-H_A)$

$$\delta S_A = \mathrm{Tr}(\delta\rho H_A) + \cancel{\delta} = \delta\langle H_A\rangle$$

$$\boxed{\delta\langle H_A\rangle = \delta S_A}$$ 1ST LAW OF ENTANGLEMENT ENTROPY

NOTE: A ... ENTIRE SPACE, $\rho_A = e^{-H/T}$

... STANDARD TDS.

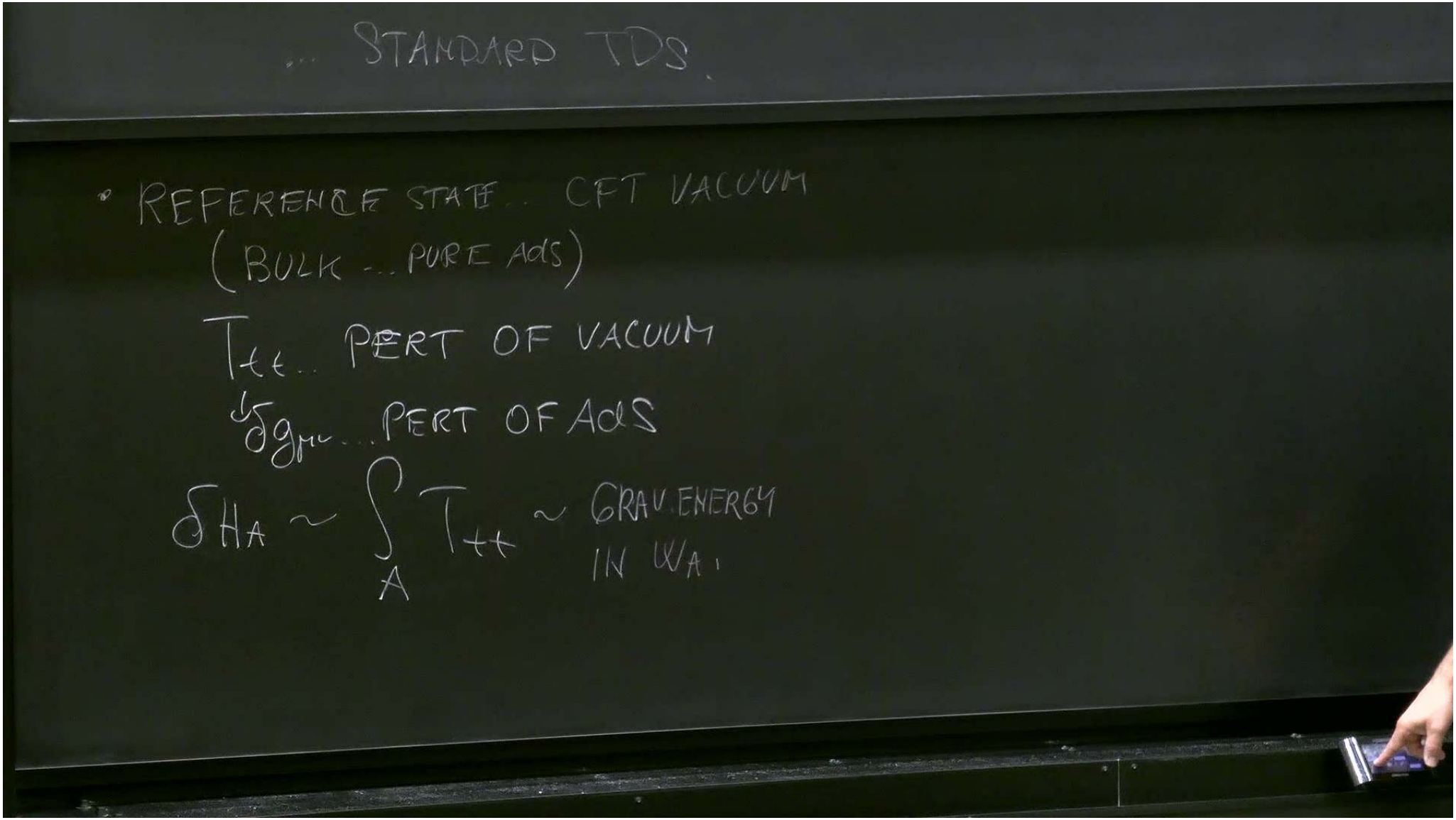
• IN GENERAL H_A IS A BIG NON-LOCAL MESS:

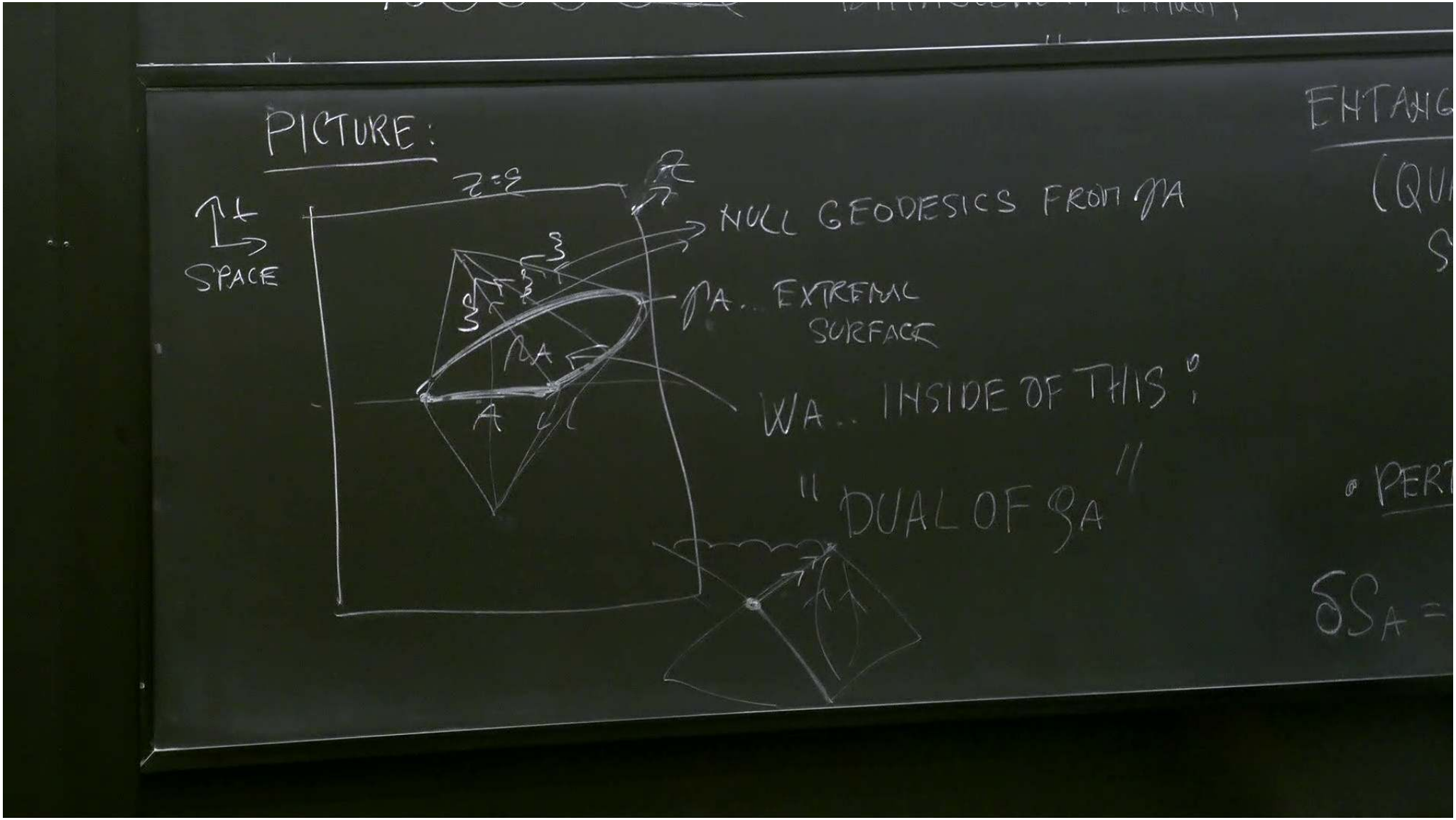
$$\left(\int dx\,dy\; O(x)\,O(y)\,K(x,y)\right)$$

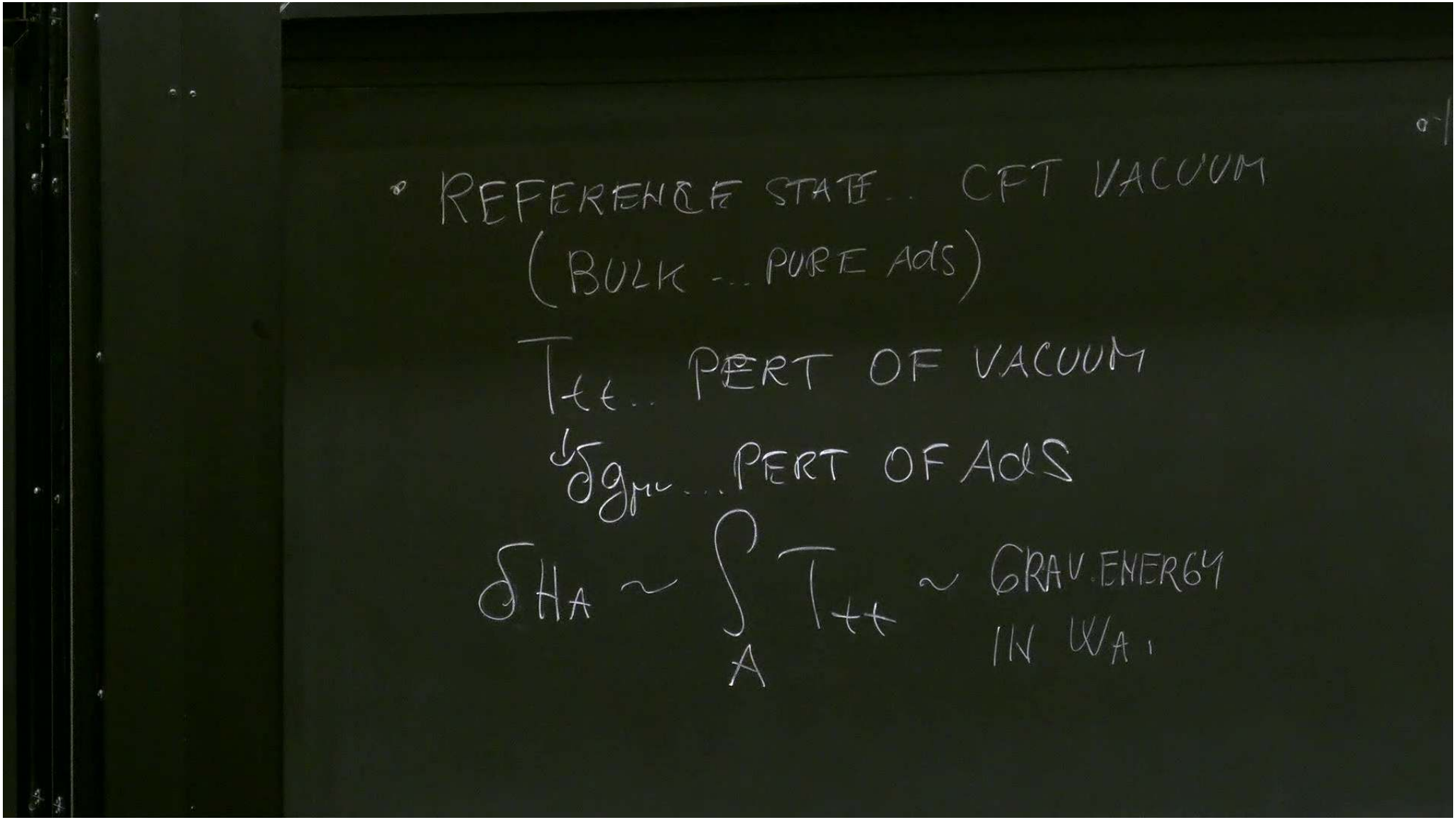
BUT: FOR A CFT & A.. SPHERICAL ENTANGLING SURFACE

$$H_A \sim \int_{|y|<R} d^{d-1}y\; \frac{R^2-|y|^2}{2R}\, T_{tt}(y)$$

$$\sim \int_A T_{tt} \quad \text{"} \quad \text{ENERGY}$$


... STANDARD TDS.
• REFERENCE STATE .. CFT VACUUM
(BULK ... PURE AdS)
T_{tt} .. PERT OF VACUUM
$\delta g_{\mu\nu}$ ... PERT OF AdS
$\delta H_A \sim \int_A T_{tt} \sim$ GRAV. ENERGY IN W_A.


PICTURE:
SPACE
NULL GEODESICS FROM $\mathcal{I}A$
$\mathcal{I}A$.. EXTREMAL SURFACE
WA.. INSIDE OF THIS
"DUAL OF $\mathcal{I}A$"
$\delta S_A =$


• REFERENCE STATE .. CFT VACUUM
(BULK ... PURE AdS)
T_{tt} .. PERT OF VACUUM
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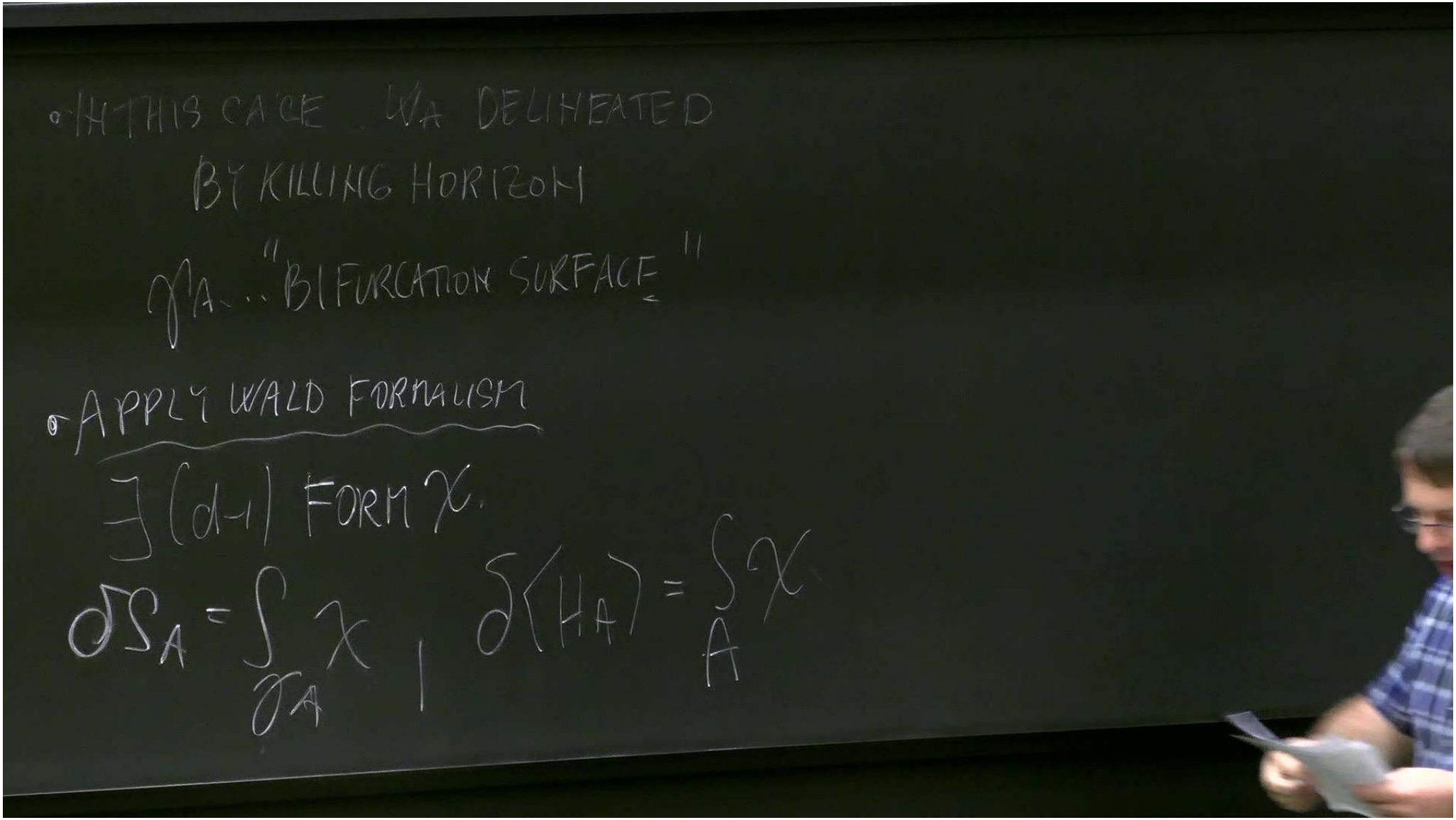
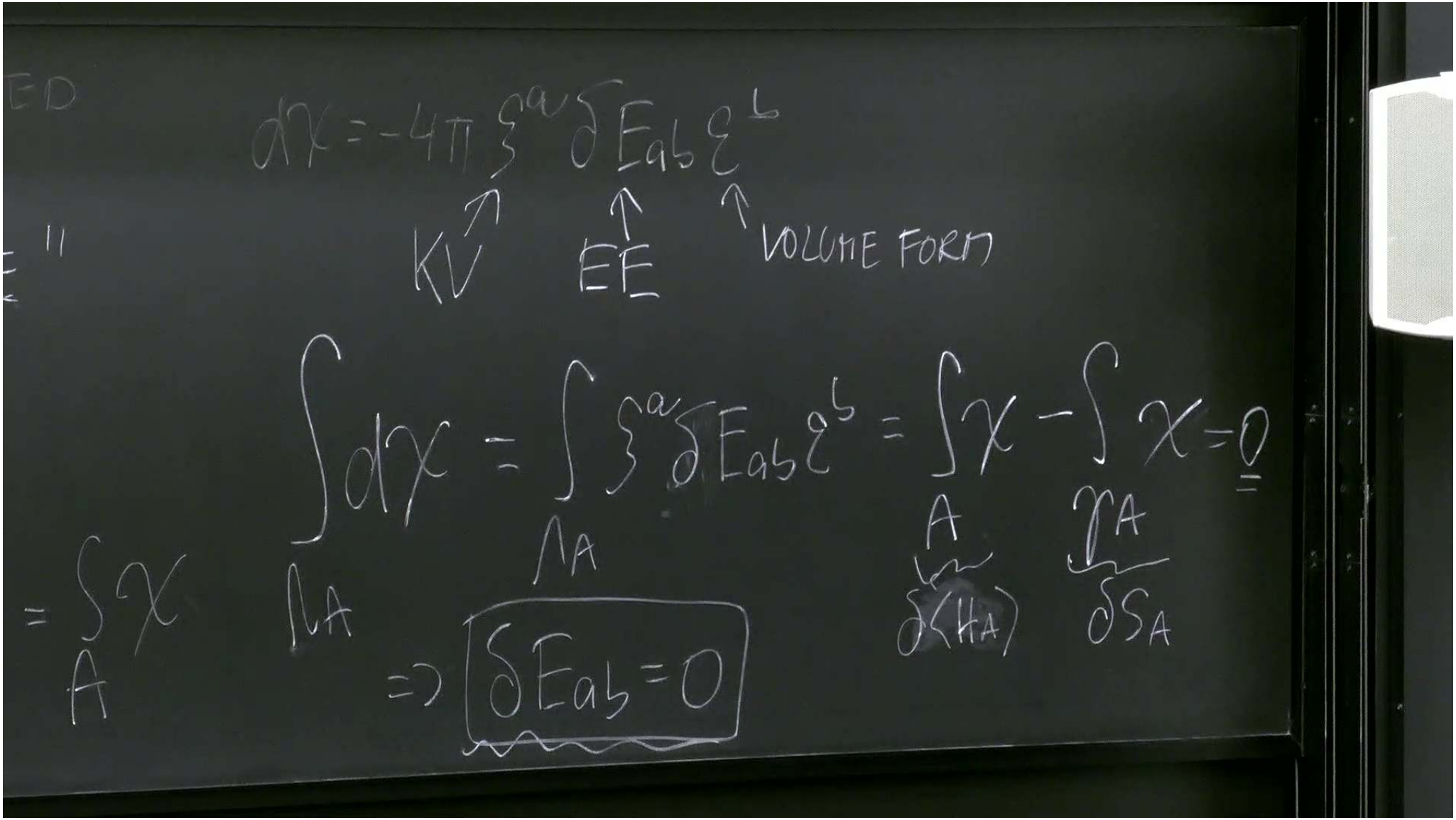

IN THIS CASE $\mathcal{W}_A$ DELINEATED
BY KILLING HORIZON
γ_A .. "BIFURCATION SURFACE"
APPLY WALD FORMALISM
$\exists$ (d-1) FORM χ.
$\delta S_A = \int_{\gamma_A} \chi$, $\delta(H_A) = \int_A \chi$


ED
$d\chi = -4\pi \xi^a \delta E_{ab} \epsilon^b$
KV
EE
VOLUME FORM
$\int d\chi = \int \xi^a \delta E_{ab} \epsilon^b = \int \chi - \int \chi = 0$
Λ_A
A
γ_A
$\delta(H_A)$
δS_A
$= \sum_A \chi$
Λ_A
$\Rightarrow \boxed{\delta E_{ab} = 0}$