Title: Lecture - AdS/CFT, PHYS 777

Speakers: David Kubiznak

Collection/Series: AdS/CFT (Elective), PHYS 777, March 31 - May 2, 2025

Subject: Quantum Fields and Strings, Quantum Gravity

Date: May 01, 2025 - 9:00 AM

URL: https://pirsa.org/25050002

L13: HOLOGRAPHIC ENTANGLEMENT ENTROPY

QM: VON NEUMANN ENTROPY:

$$\boxed{S_{VN} = -\mathrm{Tr}\,\rho \log \rho}$$

0 ... PURE STATE

MAXIMAL (EQUAL PROB)

- IF $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

REDUCED DENSITY MATRIX FOR A

$$\rho_A = \mathrm{Tr}_B\,\rho$$

• ENTANGLEMENT ENTROPY

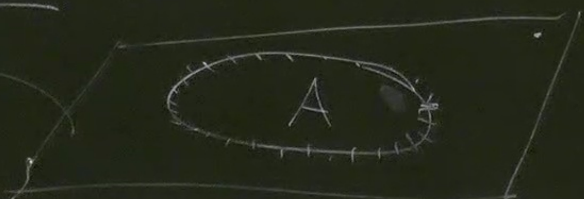
$$\boxed{S_A = -\mathrm{Tr}(\rho_A \log \rho_A)}$$

• PURE STATE IN $\mathcal{H}$, $B = \bar{A}$

$\Rightarrow \rho_A = \rho_B$

QFT: (FIX $t = t_0$) d-DIMS

$t = t_0$



$$S_A = \#(R\Lambda)^{d-2} + \#(R\Lambda)^{d-3} + \ldots$$

R .. LINEAR SIZE OF A

Λ ... UV CUTOFF

NOTE:

$$\boxed{S_A \sim \mathrm{AREA}(\partial A) \sim R^{d-2}}$$

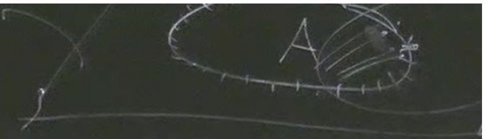
AREA LAW

SUBADDITIVITY:

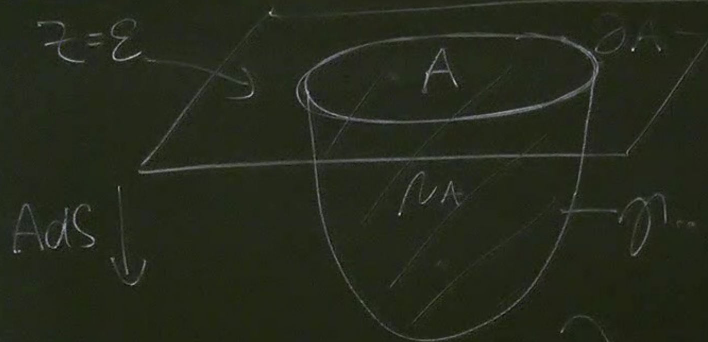
$$\boxed{S_A + S_B \geq S_{A\cup B} + S_{A\cap B}}$$

... MATRIX FOR A

$\rho_A = \mathrm{Tr}_B\, \rho$



AdS/CFT PRESCRIPTION. RYU & TAKAYANAGI (... 2006)



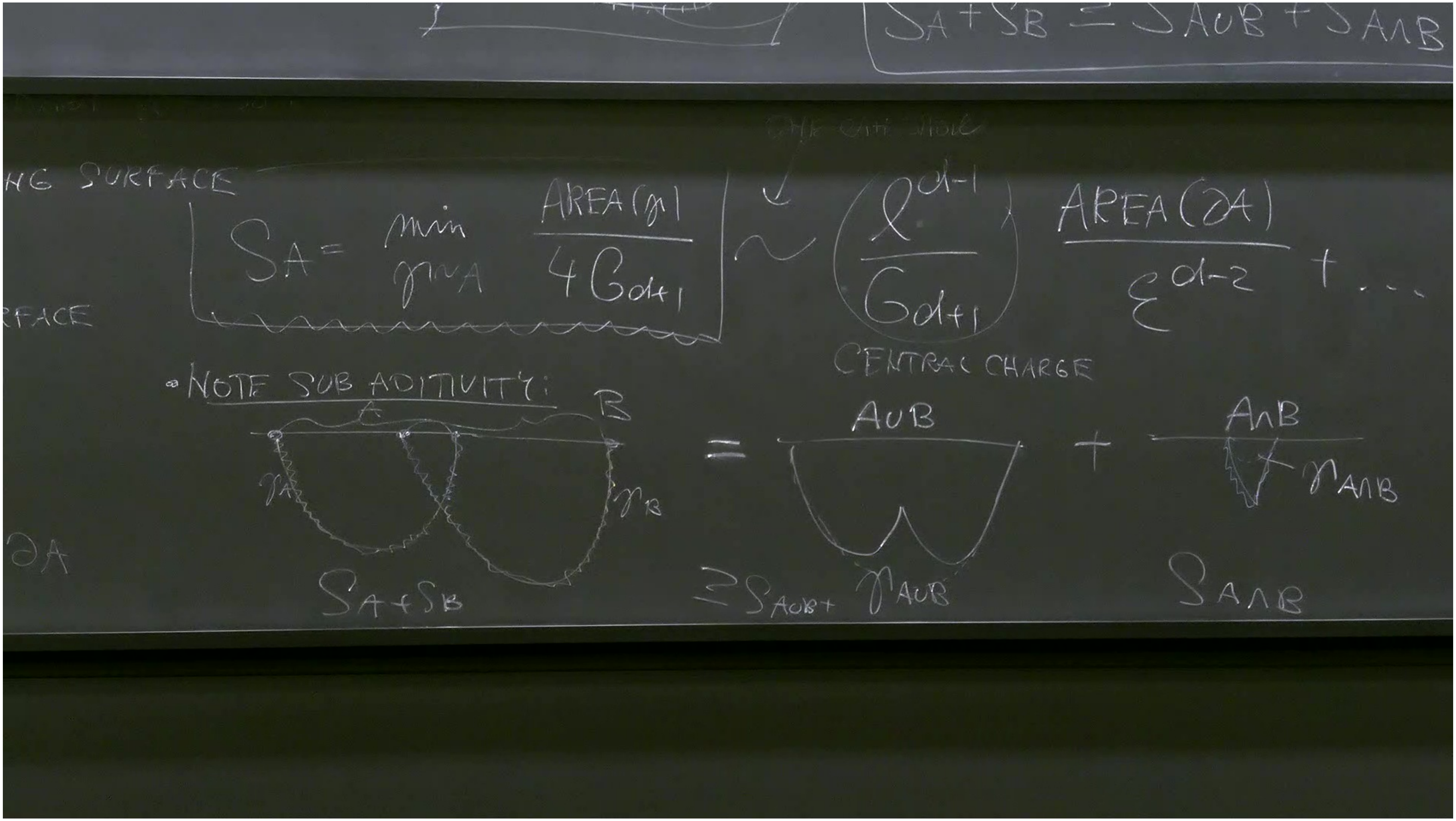
∂A — ENTANGLING SURFACE

$$\boxed{S_A = \min_{\gamma \sim A} \frac{\mathrm{AREA}(\gamma)}{4G_{d+1}}}$$

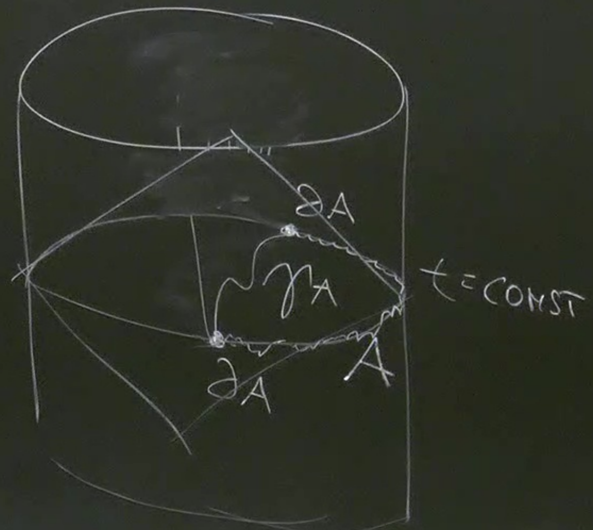
γ ... MINIMAL SURFACE IN AdS_{d+1}

$$\partial r_A = A \cup \gamma$$

γ ANCHORED ON ∂A


S_A + S_B ≥ S_{A∪B} + S_{A∩B}
SURFACE
S_A = min_{γ~A} AREA(γ) / 4 G_{d+1}
~ (ℓ^{d-1} / G_{d+1}) AREA(∂A) / ε^{d-2} + ...
CENTRAL CHARGE
• NOTE SUB ADITIVITY:
A
B
γ_A
γ_B
S_A + S_B
= A∪B
+ A∩B
γ_{A∩B}
≥ S_{A∪B} + γ_{A∪B}
S_{A∩B}
∂A

- MORE GENERALLY: IF BULK M & STATES ON ∂M ARE TIME VARYING (GEOMETRY ON ∂M FIXED)



HRT COVARIANT PRESCRIPTION.

γ MINIMAL SURFACE AT FIXED t $\longrightarrow$ γ_A ... EXTREMAL SURFACE "CAN MOVE IN TIME"

ON.

MAL SURFACE

VE IN TIME"

LOCAL PATCH.

- BULK RECONSTRUCTION:
  - AdS/CFT: KNOWING CFT ON ∂M
    $\rightarrow$ RECOVER AdS BULK
  - WHAT IF WE KNOW CFT ONLY IN THE REGION A (ρ_A)
    $\rightarrow$ RECOVER PART OF THE BULK

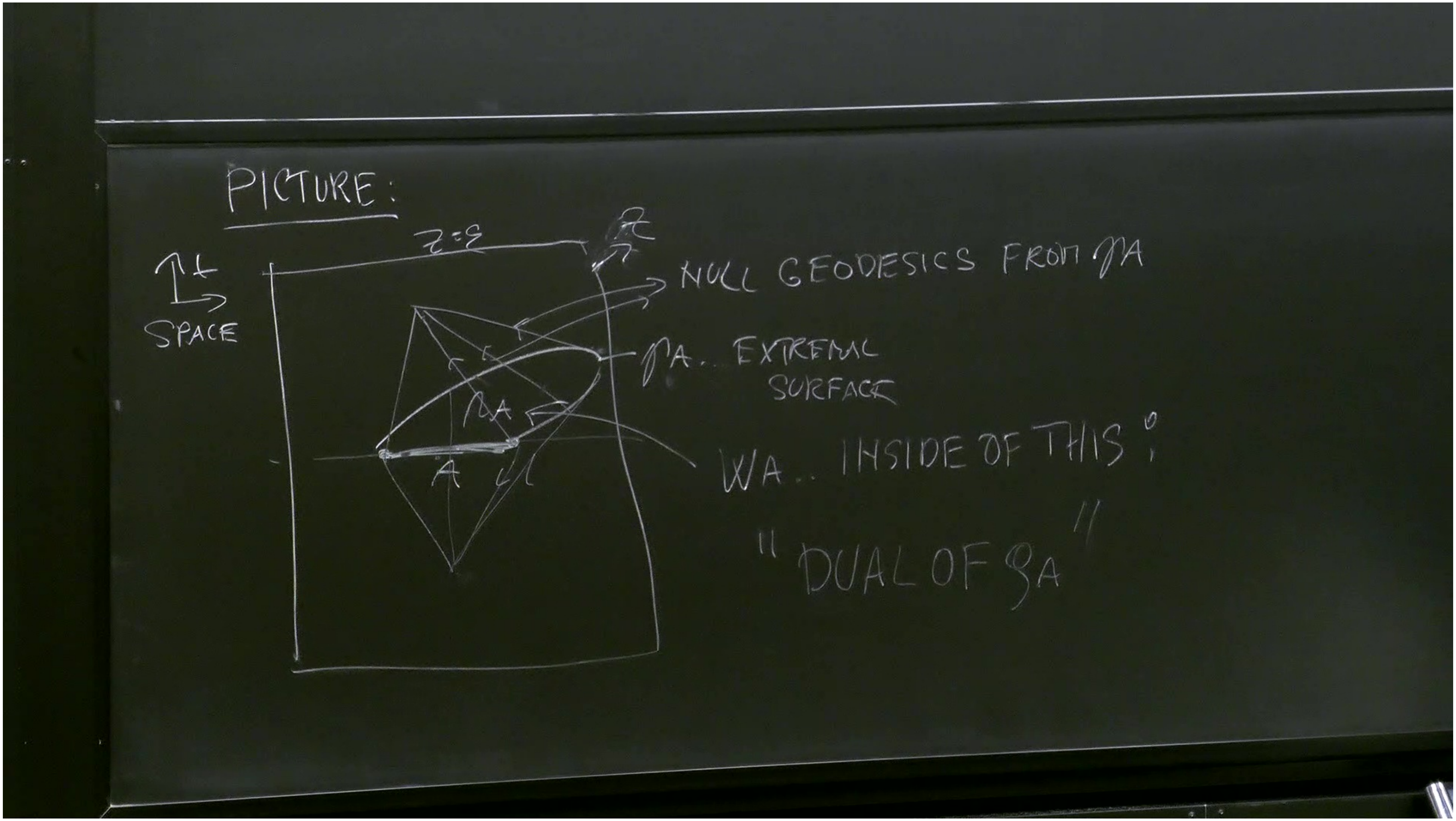
ON.

MAL SURFACE

E IN TIME"

WdW PATCH.

- **BULK RECONSTRUCTION:**
  - AdS/CFT: KNOWING CFT ON ∂M
    $\rightarrow$ RECOVER AdS BULK
  - WHAT IF WE KNOW CFT ONLY IN THE REGION A (ρ_A)
    $\rightarrow$ RECOVER PART OF THE BULK
    IN THE ENTANGLEMENT WEDGE W_A


PICTURE:
z = ε
t
SPACE
NULL GEODESICS FROM γA
γA .. EXTREMAL SURFACE
rA
A
WA .. INSIDE OF THIS ;
"DUAL OF γA"

ENTANGLEMENT = SPACETIME

(QUANTUM ENTANGLEMENT OF MICROSCOPIC DOF GIVES RISE TO SMOOTH SPACETIME GEOMETRY OF BULK GRAV. THEORY)

$$S_A = -\mathrm{Tr}\left(\rho_A \log \rho_A\right)$$

REFERENCE STATE

• PERT: $\rho_A \to \tilde{\rho}_A = \rho_A + \delta\rho \qquad (\mathrm{Tr}\,\delta\rho = 0)$

$$\delta S_A = -\mathrm{Tr}\left(\delta\rho \log \rho_A\right) - \mathrm{Tr}\left(\rho_A \frac{1}{\rho_A}\delta\rho\right) =$$

Ø

DEFINE: MODULAR HAMILTONIAN $\boxed{\rho_A \equiv \frac{1}{Z} \exp(-H_A)}$

$$\delta S_A = \mathrm{Tr}(\delta\rho\, H_A) + \mathcal{O} = \delta \langle H_A \rangle$$

$$\boxed{\delta \langle H_A \rangle = \delta S_A}$$

1ST LAW OF ENTANGLEMENT ENTROPY

NOTE: A.. ENTIRE SPACE, $\rho_A = e^{-H/T}$

... STANDARD TDS.

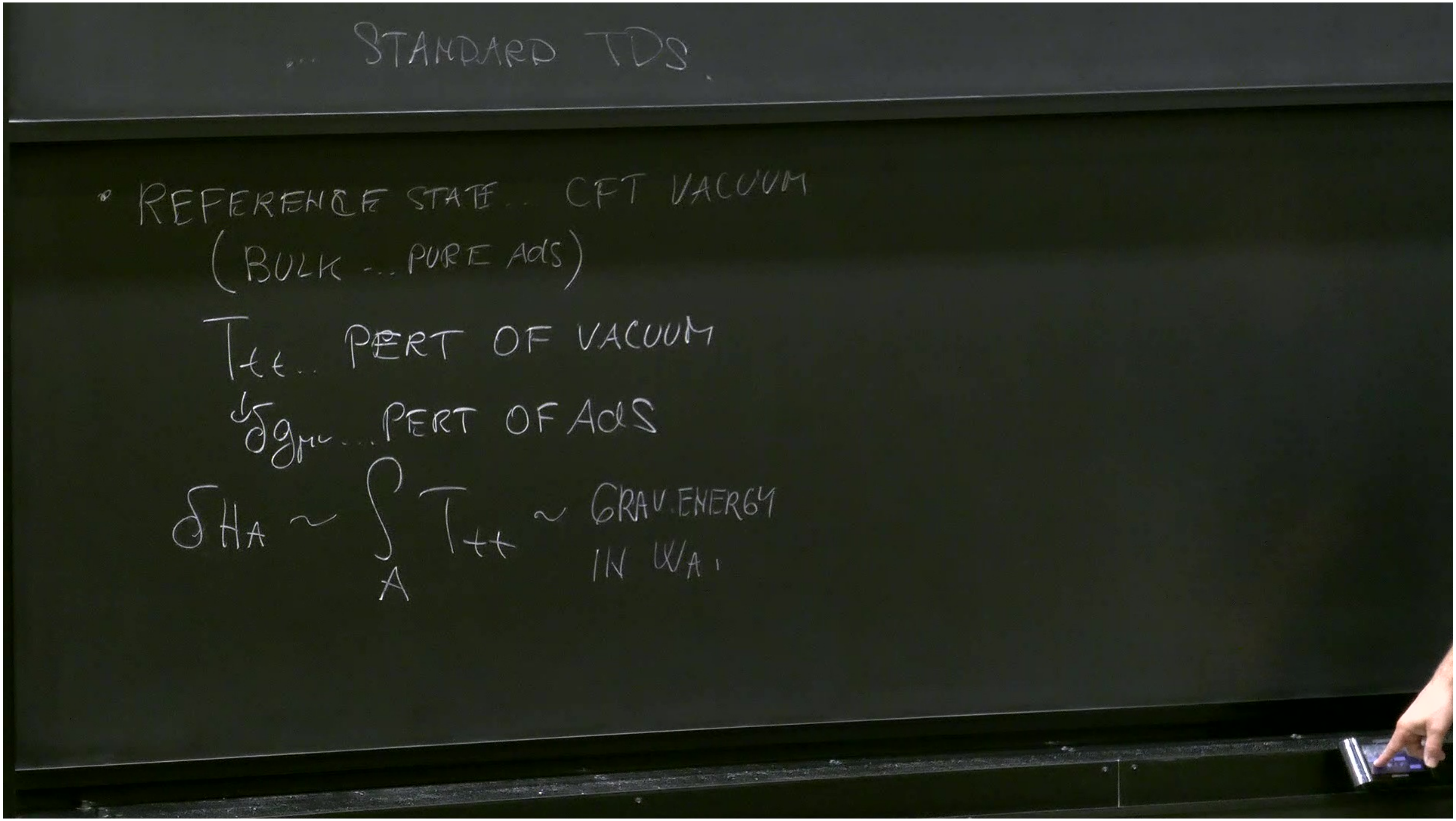
• IN GENERAL H_A IS A BIG NON-LOCAL MESS:

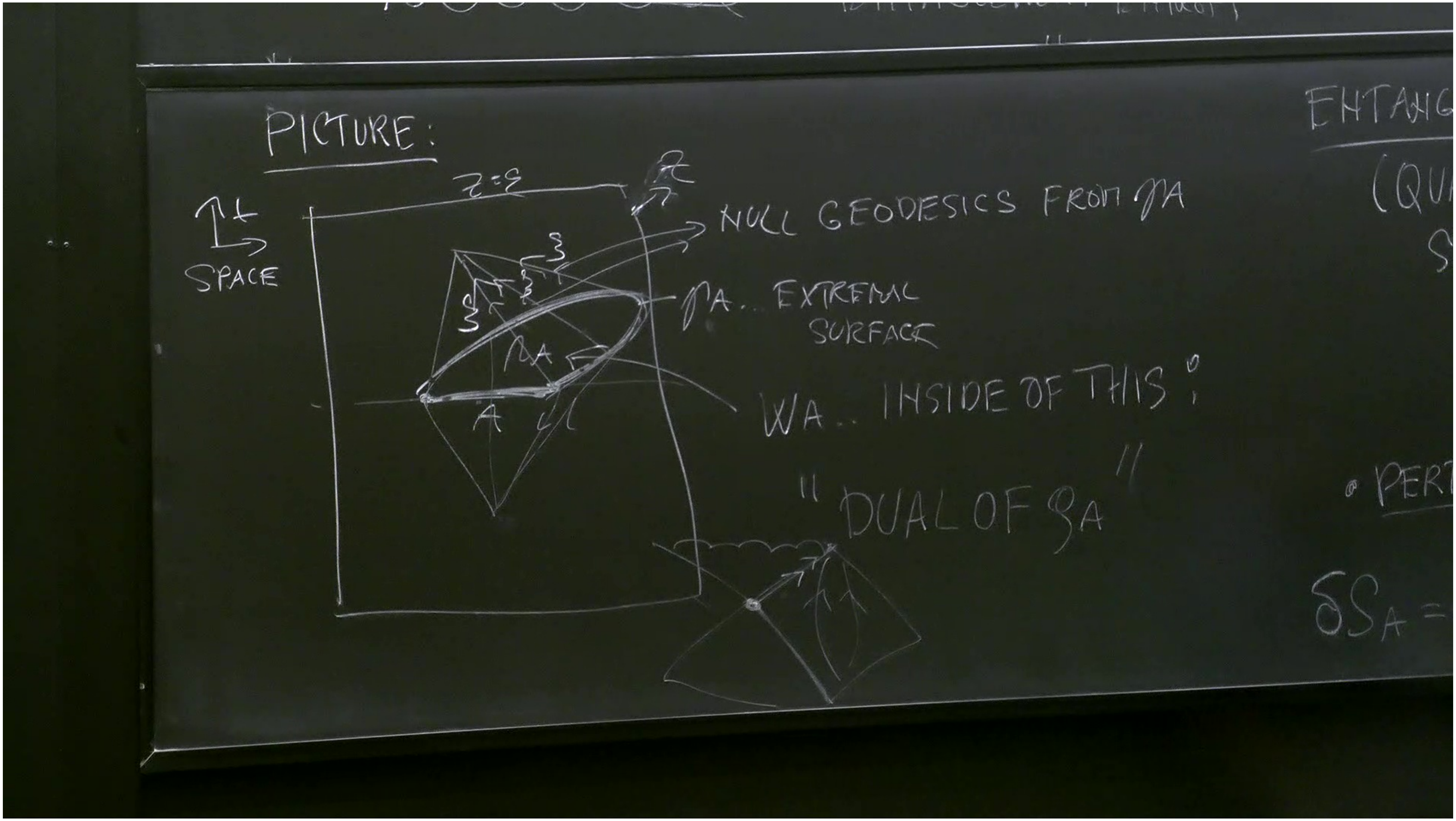
$$\left(\int dx\,dy\; O(x)\, O(y)\, K(x,y) \right)$$

<u>BUT</u>: FOR A CFT & A SPHERICAL ENTANGLING SURFACE

$$H_A \sim \int_{|y|<R} d^{d-1}y\; \frac{R^2-|y|^2}{2R}\, T_{tt}(y)$$

$$\sim \text{"}\int_A T_{tt}\text{"} \sim \text{ENERGY}$$


... STANDARD TDS.
• REFERENCE STATE ... CFT VACUUM
(BULK ... PURE AdS)
T_{tt} ... PERT OF VACUUM
$\delta g_{\mu\nu}$ ... PERT OF AdS
$\delta H_A \sim \int_A T_{tt} \sim$ GRAV. ENERGY
IN W_A.


PICTURE:
z = ε
t
SPACE
NULL GEODESICS FROM γA
γA.. EXTREMAL SURFACE
WA.. INSIDE OF THIS:
"DUAL OF ρA"
ENTANG
(QU
PERT
δSA =

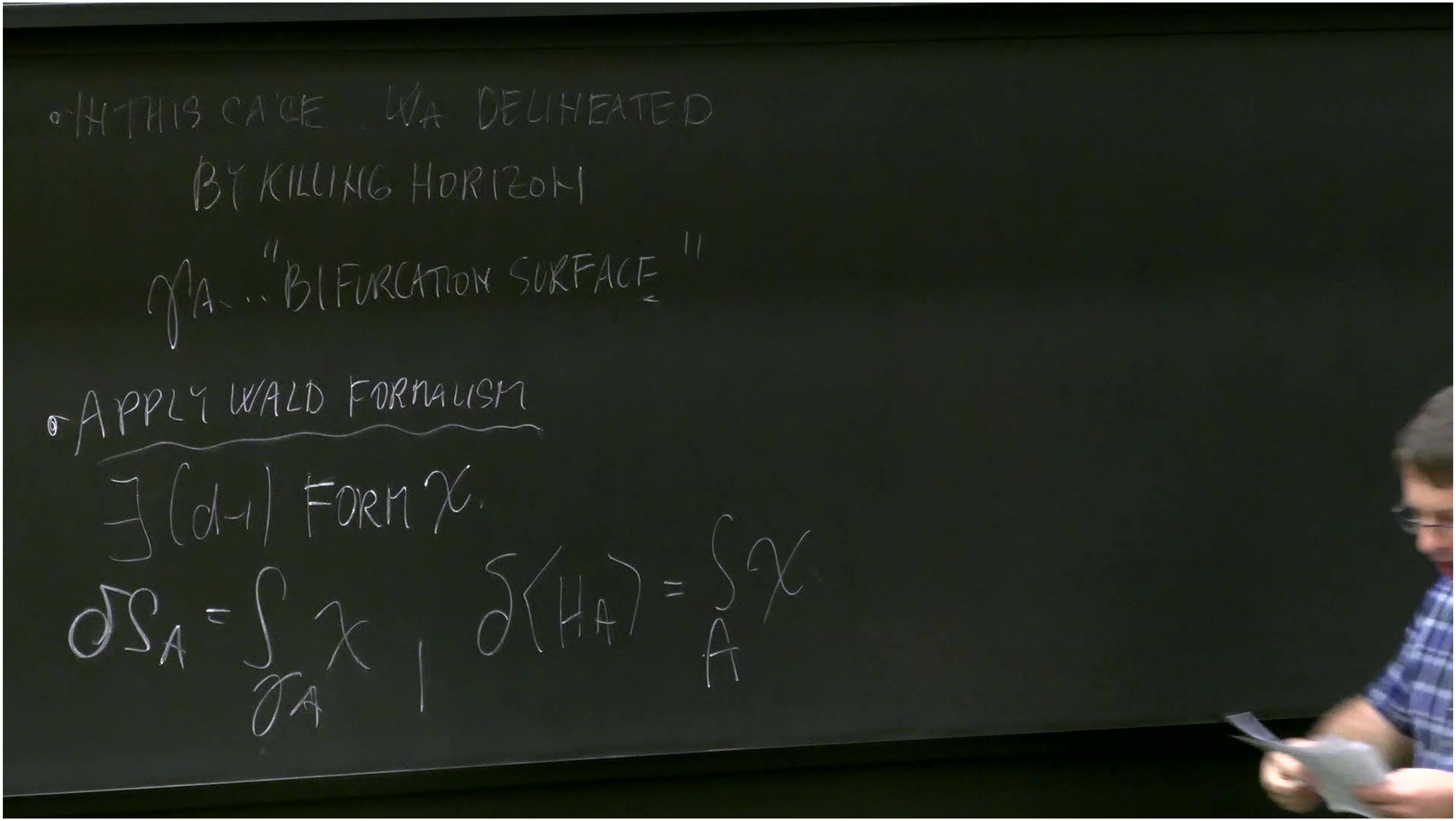
• REFERENCE STATE.. CFT VACUUM

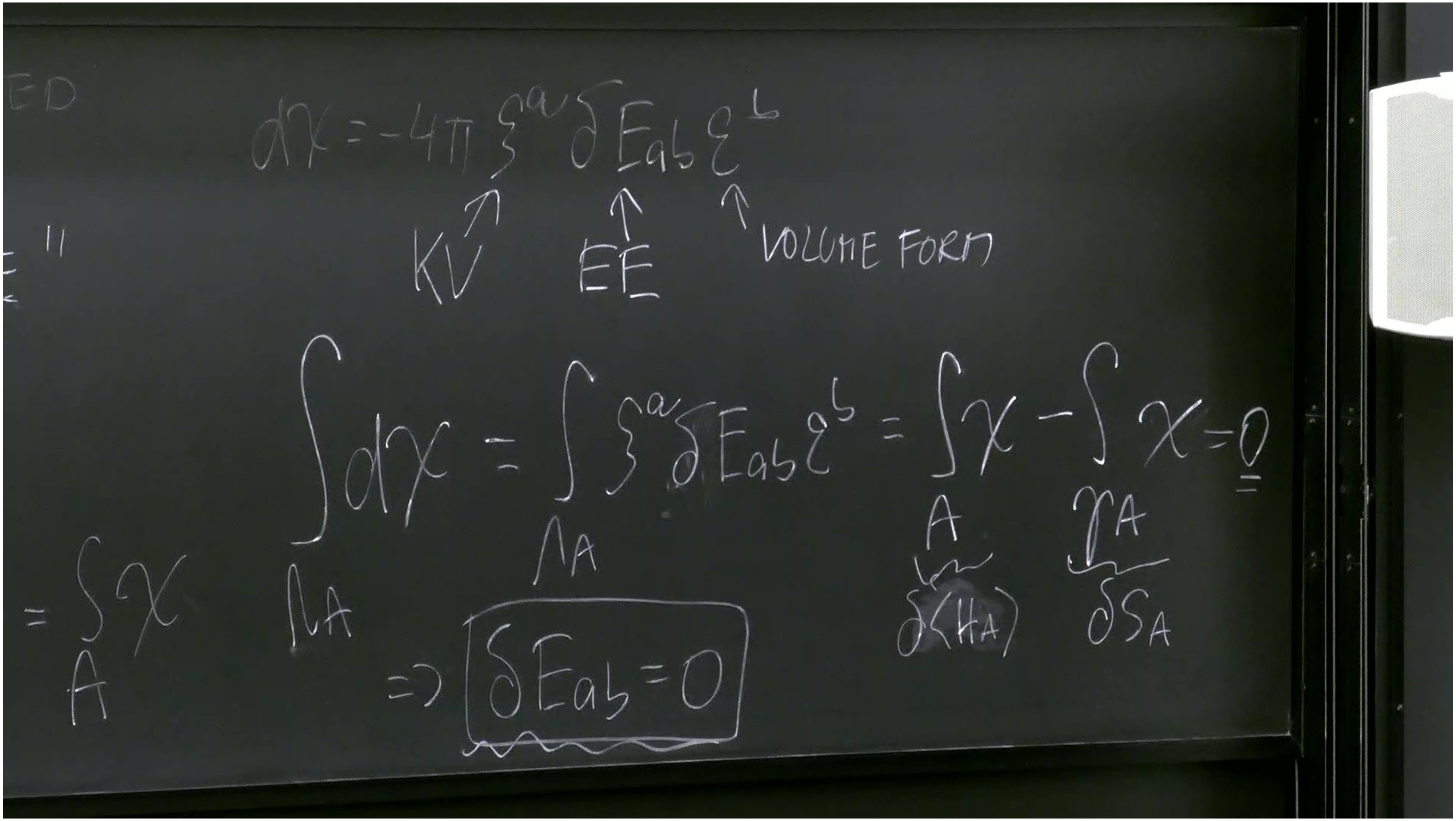
(BULK ... PURE AdS)

T_{tt}.. PERT OF VACUUM

$\delta g_{\mu\nu}$ ... PERT OF AdS

$$\delta H_A \sim \int_A T_{tt} \sim \text{GRAV. ENERGY IN } W_A .$$


• IN THIS CASE W_A DELINEATED
BY KILLING HORIZON
γ_A ... "BIFURCATION SURFACE"
• APPLY WALD FORMALISM
$\exists$ $(d-1)$ FORM χ.
$\delta S_A = \int_{\gamma_A} \chi \quad , \quad \delta \langle H_A \rangle = \int_A \chi$


$d\chi = -4\pi\, \xi^a\, \delta E_{ab}\, \varepsilon^b$
KV
EE
VOLUME FORM
$\int_{\Lambda_A} d\chi = \int_{\Lambda_A} \xi^a \delta E_{ab} \varepsilon^b = \underbrace{\int_A \chi}_{\delta \langle H_A \rangle} - \underbrace{\int_{\chi_A} \chi}_{\delta S_A} = \underline{0}$
$= \int_A \chi$
Λ_A
$\Rightarrow \boxed{\delta E_{ab} = 0}$