Title: Lecture - Quantum Matter, PHYS 777

Speakers: Chong Wang

Collection/Series: Quantum Matter (Elective), PHYS 777, March 31 - May 2, 2025

Subject: Condensed Matter

Date: May 02, 2025 - 9:00 AM

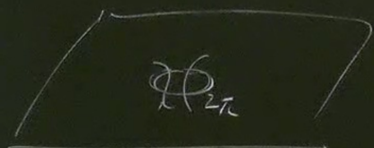
URL: https://pirsa.org/25050001

Fractional QHE.

Experimental fact: $\exists$ insulators with $\sigma_{xy} = \frac{1}{3}, \frac{1}{5} \ldots$ (in $\frac{e^2}{h} = \frac{1}{2\pi}$)

Flux threading: a local excitation with charge $q = 2\pi \sigma_{xy} = \frac{1}{3}, \frac{1}{5}$

Not locally creatable $\nexists\ O_\pi \Rightarrow \exists$ fractional excitation (anyons)



Consider $U(1)$ gauge theory.

$$Z = \int Da\, e^{-S[a]} \qquad S[a] = \frac{1}{4e^2} f_{\mu\nu}^2 \quad +$$

$$J_\mu = \frac{\epsilon^{\mu\nu\lambda}}{2\pi} \partial_\nu a_\lambda$$

$\frac{1}{3}, \frac{1}{5} \dots$ (in $\frac{e^2}{h} = \frac{1}{2\pi}$)

$2\pi \sigma_{xy} = \frac{1}{3}, \frac{1}{5}$

$\Rightarrow \exists$ fractional excitation (anyons)

Claim: this works.

$$\boxed{S[a] = \frac{1}{4e^2} f_{\mu\nu}^2 + \frac{ik}{4\pi}\epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + \frac{i\epsilon_{\mu\nu\lambda}}{2\pi} \overset{\text{background}}{A_\mu} \partial_\nu a_\lambda}$$

$k \in \mathbb{Z}$

$$J_\mu = \frac{\epsilon^{\mu\nu\lambda}}{2\pi} \partial_\nu a_\lambda$$

① Insulator (Gapped):

$$\int d^3x \, \underline{\frac{1}{4e^2}(da)^2} + \frac{i}{4\pi} k \, a da . \quad [e^2] = [p] = [E]$$

Solve e.o.m. $\Rightarrow$ a_μ is gapped with mass $\sim e^2$.

Deep IR, $e^2 \gg$ probing energy $\Rightarrow$ drop Maxwell!

$$S = -i \int d^3x \frac{k}{4\pi} a da \quad \text{e.o.m: } \epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda = 0 \Rightarrow \text{only flat connection } \text{Su}$$

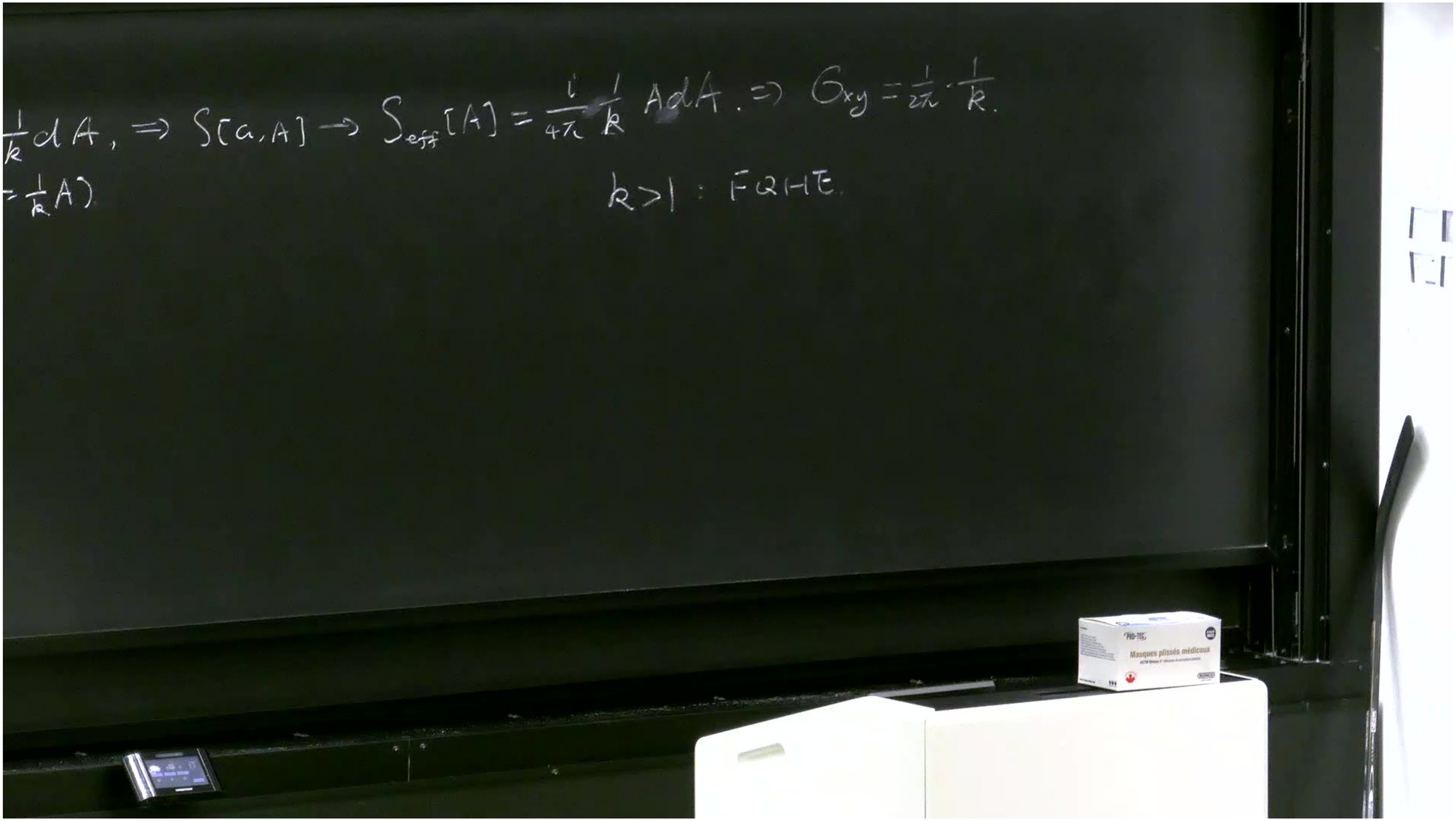
(Gapped):

$\int d^3x \; \frac{1}{4e^2}(da)^2 + \frac{i}{4\pi} k\, a da.$ $[e^2] = [p] = [E]$

Solve e.o.m. $\Rightarrow$ a_μ is gapped with mass $\sim e^2$.

Deep IR, $e^2 \gg$ probing energy $\Rightarrow$ drop Maxwell!

$S = -i \int d^3x \frac{k}{4\pi} a da$ e.o.m: $\epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda = 0 \Rightarrow$ only flat connection survive as d.o.f.


$\frac{1}{k} dA, \Rightarrow S[a, A] \rightarrow S_{eff}[A] = \frac{i}{4\pi} \frac{1}{k} AdA. \Rightarrow \sigma_{xy} = \frac{1}{2\pi} \cdot \frac{1}{k}.$
$= \frac{1}{k} A)$
$k > 1$: FQHE.
Masques plissés médicaux

... m. $da = \frac{1}{k} dA, \Rightarrow S[a,A] \to S_{eff}[A] = \frac{i}{4\pi}\frac{1}{k} AdA . \Rightarrow \sigma_{xy} = \frac{1}{2\pi}\cdot\frac{1}{k}.$

$(a = \frac{1}{k} A)$

$k > 1$: FQHE

...der a_μ .

$-\frac{i}{4\pi} k\, a da + i a_\mu \cdot j_\mu^\phi$

$|\phi|^2 + \dots$

$\delta a : \epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda = \frac{2\pi}{k} j_\mu^\phi$

$\frac{2\pi}{k}\rho = \partial \times a = b$

$\int b\, d^2x = \frac{2\pi}{k}$

$q=1$

② Frational σ_{xy}.

$S[a]$ quadratic $\longrightarrow \int Da_\mu$ is simple. e.o.m. $da = \frac{1}{k} dA, \Rightarrow S[$

$(a = \frac{1}{k} A)$

③ Anyons! Introduce gapped ϕ charged under a_μ.

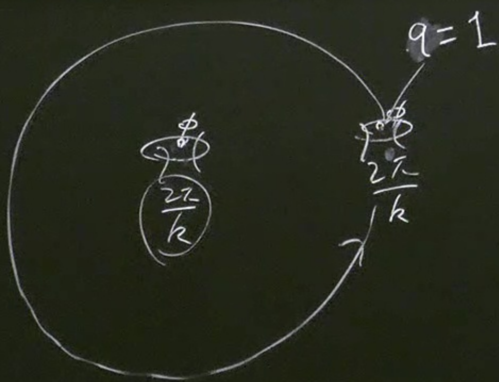
$-\frac{i}{4\pi} k \, a d$

$$S[\phi, a] = \frac{1}{2} |(\partial - ia)\phi|^2 + m^2 |\phi|^2 + \ldots$$

$\delta a : \epsilon$

- Fractional electric charge. $\frac{1}{2\pi} b = \frac{1}{k}$

- Fractional statistics.

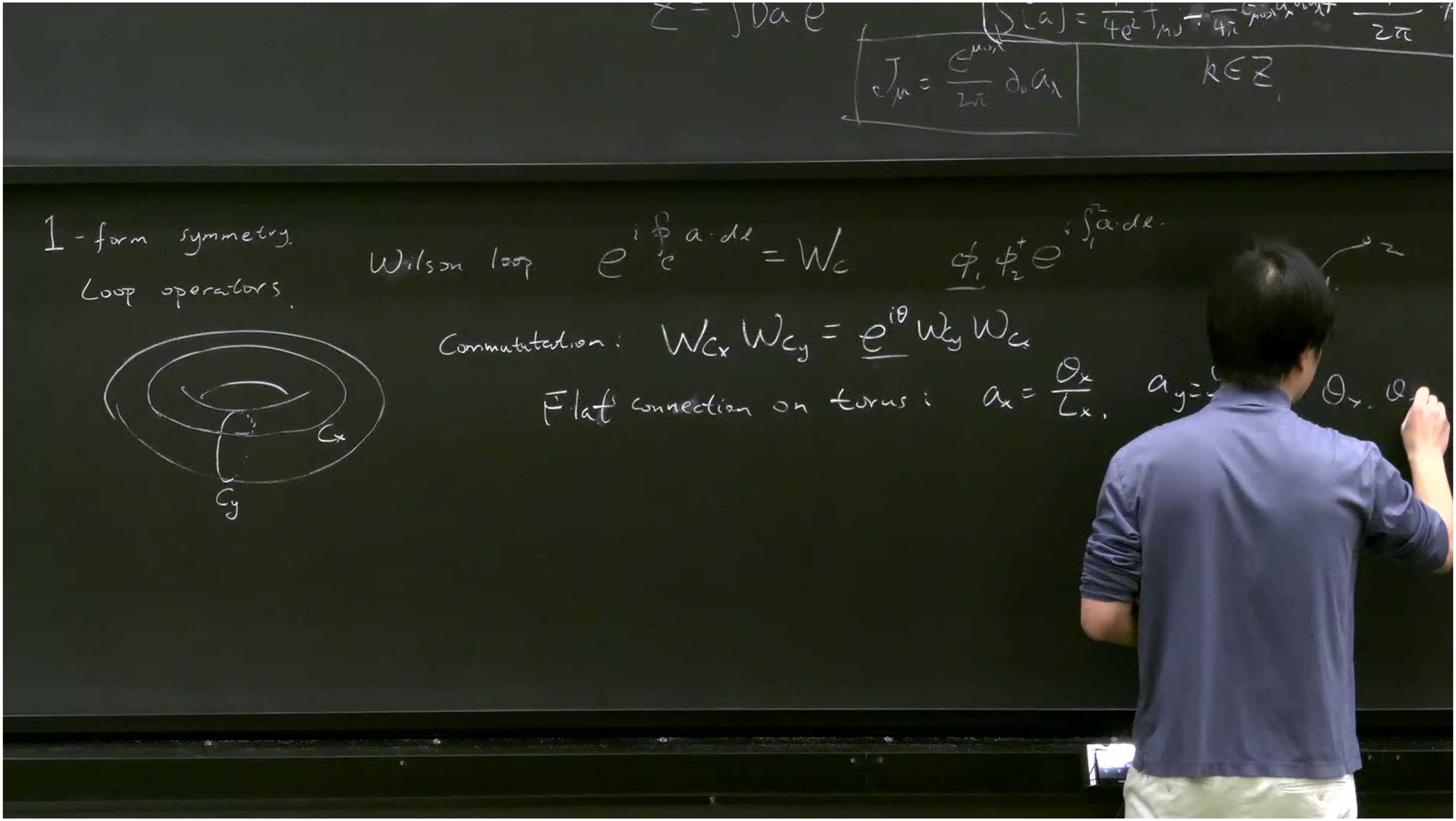


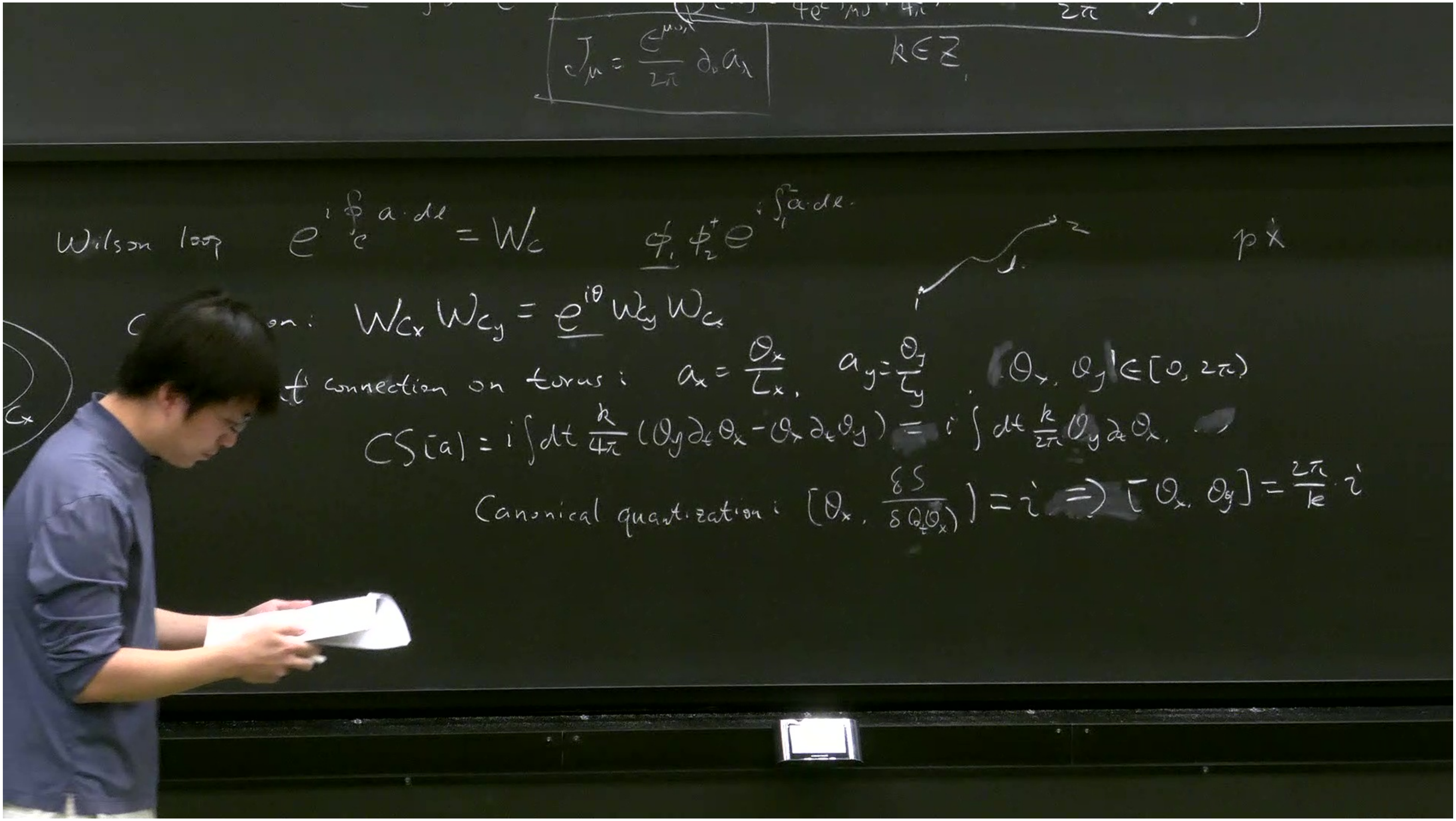
Braiding phase = AB phase = $q\cdot\Phi = \frac{2\pi}{k}$

Exchange phase = $\frac{\pi}{k}$, $k>1$: Neither boson nor fermion.

Missed two factors:

- 2 both charge see both flux
- AB phase $\int i a\cdot j$, $-\frac{i}{4\pi} a\,da \sim -\frac{i}{2} a\cdot j$ $\quad (1-\frac{1}{2}) = \frac{1}{2}$


$J_\mu = \frac{\epsilon^{\mu\nu\lambda}}{2\pi}\partial_\nu a_\lambda$
$k \in \mathbb{Z}$
1-form symmetry
Loop operators
Wilson loop $e^{i\oint_C a\cdot dl} = W_C$
$\phi_1 \phi_2^\dagger e^{i\int_1^2 a\cdot dl}$
C_x
C_y
Commutation: $W_{C_x} W_{C_y} = e^{i\theta} W_{C_y} W_{C_x}$
Flat connection on torus: $a_x = \frac{\theta_x}{L_x}$,



$J_\mu = \frac{\epsilon^{\mu\nu\lambda}}{2\pi}\partial_\nu a_\lambda$
$k \in \mathbb{Z}$
Wilson loop $e^{i\oint_c a\cdot dl} = W_c$
$\phi_1 \phi_2^\dagger e^{i\int_1^2 a\cdot dl}$
$p\ x$
on: $W_{C_x} W_{C_y} = e^{i\theta} W_{C_y} W_{C_x}$
t connection on torus: $a_x = \frac{\theta_x}{L_x}$, $a_y = \frac{\theta_y}{L_y}$, $(\theta_x, \theta_y) \in [0, 2\pi)$
$CS[a] = i\int dt \frac{k}{4\pi}(\theta_y \partial_t \theta_x - \theta_x \partial_t \theta_y) = i\int dt \frac{k}{2\pi}\theta_y \partial_t \theta_x$
Canonical quantization: $[\theta_x, \frac{\delta S}{\delta \partial_t \theta_x}] = i \Rightarrow [\theta_x, \theta_y] = \frac{2\pi}{k} i$

$$S[a] = \frac{1}{4e^2} f_{\mu\nu} \cdots - \frac{k}{4\pi} \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + \frac{\cdots}{2\pi} A_\mu \partial_\nu a_\lambda$$

$$J_\mu = \frac{\epsilon^{\mu\nu\lambda}}{2\pi} \partial_\nu a_\lambda \qquad k \in \mathbb{Z}$$

$$\phi_1 \phi_2^\dagger e^{i \int_1^2 a \cdot dl}$$

$p\dot{x}$

$$W_{C_{x,y}} = e^{i\theta_{x,y}}$$

$$\cdots e^{i\theta} W_{C_y} W_{C_x}$$

$$\boxed{W_{C_x} W_{C_y} = e^{i\frac{2\pi}{k}} W_{C_y} W_{C_x}}$$

$$\left(e^{i\theta_x} e^{i\theta_y} = e^{\frac{[\theta_x, \theta_y]}{2}} e^{i(\theta_x + \theta_y)} \right)$$

Open string braiding $e^{i\frac{2\pi}{k}}$

Exchange $e^{i\frac{\pi}{k}}$

...us: $a_x = \frac{\theta_x}{L_x}, \quad a_y = \frac{\theta_y}{L_y}, \quad \theta_x, \theta_y \in [0, 2\pi)$

$$\cdots \frac{k}{4\pi}(\theta_y \partial_t \theta_x - \theta_x \partial_t \theta_y) = i \int dt \frac{k}{2\pi} \theta_y \partial_t \theta_x$$

...cal quantization: $[\theta_x, \frac{\delta S}{\delta(\partial_t \theta_x)}] = i \Rightarrow [\theta_x, \theta_y] = \frac{2\pi}{k} i$

Braiding phase = AB phase = $q \cdot \Phi = \frac{2\pi}{k}$

Exchange phase = $\frac{\pi}{k}$, $k > 1$: Neither boson nor fermion.

$k=1$. "Anyon" has electric charge $Q=1$, statistics: $e^{i\pi} = -1 \simeq c^{\dagger}$

Nothing fractional $\Rightarrow$ IQHE.

$q=1.$

$\frac{2\pi}{k}$

1: Neither boson nor fermion.

...c charge $Q=1$, statistis: $e^{i\pi}=-1 \simeq c^{\dagger}$

QHE.

Consider ϕ^k. flux attached $= \frac{2\pi}{k}\cdot k = 2\pi$.

$\phi^k \mathcal{M}_{2\pi}$ is a <u>local</u> <u>operator</u>

monopole

$Q=1$

Exchange Statistics: $\pi k \begin{cases} \text{Fermion if } k \text{ odd} \\ \text{Boson if } k \text{ even.} \end{cases}$

Electron system: c, $ccc^{\dagger}$

$\Rightarrow k$ odd

$k=1$

Guessing a wavefunction:

Superfluid: $\frac{1}{4e^2} f_{\mu\nu}^2$.   local particle $\sim \mathcal{M}_{2\pi}$, ϕ . vortex.

Wavefunction (1st quantized, real space)

$$\Psi(\{x_i, y_i\}) = 1$$

$z_i = x_i + iy_i$

QHE: $\frac{1}{4e^2} f_{\mu\nu}^2 + \frac{i}{4\pi} k\, a\, da$ local operators $\sim \mathcal{M}_{2\pi}\phi^k \Rightarrow$ Every particle is attache

$$\Psi(\{z_i\}) = \prod_{i\neq j} (z_i - z_j)^k e^{-\sum |z_i|^2 B/4}$$

Laughlin wavefunction.

Lowest Land level, $\psi = f(\{z_i\})\, e^{-\sum |z_i|^2 B/4}$

$f \downarrow$ holomorphic