Title: De Sitter Horizon Edge Partition Functions

Speakers: Albert Law

Collection/Series: Quantum Gravity

Subject: Quantum Gravity

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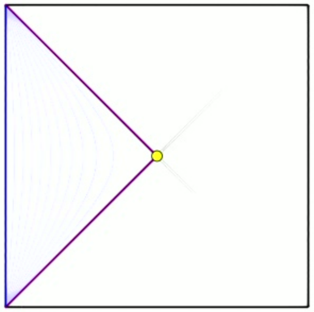
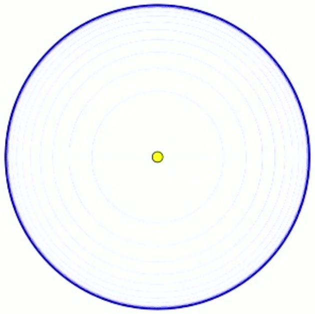
Abstract:

Discrepancies between Lorentzian and Euclidean calculations of quantum corrections to de Sitter horizon thermodynamics revealed the existence of ‘edge’ degrees of freedom on the horizon. In this talk, I will review these calculations and present updates on understanding the physics of such edge modes in both gauge theories and (higher-spin) gravity. For Einstein gravity, I will discuss a plausible interpretation involving an embedded spherical brane. Finally, I will comment on how these ideas extend to more general black hole spacetimes.

De Sitter horizon edge partition functions

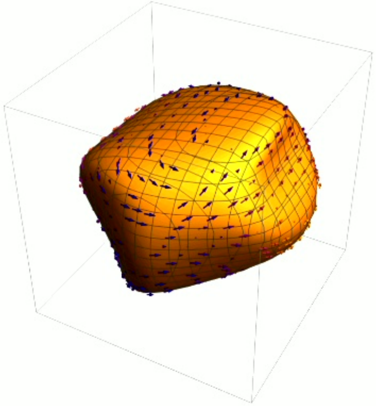




Y. T. Albert Law

Based on 2501.17912

+ work with D. Anninos, F. Denef, Z. Sun, K. Parmentier, M. Grewal, A. Ball, G. Wong



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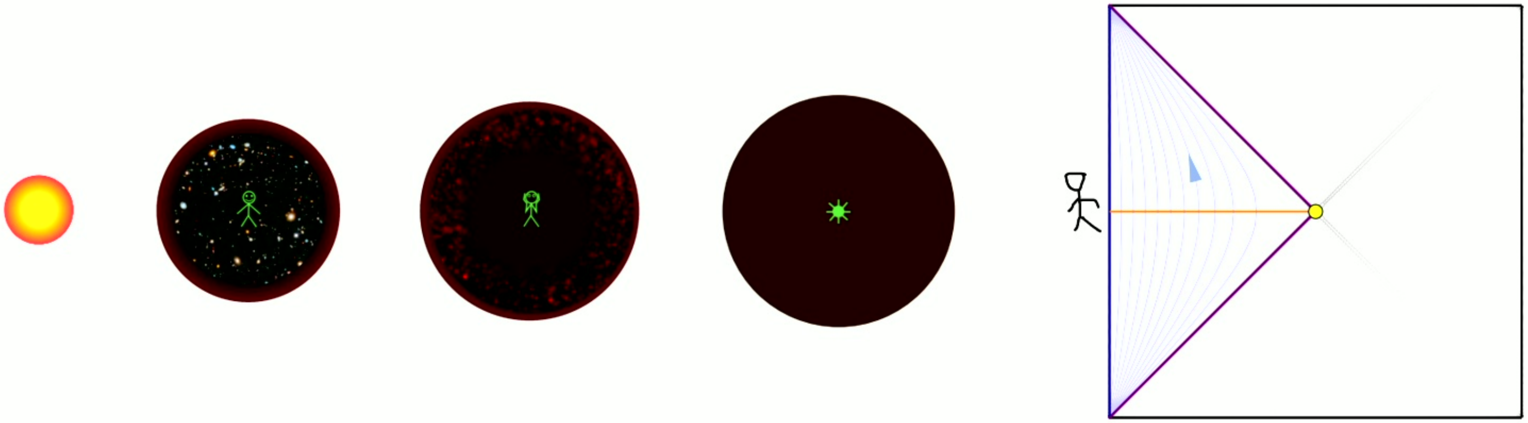
Extrapolating the current ΛCDM model, our observable universe will become indistinguishable from a static patch of dS_4



[Loeb 01, Krauss, Scherrer 07]

Analogous to black holes, a dS horizon is hypothesized to have an entropy

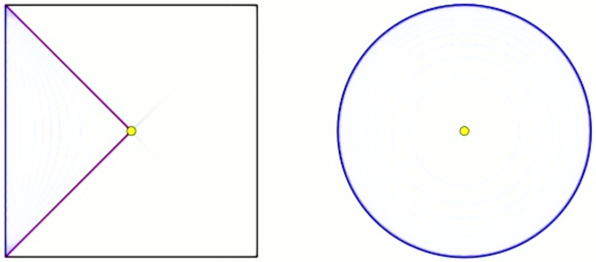
$$S = \underbrace{\frac{A}{4G_N}}_{\text{Gibbons-Hawking}} + \underbrace{\# \log \frac{A}{4G_N} + O\left(G_N^0\right)}_{\text{1-loop}} + \underbrace{\cdots}_{\text{Higher-loops + non-pert.}}$$

Quantum corrections provide precision tests for candidate microscopic models

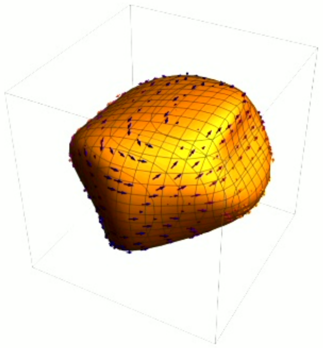
[Anninos, Denef, AL, Sun 20] [AL 20] [David, Mukherjee 21] [Anninos, Harris 21] [Anninos, Bautista, Mühlmann 21] [Anninos, Mühlmann 21] [Grewal, Parmentier 21] [Mühlmann 22] [Bobev, Hertog, Hong, Karlsson, Reys 22] [Castro, Coman, Fliss, Zukowski 23] [Bourne, Castro, Fliss 24] [Bandaru 24] [Shyam 21] [Coleman, Mazenc, Shyam, Silverstein, Soni, Torroba, Yang 21] [Collier, Eberhardt, Mühlmann 25]

I’ll review these calculations and report some progress in understanding these “edge” degrees of freedom

Plan of the talk



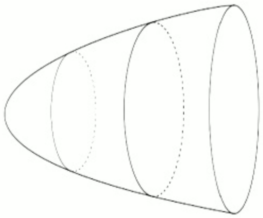
1-loop dS: Lorentzian vs Euclidean



Case study: linearized gravity

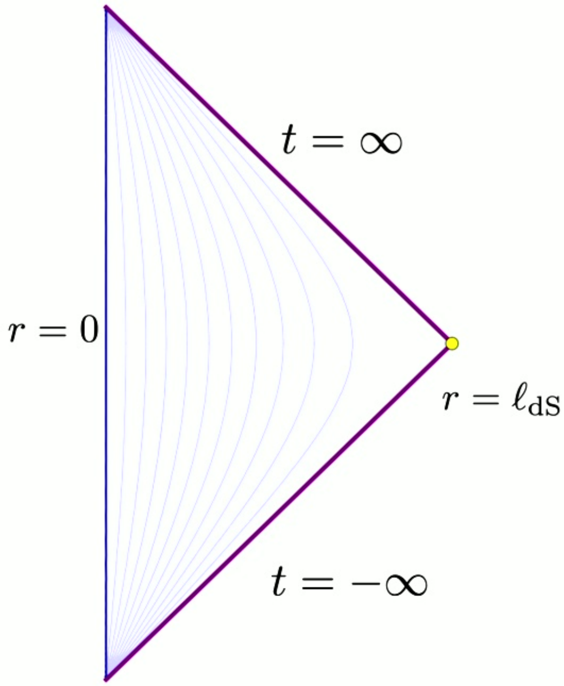
$$\phi_{\mu_1 \cdots \mu_s}$$

Remark 1: Higher-spin gauge fields



Remark 2: Black holes

We first consider a free field on a dS_{d+1} static patch

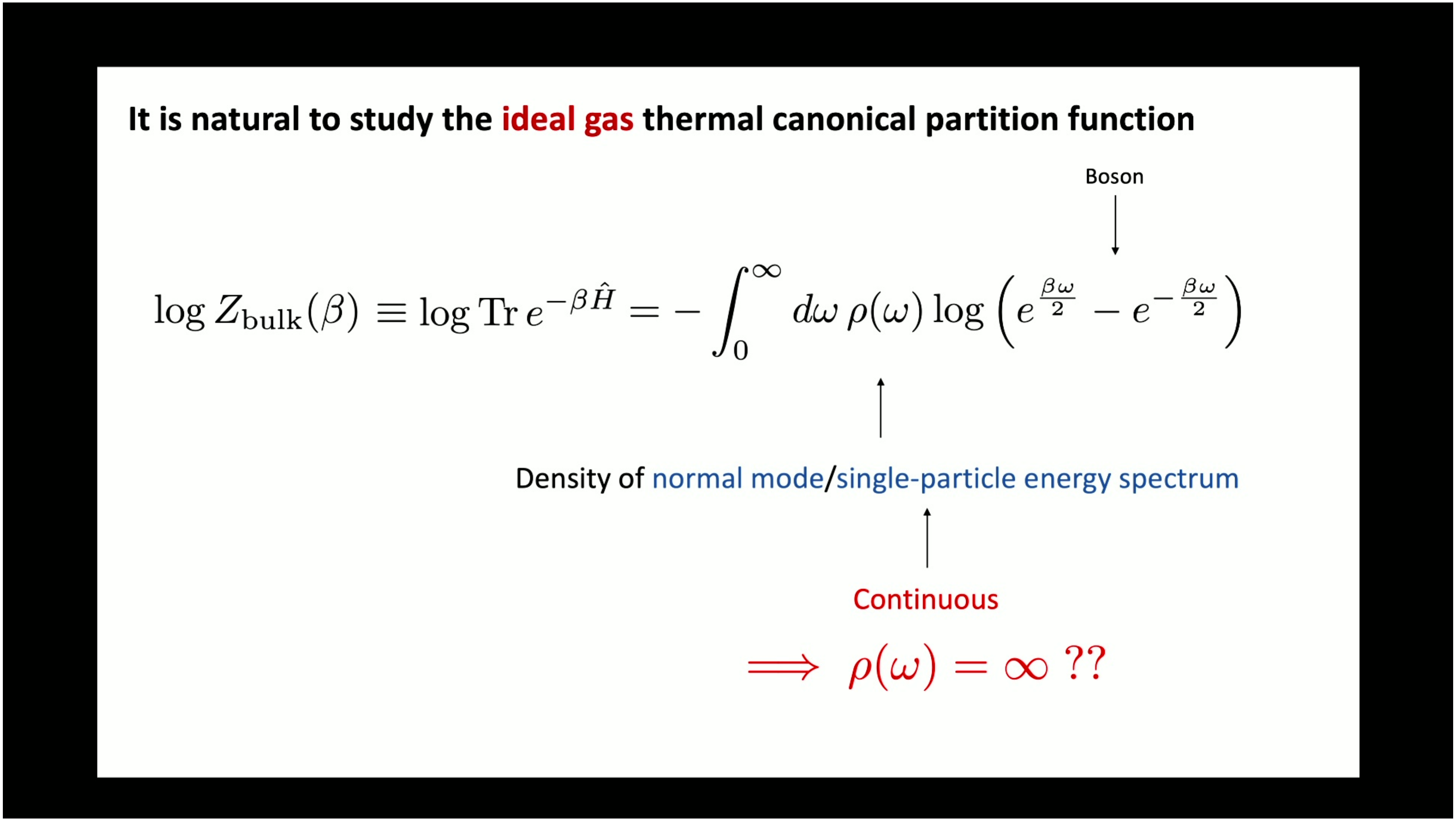


round S^{d-1}

$$ds^2 = -\left(\ell_{\rm dS}^2 - r^2\right) dt^2 + \frac{dr^2}{1 - \frac{r^2}{\ell_{\rm dS}^2}} + r^2 d\Omega^2$$

$$0 \leq r < \ell_{\rm dS} \equiv \sqrt{\frac{d(d-1)}{2\Lambda}}$$

We will set $\ell_{\rm dS} = 1$



It is natural to study the ideal gas thermal canonical partition function

Boson

$$\log Z_{\mathrm{bulk}}(\beta) \equiv \log \mathrm{Tr}\, e^{-\beta \hat{H}} = -\int_0^\infty d\omega\, \rho(\omega) \log \left(e^{\frac{\beta\omega}{2}} - e^{-\frac{\beta\omega}{2}} \right)$$

Density of normal mode/single-particle energy spectrum

Continuous

$$\Longrightarrow \quad \rho(\omega) = \infty\ ??$$

We make sense of the single-particle density of states using the **Harish-Chandra character** of the dS group $SO(1,d+1)$ [Anninos, Denef, AL, Sun 20]

$$\chi_{[\Delta,\mathbf{s}]}(t) = \mathrm{tr}_{[\Delta,\mathbf{s}]}\, e^{-iHt}$$

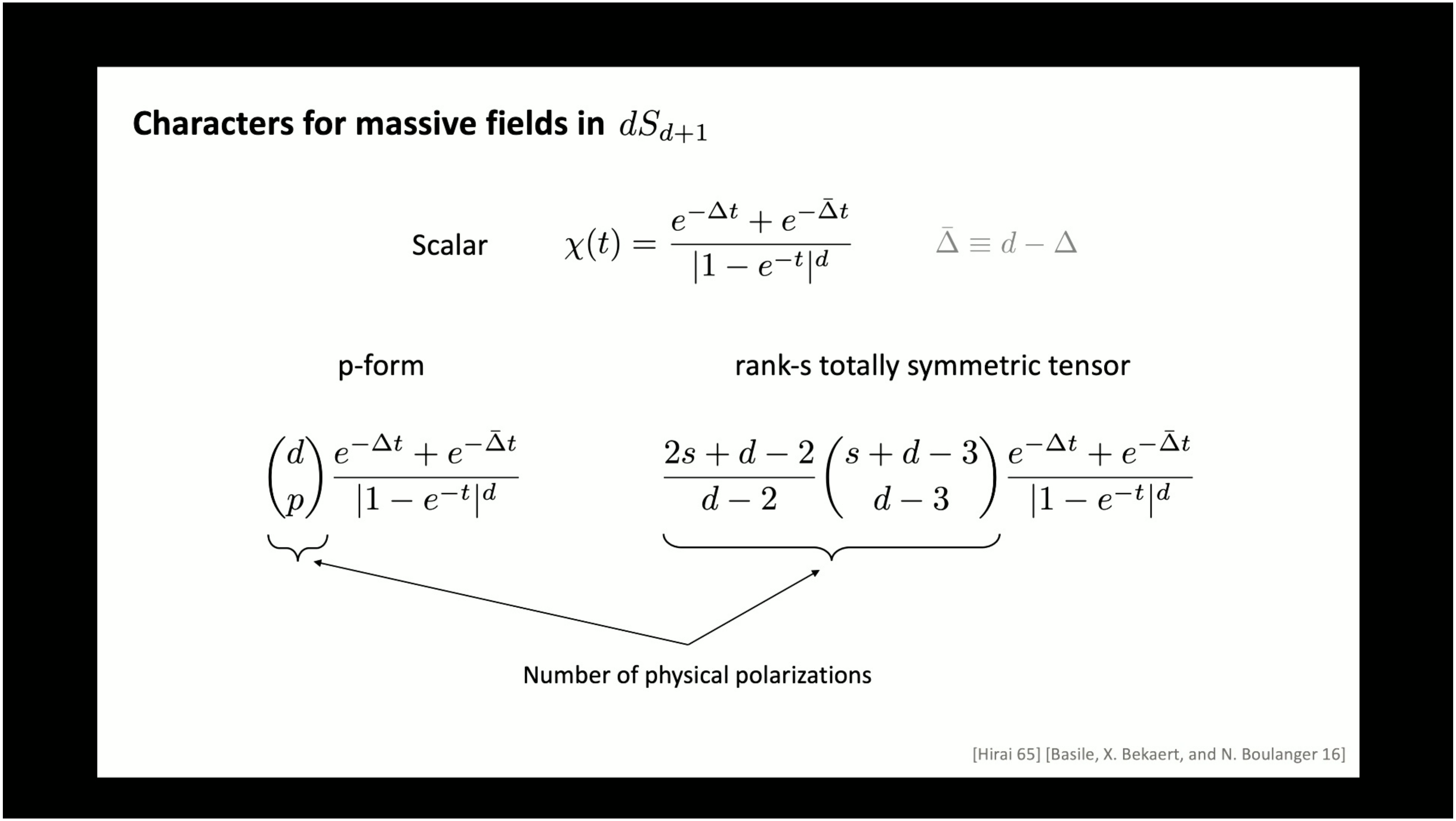
$\mathfrak{so}(1,1)$-weight
(~mass)

spin

dS boost

Rigorously defined as a distribution

See [Sun 21] for a review on the representation theory of $SO(1,d+1)$



Characters for massive fields in dS_{d+1}

Scalar $$\chi(t) = \frac{e^{-\Delta t} + e^{-\bar{\Delta} t}}{|1-e^{-t}|^d} \qquad \bar{\Delta} \equiv d - \Delta$$

p-form

$$\underbrace{\binom{d}{p}}_{} \frac{e^{-\Delta t} + e^{-\bar{\Delta} t}}{|1-e^{-t}|^d}$$

rank-s totally symmetric tensor

$$\underbrace{\frac{2s+d-2}{d-2}\binom{s+d-3}{d-3}}_{} \frac{e^{-\Delta t} + e^{-\bar{\Delta} t}}{|1-e^{-t}|^d}$$

Number of physical polarizations

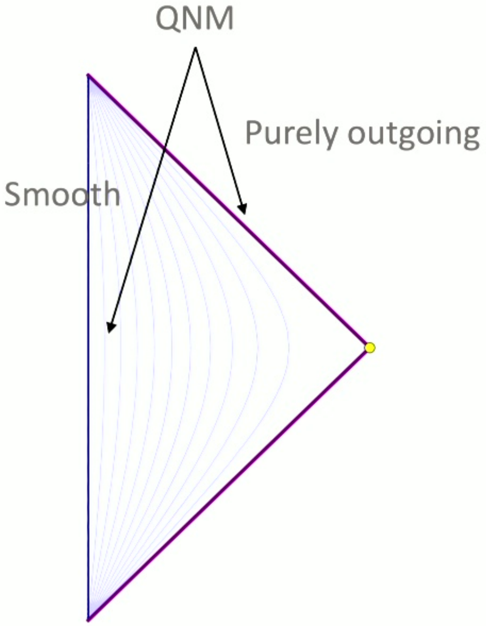
[Hirai 65] [Basile, X. Bekaert, and N. Boulanger 16]

Quasinormal mode (QNM) expansion

E.g. scalar in dS_4 with $m^2 = \Delta\bar{\Delta} = \Delta(3-\Delta)$

$$\chi(t) = \frac{e^{-\Delta t} + e^{-\bar{\Delta} t}}{|1-e^{-t}|^3}$$

$$= \sum_{n=0}^{\infty}\sum_{l=0}^{\infty} \underset{\substack{\uparrow \\ \text{degeneracy}}}{(2l+1)}\, e^{-i\overset{\text{QNM frequency}}{\underset{\uparrow}{(-i(\Delta+2n+l))}}|t|} + (\Delta \to \bar{\Delta})$$



- **Algebraic** construction of QNMs
  [Ng, Strominger 12] [Jafferis, Lupsasca, Lysov, Ng, Strominger 13] [Tanhayi 14] [Sun 20]
- Can be understood as an integrated Green function [Grewal, AL 24]

Key observation: The Fourier transform

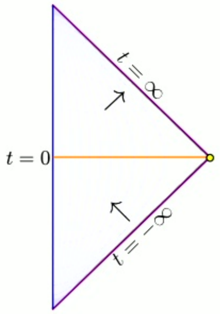
$$\tilde{\rho}(\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{-i\omega t} \chi(t)$$

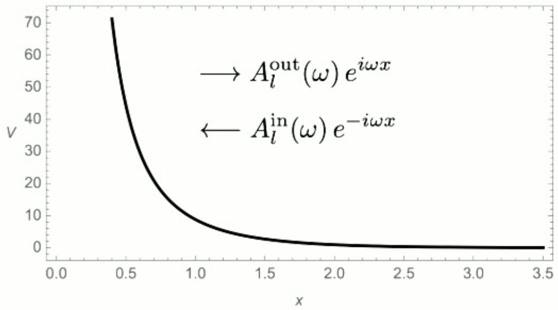
can be interpreted as a single-particle density of states

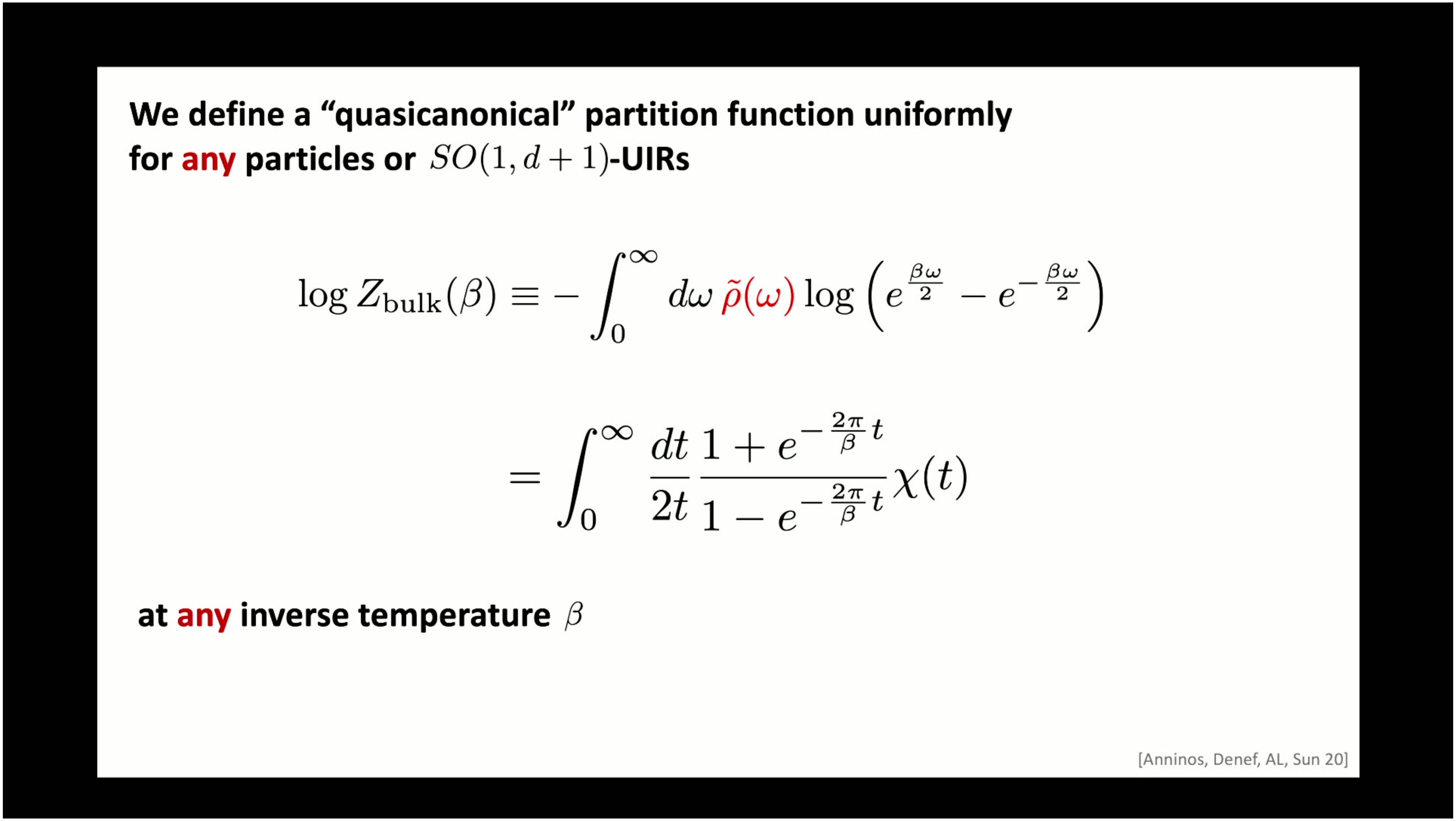
Can be understood in terms of scattering phases as a relative/renormalized DOS [AL, Parmentier 22]

$$S(\omega) \equiv \frac{A^{\text{out}}(\omega)}{A^{\text{in}}(\omega)}$$

QNMs = scattering poles







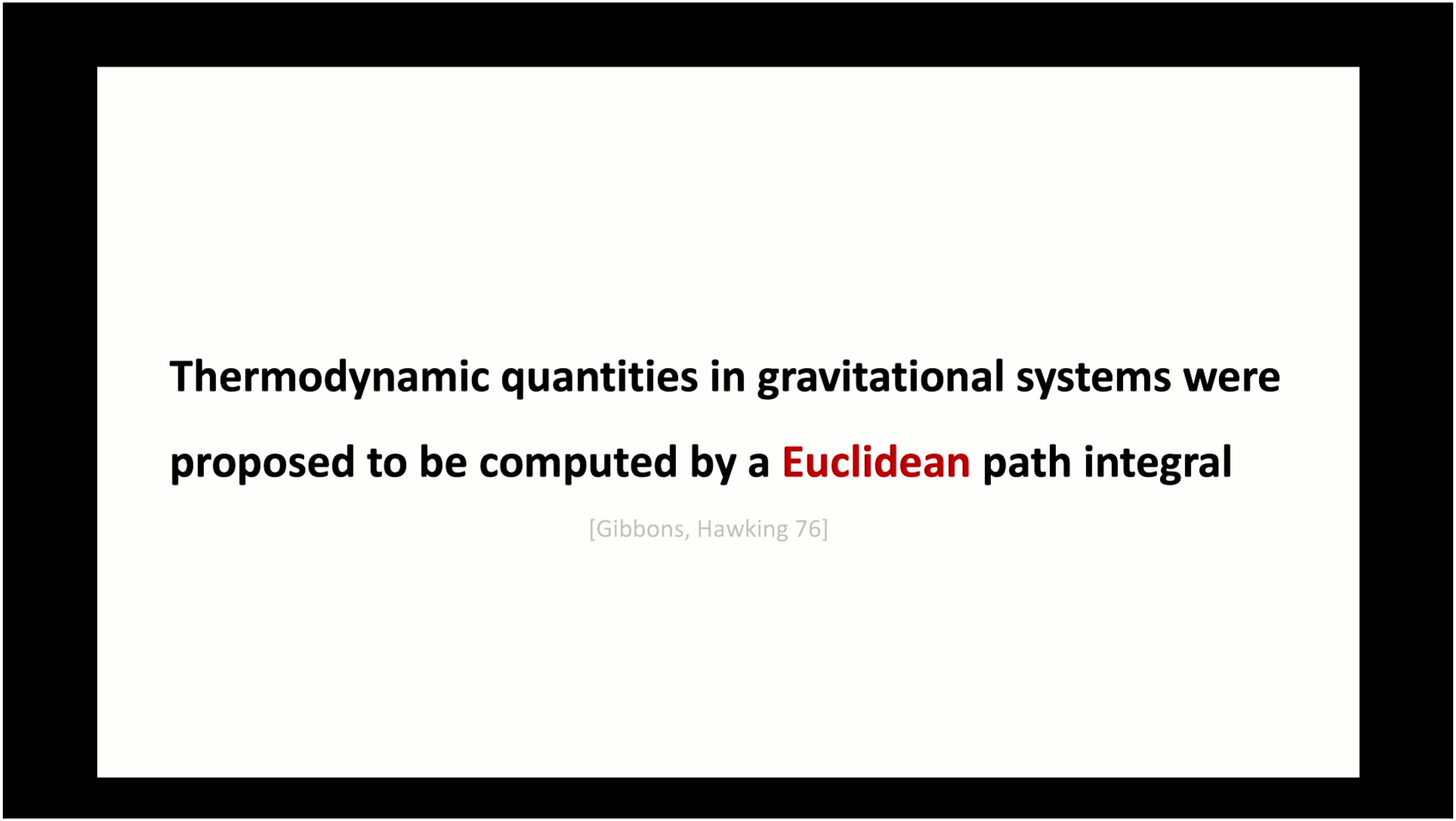
We define a “quasicanonical” partition function uniformly for **any** particles or $SO(1,d+1)$-UIRs

$$\log Z_{\text{bulk}}(\beta) \equiv -\int_0^\infty d\omega\, \tilde{\rho}(\omega) \log\left(e^{\frac{\beta\omega}{2}} - e^{-\frac{\beta\omega}{2}}\right)$$

$$= \int_0^\infty \frac{dt}{2t} \frac{1+e^{-\frac{2\pi}{\beta}t}}{1-e^{-\frac{2\pi}{\beta}t}} \chi(t)$$

at any inverse temperature β

[Anninos, Denef, AL, Sun 20]



Thermodynamic quantities in gravitational systems were proposed to be computed by a Euclidean path integral

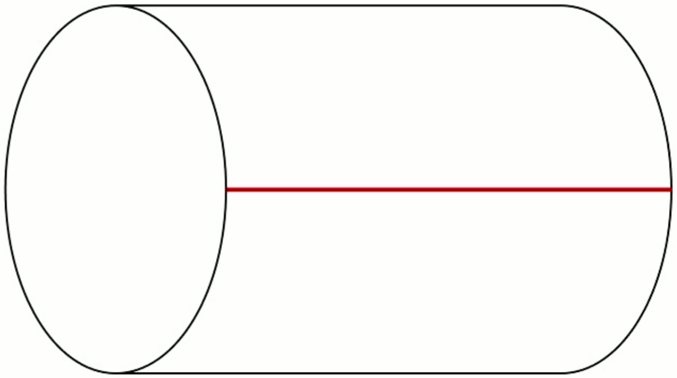
[Gibbons, Hawking 76]

Thermal QFT partition function = path integral periodic in Euclidean time

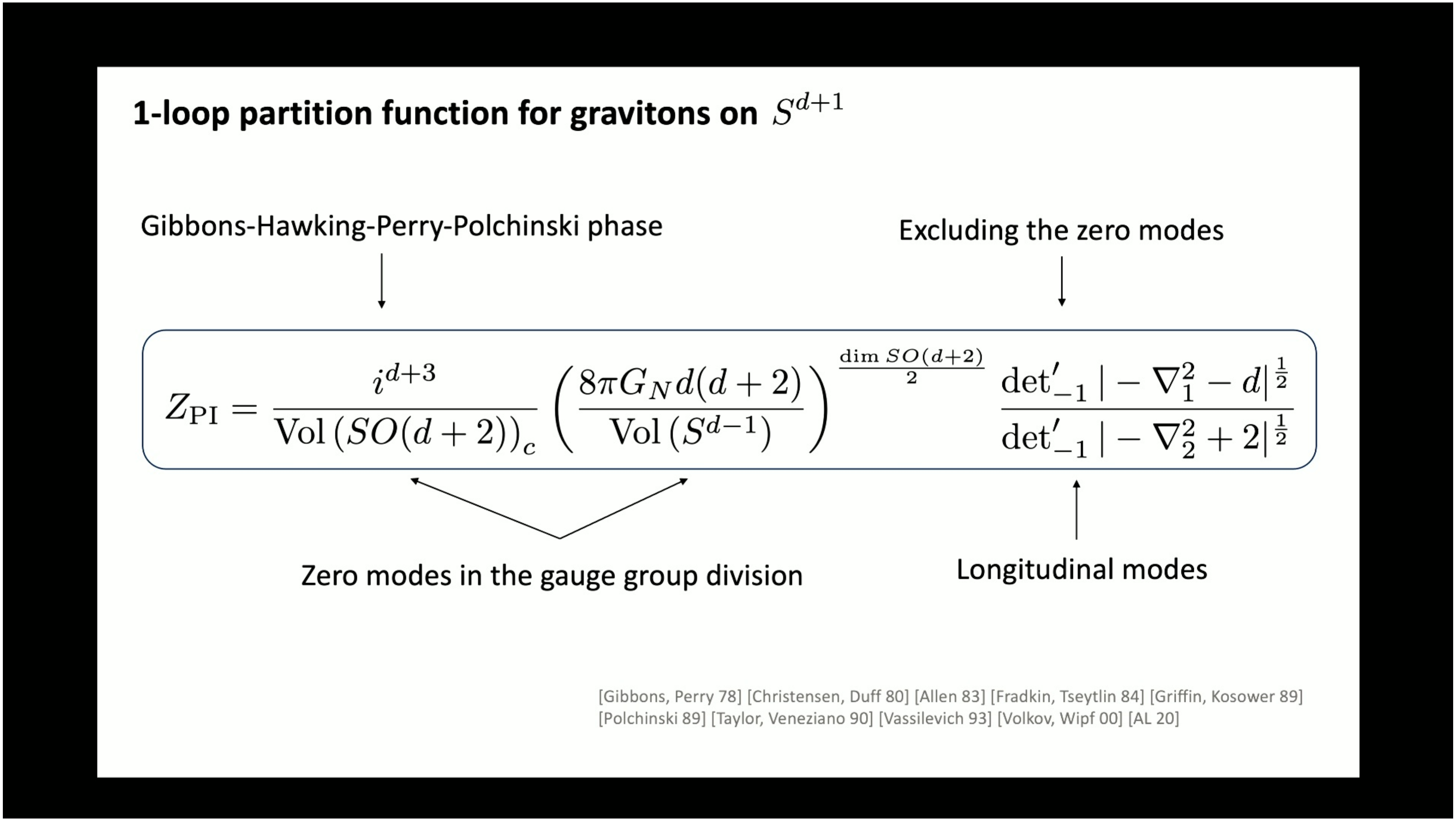
Consider a QFT at inverse temperature β on $\mathbb{R} \times \Sigma$

$\phi(\mathbf{x}) =$ field on Σ

$$\mathrm{Tr}\, e^{-\beta \hat{H}} =$$



$$\tau \simeq \tau + \beta$$



1-loop partition function for gravitons on S^{d+1}

Gibbons-Hawking-Perry-Polchinski phase

Excluding the zero modes

$$\boxed{Z_{\mathrm{PI}} = \frac{i^{d+3}}{\mathrm{Vol}\,(SO(d+2))_c} \left(\frac{8\pi G_N d(d+2)}{\mathrm{Vol}\,(S^{d-1})} \right)^{\frac{\dim SO(d+2)}{2}} \frac{\det'_{-1} |-\nabla_1^2 - d|^{\frac{1}{2}}}{\det'_{-1} |-\nabla_2^2 + 2|^{\frac{1}{2}}}}$$

Zero modes in the gauge group division

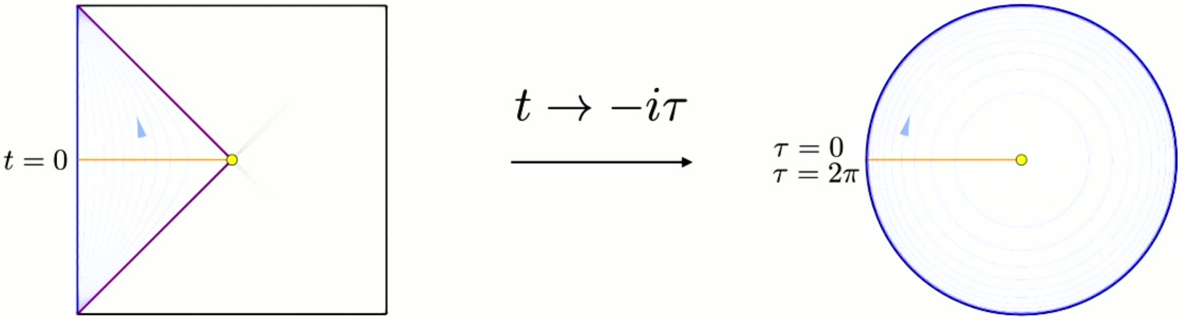
Longitudinal modes

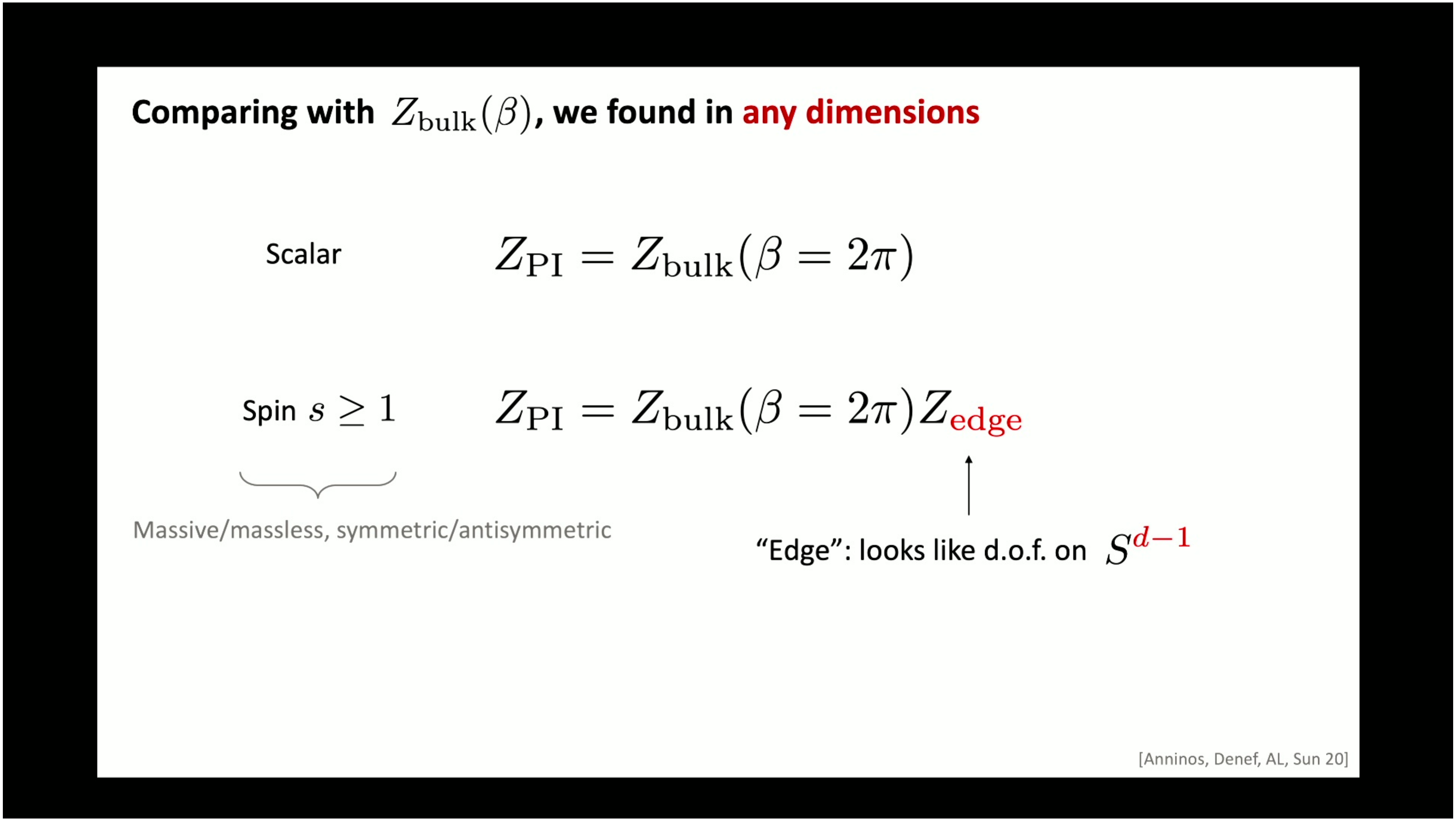
[Gibbons, Perry 78] [Christensen, Duff 80] [Allen 83] [Fradkin, Tseytlin 84] [Griffin, Kosower 89] [Polchinski 89] [Taylor, Veneziano 90] [Vassilevich 93] [Volkov, Wipf 00] [AL 20]

Euclidean gravitational path integral with $\Lambda > 0$

$$\mathcal{Z} = \int \mathcal{D}g\mathcal{D}\Phi\, e^{-S[g,\Phi]} = e^{\frac{A}{4G}}\left(\underbrace{\det(L)}_{\text{e.g. scalar: } \det\left(-\nabla_0^2 + m^2\right)^{-\frac{1}{2}}} + \cdots\right) + \cdots$$

The leading saddle is a round sphere, which is the Euclideanized static patch





Comparing with $Z_{\text{bulk}}(\beta)$, we found in any dimensions

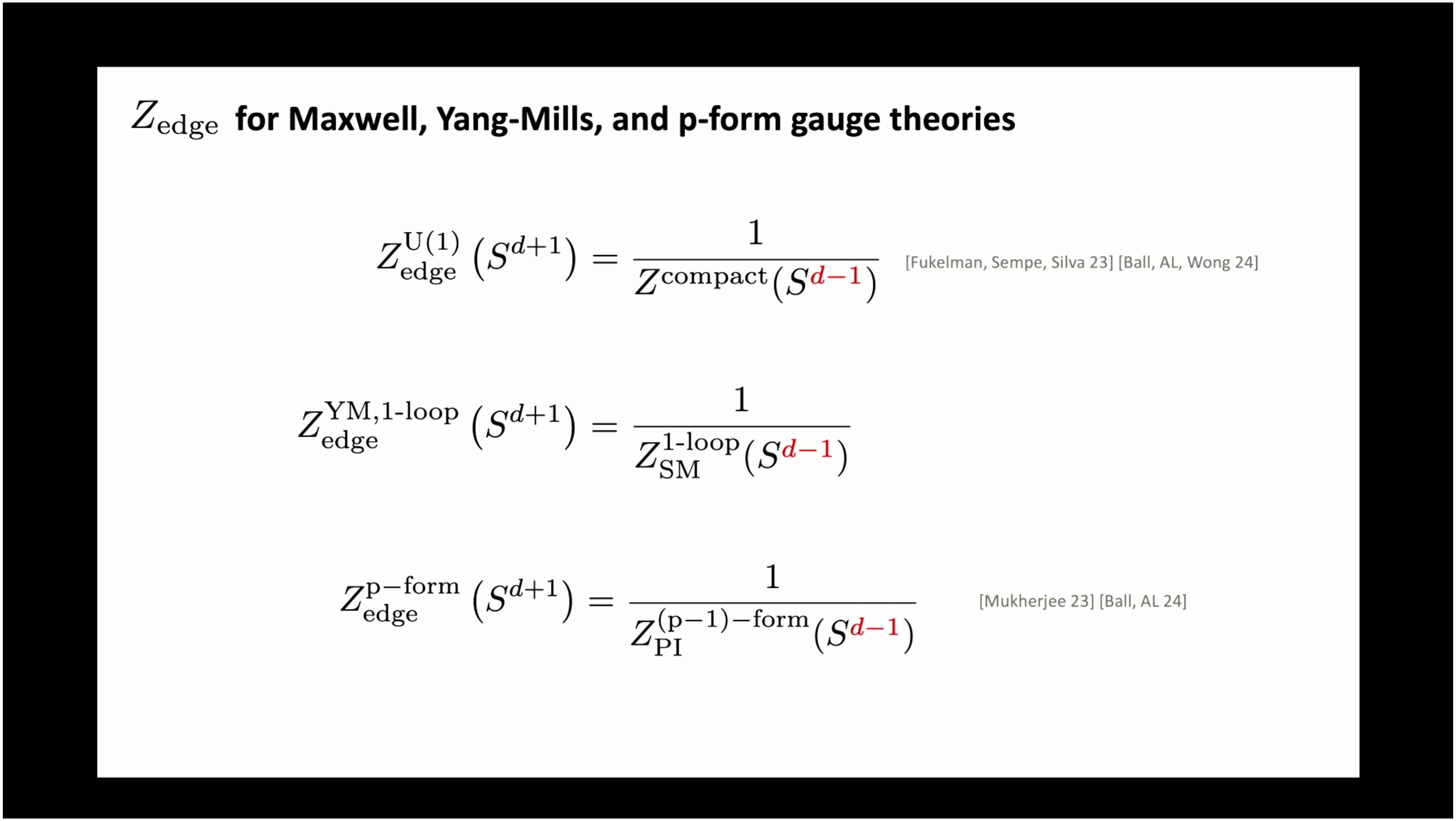
Scalar $$Z_{\text{PI}} = Z_{\text{bulk}}(\beta = 2\pi)$$

$\underbrace{\text{Spin } s \geq 1}_{}$ $$Z_{\text{PI}} = Z_{\text{bulk}}(\beta = 2\pi) Z_{\text{edge}}$$

Massive/massless, symmetric/antisymmetric

“Edge”: looks like d.o.f. on S^{d-1}

[Anninos, Denef, AL, Sun 20]



Z_{edge} for Maxwell, Yang-Mills, and p-form gauge theories

$$Z^{\mathrm{U}(1)}_{\text{edge}}\left(S^{d+1}\right) = \frac{1}{Z^{\text{compact}}\left(S^{d-1}\right)}$$

[Fukelman, Sempe, Silva 23] [Ball, AL, Wong 24]

$$Z^{\text{YM,1-loop}}_{\text{edge}}\left(S^{d+1}\right) = \frac{1}{Z^{\text{1-loop}}_{\text{SM}}\left(S^{d-1}\right)}$$

$$Z^{\mathrm{p-form}}_{\text{edge}}\left(S^{d+1}\right) = \frac{1}{Z^{(\mathrm{p}-1)-\mathrm{form}}_{\mathrm{PI}}\left(S^{d-1}\right)}$$

[Mukherjee 23] [Ball, AL 24]

This generalizes the Lorentzian vs Euclidean mismatch for 4D Maxwell to any theories in any dimensions

Expected answer from conformal anomaly, given by $\log Z_{\text{PI}}^{\text{Max}}\left(S^4\right)$

[Solodukhin 08] [Casini, Huerta, Myers 11]

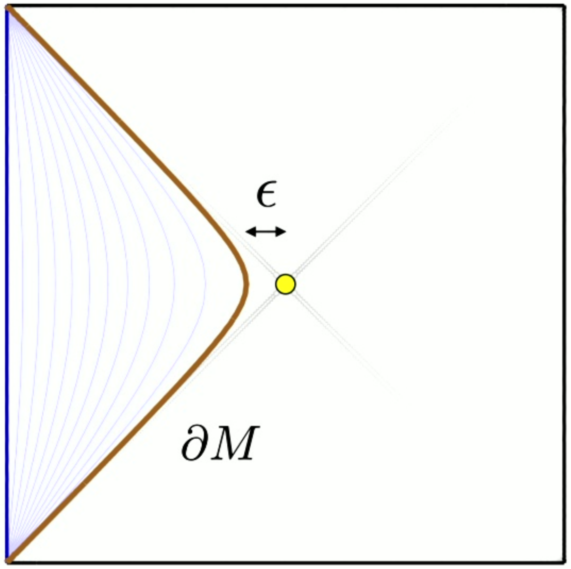
$$\overbrace{S^{\text{anom}} = -\frac{31}{45}\log\mu\ell_{\text{dS}}} = \underbrace{-\frac{16}{45}\log\mu\ell_{\text{dS}}} - \frac{1}{3}\log\mu\ell_{\text{dS}}$$

Thermal entropy in dS_4 static patch [Dowker 10] [Eling, Oz, Theison 13]

Edge modes resolve the discrepancy

[Donnelly 11, 14] [Donnelly, Wall 12, 14, 15] [Radicevic 14, 15] [Huang 14] [Ghosh, Soni, Trivedi 15] [Hung, Wan 15] [Iritani, Nozaki, Numasawa, Shiba, Tasaki 15] [Pretko, Senthil 15] [Soni, Trivedi 15, 16] [Zuo 16] [Delcamp, Dittrich, Riello 16] [Agarwal, Karabali, Nair 17] [Blommaert, Mertens, Verschelde, Zakharov 18] [Blommaert, Mertens, Verschelde 18] [Freidel, Pranzetti 18]

Dynamical Edge modes in Maxwell [Ball, AL, Wong 24]



Dynamical edge mode boundary condition

$$\boxed{A_t|_{\partial M} = 0 = n^\mu F_{\mu i}|_{\partial M}}$$

$$\downarrow \qquad\qquad \searrow$$

$$A_i = \tilde{A}_i + \partial_i \alpha \qquad E_i = \tilde{E}_i + \partial_i \beta$$

Brief recap

- We made sense of ideal gas thermal partition functions in Lorentzian dS static patch

$$\log Z_{\mathrm{bulk}}(\beta) \equiv \int_0^\infty \frac{dt}{2t} \frac{1+e^{-\frac{2\pi}{\beta}t}}{1-e^{-\frac{2\pi}{\beta}t}} \chi(t)$$

- Comparing with 1-loop Euclidean sphere partition functions, we found

$$Z_{\mathrm{PI}} = \begin{cases} Z_{\mathrm{bulk}}(\beta = 2\pi) & , \quad s = 0 \\ Z_{\mathrm{bulk}}(\beta = 2\pi) Z_{\mathrm{edge}} & , \quad s \geq 1 \end{cases}$$

- For Maxwell and p-form gauge theories, Z_{edge} have been shown to be captured by "edge modes" residing on the stretched horizon

Z_{edge} **for linearized gravity** [Anninos, Denef, AL, Sun 20]

$$\log Z_{\text{edge}} = \log \frac{i^{d+3}}{\text{Vol}(SO(d+2))_c} \left(\frac{32\pi^3 G_N}{\text{Vol}(S^{d-1})\ell_{\text{dS}}^{d-1}} \right)^{\frac{\dim SO(d+2)}{2}}$$

$$- \int_0^\infty \frac{dt}{2t} \frac{1+e^{-t}}{1-e^{-t}} \left[(d+2)\frac{e^{-(d-1)t}+e^{t}}{(1-e^{-t})^{d-2}} - \frac{e^{-dt}+e^{2t}}{(1-e^{-t})^{d-2}} \right]_+$$

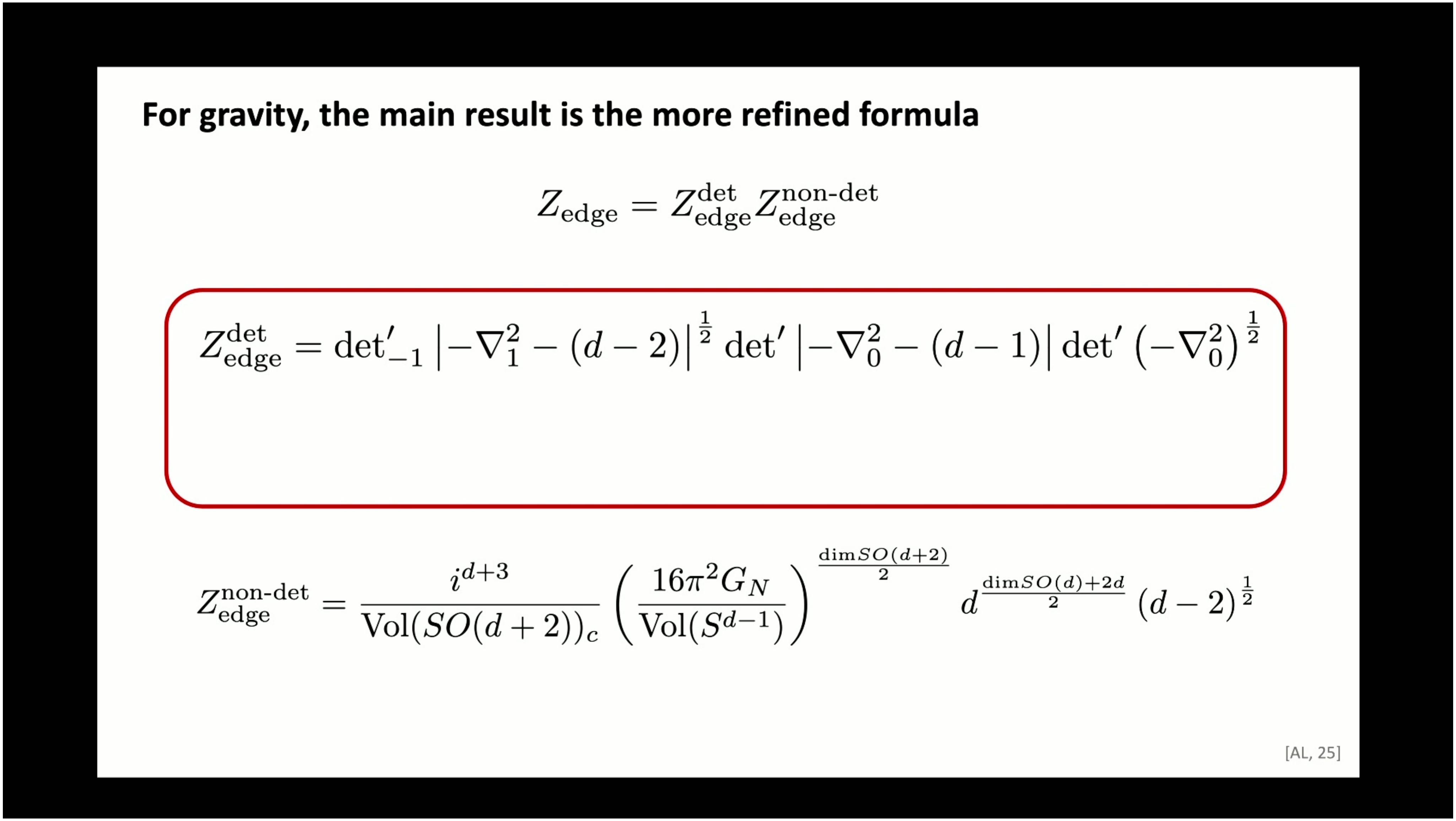
This may have an edge mode interpretation, but it is not clear at this point.

Q: what is the $SO(d)$ or S^{d-1} field content of Z_{edge}?

Notation: $\left[\sum_k c_k q^k\right]_+ \equiv \sum_{k<0} (-c_k)\, q^{-k} + \sum_{k>0} c_k q^k = \sum_k c_k q^k - c_0 - \sum_{k<0} c_k \left(q^k + q^{-k}\right)$

To analyze the $SO(d)$ or S^{d-1} field content, we unpack the sphere by applying the $SO(d+2) \to U(1) \times SO(d)$ branching rule

$$\begin{tikzcd}
\text{Spherical harmonics on } S^{d+1} \arrow[d] & \\
Z_{\mathrm{PI}} \sim \prod \left(SO(d+2)\text{-irreps}\right) \arrow[r] \arrow[d, "\text{[AL, 25]}"] & Z^{\mathrm{old}}_{\mathrm{bulk}} Z^{\mathrm{old}}_{\mathrm{edge}} \\
\prod \left(U(1) \times SO(d)\text{-irreps}\right) \arrow[r] & Z^{\mathrm{refined}}_{\mathrm{bulk}} Z^{\mathrm{refined}}_{\mathrm{edge}} \\
 & \text{Read off the } SO(d) \text{ or } S^{d-1} \text{ field content} \arrow[u]
\end{tikzcd}$$



For gravity, the main result is the more refined formula

$$Z_{\text{edge}} = Z_{\text{edge}}^{\text{det}} Z_{\text{edge}}^{\text{non-det}}$$

$$\boxed{Z_{\text{edge}}^{\text{det}} = \text{det}'_{-1} \left|-\nabla_1^2 - (d-2)\right|^{\frac{1}{2}} \text{det}' \left|-\nabla_0^2 - (d-1)\right| \text{det}' \left(-\nabla_0^2\right)^{\frac{1}{2}}}$$

$$Z_{\text{edge}}^{\text{non-det}} = \frac{i^{d+3}}{\text{Vol}(SO(d+2))_c} \left(\frac{16\pi^2 G_N}{\text{Vol}(S^{d-1})}\right)^{\frac{\dim SO(d+2)}{2}} d^{\frac{\dim SO(d)+2d}{2}} (d-2)^{\frac{1}{2}}$$

[AL, 25]

Z_{edge} for linearized gravity [Anninos, Denef, AL, Sun 20]

$$\log Z_{\text{edge}} = \log \frac{i^{d+3}}{\text{Vol}(SO(d+2))_c}\left(\frac{32\pi^3 G_N}{\text{Vol}(S^{d-1})\ell_{\text{dS}}^{d-1}}\right)^{\frac{\dim SO(d+2)}{2}}$$

$$-\int_0^\infty \frac{dt}{2t}\,\frac{1+e^{-t}}{1-e^{-t}}\left[(d+2)\frac{e^{-(d-1)t}+e^{t}}{(1-e^{-t})^{d-2}} - \frac{e^{-dt}+e^{2t}}{(1-e^{-t})^{d-2}}\right]_+$$

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Key observation: The Fourier transform

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can be interpreted as a single-particle density of states

Can be understood in terms of scattering phases as a relative/renormalized DOS [AL, Parmentier 22]

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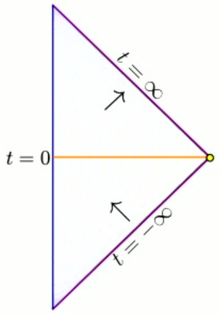
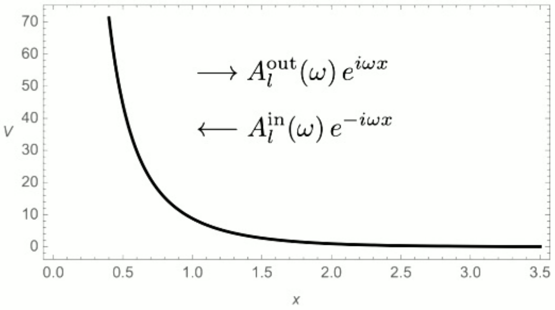
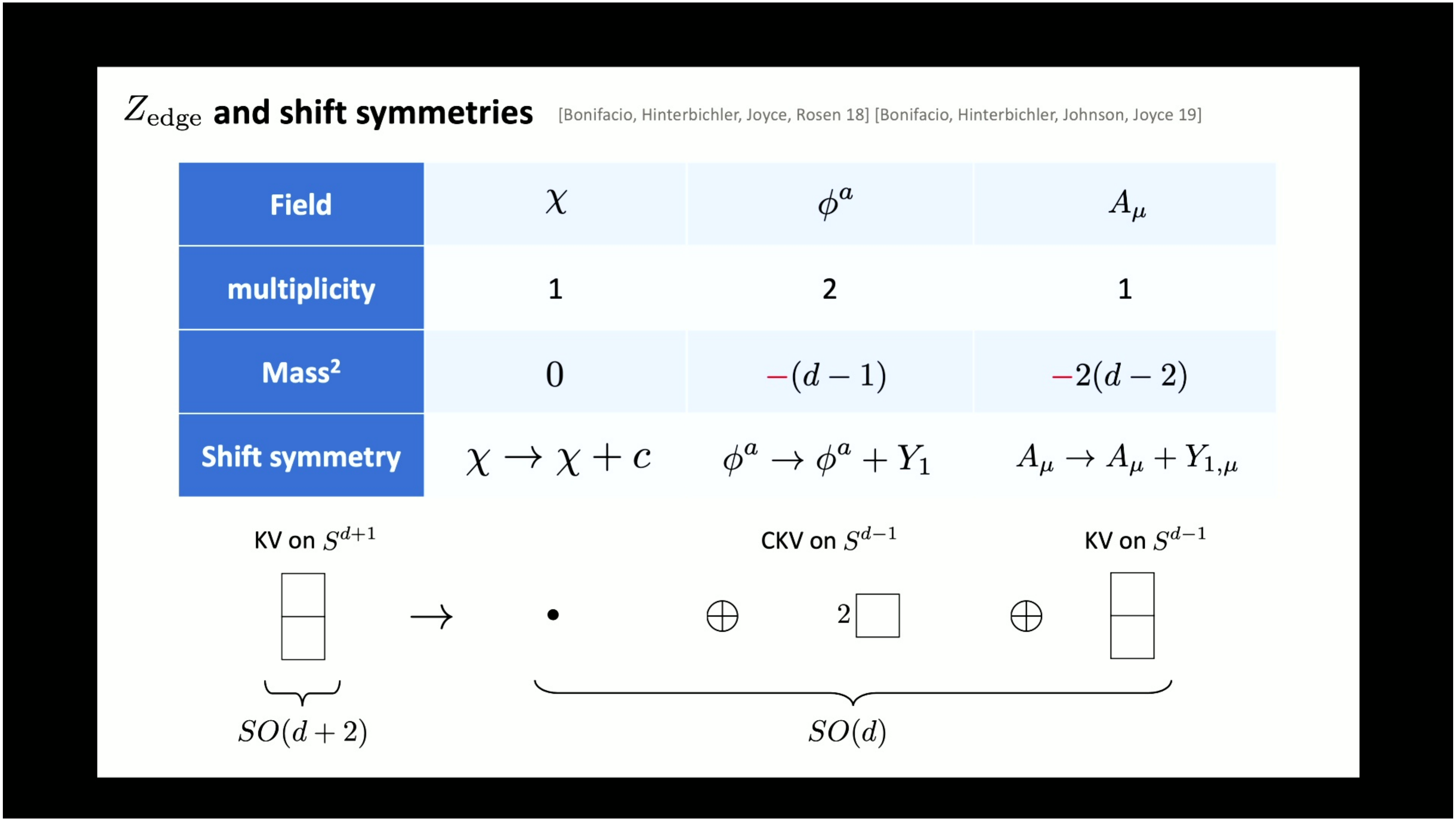






Z_{edge} and shift symmetries

[Bonifacio, Hinterbichler, Joyce, Rosen 18] [Bonifacio, Hinterbichler, Johnson, Joyce 19]

Field	χ	ϕ^a	A_μ
multiplicity	1	2	1
Mass^2	0	$-(d-1)$	$-2(d-2)$
Shift symmetry	$\chi \to \chi + c$	$\phi^a \to \phi^a + Y_1$	$A_\mu \to A_\mu + Y_{1,\mu}$

KV on S^{d+1}

CKV on S^{d-1}

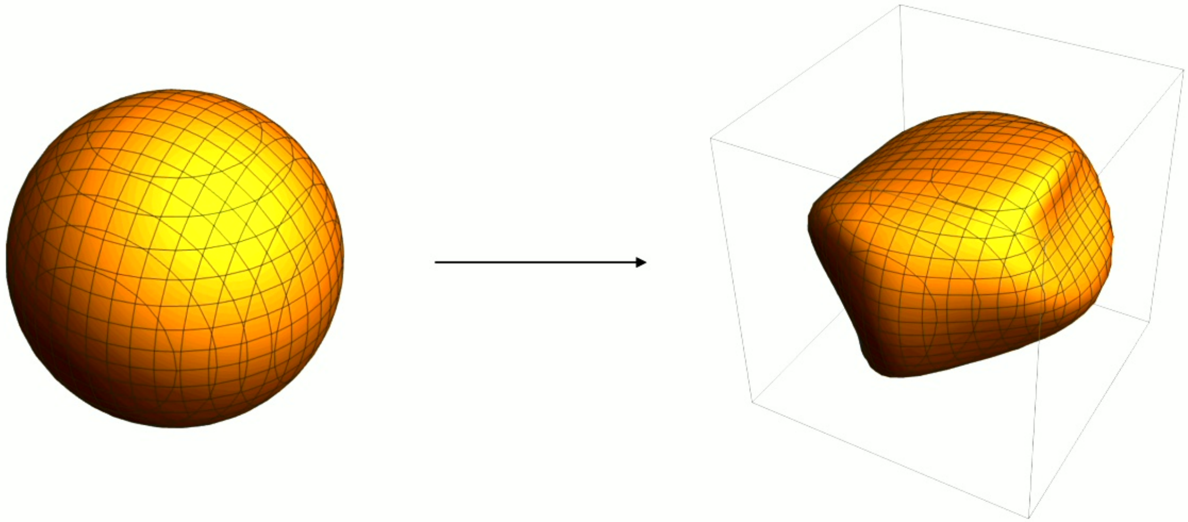
KV on S^{d-1}

$\to$ $\bullet$ $\oplus$ 2 $\oplus$

$SO(d+2)$

$SO(d)$

The two tachyonic scalars ϕ^a are known to describe small deformations of a spherical brane embedded in some ambient space



We are then led to consider a S^{d-1} brane embedded in a rigid round S^{d+1}

[Goon, Hinterbichler, Trodden 11, Burrage, de Rham, Heisenberg 11]

With the induced metric $G_{\mu\nu}[\phi^a] \equiv \delta_{AB}\partial_\mu X^A \partial_\nu X^B$ **,**

we write down the simplest worldvolume action for the S^{d-1} **brane**

$$S^{\mathrm{brane}}[\phi^a] = \frac{1}{8\pi G_N}\int_{S^{d-1}} d^{d-1}x\sqrt{G[\phi^a]}$$

$$\approx \frac{1}{8\pi G_N}\int_{S^{d-1}} \sqrt{\bar{g}}d^{d-1}x\left(1+\frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a - \frac{d-1}{2}\phi^a\phi^a + \cdots\right)$$

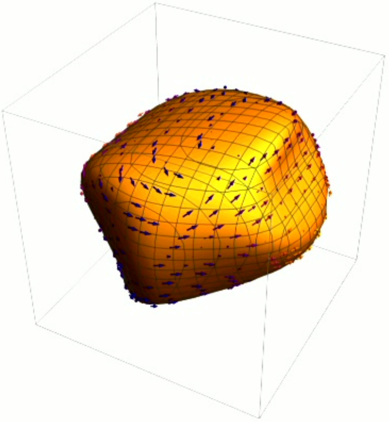
↑

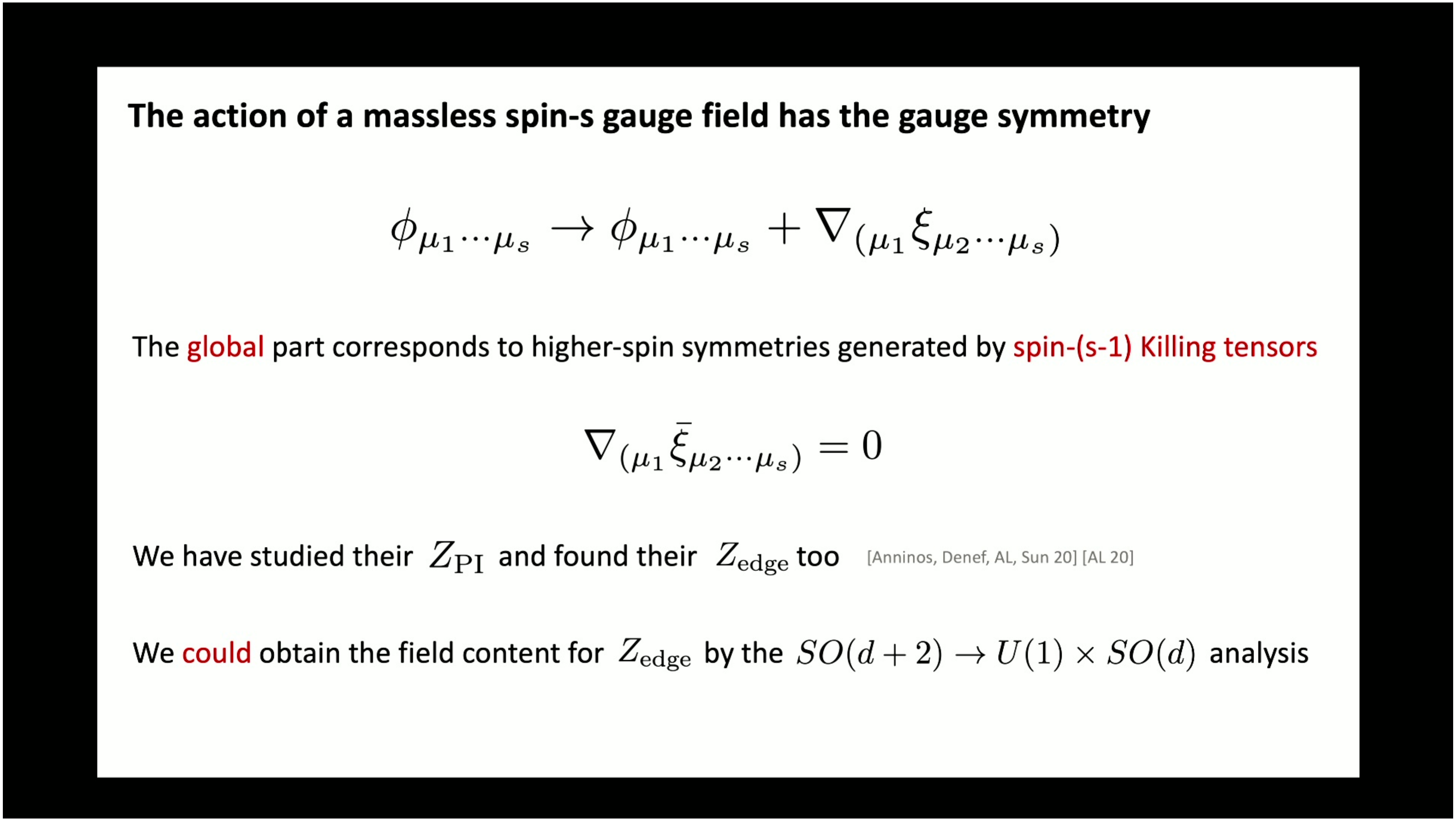
The deformations tend to decrease the size of the brane

Brief recap

- Applying the $SO(d+2) \to U(1) \times SO(d)$ branching rule, we identified the $SO(d)$ or S^{d-1} field content for Z_{edge} of linearized gravity on S^{d+1}

- The field content suggests that Z_{edge} describes a S^{d-1} brane embedded in an ambient round S^{d+1}





The action of a massless spin-s gauge field has the gauge symmetry

$$\phi_{\mu_1\cdots\mu_s} \to \phi_{\mu_1\cdots\mu_s} + \nabla_{(\mu_1}\xi_{\mu_2\cdots\mu_s)}$$

The global part corresponds to higher-spin symmetries generated by spin-(s-1) Killing tensors

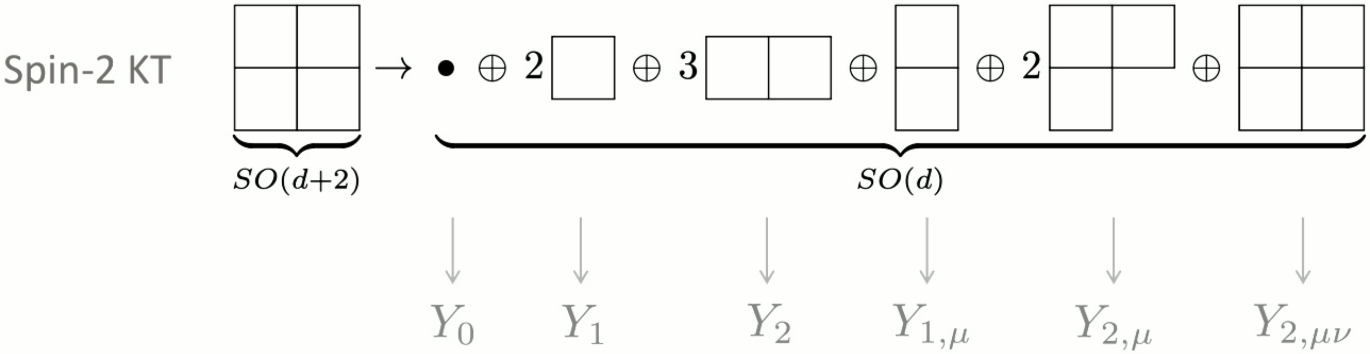
$$\nabla_{(\mu_1}\bar{\xi}_{\mu_2\cdots\mu_s)} = 0$$

We have studied their $Z_{\rm PI}$ and found their $Z_{\rm edge}$ too [Anninos, Denef, AL, Sun 20] [AL 20]

We could obtain the field content for $Z_{\rm edge}$ by the $SO(d+2) \to U(1) \times SO(d)$ analysis

We can guess the field content of Z_{edge} by demanding the nonlinear realization of the global HS symmetries

e.g. s=3



Including all (generically tachyonic) fields invariant under these shift symmetries recover (the kinematic part of) Z_{edge} found in [Anninos, Denef, AL, Sun 20] !

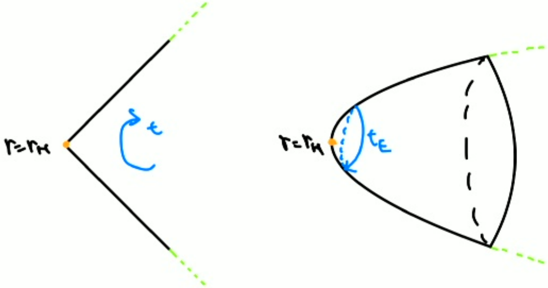
We have a wealth of phenomena awaiting further investigation!

	p-form	Spin-s totally symmetric tensor	Mixed-symmetry tensor (d>3)
Massive	Massive (p-1)-form	Massive spin ≤ s-1	??
Partially massless	╳	Massive + shift-symmetric spin ≤ s-1	??
Massless	Massless (p-1)-form	Shift-symmetric spin ≤ s-1	??

1-loop **Euclidean** partition functions on **BTZ** and **Nariai**:

Scalar $$Z_{\text{PI}} = Z_{\text{bulk}}(\beta = \beta_H)$$

Spin $s \geq 1$ $$Z_{\text{PI}} = Z_{\text{bulk}}(\beta = \beta_H)\, Z_{\text{edge}}$$



$$\log Z_{\text{bulk}}(\beta) \equiv \int_0^\infty \frac{dt}{2t} \frac{1+e^{-\frac{2\pi}{\beta}t}}{1-e^{-\frac{2\pi}{\beta}t}} \chi(t)$$

$$\boxed{\chi(t) = \sum_z N_z\, e^{-izt}}$$

QNM degeneracy

- **Perspective for Z_{edge} in terms of Denef-Hartnoll-Sachdev formula**

[Castro, Keeler, Szepietowski 17] [Keeler, Martin, Svesko 18, 19] [Grewal, AL, Parmentier 22]

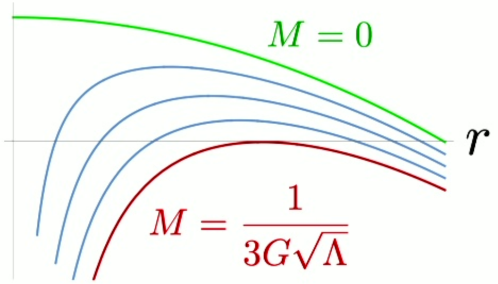
[AL, Parmentier 22] [Grewal, AL, Parmentier 22]

Nariai spacetime = the largest black hole fit inside a cosmological horizon

Schwarzschild-dS$_4$ with mass M

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

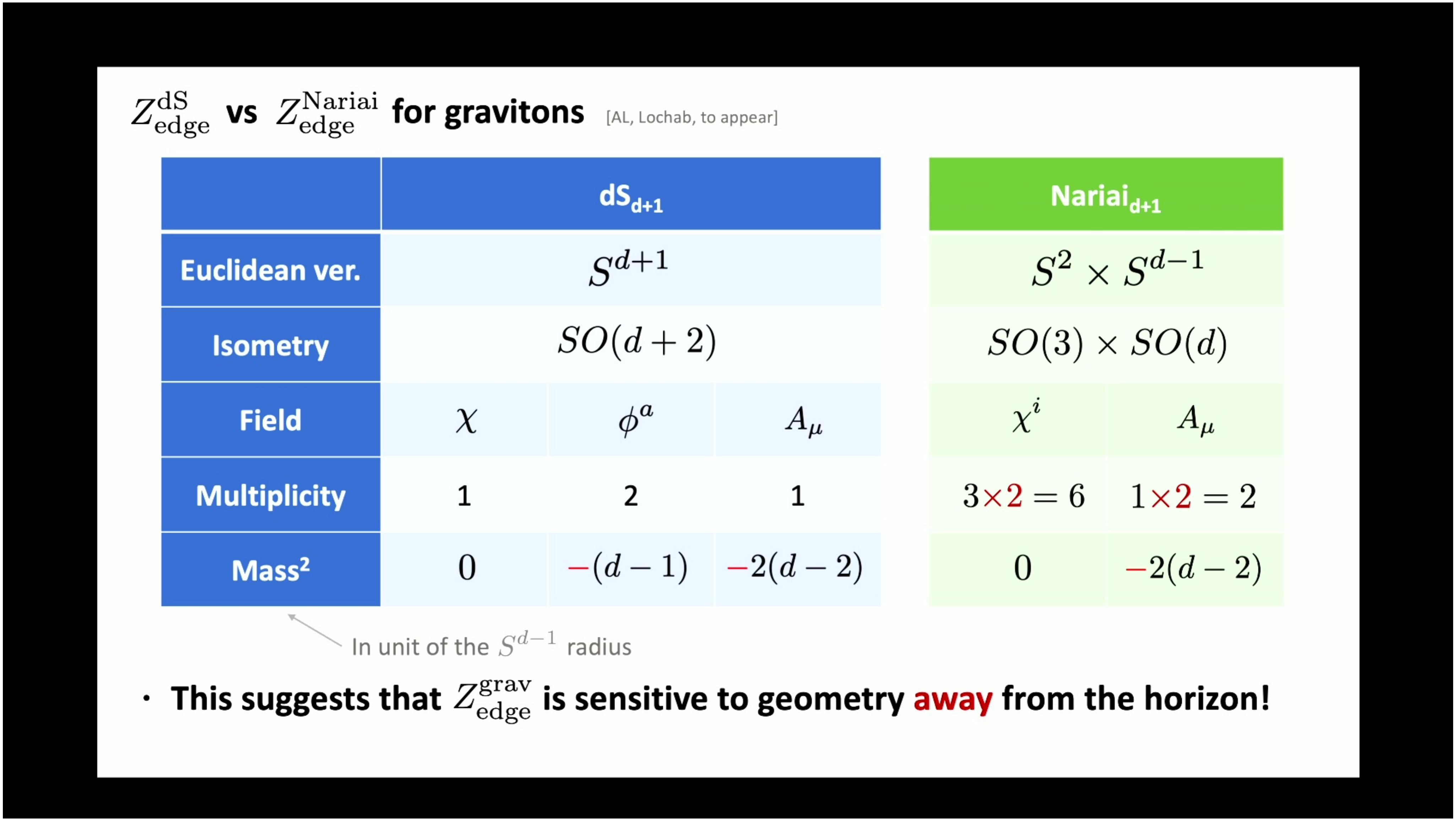
$$F(r) = 1 - \frac{2GM}{r} - \frac{\Lambda}{3}r^2$$



$$\Big\downarrow \quad M \to \frac{1}{3G\sqrt{\Lambda}}$$ + near-horizon limit

$$ds^2 = \frac{1}{\Lambda}\Big[\underbrace{-\left(1-\rho^2\right)dt^2 + \frac{d\rho^2}{1-\rho^2}}_{dS_2} \times \underbrace{+\, d\theta^2 + \sin^2\theta\, d\phi^2}_{S^2}\Big]$$

Two horizons at $\rho = \pm 1$



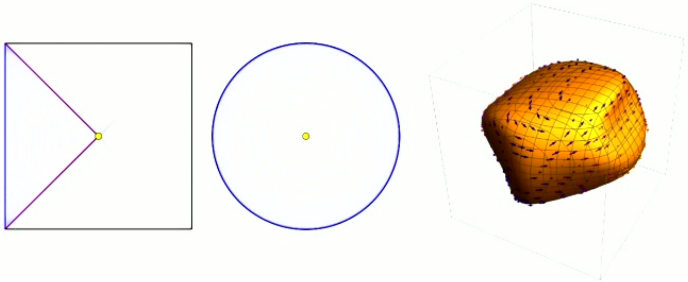
$Z^{\mathrm{dS}}_{\mathrm{edge}}$ vs $Z^{\mathrm{Nariai}}_{\mathrm{edge}}$ for gravitons [AL, Lochab, to appear]

	dS_{d+1}			Nariai_{d+1}	
Euclidean ver.	S^{d+1}			$S^2 \times S^{d-1}$	
Isometry	$SO(d+2)$			$SO(3) \times SO(d)$	
Field	χ	ϕ^a	A_μ	χ^i	A_μ
Multiplicity	1	2	1	$3\times 2 = 6$	$1\times 2 = 2$
Mass²	0	$-(d-1)$	$-2(d-2)$	0	$-2(d-2)$

In unit of the S^{d-1} radius

- **This suggests that $Z^{\mathrm{grav}}_{\mathrm{edge}}$ is sensitive to geometry away from the horizon!**

The discrepancy of Lorentzian and Euclidean calculations of 1-loop dS thermodynamics reveals co-dimension-2 degrees of freedom



	p-form	Spin-s totally symmetric
Massive	Massive (p-1)-form	Massive spin ≤ s-1
Massless	Massless (p-1)-form	Shift-symmetric spin ≤ s-1

Future directions:

1. $Z^{\text{grav}}_{\text{edge}}$ from gravitational edge modes? Direct derivation of the bulk-edge split?

2. Edge modes as Goldstones?

   Quantum reference frame, observer, timelike boundaries, …

3. What are edge modes microscopically? Log $\frac{A}{4G}$ corrections, dS_3 …