Title: Cosmological magnetic fields from electroweak symmetry breaking

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Collection/Series: Cosmology and Gravitation

Subject: Cosmology

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Abstract:

The standard models of particle physics and cosmology predict the existence of a cosmological magnetic field. Estimates of the magnetic field strength and coherence scale are derived from MHD simulations and new MHD invariants. Such magnetic fields are consistent with upper and lower bounds obtained from various cosmological observations.

Cosmological magnetic fields from electroweak symmetry breaking

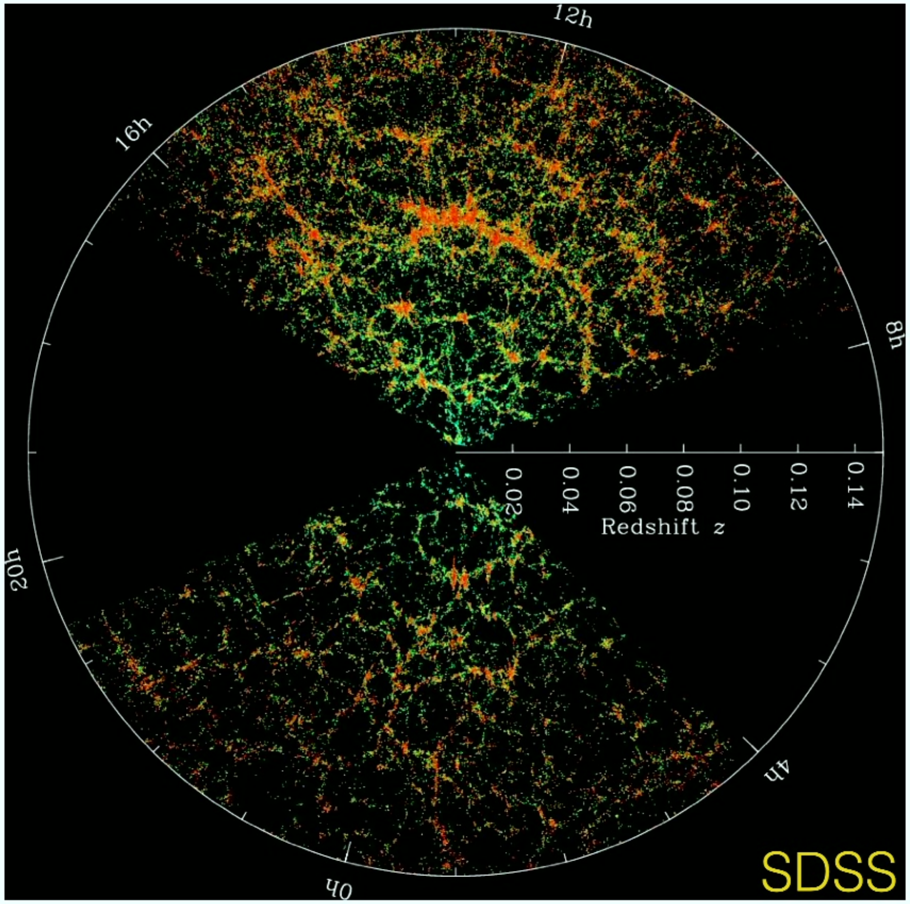
Tanmay Vachaspati

Cosmology Initiative

Arizona State University

Perimeter Institute Seminar; 29 April 2025

Context



Are voids magnetized?
Voids are ~50 Mpc in size.

Outline

1. Generation.
2. Evolution.
3. Observations.
4. Implications.

Generation during EWSB

Evolve electroweak equations through symmetry breaking.

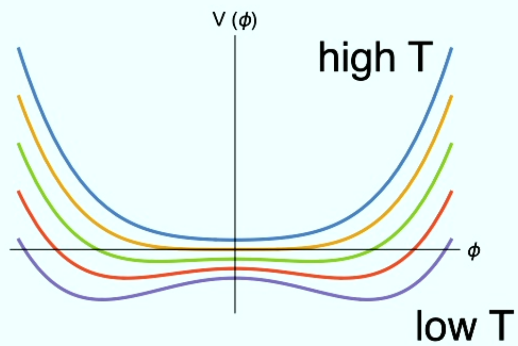
Diaz-Gil, Garcia-Bellido, Perez & Gonzalez-Arroyo, 2008

Zhang, Ferrer & TV, 2019

Zhou-Gang Mou, Teerthal Patel, Paul Saffin & TV (ongoing)

EWSB in thermal state

Zhou-Gang Mou, Teerthal Patel, Paul Saffin & TV (ongoing)



Set up "half" thermal initial conditions of Higgs and gauge fields at high temperature (T) in temporal gauge to satisfy Gauss constraints.

$$\Phi = \mathrm{Bose-Einstein\ distribution\ of\ } \Phi_{\mathbf{k}}$$

$$W_i^a = \mathrm{Bose-Einstein\ distribution\ of\ } \mathrm{W}^{\mathrm{a}}_{\mathrm{i},\mathbf{k}}$$

$$Y_i = \mathrm{Bose-Einstein\ distribution\ of\ } \mathrm{Y}_{\mathrm{i},\mathbf{k}}$$

$$\dot{\Phi} = 0 = \dot{W}_i^a = \dot{Y}_i$$

Numerical method

Equations of motion (temporal gauge):

$$\partial_0^2 \Phi = D_i D_i \Phi - 2\lambda(|\Phi|^2 - \eta^2)\Phi$$

$$\partial_0^2 Y_i = -\partial_j Y_{ij} + g' \,\mathrm{Im}[\Phi^\dagger (D_i \Phi)]$$

$$\partial_0^2 W_i^a = -\partial_j W_{ij}^a - g\epsilon^{abc} W_j^b W_{ij}^c + g\mathrm{Im}[\Phi^\dagger \sigma^a (D_i \Phi)]$$

Gauss constraints:

$$\partial_0 Y_{0i} - g' \,\mathrm{Im}[\Phi^\dagger (\partial_0 \Phi)] = 0$$

$$\partial_0 W_{0i}^a + g\epsilon^{abc} W_i^b \partial_0 W_i^c - g\mathrm{Im}[\Phi^\dagger \sigma^a (\partial_0 \Phi)] = 0$$

Numerical Relativity inspired method

TV, 2016

Define new variables: $\Xi = \partial_i Y_i, \quad \Gamma^a = \partial_i W_i^a$

Equations of motion:

$$\partial_0^2 \Phi = \nabla^2 \Phi - \gamma \Phi \partial_0 \ln|\Phi| \quad \text{(damping)}$$

$$- i\frac{g}{2}\sigma^a W_i^a \partial_i \Phi - i\frac{g'}{2} Y_i \partial_i \Phi - i\frac{g}{2}\sigma^a \Gamma^a \Phi - i\frac{g'}{2}\Xi\Phi - i\frac{g}{2}\sigma^a W_i^a (D_i\Phi) - i\frac{g'}{2} Y_i (D_i \Phi) - 2\lambda(|\Phi|^2 - \eta^2)^2 \Phi$$

(+thermal)

$$\partial_0^2 Y_i = \nabla^2 Y_i - \partial_i \Xi + g' \mathrm{Im}[\Phi^\dagger \sigma^a (D_i \Phi)]$$

$$\partial_0^2 W_i^a = \nabla^2 W_i^a + \partial_i \Gamma^a - g\epsilon^{abc}(\partial_k W_i^b) W_k^c - g\epsilon^{abc} W_i^b \Gamma^c - g\epsilon^{abc} W_k^b W_{ik}^c + g\mathrm{Im}[\Phi^\dagger \sigma^a (D_i \Phi)]$$

Additional "equations of motion":

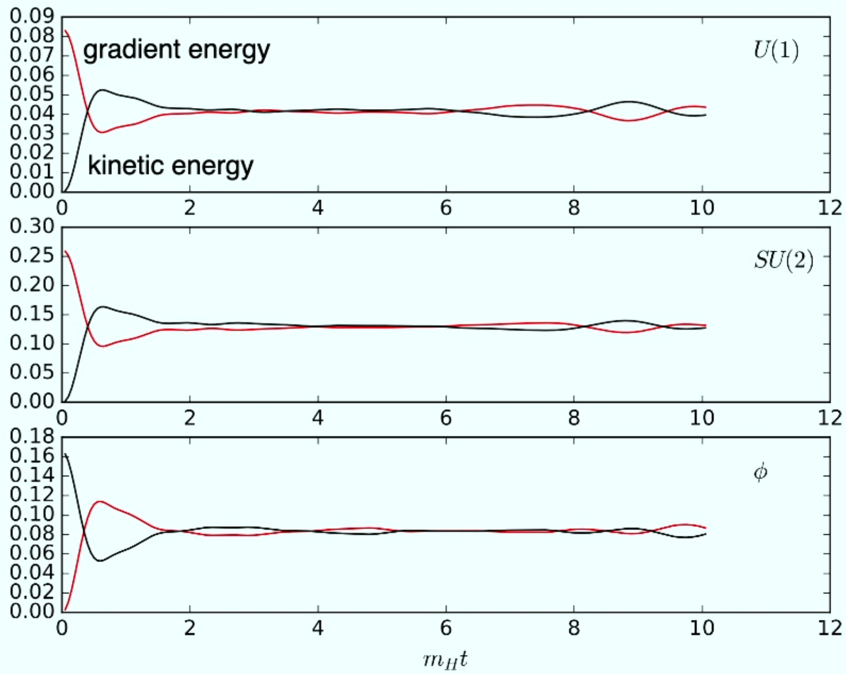
$$\partial_0 \Xi = \partial_0 Y_{0i} - g_p^2 \{\partial_0 Y_{0i} - g' \,\mathrm{Im}[\Phi^\dagger (\partial_0 \Phi)]\}$$

$$\partial_0 \Gamma^a = \partial_0 W_{0i}^a - g_p^2 \{\partial_0 W_{0i}^a + g\epsilon^{abc} W_i^b \partial_0 W_i^c - g\mathrm{Im}[\Phi^\dagger \sigma^a (\partial_0 \Phi)]\}$$

$0 < g_p^2 < 1$ is a numerical parameter.

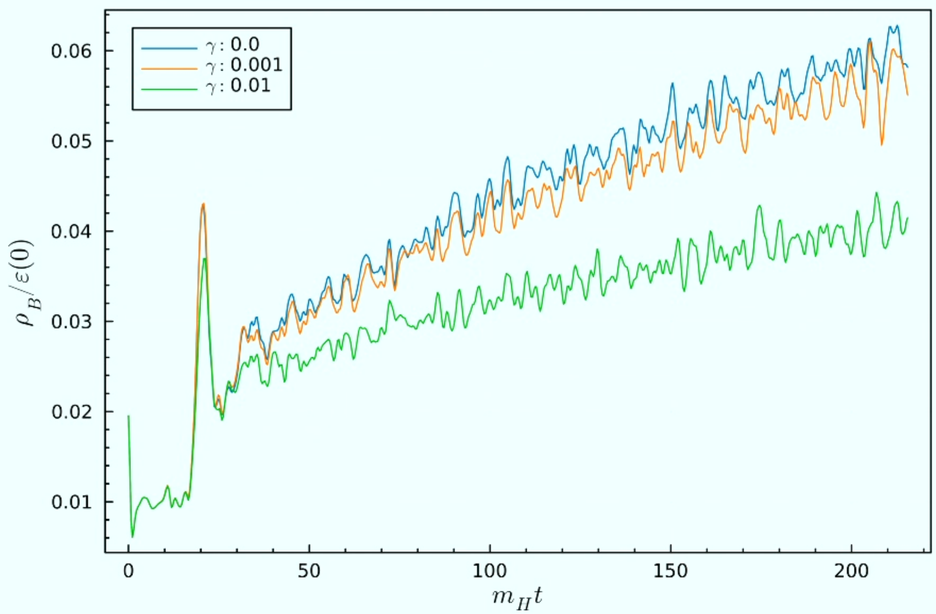
Evolution to a thermal state

Evolve with thermally corrected potential so that the system completely thermalizes.



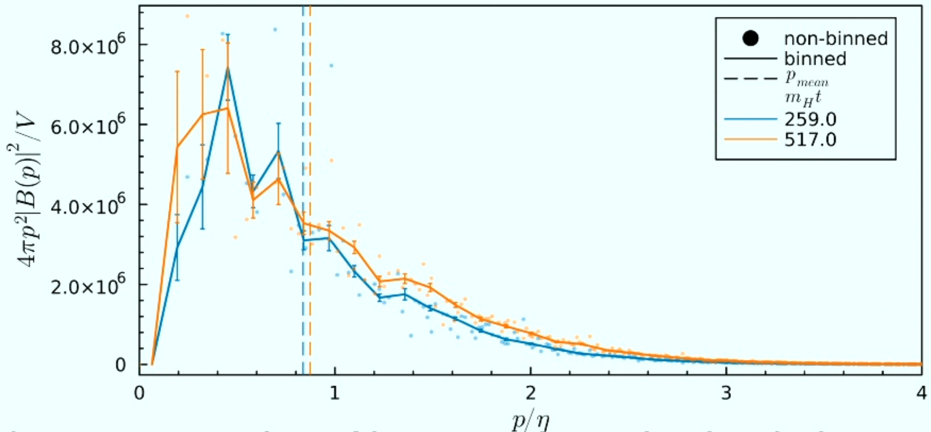
EWSB dynamics

Turn off thermal corrections to Higgs potential and evolve the equations of motion. Track the energy in electromagnetic magnetic fields for different damping parameters:



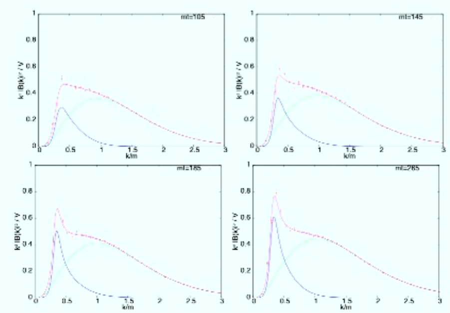
Order 5% energy density in B (but still growing).

Magnetic field power spectrum

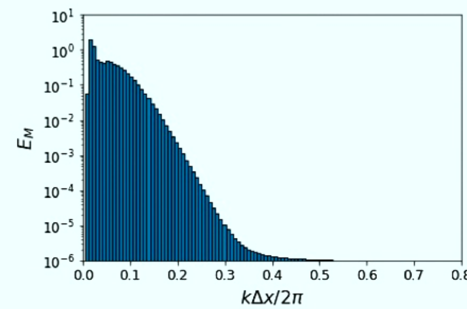


Peak at low wavenumber. Also seen in early simulations using different (non-thermal) initial conditions.

Diaz-Gil, Garcia-Bellido, Perez & Gonzalez-Arroyo, 2008



Zhang, Ferrer & TV, 2019



Understanding the generation during EWSB

Electromagnetism, non-dynamical simulations and power spectrum.

Electroweak symmetry breaking

Order parameter:

$$\Phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

Higgs field

Vacuum manifold:

$$\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = \eta^2$$

Hopf parametrization:

$$\Phi = \eta \begin{pmatrix} \cos\alpha \ e^{i\beta} \\ \sin\alpha \ e^{i\gamma} \end{pmatrix}$$

"angular coordinates on a three-sphere"

Electromagnetism

Unbroken symmetry (electromagnetism) generator Q is given by:

$$Q\Phi = 0$$

Associated gauge field is the electromagnetic gauge field,

$$A_\mu = \sin\theta_w \hat{n}^a W^a_\mu + \cos\theta_w Y_\mu \qquad \hat{n}^a = \frac{\Phi^\dagger \sigma^a \Phi}{\Phi^\dagger \Phi}$$

What is the field strength? **'t Hooft, 1974**

Two guiding principles — definition should be gauge invariant and definition should reduce to usual Maxwell expression in "unitary gauge" (Phi=constant).

$$A_{\mu\nu} \stackrel{?}{=} \sin\theta_w \partial_\mu(\hat{n}^a W^a_\nu) + \cos\theta_w \partial_\mu Y_\nu - (\mu \leftrightarrow \nu)$$ not gauge invariant

$$A_{\mu\nu} \stackrel{?}{=} \sin\theta_w \hat{n}^a W^a_{\mu\nu} + \cos\theta_w Y_{\mu\nu}$$ doesn't reduce to Maxwell in unitary gauge

Magnetic field definition

TV, 1991

$$A_{\mu\nu} = \sin\theta_w \hat{n}^a W^a_{\mu\nu} + \cos\theta_w Y_{\mu\nu} - i\frac{2\sin\theta_w}{g\eta^2}(D_\mu\Phi^\dagger D_\nu\Phi - D_\nu\Phi^\dagger D_\mu\Phi)$$

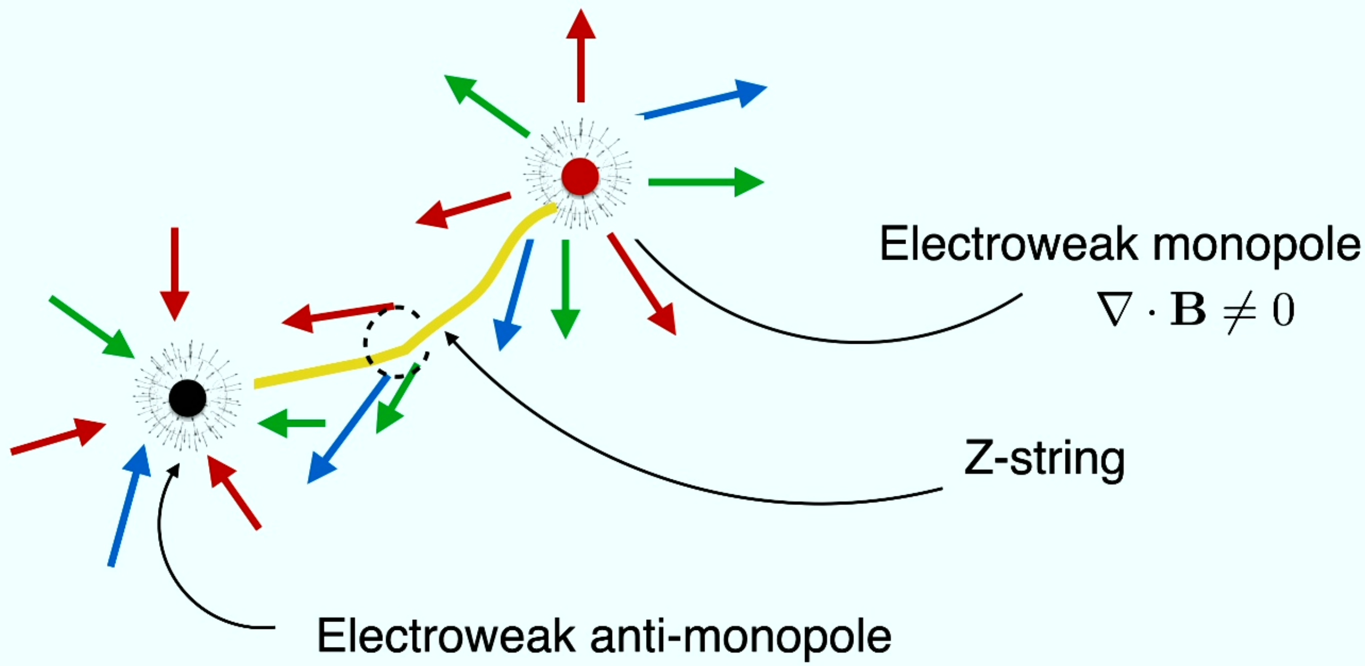
$$= \partial_\mu A_\nu - \partial_\nu A_\mu - i\frac{2\sin\theta_w}{g\eta^2}(\partial_\mu\Phi^\dagger\partial_\nu\Phi - \partial_\nu\Phi^\dagger\partial_\mu\Phi) \qquad (|\Phi| = \eta)$$

$$\mathbf{B} = \nabla\times\mathbf{A} - i\frac{2\sin\theta_w}{g\eta^2}\nabla\Phi^\dagger\times\nabla\Phi$$

Example: $\Phi = \eta\begin{pmatrix}\cos(\theta/2)\\ \sin(\theta/2)e^{i\phi}\end{pmatrix}$ ⟶ $\mathbf{B}\sim\frac{\hat{r}}{r^2}$ (magnetic monopole)

Electroweak Dumbbells

Nambu, 1977

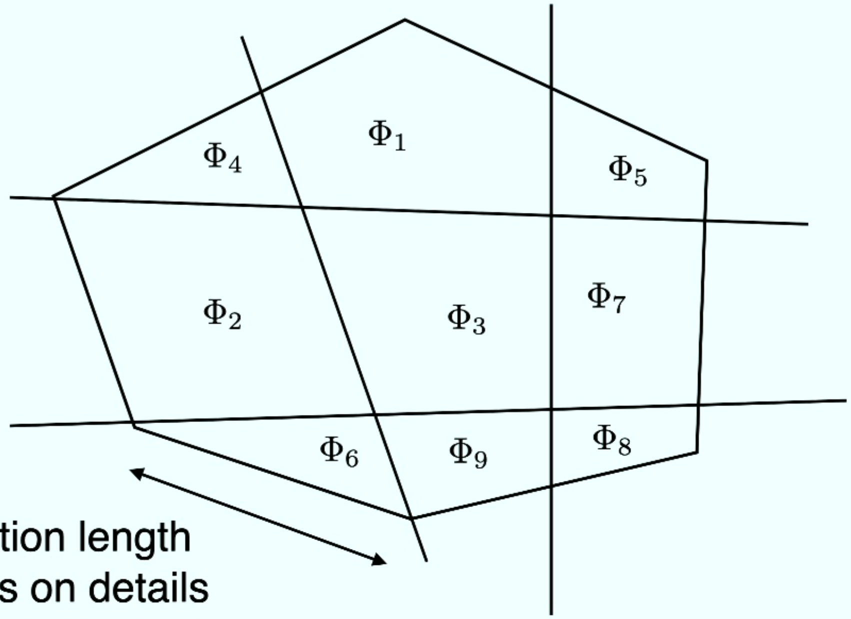


Arrows indicate points on S^2, colors indicate points on S^1.

Kibble mechanism

Kibble, 1976

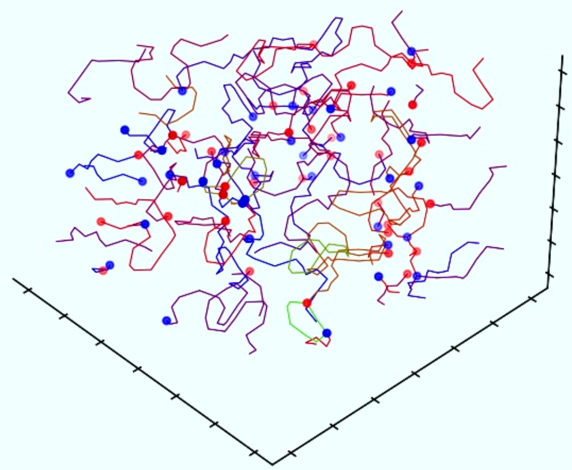
Finite size domains of ~constant order parameter.



Correlation length
(depends on details
of phase transition)

Non-dynamical simulation: singularities

T. Patel & TV, 2022



Where there are magnetic monopoles, there are magnetic fields.…

Dumbbells at large Weinberg angle, small Higgs mass

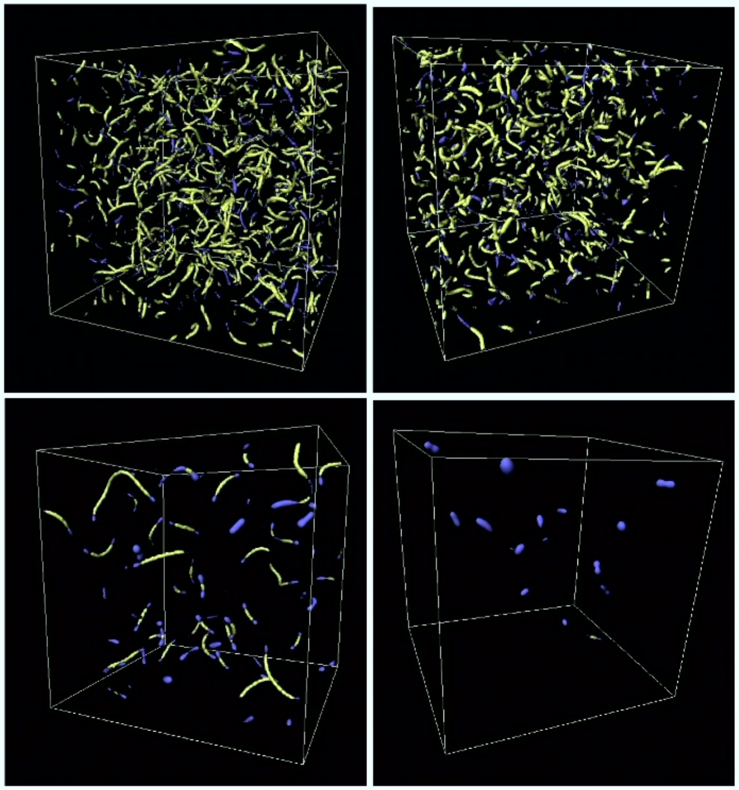
Urrestilla, Achucarro, Borrill & Liddle, 2002

Standard model but with:

$$m_H \lesssim m_Z$$

$$\sin^2 \theta_w \approx 0.995$$

yellow=Z magnetic
blue=A "magnetic" (w/o Higgs term)



Non-dynamical simulation: magnetic fields

TV & A. Brandenburg, 2025

$$\mathbf{B} = \nabla \times \mathbf{A} - i\frac{2\sin\theta_w}{g\eta^2}\nabla\Phi^\dagger \times \nabla\Phi \;\rightarrow\; -i\frac{2\sin\theta_w}{g\eta^2}\nabla\Phi^\dagger \times \nabla\Phi$$

Randomly assign Higgs field directions on a lattice:

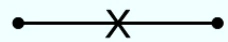
$$\Phi = \begin{pmatrix} \cos\alpha\, e^{i\beta} \\ \sin\alpha\, e^{i\gamma} \end{pmatrix} \qquad \alpha \in [0, \pi/2],\; \beta \in [0, 2\pi],\; \gamma \in [0, 2\pi]$$

Uniform volume measure: $dV = du\, d\beta\, d\gamma \qquad u = \cos(2\alpha)/2 \in [-1/2, 1/2]$

$$\mathcal{A} \equiv -i\frac{2\sin\theta_w}{g\eta^2}\Phi^\dagger\nabla\Phi = \frac{2\sin\theta_w}{g\eta^2}[\cos^2\alpha\,\nabla\beta + \sin^2\alpha\,\nabla\gamma]$$

Non-dynamical simulation: discretization

$$\mathcal{A} = \frac{2\sin\theta_w}{g\eta^2}[\cos^2\alpha\,\nabla\beta + \sin^2\alpha\,\nabla\gamma]$$



$$(\alpha_{i,j,k}, \beta_{i,j,k}, \gamma_{i,j,k}) \qquad (\alpha_{i+1,j,k}, \beta_{i+1,j,k}, \gamma_{i+1,j,k})$$

$$\Phi_{i+1/2,j,k} = \frac{\Phi_{i,j,k} + \Phi_{i+1,j,k}}{|\Phi_{i,j,k} + \Phi_{i+1,j,k}|}$$

$$(\cos\alpha)_{i+1/2,j,k} = |\Phi^{(1)}_{i+1/2,j,k}| \qquad (\sin\alpha)_{i+1/2,j,k} = |\Phi^{(2)}_{i+1/2,j,k}|$$

$$\partial_x\beta = \frac{[[\beta_{i+1,j,k} - \beta_{i,j,k}]]}{dx} \qquad \partial_x\gamma = \frac{[[\gamma_{i+1,j,k} - \gamma_{i,j,k}]]}{dx}$$

$[[\cdot]] \in [-\pi, \pi]$ “geodesic rule”

Algorithm works very well for electroweak monopole configuration.

Non-dynamical simulation: monopole annihilation

TV & Brandenburg, 2025

We would like to construct the divergence free magnetic field after the magnetic charges have annihilated. Then MHD evolution can follow.

Therefore, do not use: $\mathbf{B} = \nabla \times \mathcal{A}$

Instead find the magnetic flux through a plaquette P (with direction p):

$$\mathbf{B}\, dx^2 = \hat{p} \oint_{\partial P} \mathbf{dl} \cdot \mathcal{A}$$

Then the net magnetic flux from a cell will vanish and the constructed magnetic field will be divergence free.

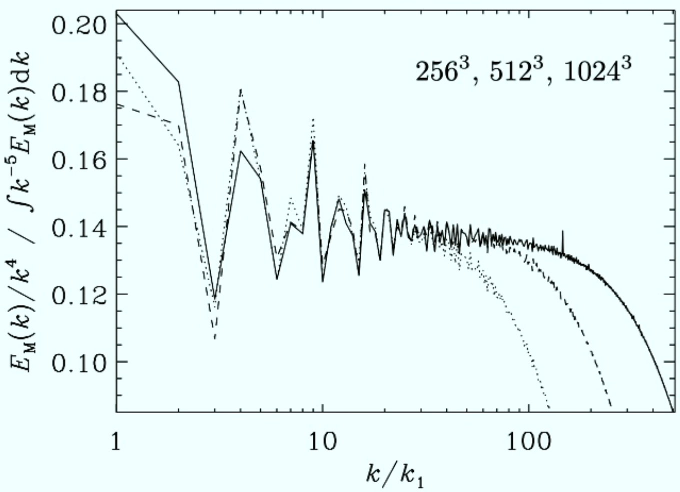
Non-dynamical simulation: spectra

TV & Brandenburg, 2025

$$\mathbf{b}(\mathbf{k}) = \int d^3x\, \mathbf{B}(\mathbf{x}) e^{+i\mathbf{k}\cdot\mathbf{x}} \qquad \langle b_i(\mathbf{k}) b_j^*(\mathbf{k}')\rangle = \left[\frac{E_M(k)}{4\pi k^2} p_{ij} + i\epsilon_{ijl} k^l \frac{H_M(k)}{8\pi k^2}\right] \times (2\pi)^6 \delta^{(3)}(\mathbf{k}-\mathbf{k}')$$

"power spectrum" "helicity power spectrum"

$$p_{ij} = \delta_{ij} - \hat{k}_i \hat{k}_j \qquad E_M(k) = \frac{k^2}{(2\pi)^2 V} \langle |\mathbf{b}(\mathbf{k})|^2 \rangle$$



$E_M(k) \propto k^4$ for small k.

"Batchelor spectrum"

Limitation: only due to Higgs variation; doesn't give power on small scales/early times.

Non-dynamical simulation: magnetic fields

TV & A. Brandenburg, 2025

$$\mathbf{B} = \nabla \times \mathbf{A} - i\frac{2\sin\theta_w}{g\eta^2}\nabla\Phi^\dagger \times \nabla\Phi \;\rightarrow\; -i\frac{2\sin\theta_w}{g\eta^2}\nabla\Phi^\dagger \times \nabla\Phi$$

Randomly assign Higgs field directions on a lattice:

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Non-dynamical simulation: spectra

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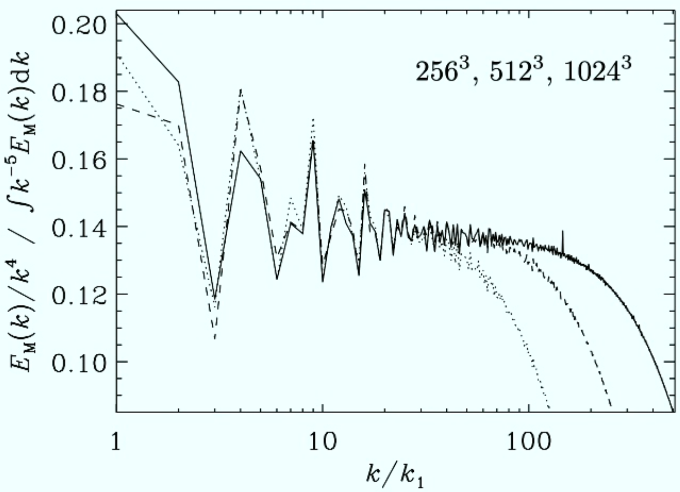
$$\mathbf{b}(\mathbf{k}) = \int d^3x\, \mathbf{B}(\mathbf{x}) e^{+i\mathbf{k}\cdot\mathbf{x}} \qquad \langle b_i(\mathbf{k}) b_j^*(\mathbf{k}')\rangle = \left[\frac{E_M(k)}{4\pi k^2} p_{ij} + i\epsilon_{ijl} k^l \frac{H_M(k)}{8\pi k^2}\right] \times (2\pi)^6 \delta^{(3)}(\mathbf{k}-\mathbf{k}')$$

"power spectrum" "helicity power spectrum"

$$p_{ij} = \delta_{ij} - \hat{k}_i \hat{k}_j \qquad E_M(k) = \frac{k^2}{(2\pi)^2 V} \langle |\mathbf{b}(\mathbf{k})|^2 \rangle$$



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$$\mathbf{B} = \nabla \times \mathbf{A} - i\frac{2\sin\theta_w}{g\eta^2}\nabla\Phi^\dagger \times \nabla\Phi \;\to\; -i\frac{2\sin\theta_w}{g\eta^2}\nabla\Phi^\dagger \times \nabla\Phi$$

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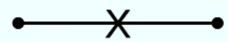
Non-dynamical simulation: discretization

$$\mathcal{A} = \frac{2\sin\theta_w}{g\eta^2}[\cos^2\alpha\,\nabla\beta + \sin^2\alpha\,\nabla\gamma]$$



$(\alpha_{i,j,k}, \beta_{i,j,k}, \gamma_{i,j,k})$ $(\alpha_{i+1,j,k}, \beta_{i+1,j,k}, \gamma_{i+1,j,k})$

$$\Phi_{i+1/2,j,k} = \frac{\Phi_{i,j,k} + \Phi_{i+1,j,k}}{|\Phi_{i,j,k} + \Phi_{i+1,j,k}|}$$

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$$\partial_x\beta = \frac{[[\beta_{i+1,j,k} - \beta_{i,j,k}]]}{dx} \qquad \partial_x\gamma = \frac{[[\gamma_{i+1,j,k} - \gamma_{i,j,k}]]}{dx}$$

$[[\cdot]] \in [-\pi, \pi]$ “geodesic rule”

Algorithm works very well for electroweak monopole configuration.

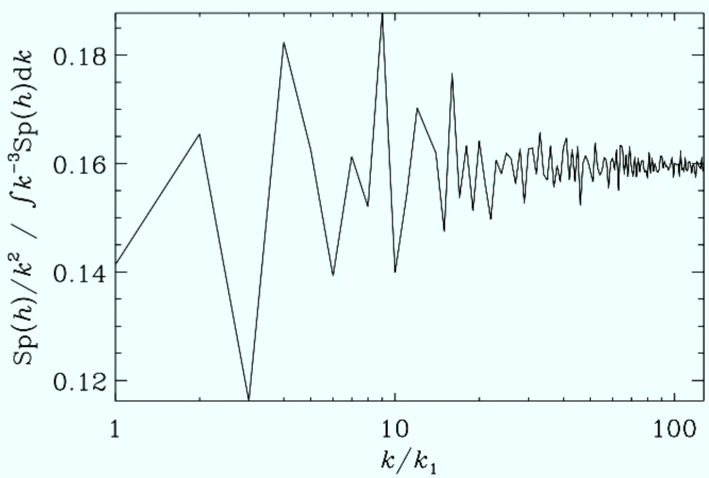
Non-dynamical simulation: helicity

$$H_M(k) = \frac{k^2}{(2\pi)^2 V} \langle \mathbf{a}(\mathbf{k}) \cdot \mathbf{b}^*(\mathbf{k}) \rangle$$

$H_M(k) = 0$ (no parity violation)

Helicity fluctuations: $\mathrm{Sp}(h) = \frac{k^2}{8\pi^3 V} \oint d\Omega_k |\tilde{h}|^2$ $\quad h = \mathbf{A} \cdot \mathbf{B}$

Hosking integral: $I_H = \frac{2\pi^2}{k^2} \mathrm{Sp}(h) \Big|_{k \to 0}$



MHD Evolution: Simulations

Decay of peak:

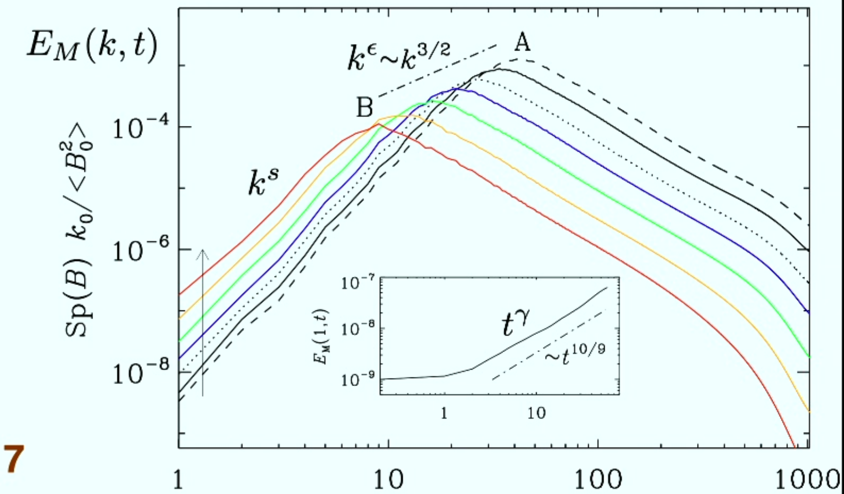
$$E_M(k_B,\tau_0) = E_M(k_A,\tau_{EW})\left(\frac{k_B}{k_A}\right)^{\epsilon}$$

Spectral decay & inverse cascade:

$$E_M(k_B,\tau_0) = E_M(k_A,\tau_{EW})\left(\frac{k_B}{k_A}\right)^{s}\left(\frac{\tau_0}{\tau_{EW}}\right)^{\gamma}$$

$$\gamma = \frac{2(s-\epsilon)}{\epsilon+3}$$

Brandenburg & Kahniashvili, 2017



Therefore,

$$k_B = k_A\left(\frac{\tau_{EW}}{\tau_0}\right)^{2/(\epsilon+3)}$$

$$E_M(k_B,\tau_0) = E_M(k_A,\tau_{EW})\left(\frac{\tau_{EW}}{\tau_0}\right)^{2\epsilon/(\epsilon+3)}$$

Electroweak magnetic fields

$$B_\lambda^{\rm conf}(\tau) = \sqrt{2kE_M(k,\tau)}$$

$$B^{\rm phys}(\tau_0) = B^{\rm phys}(\tau_{EW}) \left(\frac{\tau_{EW}}{\tau_0}\right)^{(\epsilon+1)/(\epsilon+3)} \left(\frac{T_0}{T_{EW}}\right)^2$$

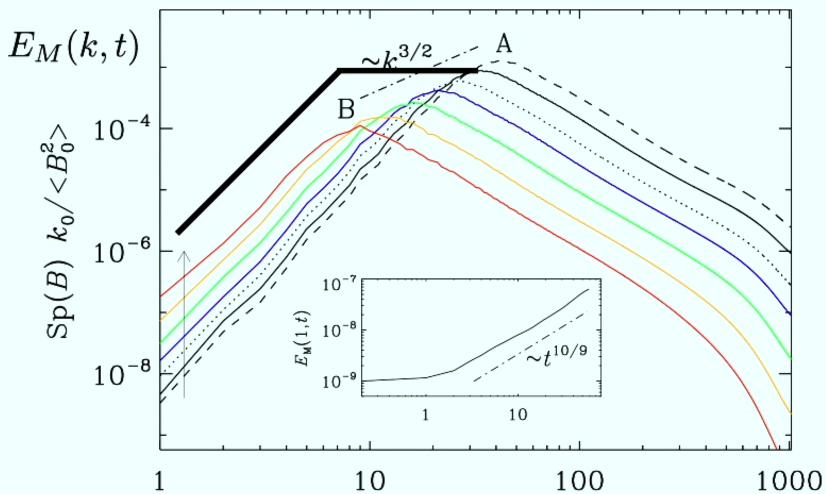
$$\frac{\tau_0}{\tau_{EW}} = \frac{T_{EW}}{T_{eq}}\sqrt{\frac{T_{eq}}{T_0}} = 10^{13} \qquad \epsilon = 3/2$$

$$k_B = k_A \left(\frac{\tau_{EW}}{\tau_0}\right)^{2/(\epsilon+3)}$$

Initial magnetic field at electroweak? Assume few percent of total energy density in magnetic fields, coherent on the electroweak horizon scale. Then,

$$k_B^{\rm phys} \sim (1\ {\rm kpc})^{-1},\ B_{\rm kpc}^{\rm phys}(t_0) \sim 10^{-13}\,{\rm G}$$

Helical MHD Evolution

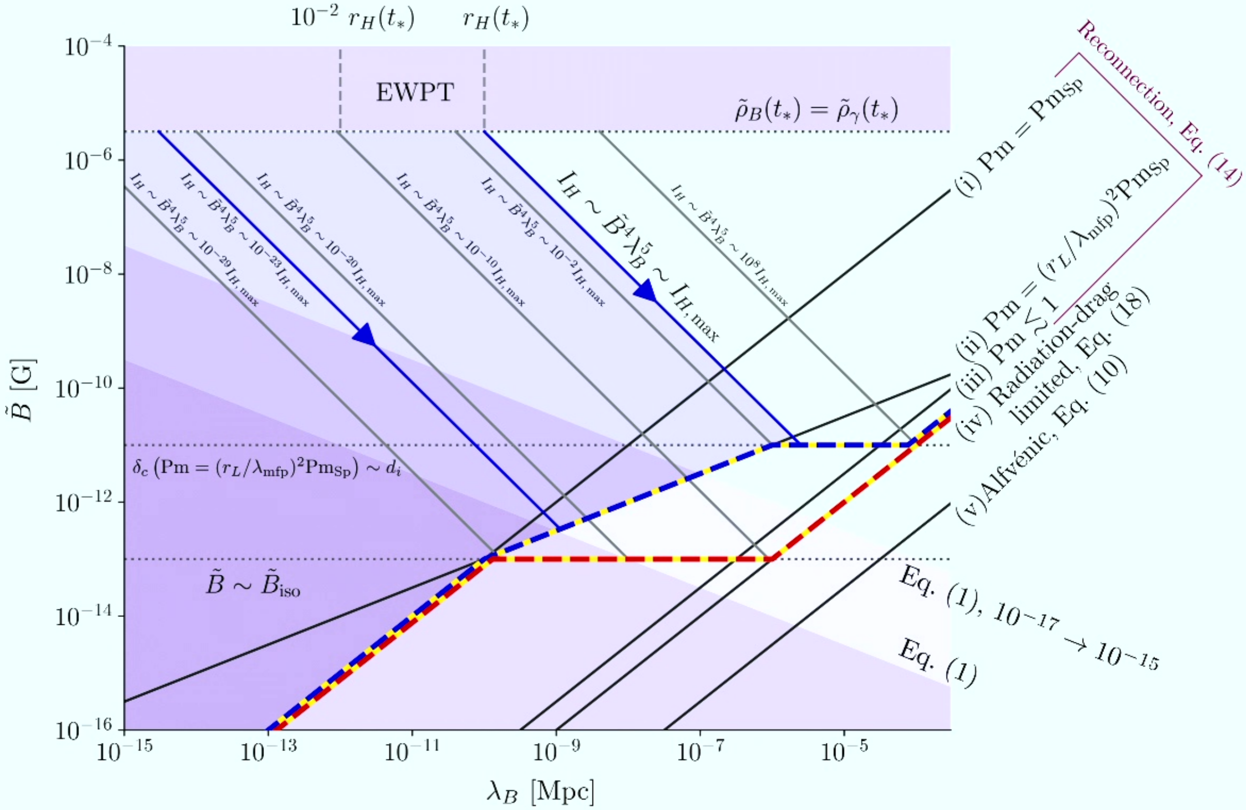


$$k_B^{\mathrm{phys}} \sim (1\ \mathrm{Mpc})^{-1},\ B_{\mathrm{kpc}}^{\mathrm{phys}}(t_0) \sim 10^{-10}\,\mathrm{G}$$

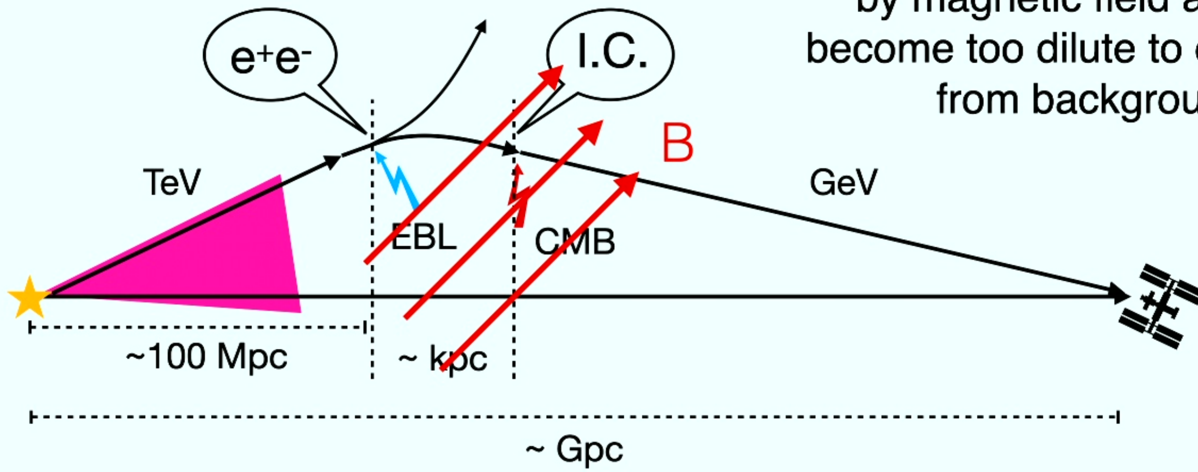
Analytic arguments

Hosking & Schekochihin, 2023

Conservation of the "Hosking integral" (fluctuations of magnetic helicity).



Blazar Cascades + B



GeV flux gets spread out by magnetic field and may become too dilute to distinguish from background.

Missing GeV photons attributed to $B > 10^{-19}$-10^{-16} Gauss

(B assumed smooth on Mpc scales.)

Magnetic field lower bounds from TeV blazars:

Neronov & Vovk, 1006.3504
Tavecchio, Ghisellini, Foschini, Bonnoli, Ghirlanda & Coppi, 1004.1329
Dolag, Kachelriess, Ostapenko, Tomas, 1009.1782
Dermer, Cavadini, Razzaque, Finke, Chiang & Lott, 1011.6660
Essey, Ando & Kusenko, 1012.5313
Taylor, Vovk & Neronov, 1101.0932
Huan, Weisberger, Arlen & Wakely, 1106.1218
Takahashi, Mori, Ichiki, Inoue & Takami, 1303.3069
Finke et al, 1510.02485
Ackermannn et al (Fermi-Lat), 1804.08035
Podlesnyi, Dzhatdoev & Galkin, 2204.11110
Acciari et al (MAGIC), 2210.03321
Aharonian et al (H.E.S.S. & Fermi-LAT), 2306.05132
…

Missing GeV photons attributed to $B > 10^{-19}$-10^{-16} Gauss if with Mpc coherence.

Plasma instability timescale debate:

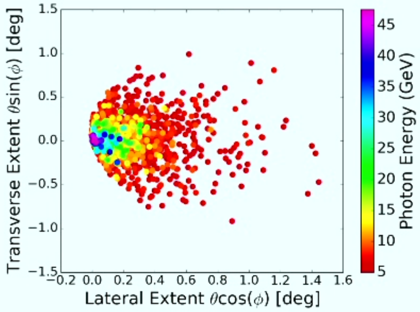
Broderick, Chang & Pfrommer, 1106.5494, …
Schlickeiser, Ibscher & Supsar, Ap. J. 758, 102 (2012).
Miniati & Elyiv, 1208.1761
Batista, Saveliev & Dal Pino, 1904.13345
…

Instability cannot apply to TeV GRBs as these are very short duration.

Magnetic field lower bounds from GRB:
Vovk, Korochkin, Neronov & Semikoz, 2306.07672
Manuel Meyer seminar, https://indico.cern.ch/event/1334236/contributions/5759310/

Magnetic field lower bounds from neutrino event:
Fang, Halzen & Hooper, 2502.09545
Crnogorcevic, Blanco & Linden, 2503.16606

Future: Cerenkov Telescope Array (CTA):
Should see the cascade halo.

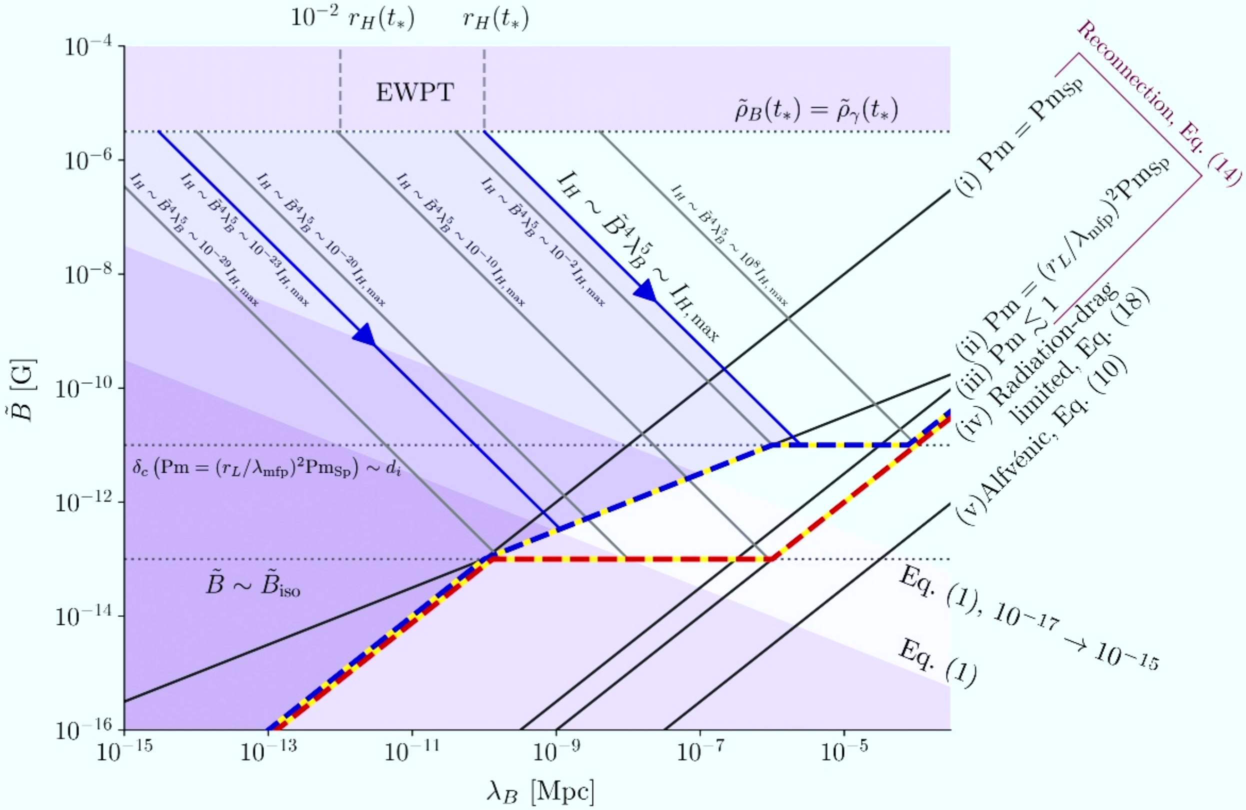


Long & TV, 2015
Duplessis & TV, 2017
+ …

Analytic arguments

Hosking & Schekochihin, 2023

Conservation of the "Hosking integral" (fluctuations of magnetic helicity).



Conclusions

1. Standard model particle physics + standard FRW cosmology predicts cosmological magnetic fields with Batchelor spectrum.

2. MHD evolution is governed by conservation of magnetic helicity/Hosking integral and magnetic reconnection (some debate).

3. Lower bounds on cosmological magnetic fields have now been obtained from absence of cascade emission from TeV blazars, TeV GRB, and possibly, PeV neutrino event.

4. Cosmological magnetic fields have implications for cosmological recombination and cosmological parameter estimation from CMB observations.

Cosmological magnetic fields and recombination:

Pogosian & Jedamzik, 2004.09487

Jedamzik, Pogosian & Abel, 2503.09599