Title: Entanglement Bootstrap, a perspective on quantum field theory

Speakers: John McGreevy

Collection/Series: Colloquium

Subject: Other

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Abstract:

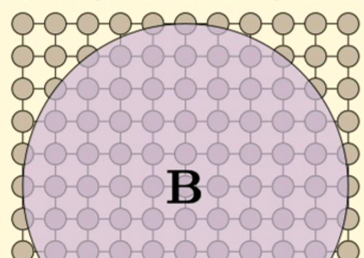
I will introduce the Entanglement Bootstrap, a program to extract and understand the universal information characterizing a state of matter, starting from the local entanglement structure of a single representative state. This universal information is usually packaged in the form of a quantum field theory; the program therefore provides a surprising new perspective on quantum field theory. I will discuss what we can learn about gapped topological phases and their associated topological field theories, and about quantum critical points and their associated conformal field theories.

Entanglement Bootstrap

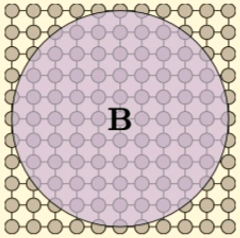
a perspective on quantum field theory

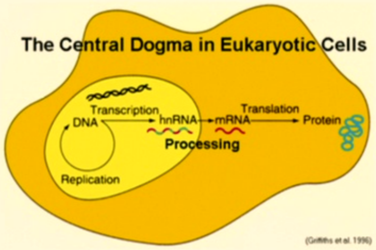
John McGreevy (UCSD)

based on 2112.08398, 2301.07119 with
Bowen Shi and **Jin-Long Huang**
2303.05444 with **Ting-Chun (David) Lin** (UCSD)
2403.18410 2404.03725 2408.10306 with BS, TCL, **Xiang Li** (UCSD), Isaac Kim,
and work in progress with **Xiang Li, Ting-Chun (David) Lin, Rolando Ramirez Camasca** (UCSD)



Central Dogma of Entanglement Bootstrap: All the universal properties of a state of matter are encoded in a single representative density matrix on a ball.





What is quantum field theory (QFT)?

A *field theory* is a description of the mechanics of degrees of freedom spread over space, with local interactions.

e.g. electromagnetic field

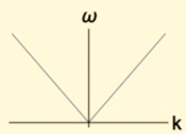
A QFT is a *quantum* description of the mechanics of degrees of freedom spread over space, with local interactions.

electromagnetic field $\rightarrow$ photons.

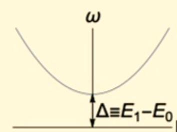
In fact, all particles arise this way, as quanta of some field.

Diversity of QFT:

A QFT can be gapless:

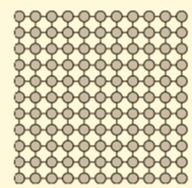


or gapped:



A QFT can be weakly coupled or strongly coupled.

A QFT can live in the continuum or on a lattice.

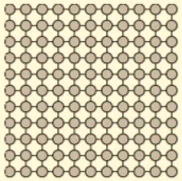


Necessary metaphysics: Renormalization Group idea

[Kadanoff, Wilson 68-72]

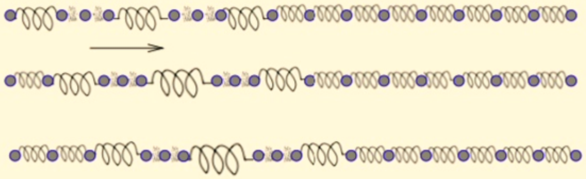
Each material is its own universe

A model of condensed matter is a lattice QFT.



Each material is its own universe, with its own laws of physics.

Some of them have particle physics (of *quasiparticles*),



some don't! (eg CFT)

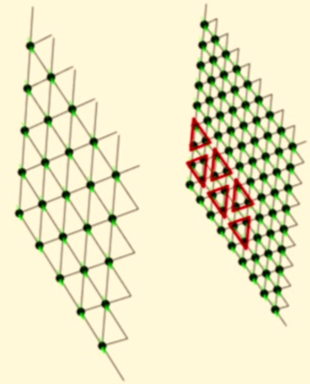
What is a QFT?

... the language of Nature.

Necessary metaphysics: Renormalization Group idea

[Kadanoff, Wilson 68-72]

$$eg: \; s_i = \pm 1 \quad H = \sum_{\text{neighbors, } \langle ij \rangle} J_{ij} s_i s_j + \sum_{\text{next neighbors, } \langle\langle ij \rangle\rangle} K_{ij} s_i s_j + \ldots$$



Idea: measure the system with coarser and coarser rulers.

eg: Let 'block spin' = majority value of spins in block.

Necessary metaphysics: Renormalization Group idea

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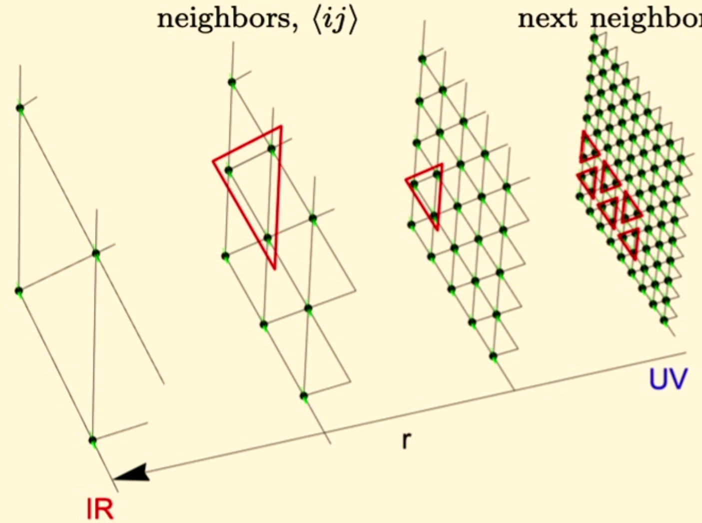


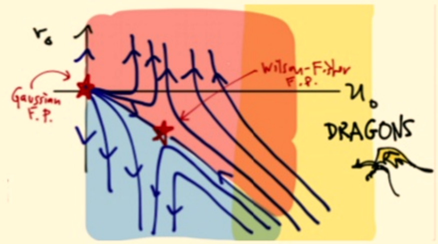
Idea: measure the system with coarser and coarser rulers.

eg: Let 'block spin' = majority value of spins in block.

Define a Hamiltonian $H(r)$ for block spins so long-wavelength observables are the same. eg: $Z = \text{tr}e^{-\beta H} = e^{-\beta F}$

$\longrightarrow$ Renormalization Group (RG) flow on the space of Hamiltonians: $\vec{J}(r)$

Fixed points of the RG



When we encounter a dynamical system like this

$$\vec{J}_{r+1} = \beta(\vec{J}_r)$$

the first question to ask is:

What are the *fixed points*? $\vec{J}_\star = \beta(\vec{J}_\star)$

The fixed point theory is scale-invariant:

if you change your resolution you get the same picture back.

Under the RG transformation, the gap transforms like $\Delta \to a\Delta$.

At a fixed point, $\Delta \overset{!}{=} a\Delta$.

This requires either $\Delta = \infty$ (gapped) or $\Delta = 0$ (gapless).

CFT

Two kinds of RG fixed points: gapped $\Delta = \infty$ and gapless $\Delta = 0$.

A gapped fixed point ("IR fixed point") has no local dofs left, is attractive from all directions.

The former ("UV fixed point") is a more interesting QFT, with lots of gapless stuff.

UV fixed points have at least one unstable direction:
That is, they arise at *critical points* between phases, by tuning a parameter.

Often, in addition to scale invariance, such a field theory has a larger symmetry called conformal invariance, it is a conformal field theory (CFT).

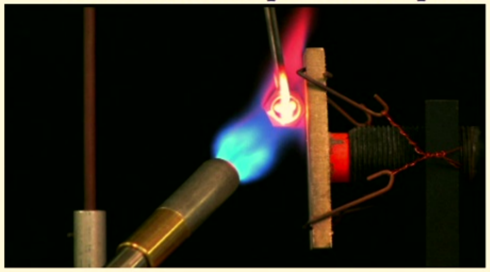
Universality as an experimental phenomenon

Experimental observation [1960s]:

same critical exponents from (microscopically) very different systems.

Near a critical point (at $T = T_c$), scaling laws:

observables depend like power laws on the deviation from the critical point.



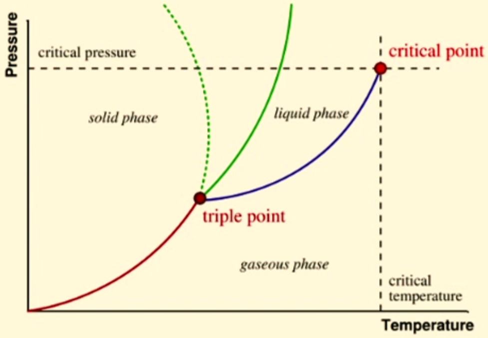
[MIT TSG]

e.g. ferromagnet near the Curie transition

(let $t \equiv \frac{T_c - T}{T_c}$)

specific heat: $c_v \sim t^{-\alpha}$

magnetic susceptibility: $\sim t^{-\gamma}$



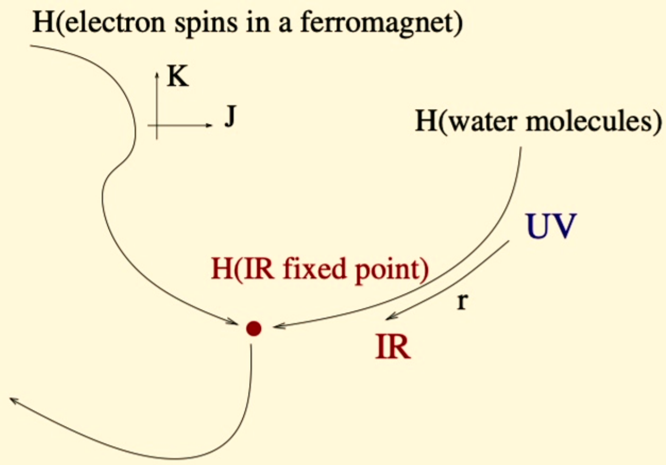
Water near its liquid-gas critical point:

specific heat: $c_v \sim t^{-\alpha}$

compressibility: $\sim t^{-\gamma}$

with the same α, γ!

RG fixed points give universal physics



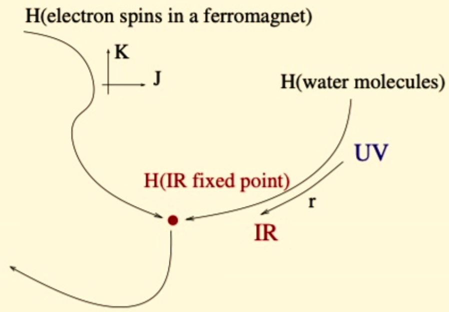
Universality is explained by the fact that fixed points are rare.

Many microscopic theories will flow to the same fixed-point.

same CFT $\Longrightarrow$
same critical exponents.

Def: a *universal* property is a property of an RG fixed point theory.

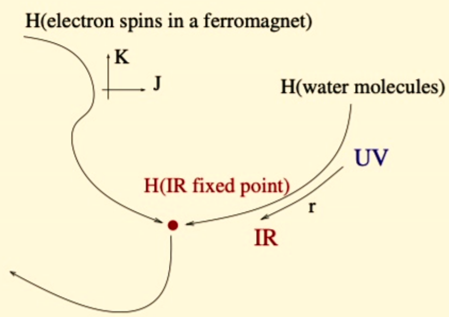
Low-energy bias

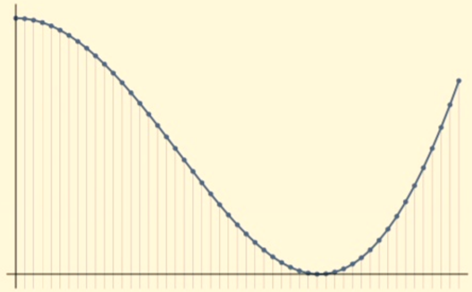


Even in High-Energy Physics there is a relentless focus on low-energy physics: the groundstate and zero-energy-density excitations above it.
The language betrays this bias: attractive directions are called 'irrelevant' and repulsive directions are called 'relevant'.

What do I want to know: given some microscopic definition of the system, what is the universal low-energy physics?

Low-energy bias





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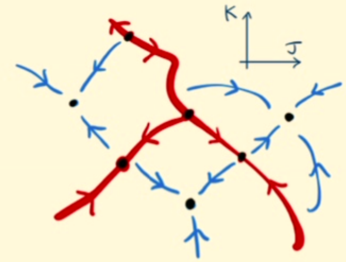
Often, this can be encoded in a continuum QFT.

Phases of Matter

Big goal of cond-mat: understand what are the possible *phases of matter*.

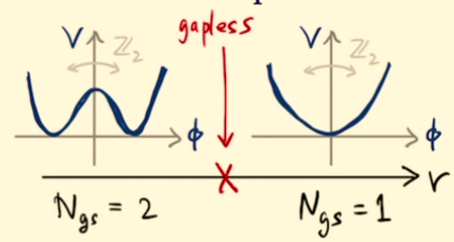
Def: a phase of matter is the basin of attraction of an RG fixed point.



Q: how to tell if two states are in the same phase?

A: find invariants – quantities that do not change within a phase. Such quantities are universal, since they are properties of the RG fixed point.

e.g. If a phase spontaneously breaks a discrete symmetry:



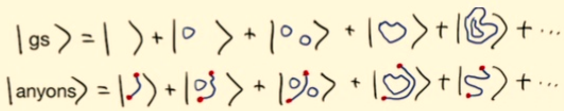
Such universal data can also be called *topological*.

Not all states are distinguished by (ordinary) symmetry.

Topological order

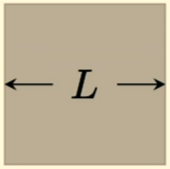
Phases of matter can be distinguished from each other by data other than simple symmetries.

Topological order [Wen]: localized excitations that can't be created by any local operator (anyons).

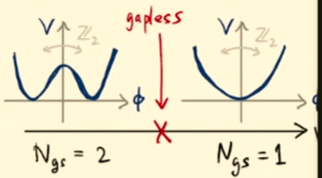


e.g.: fractional quantum Hall (FQH) states, gapped spin liquids
These excitations can have fractional charge and fractional (or even non-abelian) statistics, characterized by a *quantum dimension* d_a (with $N \gg 1$ anyons of type a, we get d_a^N states) and *fusion rules* N_{ab}^c.
We believe the low energy description is a *topological quantum field theory* (TQFT) – a $\Delta = \infty$ fixed point.

Perils of the thermodynamic limit



The thermodynamic limit ($L \to \infty$) is where the magic happens
(e.g. where phases are sharply defined),



but it has a price:
Hilbert space is BIG. The wavefunction of ~ 100 spins is more numbers than atoms in the Earth.

Usually, when confronted with a big problem, the useful strategy is to break it into smaller parts.
Obstacle:
quantumly, a small part doesn't even have a definite wavefunction.

A basis of states on AB is $|a_A b_B\rangle$,

Entanglement

e.g.: two qubits $|\psi\rangle = \left(|0_A 0_B\rangle + |1_A 1_B\rangle\right)/\sqrt{2}$.
All observables in subsystem A are encoded in its *reduced density matrix*:

$$\rho_A = \mathrm{tr}_B |\psi\rangle\langle\psi| = \frac{1}{2}\left(|0\rangle\langle 0|_A + |1\rangle\langle 1|_A\right) = 1\!\!1_A/2.$$

$$\langle\psi|\mathcal{O}_A|\psi\rangle = \mathrm{tr}_{AB}\mathcal{O}_A|\psi\rangle\langle\psi| = \mathrm{tr}_A \rho_A \mathcal{O}_A$$

(In the example we know nothing if we only have access to A.)

$$\rho_A = \sum_s \lambda_s |\psi_s\rangle\langle\psi_s| \text{ says}$$

the wavefunction of A is $|\psi_s\rangle$ with probability $\lambda_s \in [0,1]$.

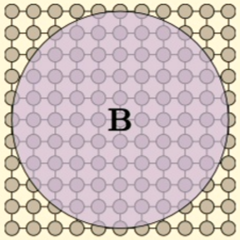
We can quantify the amount of this *entanglement* by the entanglement entropy (EE):

$$S(\rho_A) \equiv -\mathrm{tr}_A \rho_A \log \rho_A = -\sum_s \lambda_s \log \lambda_s$$

(maximal in the example).

So this entanglement between the dofs in a quantum many body state seems like a problem...

Reasons to believe the Central Dogma



Central Dogma: All the universal properties of a state of matter are encoded in a single representative density matrix on a ball.

Some evidence: (Not mathematical physics)

- In a groundstate of a local Hamiltonian, the EE satisfies an *area law*, $S_B \sim |\partial B|$ (modulo some interesting exceptions and pathologies)

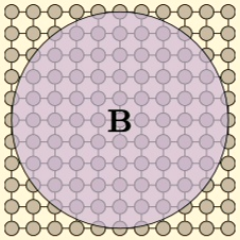
- Equal time correlations of local operators know about the gap:

$$\langle\psi|\mathcal{O}(x)^\dagger\mathcal{O}(y)|\psi\rangle = \mathrm{tr}_B \rho_B \mathcal{O}(x)^\dagger\mathcal{O}(y) \sim \begin{cases} e^{-|x-y|/\xi}, & \text{gapped} \\ \frac{1}{|x-y|^{2\alpha}}, & \text{gapless} \end{cases}$$

- In a groundstate with liquid topological order, for $B \sim$ disk, $S_B = \frac{|\partial B|}{\epsilon} - \gamma, \gamma =$ topo. entanglement entropy (TEE) extracts anyon data

[Levin-Wen, Kitaev-Preskill 06, Grover-Turner-Vishwanath]

Reasons to believe the Central Dogma



Central Dogma: All the universal properties of a state of matter are encoded in a single representative density matrix on a ball.

More reasons to believe it:

- In the groundstate of a 1+1d CFT, $B \sim$ interval,

$$S_B = -\text{tr}\rho_B \log \rho_B = \frac{c}{6} \log \frac{|B|}{\epsilon}$$

$c =$ *central charge*, a measure of the number of dofs per site.
(There is a similar formula using round balls in higher dimensions.)
[Cardy-Calabrese 05, Casini-Huerta, Myers-Sinha...]

- There is more info in ρ_B than entropies: $K_X \equiv -\log \rho_X$
entanglement Hamiltonian $\rho_X = e^{-K_X}$

$$J_\psi(A, B, C) \equiv \mathbf{i}\langle\psi|[K_{AB}, K_{BC}]|\psi\rangle = \frac{\pi c_-}{3}$$

chiral central charge, the net number of edge modes of a FQH state
[Kim-Shi-Kato-Albert 21, Zou-Shi-Sorce-Lim-Kim 22] relatedly: [Li-Haldane 10]

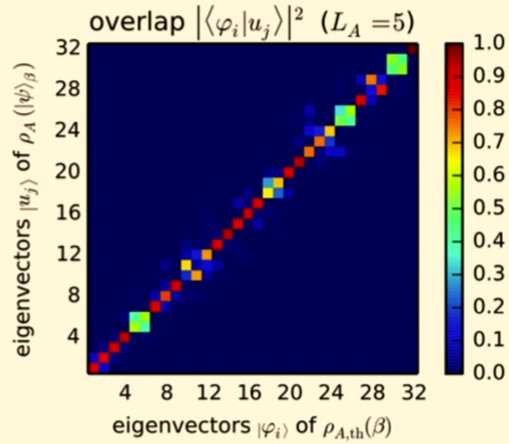
Central Dogma is more general than just groundstates

- Eigenstate Thermalization Hypothesis (ETH):

A single finite-energy density eigenstate $|E\rangle$ of an interacting H is enough to extract H:

$$\rho_A(E) = \mathrm{tr}_{\bar{A}} |E\rangle\langle E| \approx e^{-\beta(E) H_A}/Z$$

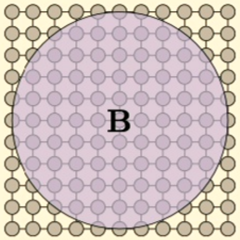
[Srednicki 93, Garrison-Grover 15]



[Garrison-Grover 1503.00729]

Local conditions

A consequence of the above Dogma is that there should be *local* conditions on a wavefunction that tell us which category of state it represents:
Is it topologically ordered? Is it a CFT groundstate?
Is it a fracton state?



Once we identify these conditions for a category of states, they can be used in (at least) two ways ('bootstrap'):

1. impose the conditions as axioms and prove structural properties of the universal data.
2. directly look for states satisfying the axioms.

So the goal of Entanglement Bootstrap is to understand the structure of this universal info and learn to extract it from the local density matrix.
Because this universal info is usually encoded in a QFT, this can teach us about QFT.

Entanglement Bootstrap for liquid topological order

[Topological order: a gapped phase of matter with local **excitations** not creatable by any local operator (anyons).

Liquid: the set of anyons does not depend on geometry (no fractons).]

$\Longrightarrow$ we expect a low-energy *topological quantum field theory* (TQFT).

Mathematicians have some opinions about what that is.

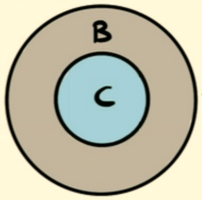
Entanglement Bootstrap axioms

[Shi-Kato-Kim 1906.09376]

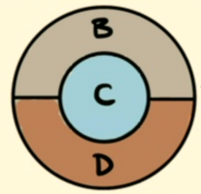
Q: When is σ a 'liquid topologically-ordered groundstate'?

A: *Local* axioms that imply an exact area law for the entanglement.

Axioms:



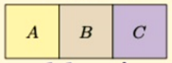
A0: $0 = \Delta(B, C) \equiv (S_{BC} + S_C - S_B)_\sigma$



A1: $0 = \Delta(B, C, D) \equiv (S_{BC} + S_{CD} - S_B - S_D)_\sigma$

on any disk of radius a few lattice sites. $\sigma \equiv \mathrm{tr}_{\overline{\mathrm{disk}}} |\psi\rangle\langle\psi|$.

$\Longrightarrow$ for any (large-enough) regions of the given topology.

- **A0, A1** $\Longrightarrow$ area law for all regions, universal subleading term.
- **A1** $\Longrightarrow$ $0 = I(A : C|B)_\sigma \equiv S_{AB} + S_{BC} - S_B - S_{ABC}$ (quantum Markov chain). 

  Solves quantum marginal problem!
- **A0** $\Longrightarrow$ $0 = I(A : C)_\sigma \equiv S_A + S_C - S_{AC}$. A = any region outside.

Entanglement Bootstrap axioms

[Shi-Kato-Kim 1906.09376]

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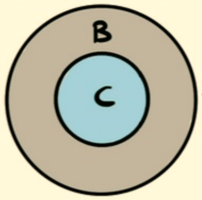
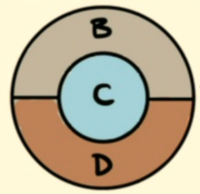


A0: $0 = \Delta(B, C) \equiv (S_{BC} + S_C - S_B)_\sigma$



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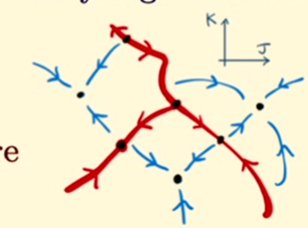
A	B	C

  Solves quantum marginal problem!
- **A0** $\Longrightarrow$ $0 = I(A : C)_\sigma \equiv S_A + S_C - S_{AC}$. A = any region outside.
- This is too strong.
  Exactly true at $\Delta = \infty$ fixed points.
  RG picture: Expect approximate axioms are enough, by zooming out.

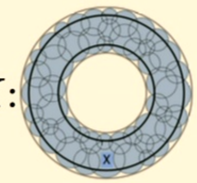


Information convex set

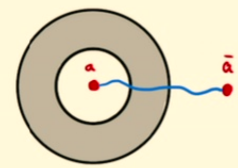
[Isaac Kim Bowen Shi Kohtaro Kato Yuan-Ming Lu]

Associate to each region of space X a convex set of density matrices $\Sigma(X)$: those density matrices which are locally indistinguishable from the reference state σ in (a slight thickening of) X.

X:

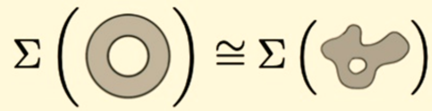


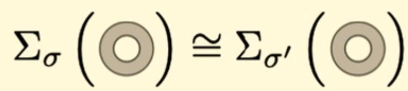
This is an interesting thing because it allows for the possibility of topological excitations *outside* X:



Axioms $\implies$ $\Sigma(X)$ is a topological invariant:

- it is unchanged by smooth deformations of X ('Isomorphism Theorem')
- it should be insensitive to small changes of the reference state within the phase by the argument of Kitaev-Preskill 06.





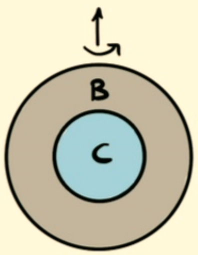
$\cong$ preserves entropy differences, fidelity.

The structure of $\Sigma(X)$ for various X extracts universal data.

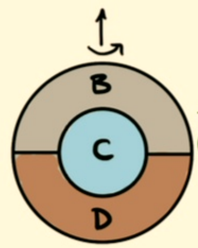
In higher dimensions...

[Jin-Long Huang, JM, Bowen Shi 2112.08398]

Axioms:



A0:
$0 = (S_{BC} + S_C - S_B)_\sigma$



A1:
$0 = (S_{BC} + S_{CD} - S_B - S_D)_\sigma$

Basic results still hold (axioms are self-reproducing, $\Sigma(X)$ is an invariant).

Many more possibilities for X!

eg Excitations along knots and links are detected by Σ(knot or link complement).

They can carry *quantum* information, like a pair of non-abelian anyons.

Can we derive TQFT?

[Huang-Shi-JM 2301.07119]

Apparent obstacle ☹: TQFT eats interesting manifolds and spits out invariants. How can we access these with just single state on a ball?

A method to partially overcome the obstacle ☹ within Entanglement Bootstrap:

Kirby torus trick.

We can use this idea to understand general **braiding non-degeneracy**:
any topological excitation
$\equiv$ cannot be created locally (in 2+1d: 'anyon')
can be detected remotely by some topological excitation
i.e. participates in a unitary S-matrix.

This is an axiom of TQFT, but a theorem of Entanglement Bootstrap.

Entanglement Bootstrap for 1+1d CFT [Ting-Chun David Lin, JM 2303.05444]

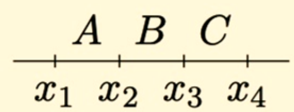
The behavior of the EE of an interval A is $S_A = \frac{c}{3}\log|A|/\epsilon$ [Holzhey et al, Cardy-Calabrese] where c is the *central charge*, a universal measure of the number of dofs per site. Not quite enough.

We know the entanglement Hamiltonian of a single interval for 1+1d CFT with groundstate $|\psi\rangle$ [Cardy-Tonni 16]

$$K_A \equiv -\log\rho_A = \int_A \beta_A(x)\mathbf{h}(x) + S_A \mathbb{1}$$

where $\mathbf{h}(x)$ is the Hamiltonian density.

$$\beta_{[x_1,x_2]}(x) = \frac{c}{6}\frac{(x-x_1)(x_2-x)}{x_2-x_1}\delta_{x\in[x_1,x_2]}$$



$$\eta = \frac{(x_2-x_1)(x_4-x_3)}{(x_3-x_1)(x_4-x_2)}$$

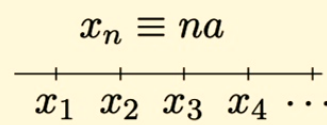
$$h(\eta) = -\eta\log\eta - (1-\eta)\log(1-\eta)$$

Fact: $\beta_{AB}(x) + \beta_{BC}(x) - \eta(\beta_A + \beta_C) - (1-\eta)(\beta_B + \beta_{ABC}) = 0, \ \forall x.$

Entanglement Bootstrap for 1+1d CFT

[Ting-Chun David Lin, JM 2303.05444]

So far: continuum formulae. To compare with quantum critical lattice models, divide up the circle into equally-spaced intervals; regard each as a 'site' (with infinite-dimensional $\mathcal{H}$).



In a finite-dimensional $\mathcal{H}$, the operator equation above does not hold.

The *vector fixed-point equation*

$$\boxed{K_\triangle|\psi\rangle = \frac{c}{3}h(\eta)|\psi\rangle} \text{ (VFPE)}$$

is a more robust statement, but still a very strong constraint (N equations on N components).

Entanglement Bootstrap for 1+1d CFT

Claim: all universal data encoded in local density matrix of a ground state.

But what is the full set of universal data?

Hack: Reconstruct the full **groundstate** and a parent **Hamiltonian** from the local reduced density matrix.

Reconstruct the **groundstate** (merging): solve the VFPE for K_{ABC}:

$$\tilde{K}_{ABC} = (\tilde{K}_{AB} - \tilde{K}_B + \tilde{K}_{BC}) - \frac{\eta}{1-\eta}\left(\tilde{K}_{AB} + \tilde{K}_{BC} - \tilde{K}_A - \tilde{K}_C\right), \quad \tilde{K} \equiv K - \langle K \rangle.$$

$\implies$ We can reconstruct ρ_{ABC} given ρ_{AB} and ρ_{BC}.

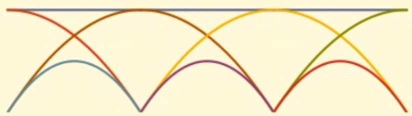
Reduces to Markov/Petz reconstruction when $\eta = 0$.

What about H?

Entanglement Bootstrap for 1+1d CFT [Ting-Chun David Lin, JM 2303.05444]

The density matrix on ABC can be used to reconstruct a lattice **Hamiltonian** for the CFT

$$H_{\text{rec}} = \sum_{i=-\infty}^{\infty} \left(K_{[i,i+2]} - K_{[i,i+1]} \right) = \int dx \mathbf{h}(x)$$

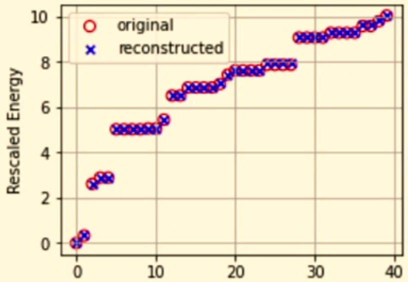


$H_{\text{rec}} = \sum_i \hat{\Delta}(i-1, i, i+1, i+2) \geq 0$ by operator weak monotonicity [Ting-Chun Lin, Kim, Hsieh 22]

$|\psi\rangle$ satisfies VFPE $\implies$ $|\psi\rangle$ is the groundstate of H_{rec}.

$\exists$ a similar formula for the rest of the global conformal generators!

It works even when the local Hilbert space is small:



[e.g., Heisenberg chain on 8 sites]

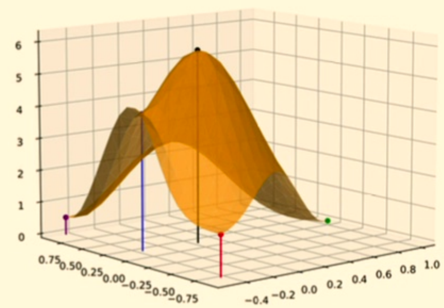
Entanglement Bootstrap for 1+1d CFT [Ting-Chun David Lin, JM 2303.05444]

Wonderful surprise: VFPE is the condition to extremize the function

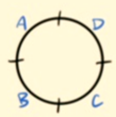
$$\boxed{S_\Delta(|\psi\rangle) \equiv S_{AB} + S_{BC} - \eta(S_A + S_C) - (1-\eta)(S_B + S_{ABC})} \geq 0\ .$$

Key: $dS = \langle d\psi|K|\psi\rangle + \langle\psi|K|d\psi\rangle$

The value of this function at the critical point is the central charge (times $h(\eta)/3$).



Conjecture: critical points of this function $\leftrightarrow$ RG fixed points



Moreover: it is numerically effective even for tiny systems! In the table at right A, B, C (and their complement on S^1) is each a single, measly qubit:

c	Description	Explicit form up to on-site unitary
0	cat states	$a\|0000\rangle + b\|1111\rangle$
0.526	Ising CFT	
1.132	W state	$\frac{1}{2}(\|0001\rangle + \|0010\rangle + \|0100\rangle + \|1000\rangle)$
1.211	XX model	$-\frac{1}{2}(\|0101\rangle + \|1010\rangle) + \frac{1}{2\sqrt{2}}(\|0011\rangle + \|0110\rangle + \|1100\rangle + \|1001\rangle)$
1.245	Heisenberg	$-\frac{1}{\sqrt{3}}(\|0101\rangle + \|1010\rangle) + \frac{1}{2\sqrt{3}}(\|0011\rangle + \|0110\rangle + \|1100\rangle + \|1001\rangle)$
3.510	ferromagnet	$\frac{1}{\sqrt{3}}\|0000\rangle + \frac{1}{\sqrt{6}}(\|0111\rangle + \|1011\rangle + \|1101\rangle + \|1110\rangle)$
4.165	Fibonacci chain	$\frac{1}{\sqrt[4]{5}}\left(\|0000\rangle + \sqrt{\frac{\sqrt{5}-1}{2}}(\|0101\rangle + \|1010\rangle)\right)$
6.000	maximum point	$\frac{1}{2}(\|0000\rangle + \|1111\rangle + \|0101\rangle + \|1010\rangle)$

Final words.

Central Dogma of Entanglement Bootstrap: all the universal data about a state of matter is encoded in a local region of a single representative wavefunction.

So far: liquid bulk topological order, gapped interfaces between topological orders, 1+1d CFT, 2+1d chiral states.

These ideas complement nicely other bootstrap strategies.

Future: Higher-dimensional CFT. Non-relativistic CFT? Non-unitary CFT? Fractons? Thermal states? Non-equilibrium steady states? String theory?

Entanglement Bootstrap for 1+1d CFT [Ting-Chun David Lin, JM 2303.05444]

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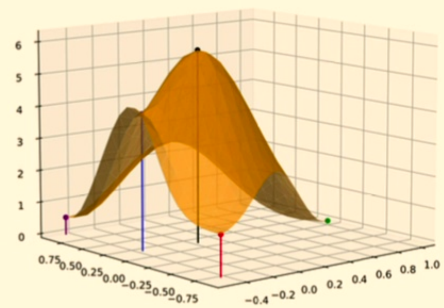

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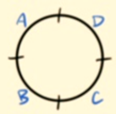

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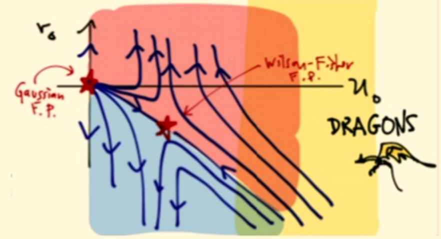
Fixed points of the RG



When we encounter a dynamical system like this

$$\vec{J}_{r+1} = \beta(\vec{J}_r)$$

the first question to ask is:

What are the *fixed points*? $\vec{J}_\star = \beta(\vec{J}_\star)$

The fixed point theory is scale-invariant:

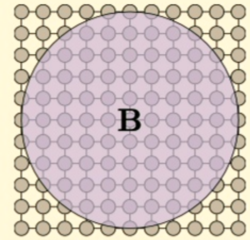
if you change your resolution you get the same picture back.

Under the RG transformation, the gap transforms like $\Delta \to a\Delta$.

At a fixed point, $\Delta \stackrel{!}{=} a\Delta$.

This requires either $\Delta = \infty$ (gapped) or $\Delta = 0$ (gapless).

Reasons to believe the Central Dogma



Central Dogma: All the universal properties of a state of matter are encoded in a single representative density matrix on a ball.

Some evidence: (Not mathematical physics)

- In a groundstate of a local Hamiltonian, the EE satisfies an *area law*, $S_B \sim |\partial B|$ (modulo some interesting exceptions and pathologies)

- Equal time correlations of local operators know about the gap:

$$\langle\psi|\mathcal{O}(x)^\dagger\mathcal{O}(y)|\psi\rangle = \text{tr}_B \rho_B \mathcal{O}(x)^\dagger \mathcal{O}(y) \sim \begin{cases} e^{-|x-y|/\xi}, & \text{gapped} \\ \frac{1}{|x-y|^{2\alpha}}, & \text{gapless} \end{cases}$$

- In a groundstate with liquid topological order, for $B \sim$ disk, $S_B = \frac{|\partial B|}{\epsilon} - \gamma, \gamma =$ topo. entanglement entropy (TEE) extracts anyon data

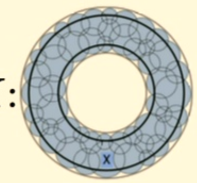
[Levin-Wen, Kitaev-Preskill 06, Grover-Turner-Vishwanath]

Information convex set

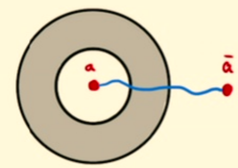
[Isaac Kim Bowen Shi Kohtaro Kato Yuan-Ming Lu]

Associate to each region of space X a convex set of density matrices $\Sigma(X)$: those density matrices which are locally indistinguishable from the reference state σ in (a slight thickening of) X.

X:

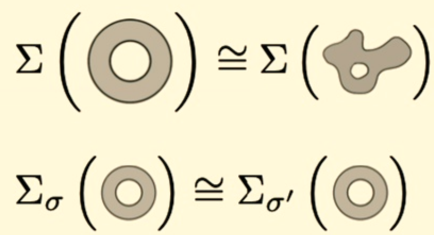


This is an interesting thing because it allows for the possibility of topological excitations *outside* X:



Axioms $\implies$ $\Sigma(X)$ is a topological invariant:

- it is unchanged by smooth deformations of X ('Isomorphism Theorem')
- it should be insensitive to small changes of the reference state within the phase by the argument of Kitaev-Preskill 06.



$\cong$ preserves entropy differences, fidelity.

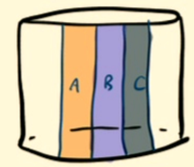
Entanglement Bootstrap for 2+1d chiral states

[Isaac Kim, Xiang Li, Ting-Chun David Lin, JM, Bowen Shi, in progress]

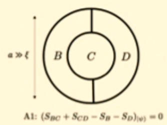
Next challenge: gapped states in 2+1d with gapless boundaries. *e.g.* $c_- \neq 0$.

Consider such a chiral state on a cylinder. By dimensional reduction, this is a (non-chiral) 1+1d CFT.

$$(\star) \implies \left(\eta \hat{\Delta}(A,B,C) + (1-\eta)\hat{I}(A:C|B)\right)|\psi\rangle = \frac{c}{3}h(\eta)|\psi\rangle$$



$$\hat{\Delta}(A,B,C) \equiv K_{AB} + K_{BC} - K_A - K_C$$
$$\hat{I}(A:C|B) \equiv K_{AB} + K_{BC} - K_B - K_{ABC}.$$



Bulk A1 plus some weak assumptions $\implies$ fixed point equation on a single boundary:

$$\boxed{\left(\eta \hat{\Delta}(AA',B,CC') + (1-\eta)\hat{I}(A:C|B)\right)|\psi\rangle = \frac{c}{3}h(\eta)|\psi\rangle.}$$