Title: On BRST Complexes coming from 4d N=2 SCFTs

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Abstract:

Vertex operator algebras (VOAs) arise in many corners of supersymmetric quantum field theory. One particularly influential instance is in 4d N=2 superconformal field theories, whereby the VOA is realized as the cohomology of a suitable supercharge. Unitarity of the underlying SCFT imposes strong constraints on the structure of the resulting VOA. In this talk, I will describe one aspect of how the unitary of the underlying SCFT constrains this VOA: in the context of superconformal gauge theories, the resulting BRST complex shares a striking resemblance to the de Rham complex of a compact Kähler manifolds. I will finish with several consequences of this observation, e.g. the formality of these BRST complexes as in the work of Deligne-Griffiths-Morgan-Sullivan on compact Kähler manifold. This is based on work in progress with C. Beem.

$Z_i = X(z_i) + u_i Y(z_i)$; $Z^a_{1\,b} Z^c_{2\,d} \sim \frac{1}{\ldots}\frac{u_2 - u_1}{\ldots} \delta^a_d \delta^c_b$ $T(z) J^b(0) \sim$

On BRST Complexes coming from 4d $\mathcal{N}=2$ SCFTs

jt WIP with C. Beem

Review of SCFT/VOA correspondence

BLLPRvR '13

$\mathbb{V}$

jt WIP with C. Beem

Review of SCFT/VOA correspondence

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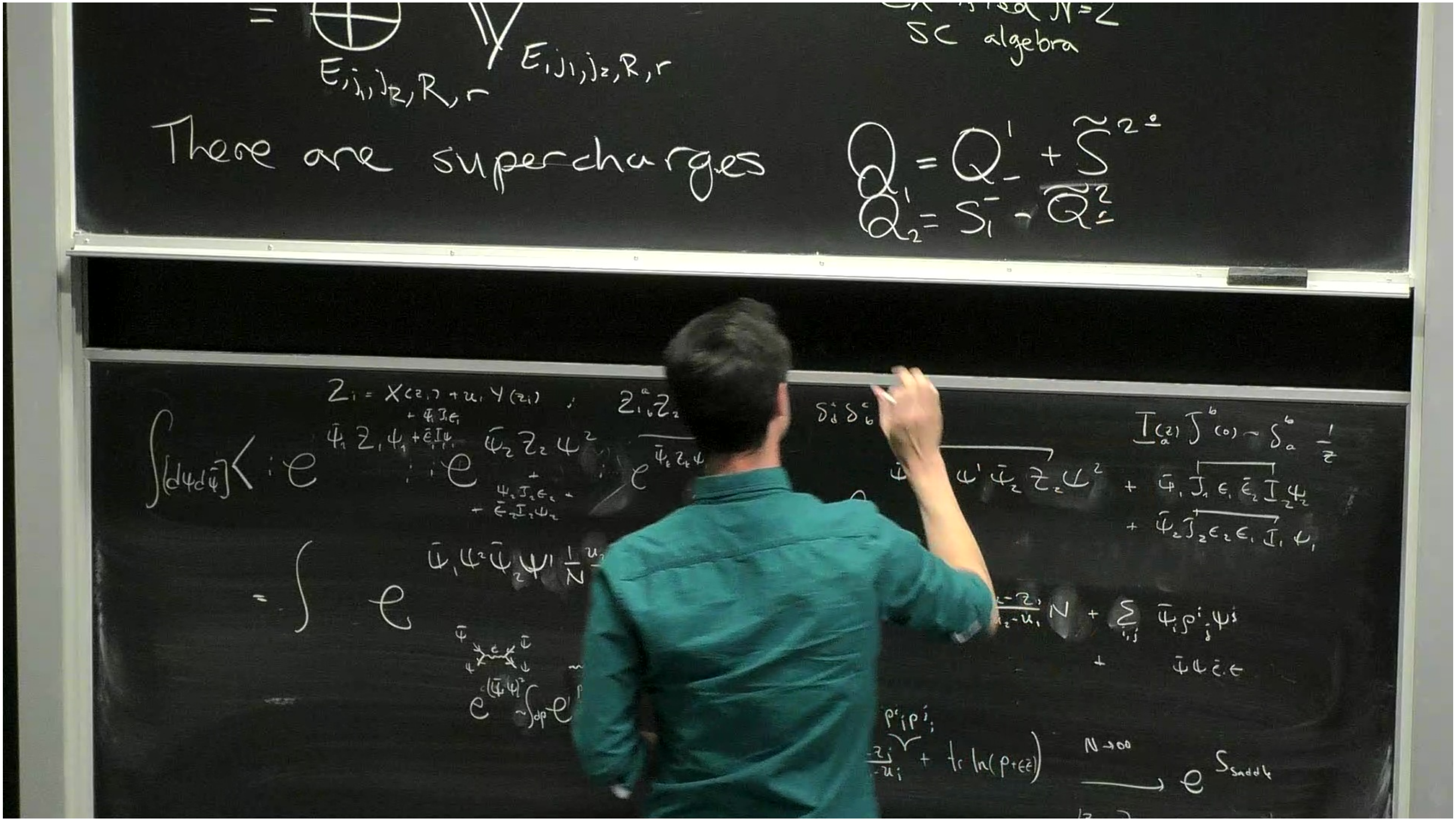
$$\mathbb{V}_{\mathcal{T}} = \left\{ \begin{array}{l} \text{vec. space of} \\ \text{local operators in } \mathcal{T} \end{array} \right\} \circlearrowleft \underbrace{\mathfrak{sl}(4|2)}_{\substack{\text{CX-ified } \mathcal{N}=2 \\ \text{SC algebra}}}$$

$$= \bigoplus_{E, j_1, j_2, R, r} \mathbb{V}_{E, j_1, j_2, R, r}$$


E, j1, j2, R, r
SC algebra
There are supercharges

$$Q_2 = S_1^- - \widetilde{Q}^2_-$$

Such that:

1) square-zero

generate conf of $\mathbb{C}\times 0 \subset \mathbb{C}^2$ $\quad$ $sl(2)_R$

2) commute with $L_{0,\pm 1}$, $\widehat{L}_{0,\pm 1} = \overline{L}_{0,\pm 1} - R_{0,\pm 1}$

3) $\widehat{L}_{0,\pm 1}$ are Q_i-exact

$u(1)_r$ $\quad$ rotations $\perp$ to $\mathbb{C}\times 0 \subset \mathbb{C}^2$

4) $\{Q_1, Q_2\} = \mathcal{Z} = r + M^\perp$

Operators surviving Q_1 or Q_2 coho. satisfy

$$\frac{1}{2}\left(E-(j_1+j_2)\right)-R=0 \qquad r+j_1-j_2=0$$

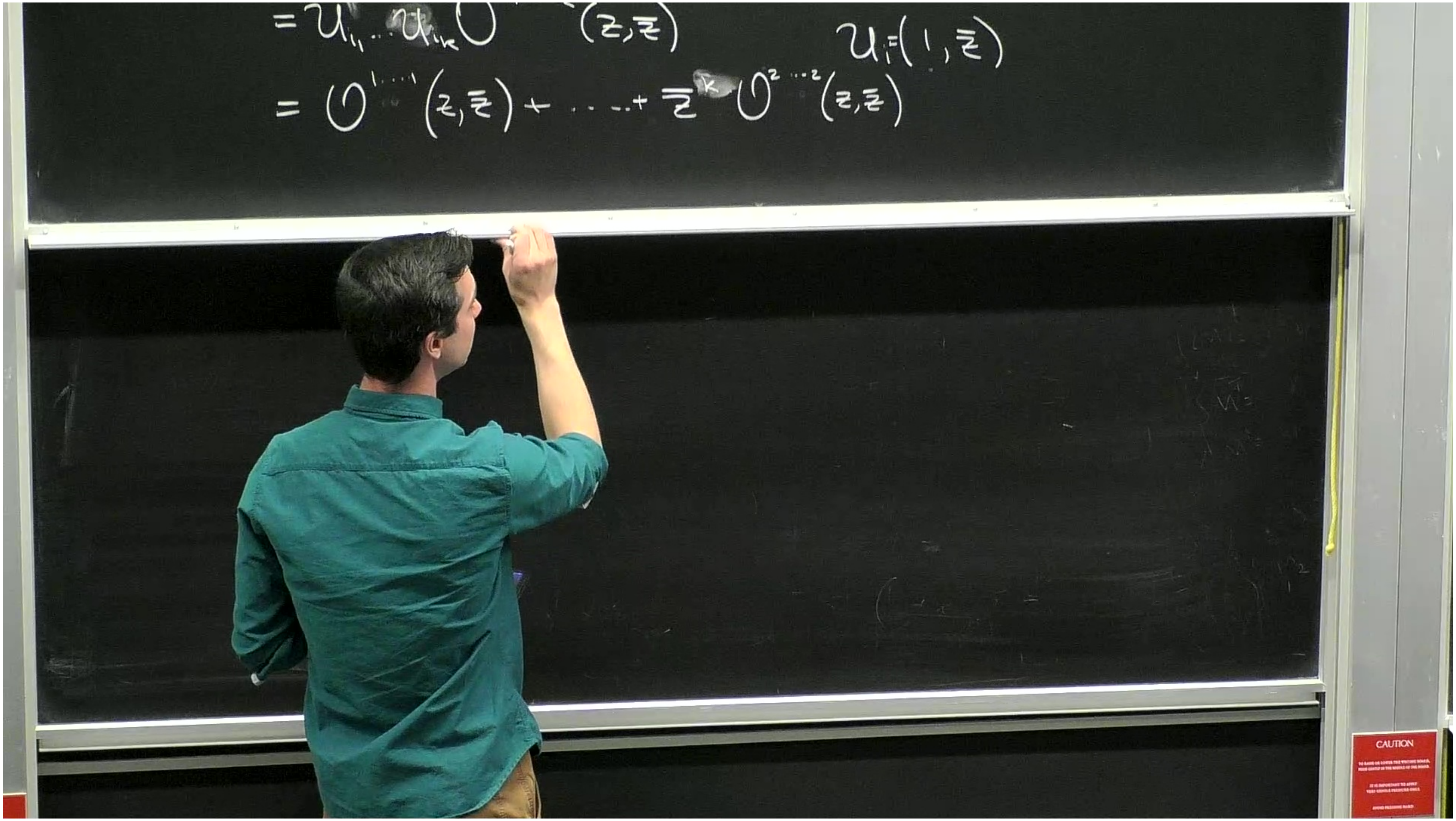
$\frac{1}{2}\left(E-(j_1+j_2)\right)-R=0$ $\qquad r+j_1-j_2=0$ $\qquad$ operators

unitarity implies Schur ops are HW vectors w.r.t. $sl(2)_R$

2) allows us to "twisted translate"

$$\mathcal{O}(z,\bar z) = e^{zL_{-1}+\bar z\hat{L}_{-1}}\,\mathcal{O}^{1\dots1}(0)\,e^{-zL_{-1}-\bar z\hat{L}_{-1}}$$

$$= u_{i_1}\cdots u_{i_k}\,\mathcal{O}^{i_1\dots i_k}(z,\bar z) \qquad u_i=(1,\bar z)$$


$= u_{i_1} \cdots u_{i_k} \mathcal{O}(z, \bar{z})$
$u_i = (1, \bar{z})$
$= \mathcal{O}^{1 \cdots 1}(z, \bar{z}) + \cdots + \bar{z}^k \mathcal{O}^{2 \cdots 2}(z, \bar{z})$
CAUTION

$[\mathcal{O}(z,\bar{z})]_{Q_i}$ is inped. of $\bar{z}$ (due to 3)

$$V_T = H^0(\mathbb{V}_T, Q_1)$$

$$\simeq H^0(\mathbb{V}_T, Q_2)$$

vertex algebra

$[\mathcal{O}(z,\bar{z})]_{Q_i}$ is inped. of $\bar{z}$ (due to 3)

$$\mathcal{V}_T = H^{\overset{U(1)_r}{\bullet}}(\mathbb{V}_T, Q_1)$$

vertex algebra

$$\simeq H^{\bullet}(\mathbb{V}_T, Q_2)$$

For a SC gauging of $T_{matter} \rightsquigarrow \mathcal{V}_{matter}$

$$\mathcal{V} = H^{\bullet+\frac{\infty}{2}}(\hat{\mathfrak{g}}_{-\kappa}, \mathfrak{g}, \mathcal{V}_{matter})$$

relative BRST cohomology

$$= H^{\bullet}((Sf[\mathfrak{g}^{\oplus 2}] \otimes \mathcal{V}_{matter})^{G}, Q)$$

$$Q^{\pm} = \sum_{n\neq 0} \frac{1}{n} :\eta^{\pm,A}_{-n} J_{A,n}: + \sum_{\substack{n,m\neq 0\\ n\neq m}} \frac{f_{ABC}}{2mn} :\eta^{\pm A}_{-n}\,\eta^{\pm B}_{m}\,\eta^{\mp C}_{n-m}:$$

$$k = -2h^{\vee}$$

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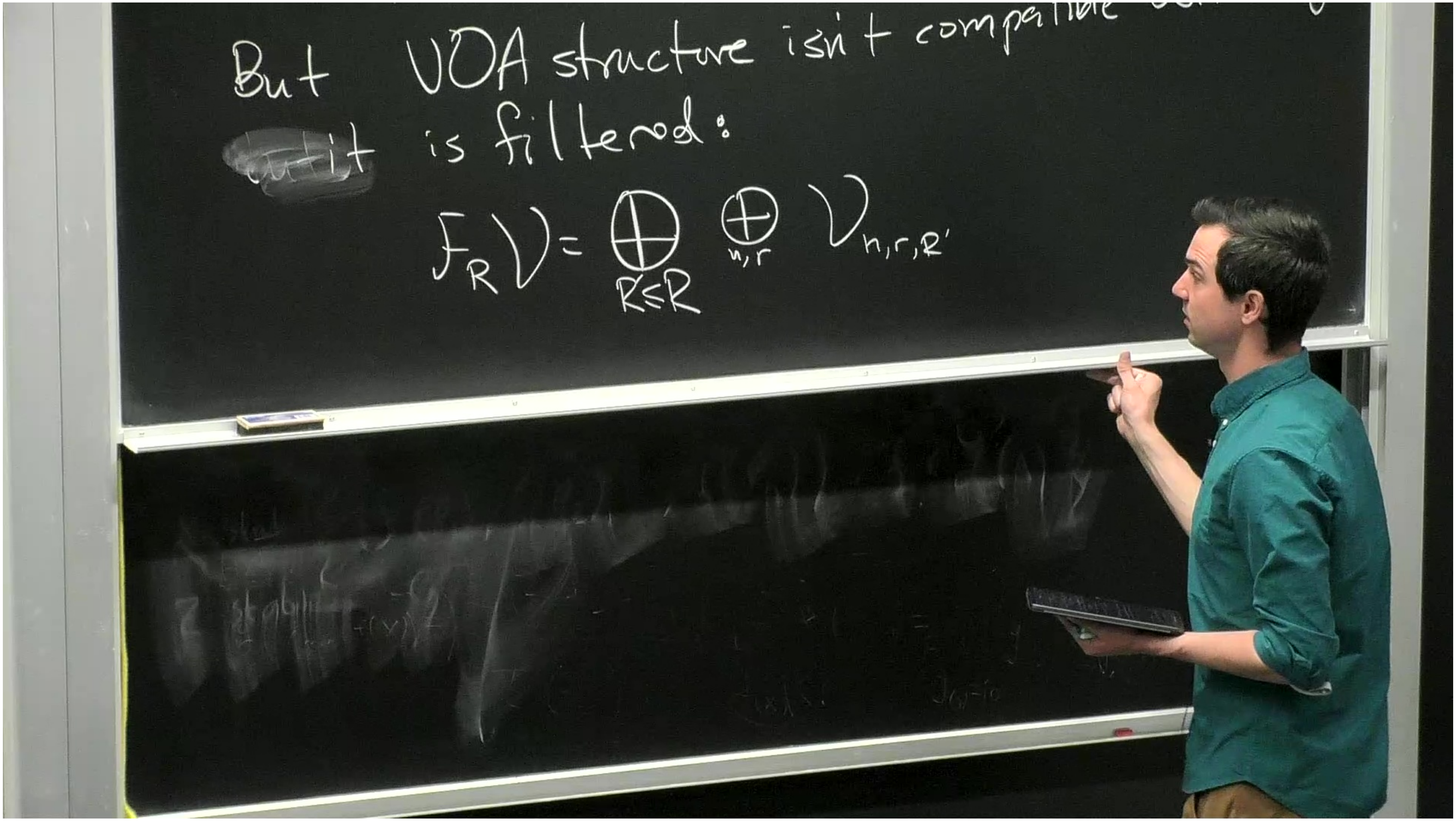
$h = \frac{1}{2}(E + j_1 + j_2)$

$$V = \bigoplus_{h,r,R} V_{h,r,R}$$

But VOA structure isn't compatible with R-grading


But VOA structure isn't
it is filtered:
$F_R V = \bigoplus_{R \leq R} \bigoplus_{n,r} V_{n,r,R'}$

this filtration is good [Li '04]

$a \in \mathcal{F}_{R_a} V, \ b \in \mathcal{F}_{R_b} V$

$$a_n b \in \begin{cases} \mathcal{F}_{R_a+R_b} V & n+h_a > 0 \\ \mathcal{F}_{R_a+R_b-1} V & n+h_a \leq 0 \end{cases}$$

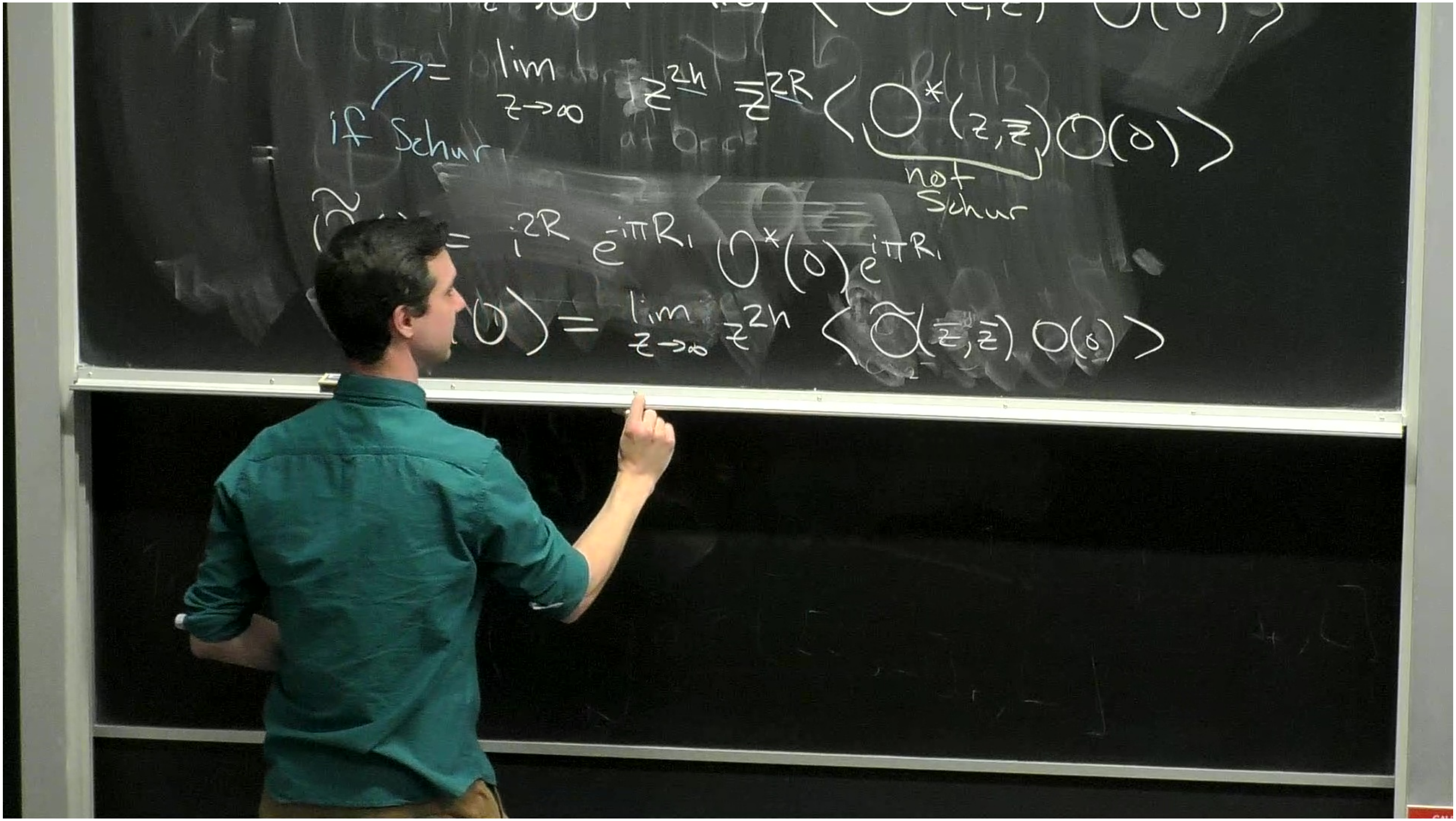
singular terms must lower filtration

Graded Unitarity WIP ABLR

In radial quantization:

$$\langle \mathcal{O} | \mathcal{O} \rangle = \lim_{z \to \infty} |z|^{\underline{2E}} \left(z/\bar{z}\right)^{\underline{j_1+j_2}} \langle \mathcal{O}^*(z,\bar{z})\ \mathcal{O}(0) \rangle$$

$$\overset{\text{if Schur}}{=} \lim_{z \to \infty} z^{\underline{2h}}\ \bar{z}^{\underline{2R}} \langle \mathcal{O}^*(z,\bar{z}) \mathcal{O}(0) \rangle$$



$= \lim_{z\to\infty} z^{2h} \bar{z}^{2R} \langle O^*(z,\bar{z}) O(0) \rangle$
if Schur
not Schur
$\tilde{O}(0) = i^{2R} e^{-i\pi R_1} O^*(0) e^{i\pi R_1}$
$(O) = \lim_{z\to\infty} z^{2h} \langle \tilde{O}(\bar{z},z) O(0) \rangle$

$$\rho : \mathcal{O}(0) \longrightarrow e^{-i\pi R_1}\, \mathcal{O}^*(0)\, e^{i\pi R_1}$$

anti-linear map of schur operators

$$\rho(a_n b) = \rho(a)_n \, \rho(b)$$

auto-morphism of V

$$\rho^2 = (-1)^{2R}$$

$$(V, F, \rho)$$

graded unitary VOA

Ex: hypermultiplet / S_b

$X \longrightarrow \bar{X} \longrightarrow -iY$

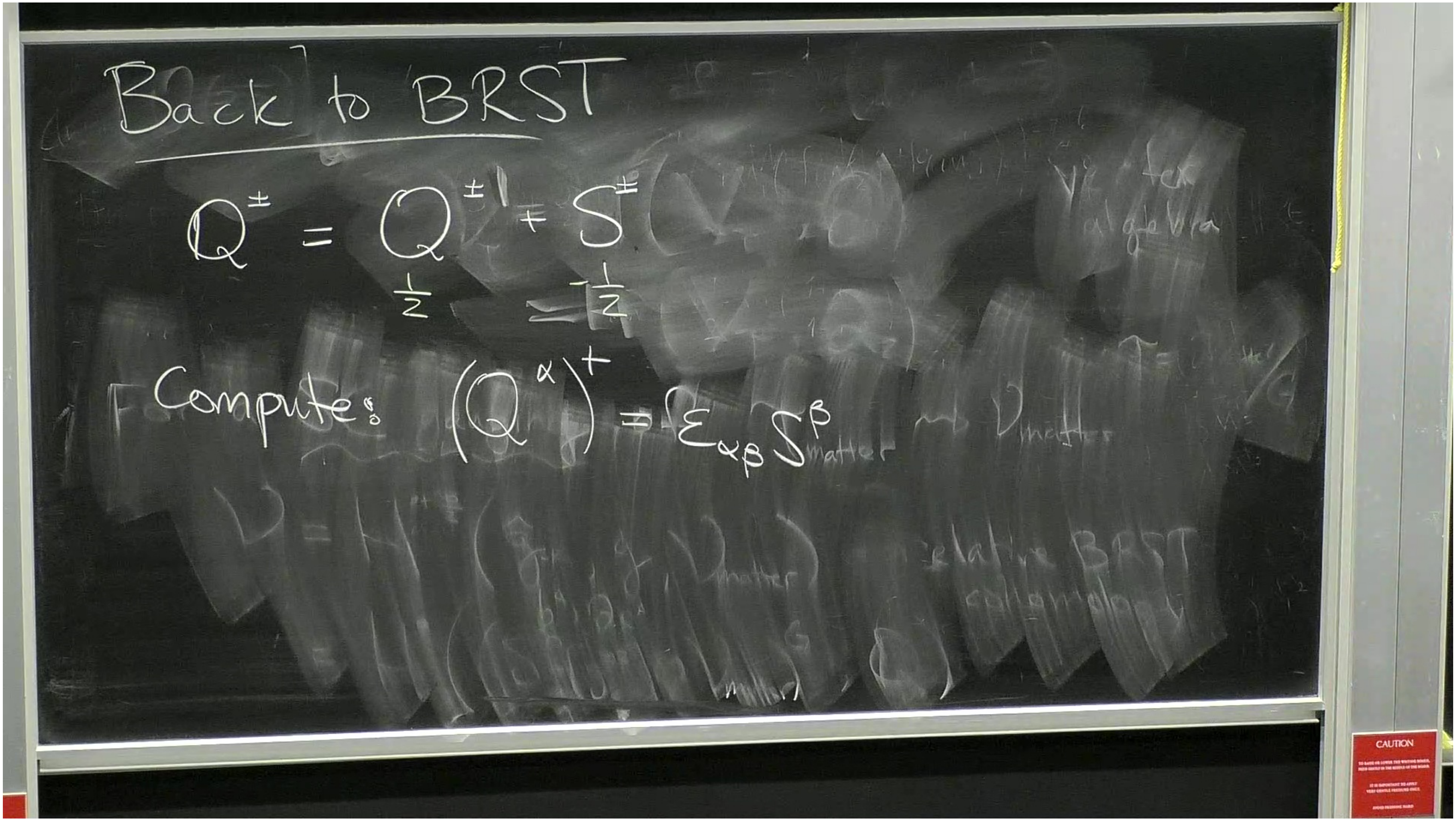
$Y \longrightarrow \bar{Y} \longrightarrow iX$

$$\begin{pmatrix} X & Y \\ -\bar{Y} & \bar{X} \end{pmatrix}$$

$$Y \to \bar{Y} \to iX$$

$$(X_{n-\frac{1}{2}})^\dagger = \begin{cases} Y_{n+\frac{1}{2}} & n \geq 0 \\ -Y_{n+\frac{1}{2}} & n < 0 \end{cases}$$

$$(Y_{-n-\frac{1}{2}})^\dagger = \begin{cases} -X_{n+\frac{1}{2}} & n \geq 0 \\ X_{n+\frac{1}{2}} & n < 0 \end{cases}$$


Back to BRST
$Q^{\pm} = Q^{\pm\prime}_{\frac{1}{2}} + S^{\pm}_{-\frac{1}{2}}$
Compute: $(Q^{\alpha})^{\dagger} = \epsilon_{\alpha\beta} S^{\beta}_{matter}$

$Q = \sum_{n\neq 0} n \ldots \;\; {}_{-n} \; J_{A,n} \quad \sum_{\substack{n,m\neq 0\\ n\neq m}} \frac{\ldots}{2mn} \;\; {}_{-n} \ldots$

$d^c = i(\partial - \bar{\partial})$

$(\mathbb{Q}^\alpha)^\dagger = \bar{\mathbb{Q}}_\alpha$

1) $\{\overset{d,\,d^c}{\mathbb{Q}^\alpha, \mathbb{Q}^\beta}\} = 0 = \{\overset{d^\dagger,\,d^{c\dagger}}{\bar{\mathbb{Q}}_\alpha, \bar{\mathbb{Q}}_\beta}\}$

2) $\{\mathbb{Q}^\alpha, \bar{\mathbb{Q}}_\beta\} = \delta^\alpha_\beta \, \Delta$

1d $N=4$ SQM

$V = \ker\Delta \oplus \mathrm{im}\,\mathbb{Q}^+ \oplus \mathrm{im}\,\bar{\mathbb{Q}}_+$

Consequences

1)

$$\begin{tikzcd} & (\ker Q^{+}, Q^{-}) \arrow[dl] \arrow[dr] & \\ (V, Q^{-}) & & \left(H^{\bullet}(V, Q^{+}),\ \overset{0}{\overset{\shortparallel}{Q^{-*}}}\right) \end{tikzcd}$$

quasi-morphisms of VOA

cf DGMS '75

2)

$$\frac{\ker Q^-}{\mathrm{im}\, Q^-} \simeq \frac{\ker Q^- \cap \ker Q^+}{\mathrm{im}\, Q^- Q^+}$$

$$\uparrow$$

$$H^0(V, Q^-) \circlearrowleft Sp(2)$$

3) $\mathfrak{g} = \mathfrak{g}_1 \oplus \mathfrak{g}_2$

$$\left(V, Q^-\right) \leftarrow \left(\ker Q_1^+, Q_1^-\right) \rightarrow \left(H^0(V, Q_1^+), \underset{\displaystyle Q_2^{-*}}{\overset{\|}{Q^{-*}}}\right) \leftarrow \left(\ker Q_2^{+*}, Q_2^{-*}\right) \rightarrow \left(H^0(H^0(V, Q_1^+), Q_2^{+*}), 0\right)$$

$$H^0(V, Q^-) \simeq H^0(H^0(V, Q_1^-), Q_2^{-*})$$