Title: Causal Inference Meets Quantum Physics

Speakers: Robert Spekkens

Collection/Series: Theory + AI Workshop: Theoretical Physics for AI

Date: April 09, 2025 - 11:00 AM

URL: https://pirsa.org/25040086

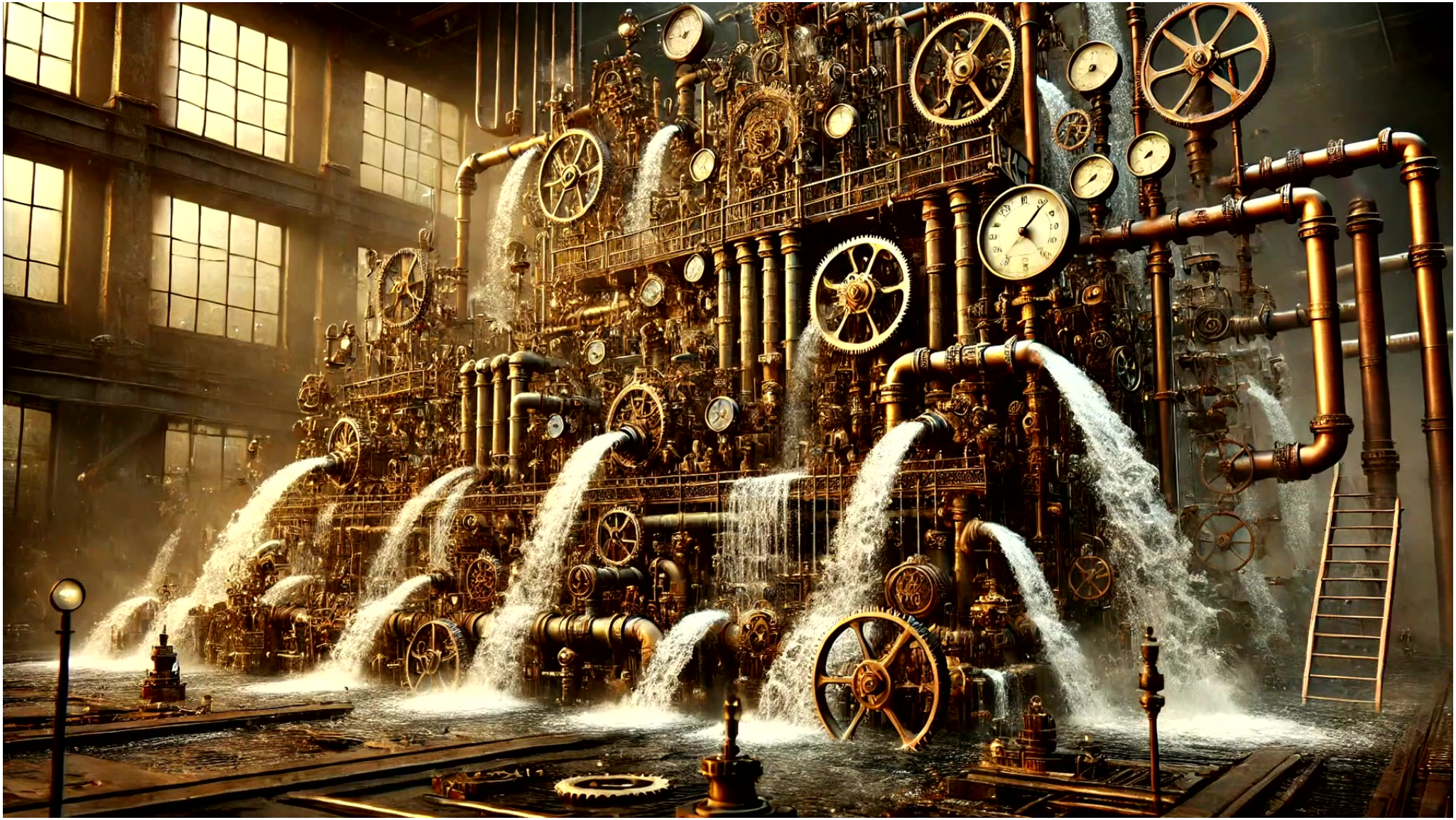
Abstract:

Can the effectiveness of a medical treatment be determined without the expense of a randomized controlled trial? Can the impact of a new policy be disentangled from other factors that happen to vary at the same time? Questions such as these are the purview of the field of causal inference, a general-purpose science of cause and effect, applicable in domains ranging from epidemiology to economics. Researchers in this field seek in particular to find techniques for extracting causal conclusions from statistical data. Meanwhile, one of the most significant results in the foundations of quantum theory—Bell's theorem—can also be understood as an attempt to disentangle correlation and causation. Recently, it has been recognized that Bell's result is an early foray into the field of causal inference and that the insights derived from 60 years of research on his theorem can supplement and improve upon state-of-the-art causal inference techniques. In the other direction, the conceptual framework developed by causal inference researchers provides a fruitful new perspective on what could possibly count as a satisfactory causal explanation of the quantum correlations observed in Bell experiments. Efforts to elaborate upon these connections have led to an exciting flow of techniques and insights across the disciplinary divide. This talk will highlight some of what is happening at the intersection of these two fields.

Causal Inference Meets Quantum Physics

Robert Spekkens
Perimeter Institute for Theoretical Physics
Waterloo, Canada

Theory + AI workshop, Perimeter Institute, Waterloo
April 9, 2025



Dynamicist vs pragmatist traditions in physics

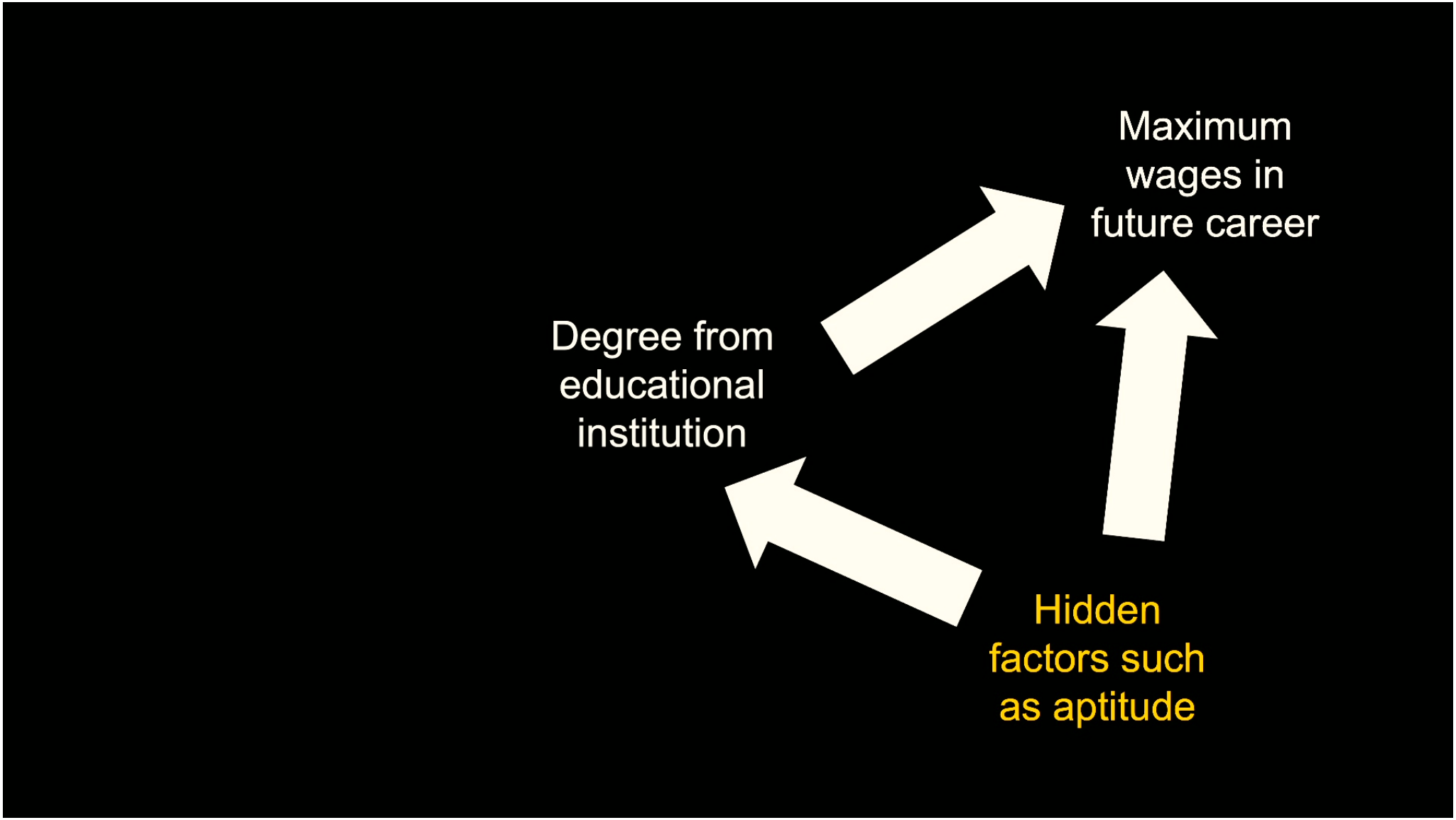
Dynamicist: the physicist's job is to describe the natural dynamical behaviour of a system, without reference to agents or their purposes

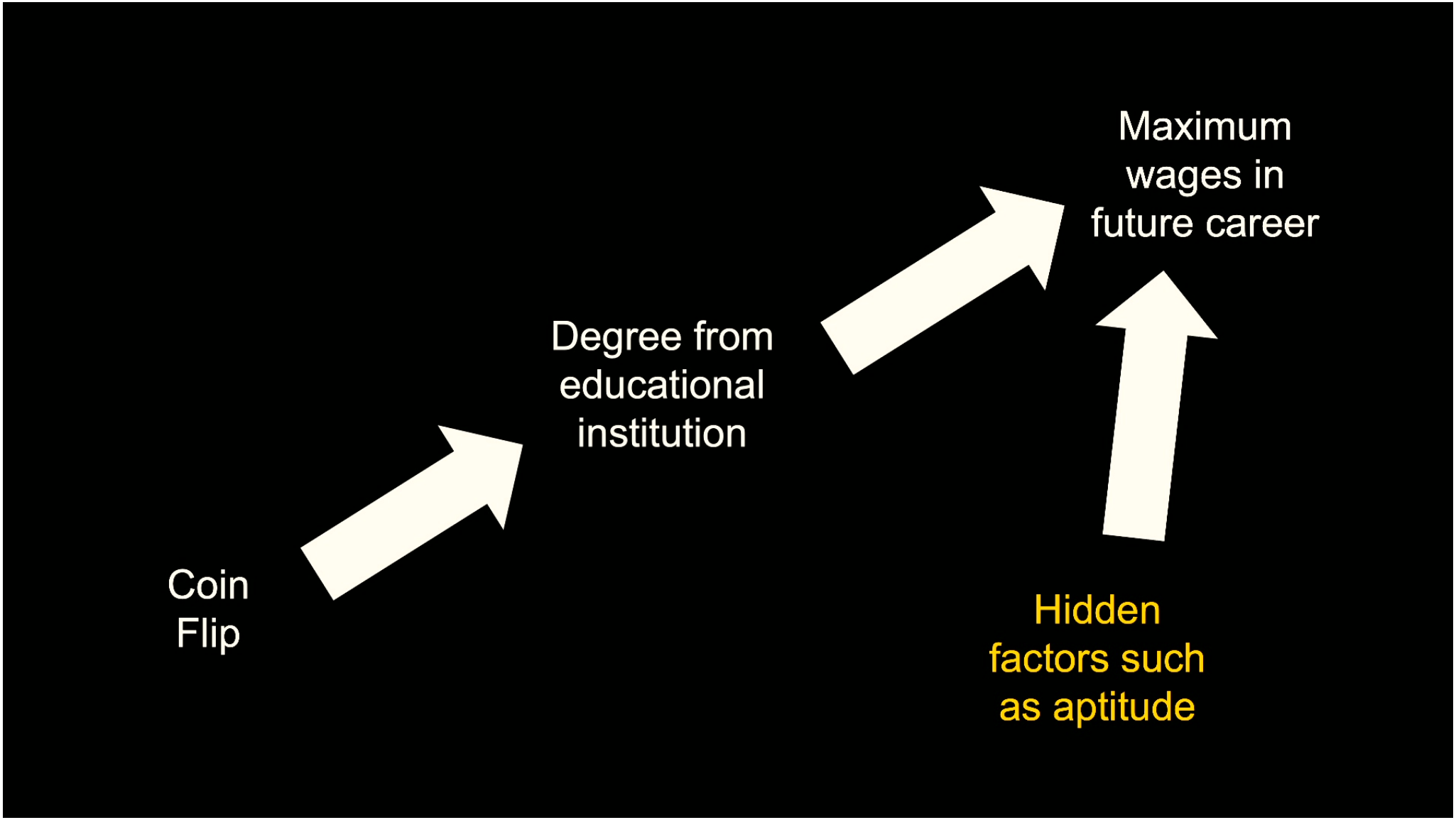
Pragmatist: the laws of physics are characterized in terms of the extent to which agents can achieve various goals within a universe obeying these laws

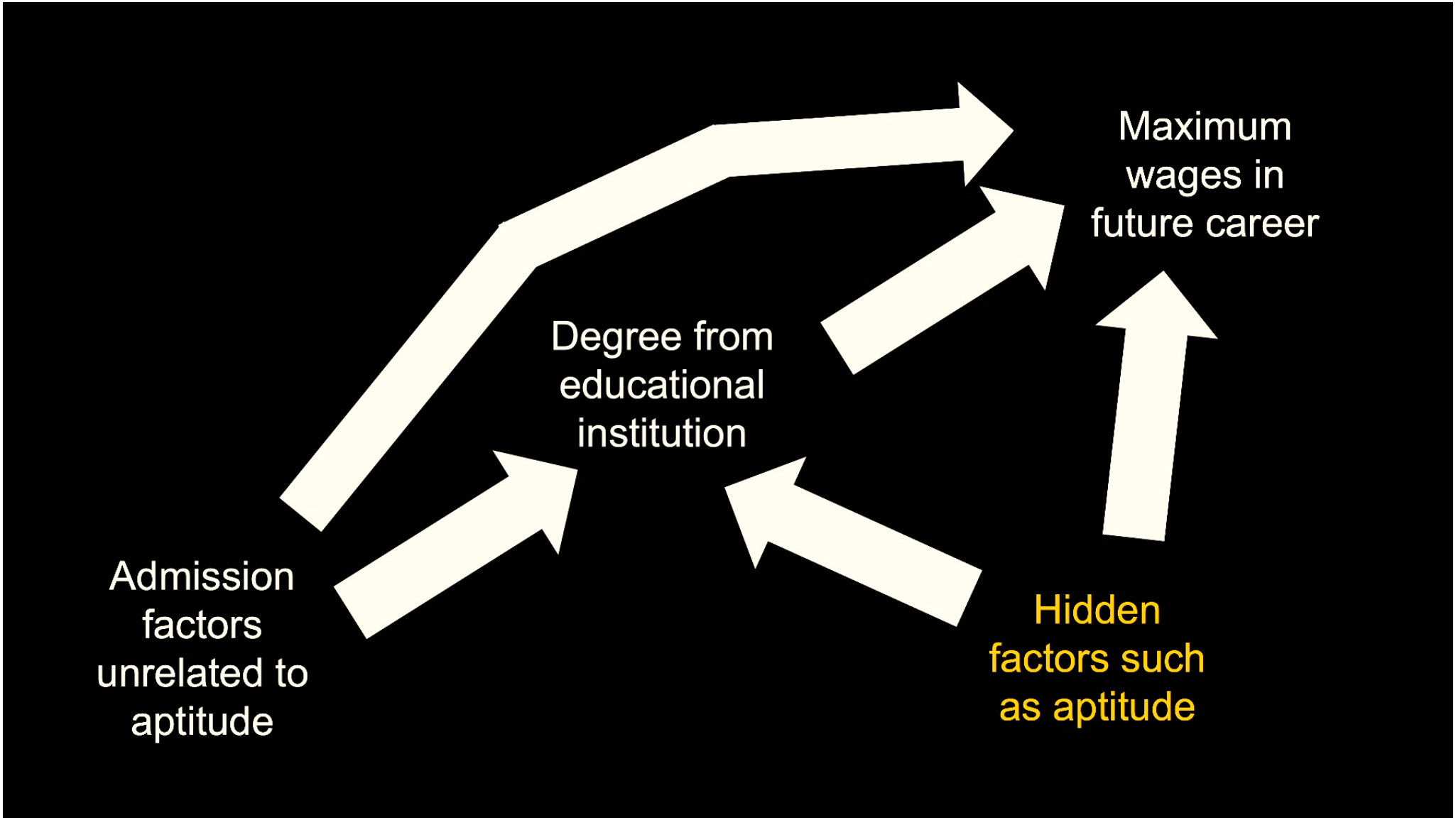
Maximum wages in future career above some threshold?

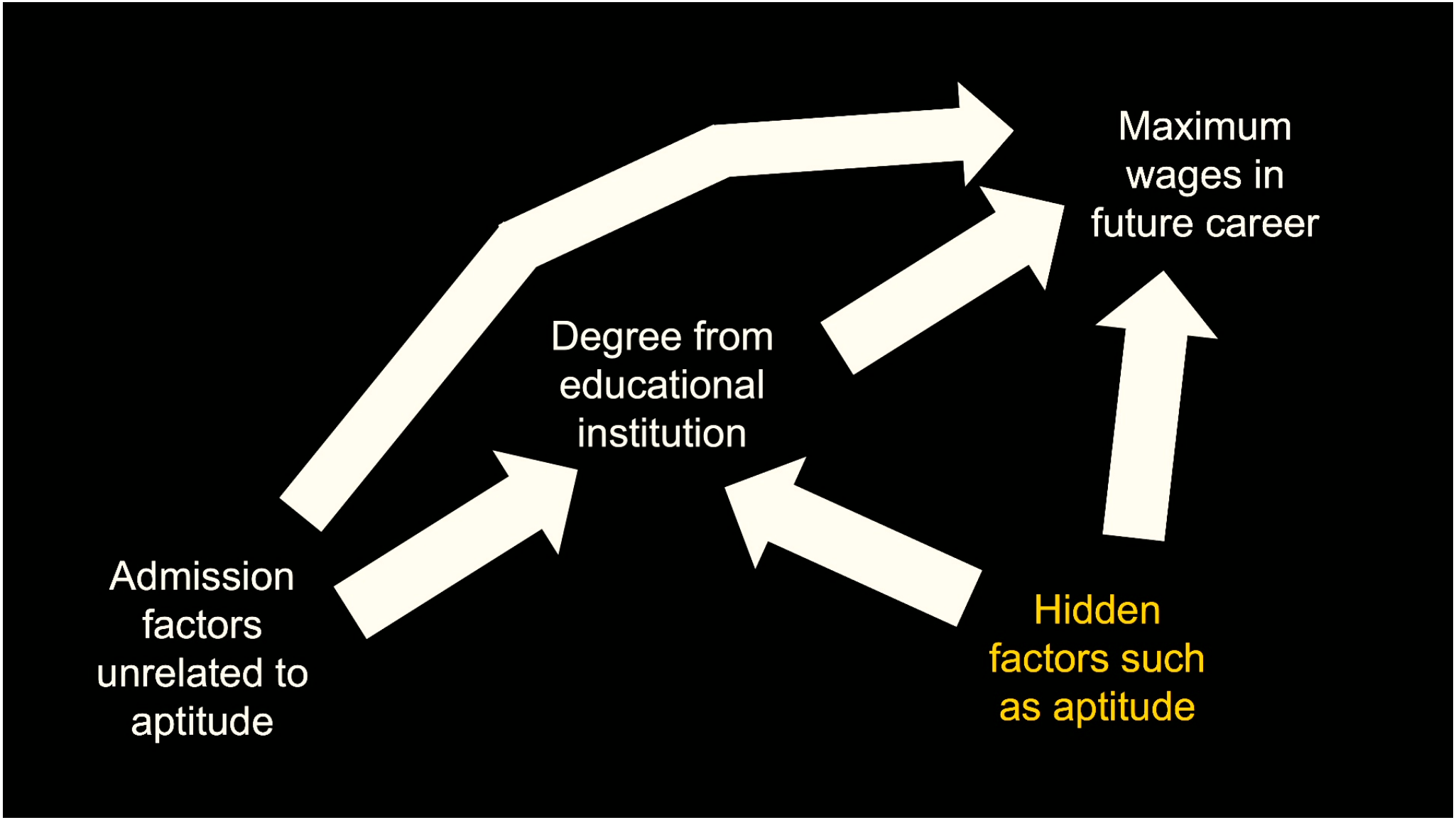
Degree from educational institution?

	Yes	No
Yes	79%	21%
No	43%	57%

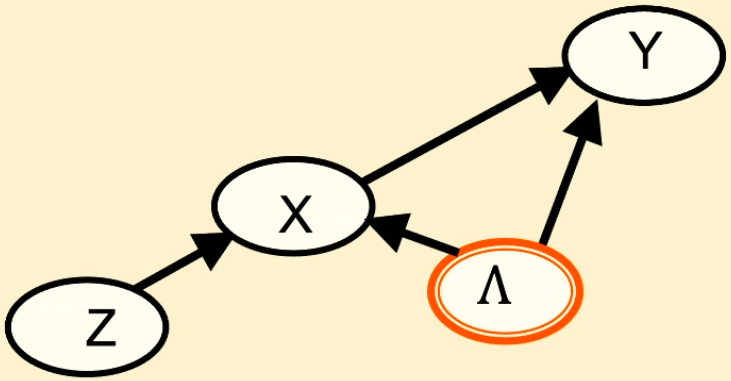

Maximum wages in future career
Degree from educational institution
Hidden factors such as aptitude


Maximum wages in future career
Degree from educational institution
Coin Flip
Hidden factors such as aptitude


Maximum wages in future career
Degree from educational institution
Admission factors unrelated to aptitude
Hidden factors such as aptitude


Maximum wages in future career
Degree from educational institution
Admission factors unrelated to aptitude
Hidden factors such as aptitude

Causal structure



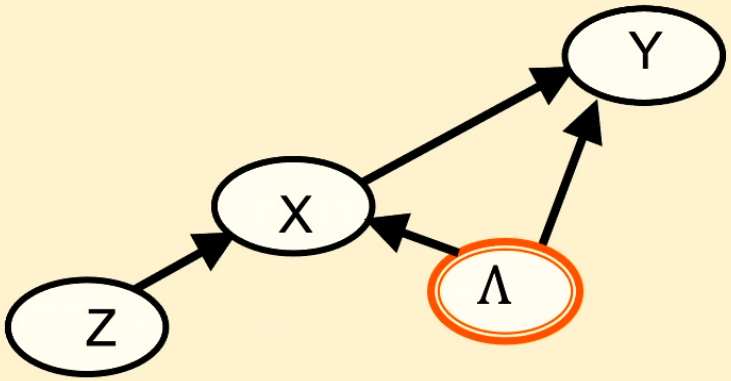
Parameters

$$P_{X|\Lambda Z}$$
$$P_{Y|\Lambda X}$$
$$P_{\Lambda}$$
$$P_{Z}$$

$$P_{XYZ} = \sum_{\Lambda} P_{Y|X\Lambda} P_{X|Z\Lambda} P_{\Lambda} P_{Z}$$

A distribution is said to be compatible with a given causal structure if there are parameters that yield it

Causal structure



Parameters

$$P_{X|\Lambda Z}$$
$$P_{Y|\Lambda X}$$
$$P_{\Lambda}$$
$$P_{Z}$$

$$P_{XYZ} = \sum_{\Lambda} P_{Y|X\Lambda} P_{X|Z\Lambda} P_{\Lambda} P_{Z}$$

Example of causal compatibility constraint:

$$P_{XY|Z}(00|0) + P_{XY|Z}(01|1) \leq 1$$

Pearl, 1993

The evidence

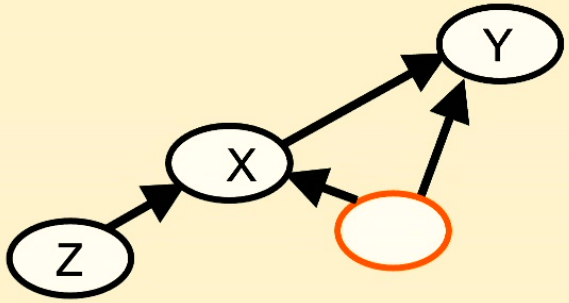
Z=0

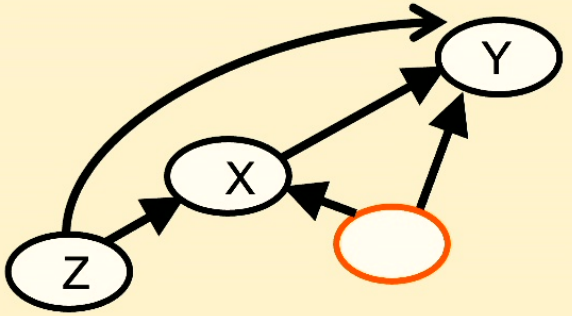
	Y=0	Y=1
X=0	0.79	0.21
X=1	0.43	0.57

Z=1

	Y=0	Y=1
X=0	0.59	0.41
X=1	0.39	0.61

The hypotheses





The evidence

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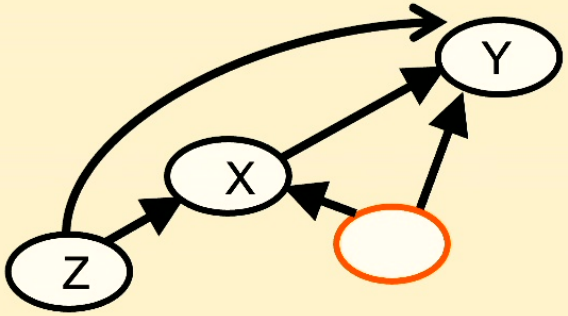
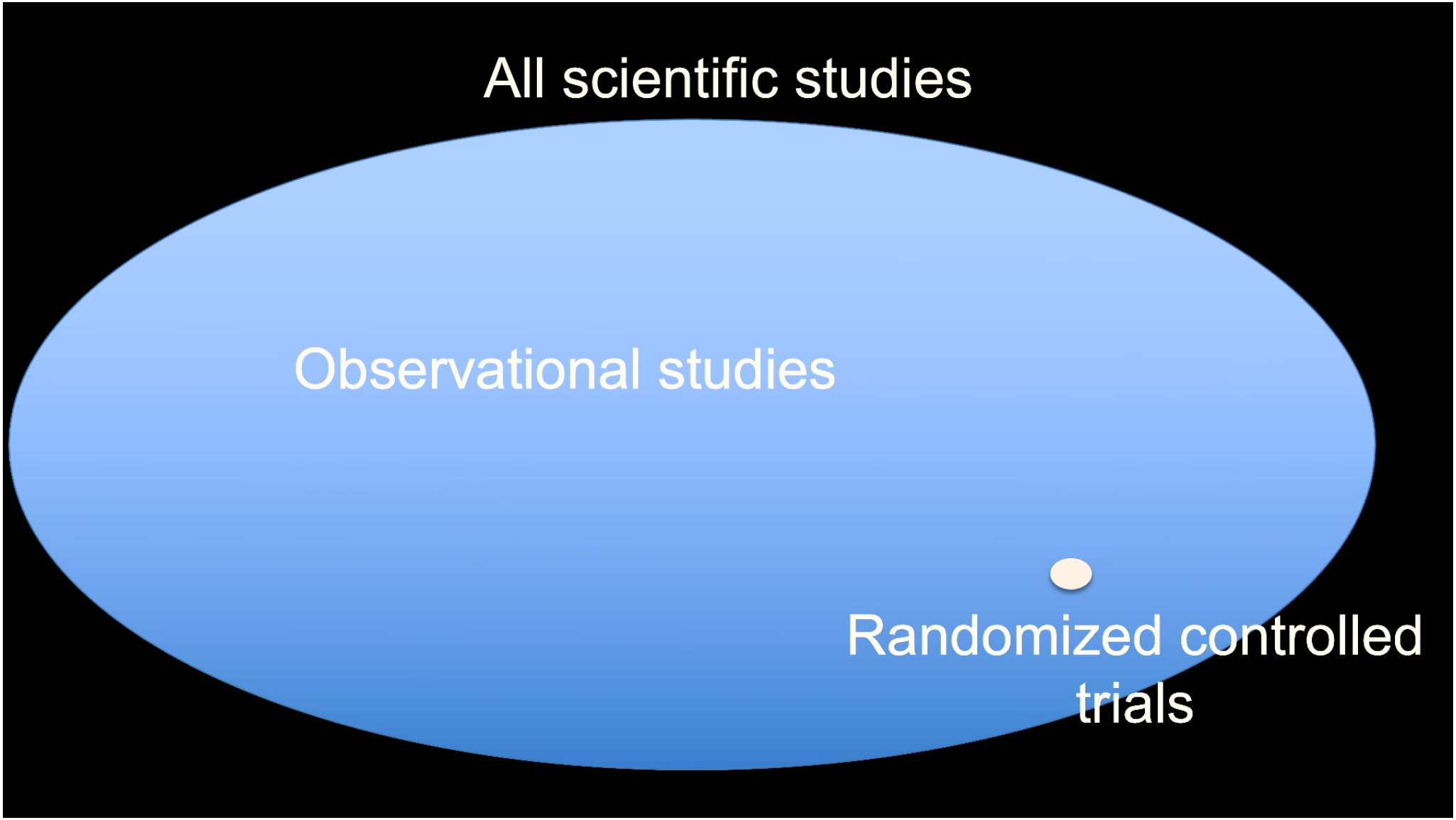
Violates the instrumental inequality!

The hypotheses

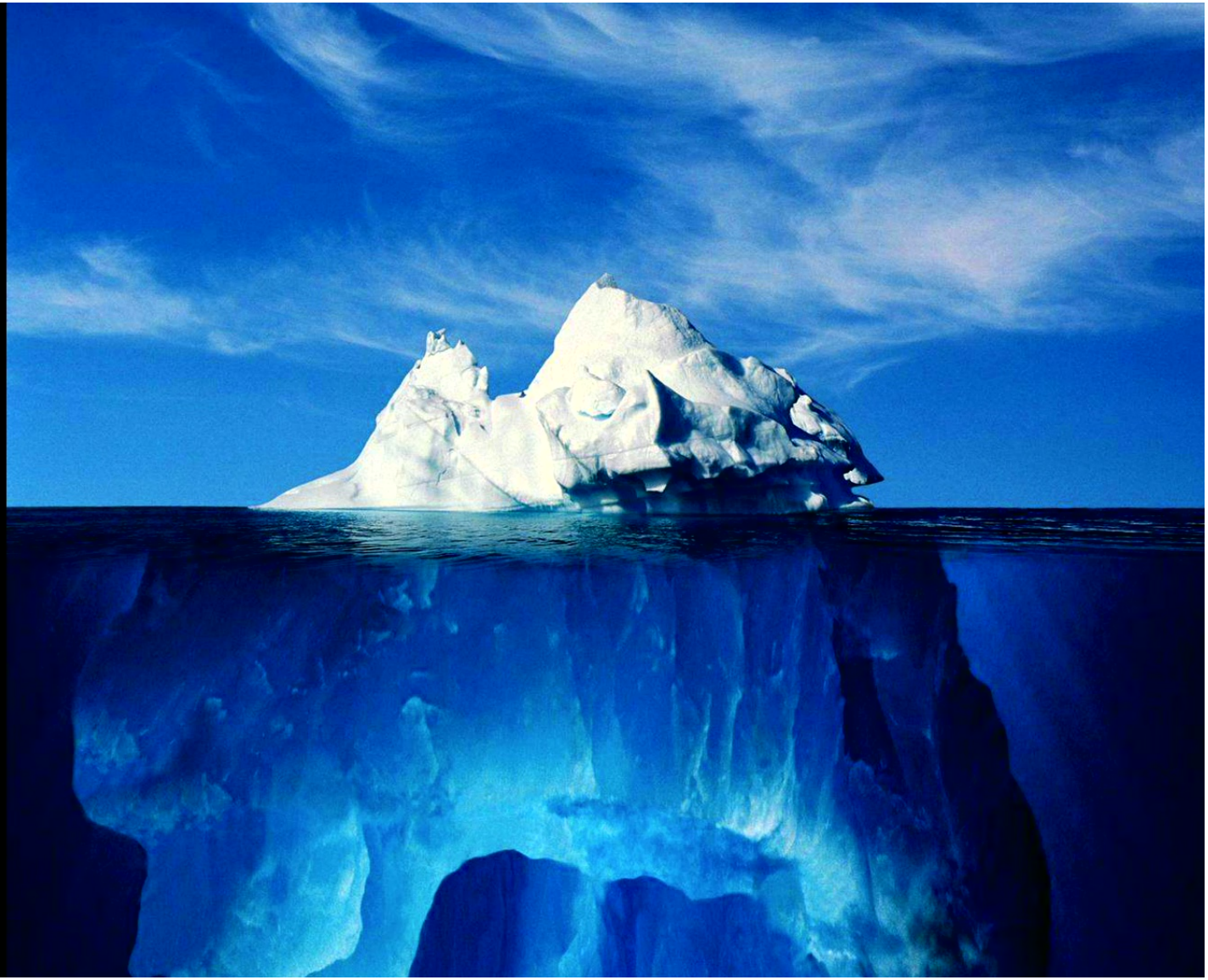
Implies a constraint:
the instrumental inequality

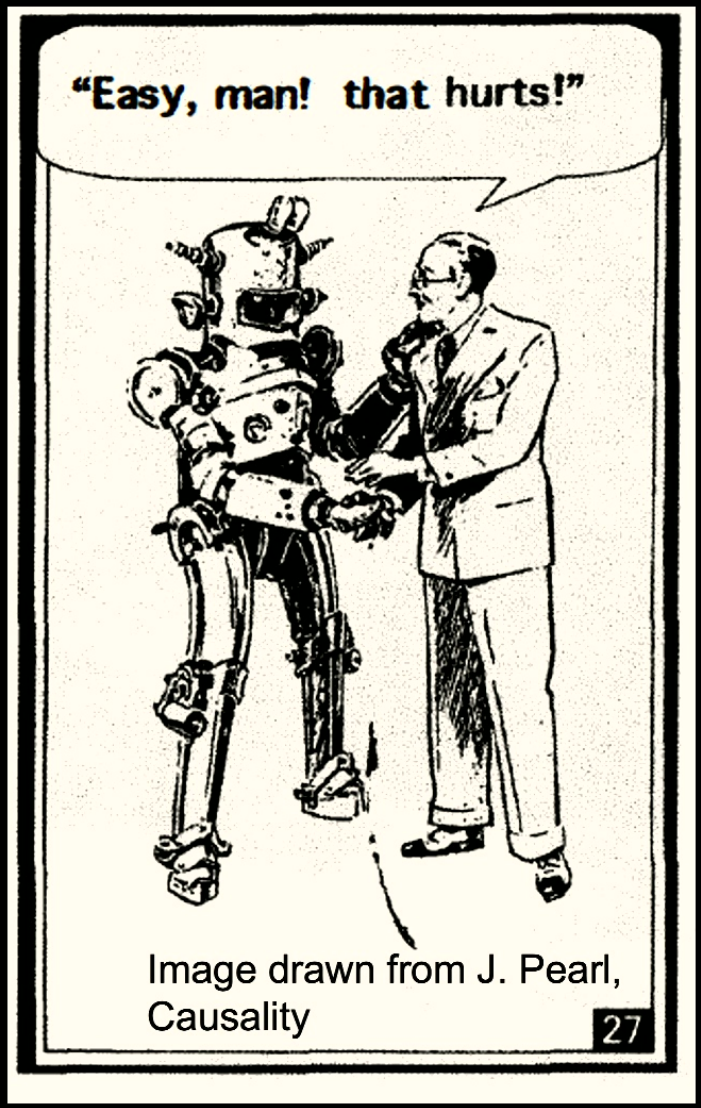


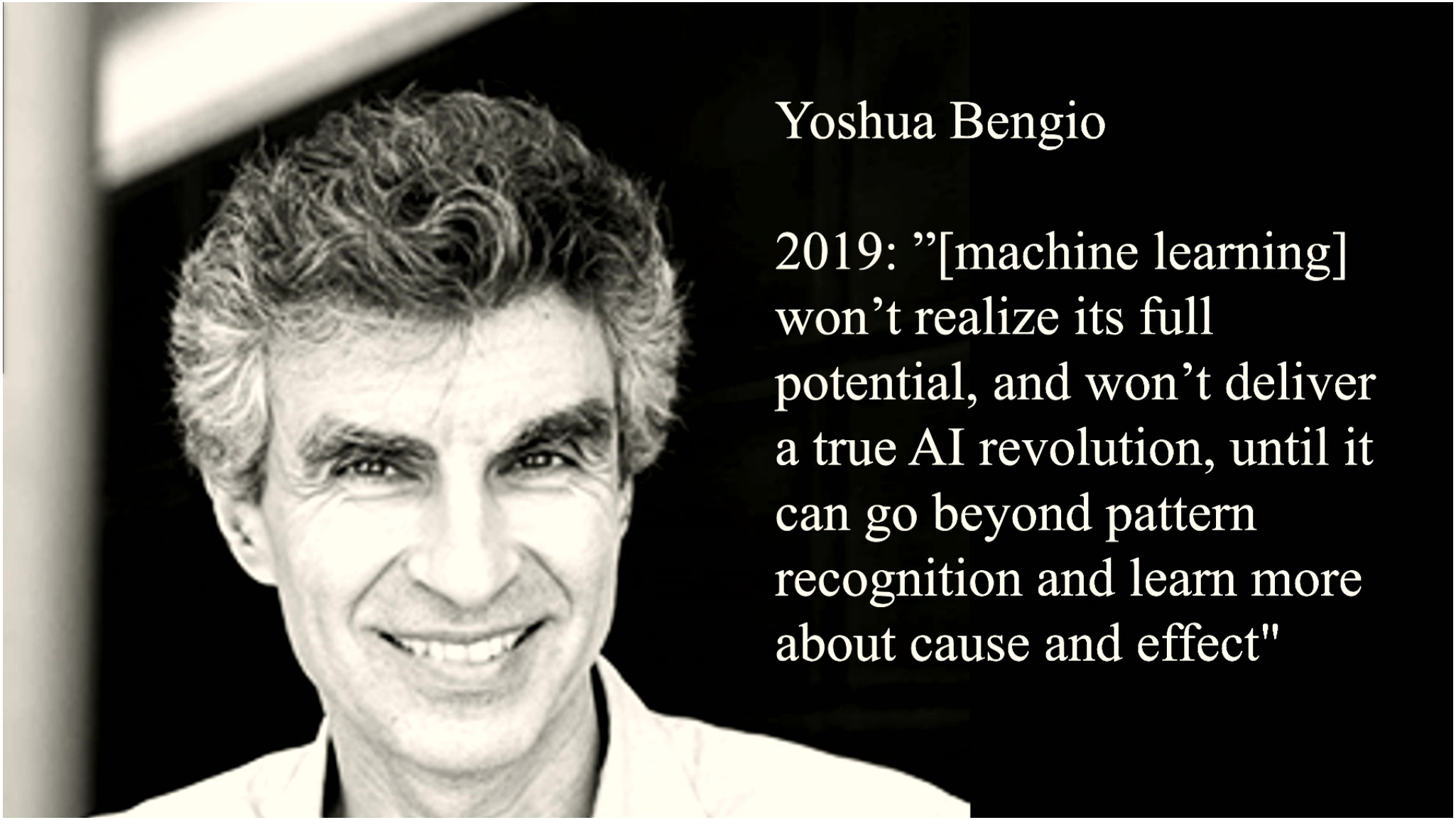

CAUSALITY
MODELS, REASONING, AND INFERENCE
JUDEA PEARL
Causation, Prediction, and Search
second edition
Peter Spirtes, Clark Glymour, and Richard Scheines
CAUSAL INFERENCE IN STATISTICS
A Primer
Judea Pearl
Madelyn Glymour
Nicholas P. Jewell
WILEY
Jonas Peters, Dominik Janzing, and Bernhard Schölkopf
Elements of Causal Inference
Foundations and Learning Algorithms
EXPLANATION IN CAUSAL INFERENCE
Methods for Mediation and Interaction
TYLER J. VANDERWEELE
OXFORD
CAUSAL INFERENCE
FOR
STATISTICS, SOCIAL, AND BIOMEDICAL SCIENCES
AN INTRODUCTION
GUIDO W. IMBENS
DONALD B. RUBIN
ICSA Book Series in Statistics
Hua He
Pan Wu
Ding-Geng (Din) Chen Editors
Statistical Causal Inferences and Their Applications in Public Health Research
Springer
ANALYTICAL METHODS FOR SOCIAL RESEARCH
Counterfactuals and Causal Inference
Methods and Principles for Social Research
SECOND EDITION
STEPHEN L. MORGAN
CHRISTOPHER WINSHIP

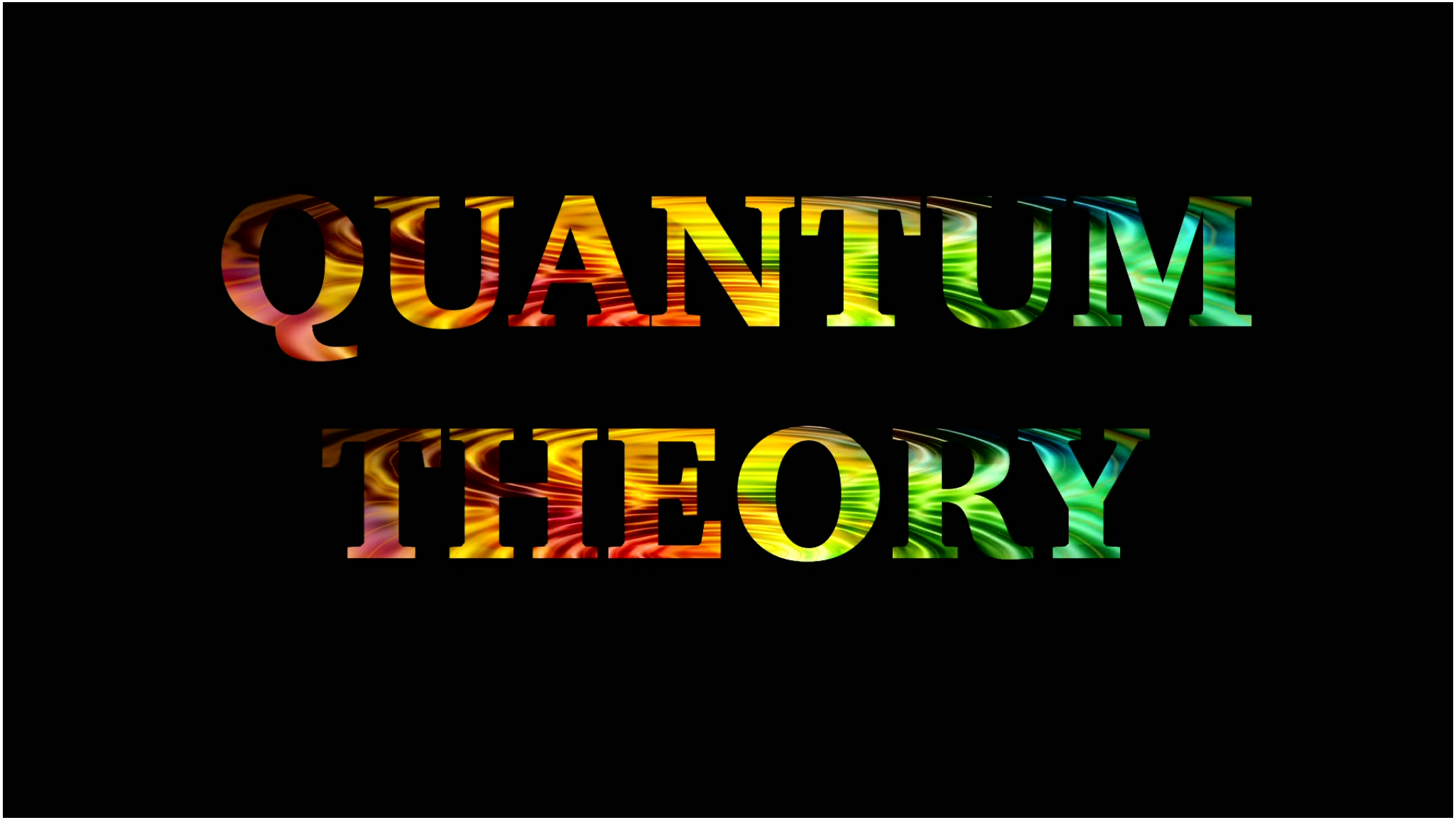

All scientific studies
Observational studies
Randomized controlled trials

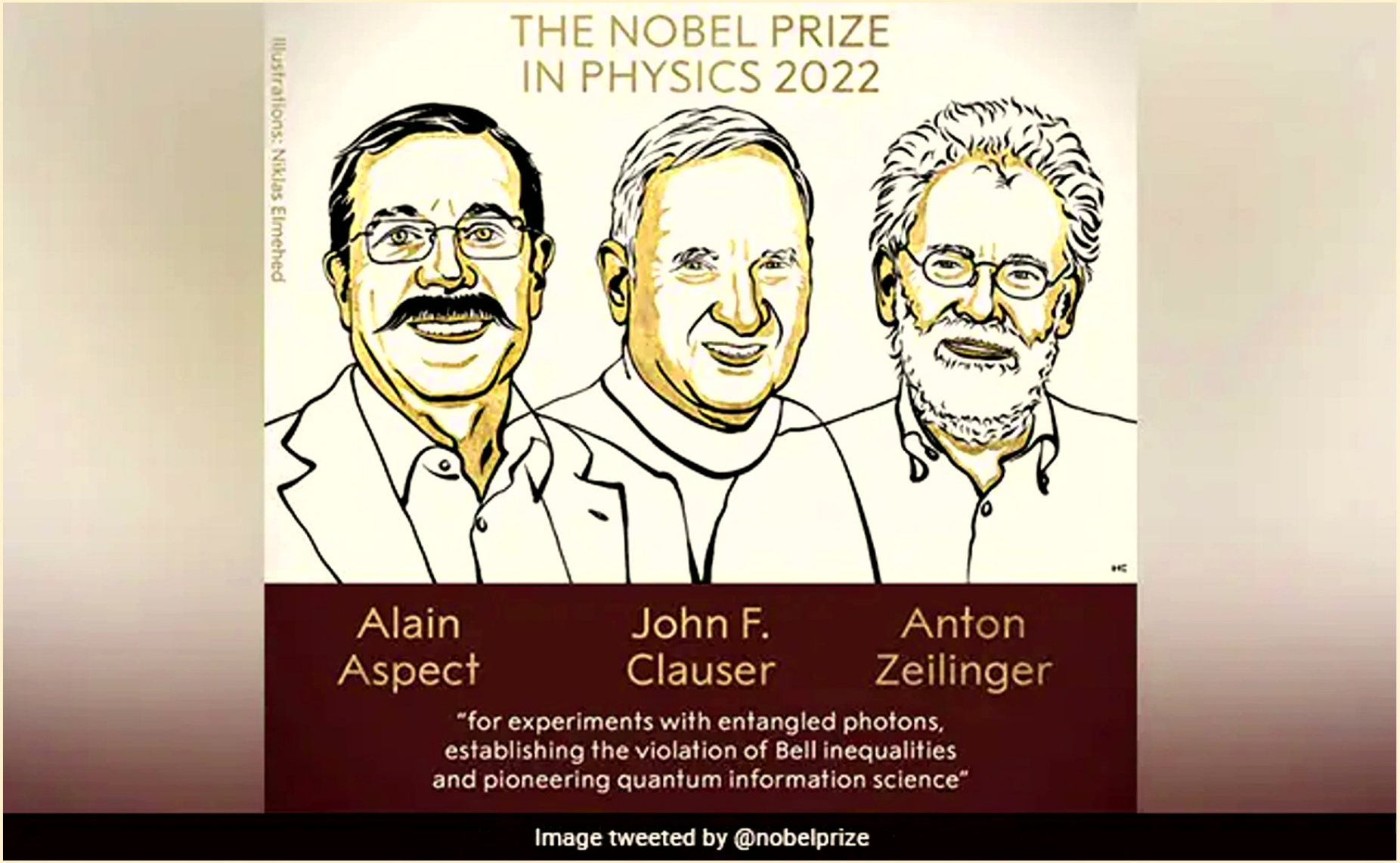
Causal inference techniques

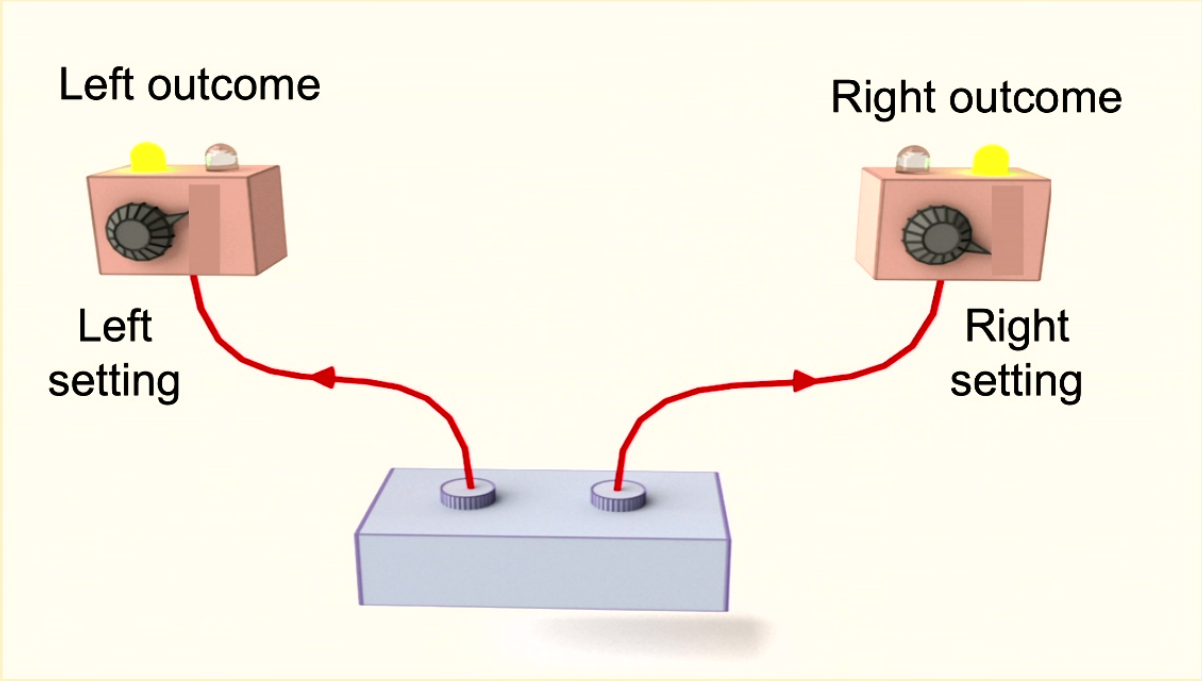



"Easy, man! that hurts!"
Image drawn from J. Pearl, Causality
27


Yoshua Bengio
2019: ”[machine learning] won’t realize its full potential, and won’t deliver a true AI revolution, until it can go beyond pattern recognition and learn more about cause and effect"


QUANTUM
THEORY


THE NOBEL PRIZE
IN PHYSICS 2022
Illustrations: Niklas Elmehed
Alain Aspect
John F. Clauser
Anton Zeilinger
"for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"
Image tweeted by @nobelprize


Left outcome
Right outcome
Left
setting
Right
setting

Left setting and Right setting	Left outcome and Right outcome			
	0 and 0	0 and 1	1 and 0	1 and 1
0 and 0	43%	7%	7%	43%
0 and 1	43%	7%	7%	43%
1 and 0	43%	7%	7%	43%
1 and 1	7%	43%	43%	7%

The evidence

Left setting and Right setting	Left outcome and Right outcome: 0 and 0	0 and 1	1 and 0	1 and 1
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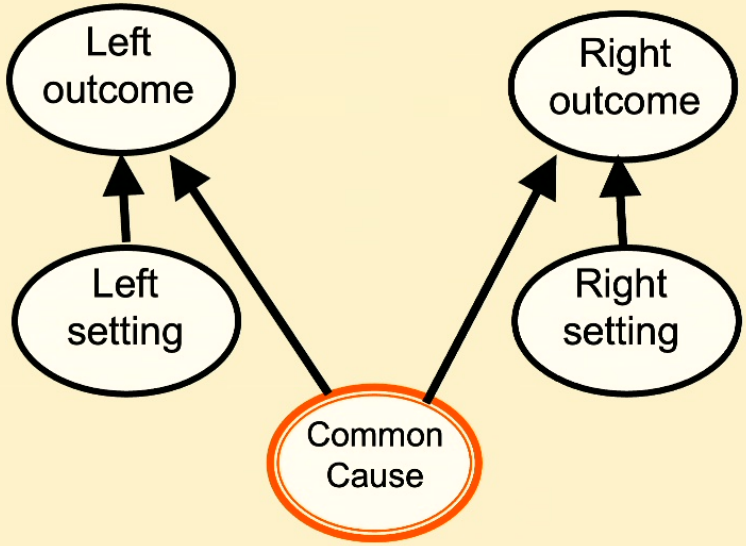
1 and 0	43%	7%	7%	43%
1 and 1	7%	43%	43%	7%

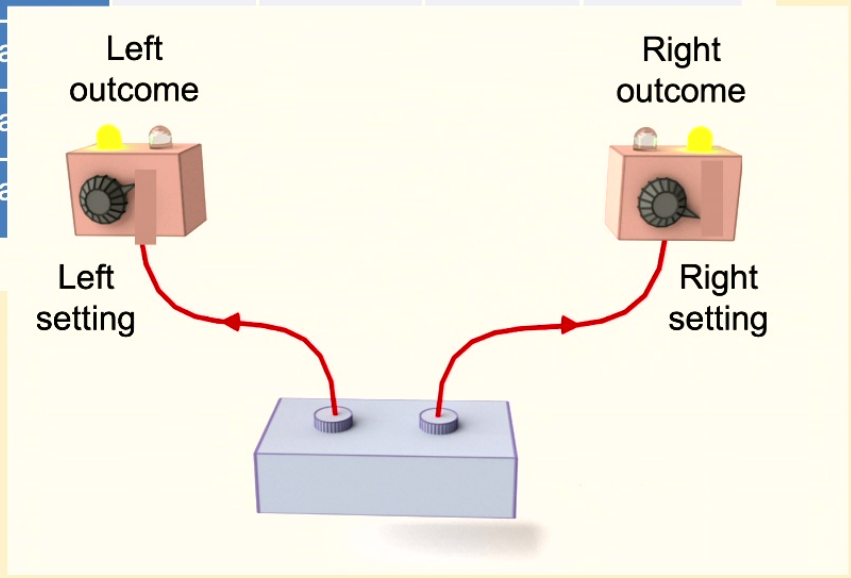
The natural hypothesis



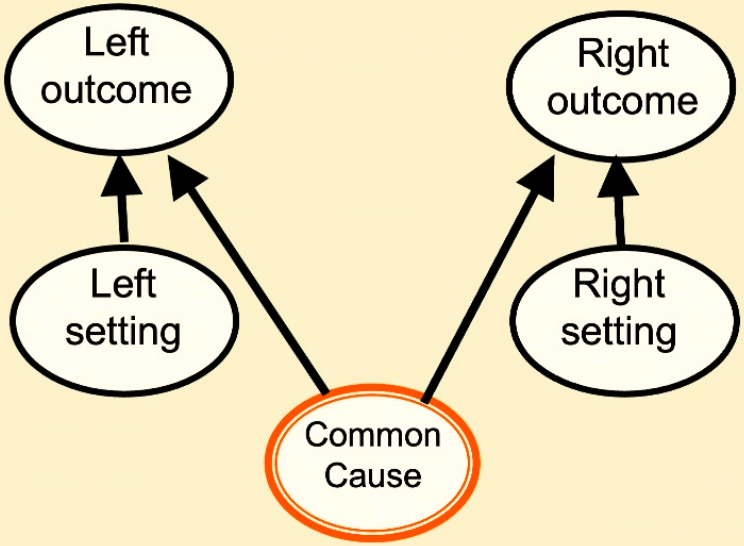
The evidence

Left outcome and Right outcome

Left setting and Right setting	0 and 0	0 and 1	1 and 0	1 and 1
0 and 0	43%	7%	7%	43%
0 a				
1 a				
1 a				



The natural hypothesis



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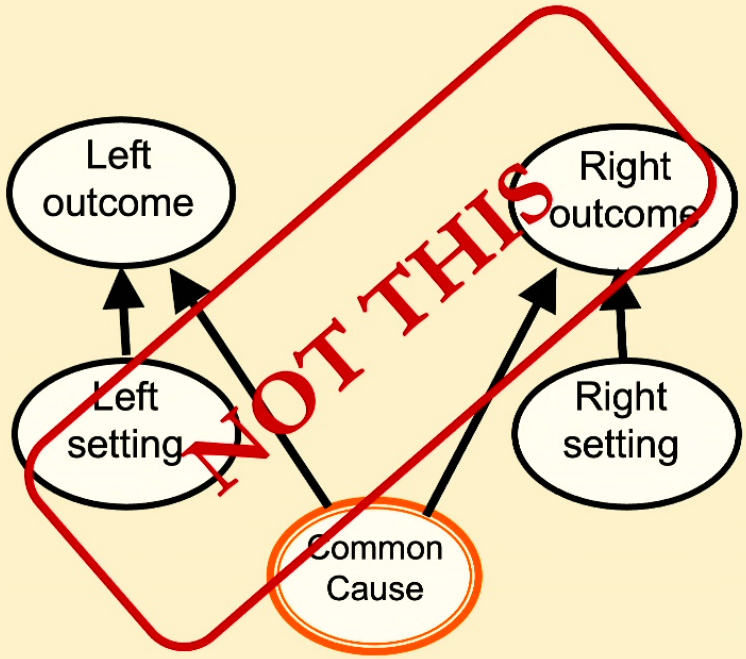
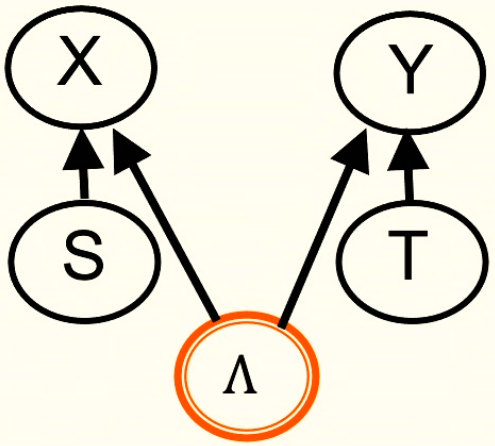


Violates the
Bell Inequalities!

Implies a constraint:
Bell Inequalities

Incompatible

Causal structure



Parameters

$$P_{X|S\Lambda}$$
$$P_{Y|T\Lambda}$$
$$P_{\Lambda}$$

$$P_{XY|ST} = \sum_{\Lambda} P_{Y|T\Lambda} P_{X|S\Lambda} P_{\Lambda}$$

Examples of causal compatiblity constraints:

$$P_{X|ST} = P_{X|S}$$
$$P_{Y|ST} = P_{Y|T}$$

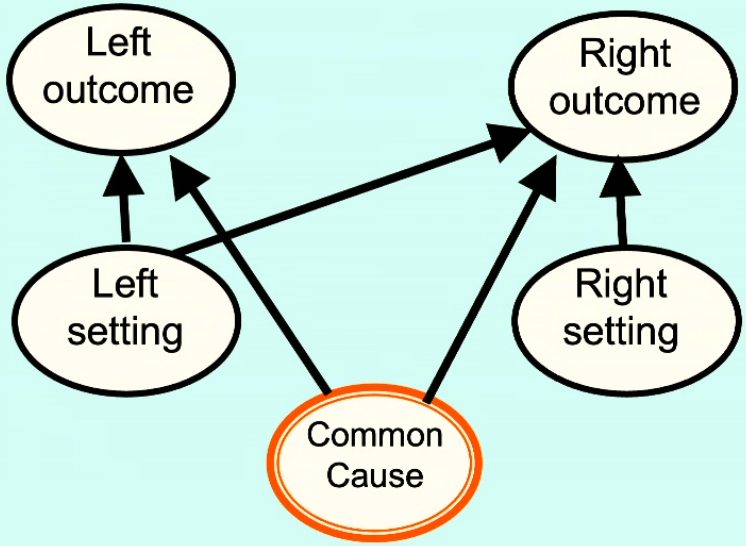
$$\frac{1}{4}\sum_{x=y} P_{XY|ST}(xy|00) + \frac{1}{4}\sum_{x=y} P_{XY|ST}(xy|01) + \frac{1}{4}\sum_{x=y} P_{XY|ST}(xy|10) + \frac{1}{4}\sum_{x\neq y} P_{XY|ST}(xy|11) \leq \frac{3}{4}$$

Clauser, Horne, Shimony and Holte, Phys. Rev. Lett.23, 880 (1967)

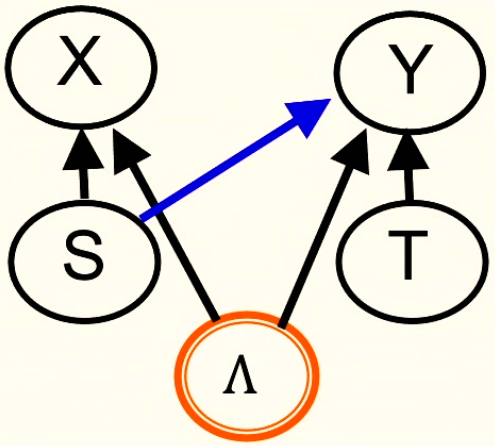
The evidence

Left setting and Right setting	Left outcome and Right outcome: 0 and 0	0 and 1	1 and 0	1 and 1
0 and 0	43%	7%	7%	43%
0 and 1	43%	7%	7%	43%
1 and 0	43%	7%	7%	43%
1 and 1	7%	43%	43%	7%

The 2nd possibility



Causal structure



Parameters

$P_{X|S\Lambda}$

$P_{Y|ST\Lambda}$

P_{Λ}

$$P_{XY|ST} = \sum_{\Lambda} P_{Y|ST\Lambda} P_{X|S\Lambda} P_{\Lambda}$$

Causal compatibility constraints:

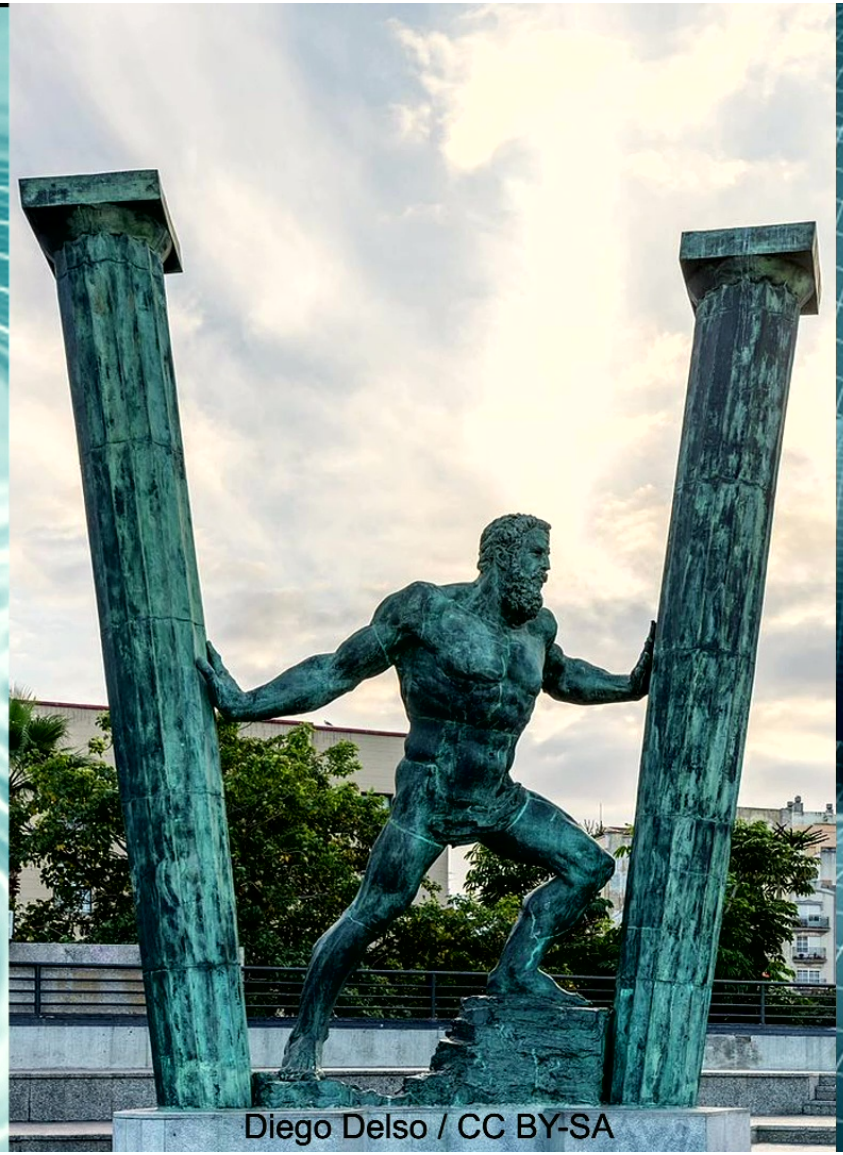
$P_{X|ST} = P_{X|S}$

But the data *also* satisfies $P_{Y|ST} = P_{Y|T}$

Reproducing this requires fine-tuning

Wood and RWS, New J. Phys. 17, 033002 (2015)

QUANTUM THEORY

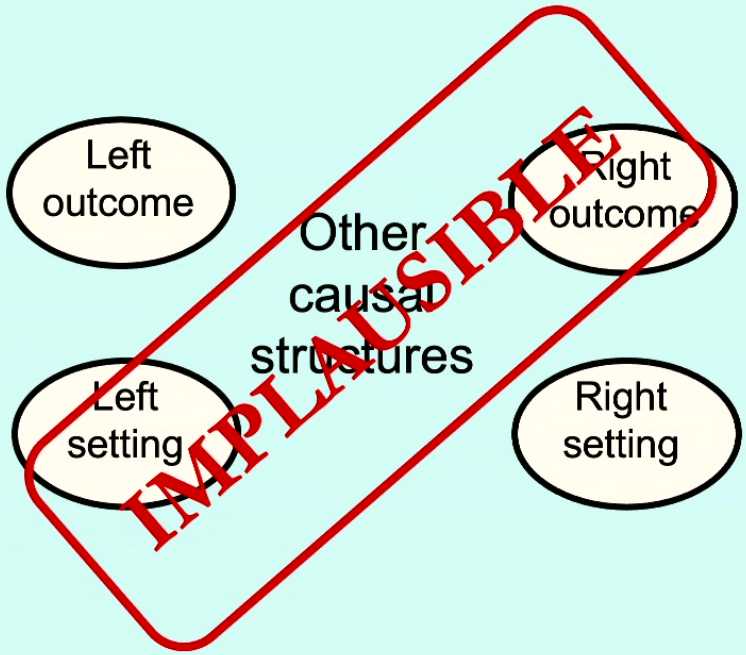


RELATIVITY

The evidence

Left setting and Right setting	Left outcome and Right outcome: 0 and 0	0 and 1	1 and 0	1 and 1
0 and 0	43%	7%	7%	43%
0 and 1	43%	7%	7%	43%
1 and 0	43%	7%	7%	43%
1 and 1	7%	43%	43%	7%

The usual suspects



Wood and RWS, New J. Phys. 17, 033002 (2015)

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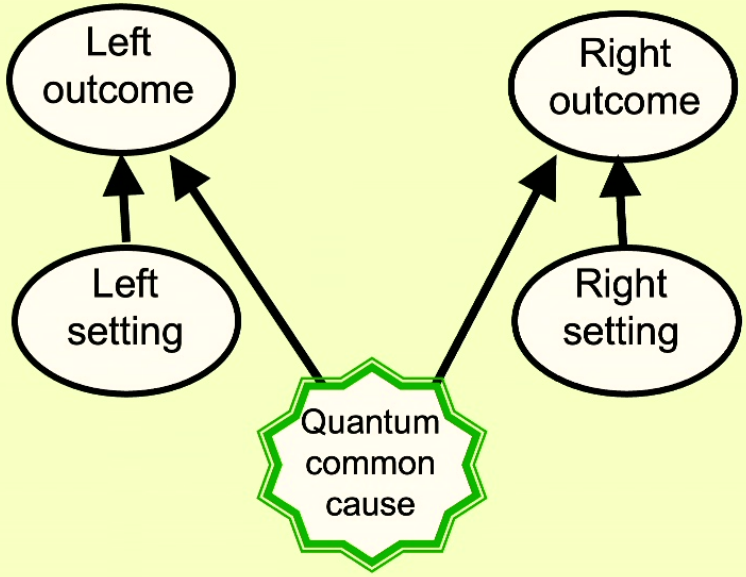
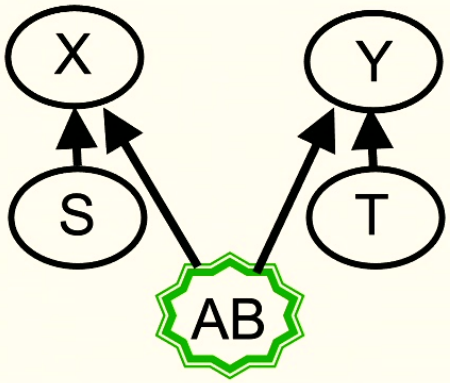
A new possibility



Quantum causal model



$\{E^A_{x|s}\}_x$ for each s

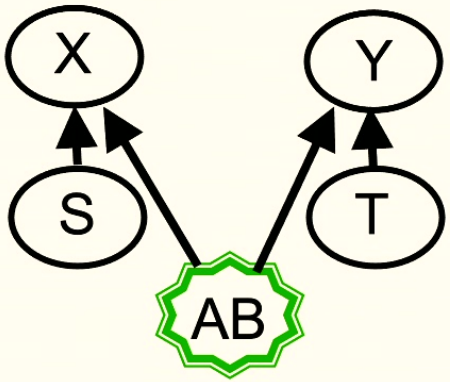
$\{E^B_{y|t}\}_y$ for each t

ρ_{AB}

$$P_{XY|ST}(xy|st) = \mathsf{Tr}_{AB}((E^A_{x|s} \otimes E^B_{y|t})\rho_{AB})$$

Leifer and RWS, PRA 88, 052130 (2013)
Henson, Lal & Pusey NJP 16, 113043 (2014)
Costa, Shrapnel NJP 18(6) (2016)
Allen, Barrett, Horsman, Lee & RWS, PRX 7, 031021 (2017)
Barrett, Lorenz, Oreshkov, arXiv:1906.10726

Quantum causal model



$\rho_{X|SA}$

$\rho_{Y|TB}$

ρ_{AB}

$$P_{XY|ST} = \mathsf{Tr}_{AB}(\rho_{X|SA}\rho_{Y|TB}\rho_{AB})$$

Causal compatibility constraints:

$$P_{X|ST} = P_{X|S}$$
$$P_{Y|ST} = P_{Y|T}$$

$$\tfrac{1}{4}\sum_{x=y} P_{XY|ST}(xy|00) + \tfrac{1}{4}\sum_{x=y} P_{XY|ST}(xy|01)$$
$$\tfrac{1}{4}\sum_{x=y} P_{XY|ST}(xy|10) + \tfrac{1}{4}\sum_{x\neq y} P_{XY|ST}(xy|11) \leq \tfrac{1}{2} + \tfrac{1}{2\sqrt{2}}$$

Tsirelson, Lett. Math. Phys. 4, 93 (1980)

The evidence

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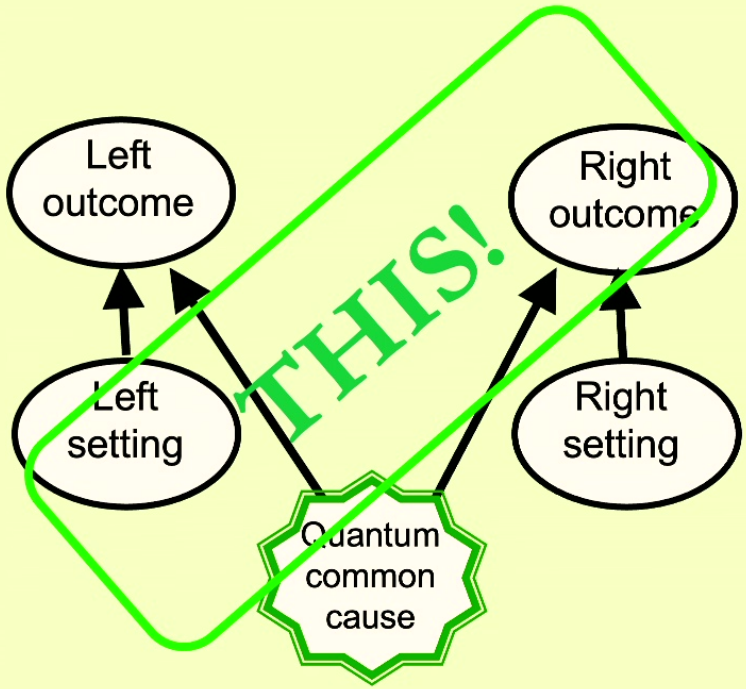
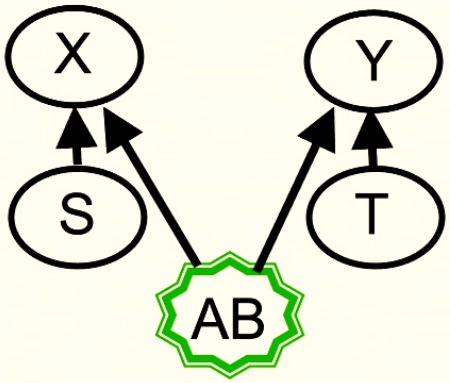
A new possibility



Compatible

Quantum causal model



$\rho_{X|SA}$

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$$P_{XY|ST} = \mathrm{Tr}_{AB}(\rho_{X|SA}\rho_{Y|TB}\rho_{AB})$$

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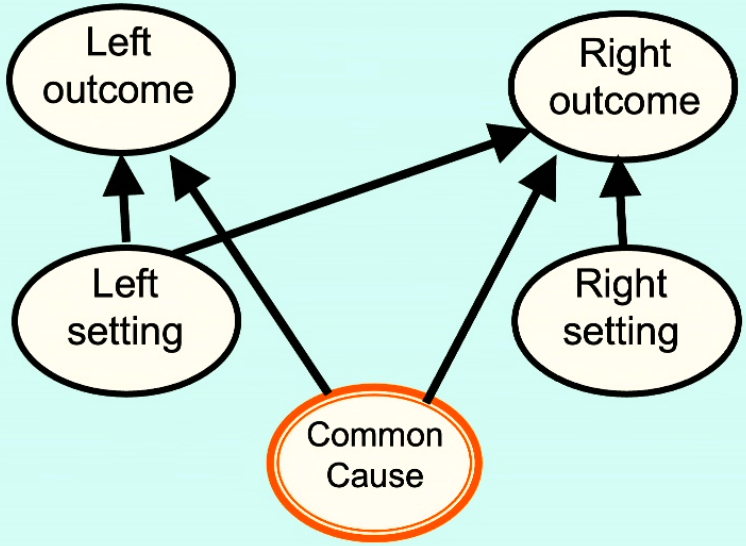
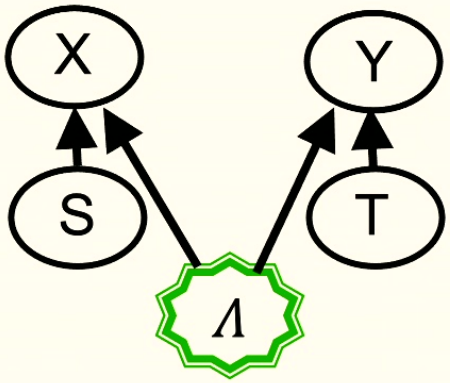


Quantum causal model



$\rho_{X|S\Lambda}$

$\rho_{Y|T\Lambda}$

ρ_{Λ}

$$[\rho_{X|S\Lambda}, \rho_{Y|T\Lambda}] = 0$$

$$P_{XY|ST} = \mathrm{Tr}_{\Lambda}(\rho_{X|S\Lambda}\rho_{Y|T\Lambda}\rho_{\Lambda})$$

Leifer and RWS, PRA 88, 052130 (2013)
Henson, Lal & Pusey NJP 16, 113043 (2014)
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Allen, Barrett, Horsman, Lee & RWS, PRX 7, 031021 (2017)
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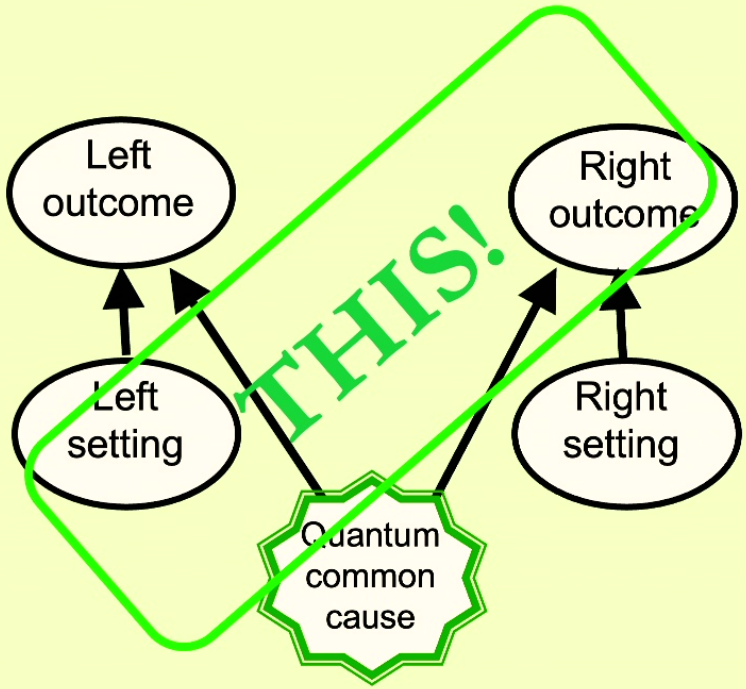
1 and 1	7%	43%	43%	7%

A new possibility

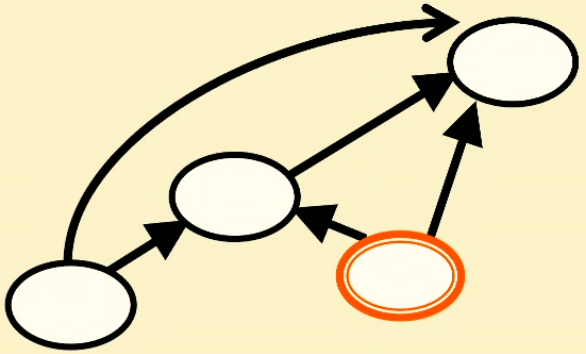


Compatible

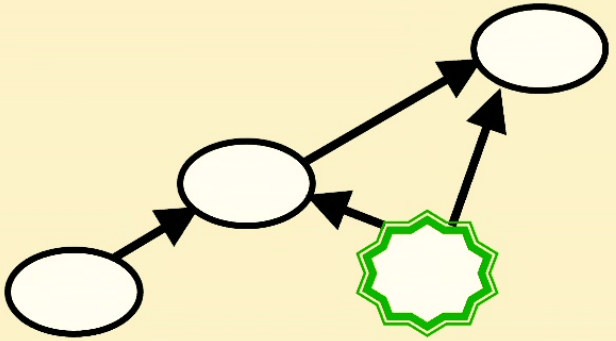
Violation of Instrumental Inequalities

or

Witnessing need for different structure

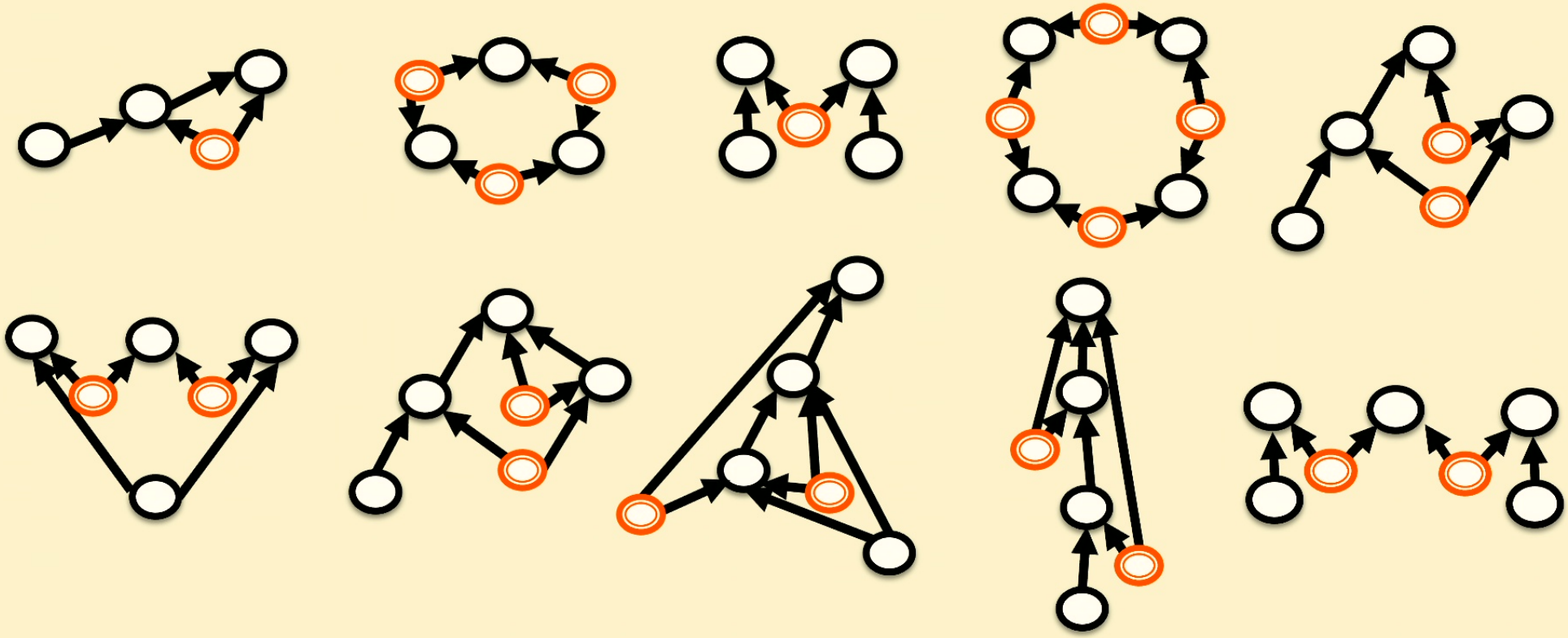


Witnessing quantumness



van Himbeeck, Bohr Brask, Pironio, Ramanathan, Sainz & Wolfe, *Quantum* 3, 186 (2019)
Chaves, Carvacho, Agresti, Di Giulio, Aolita, Giacomini & Sciarrino, *Nat. Phys.* 47, 291296 (2018)

Some causal structures that admit of inequality constraints

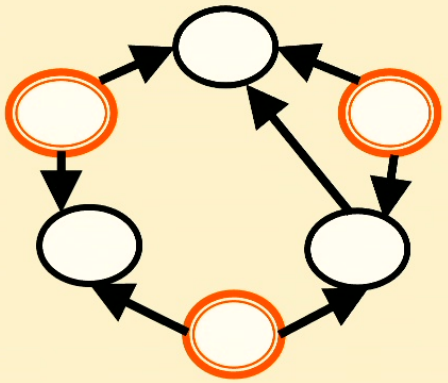


They are ubiquitous: Ansanelli, Wolfe, RWS, arXiv:2502.07891

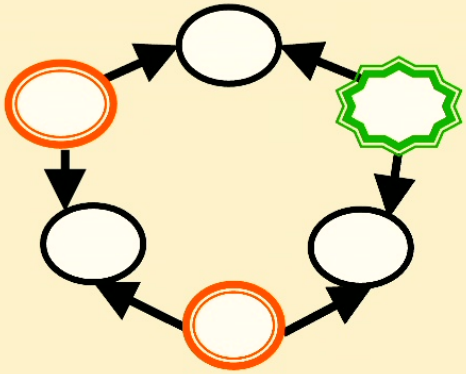
Violation of certain causal compatibility inequalities

or

Witnessing need for different structure

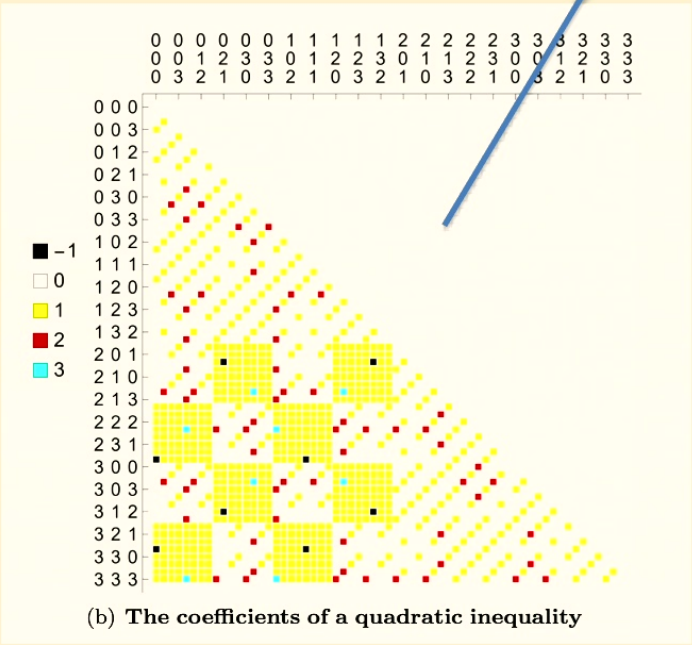


Witnessing quantumness



Data-seeded inflation technique

$$V := \sum_{a,b,c,a',b',c'} y_{abca'b'c'} P_{ABC}(abc) P_{ABC}(a'b'c') \geq 0$$

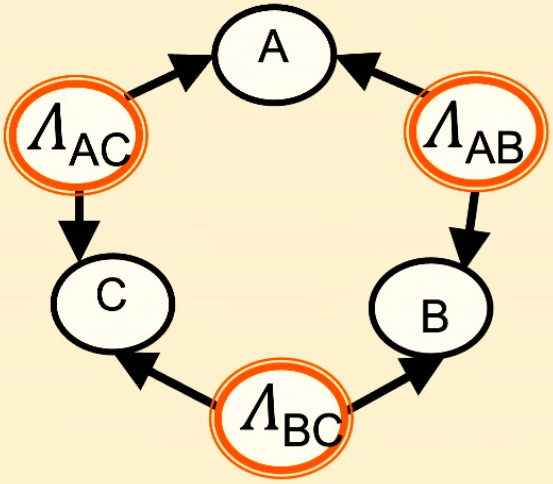


(b) **The coefficients of a quadratic inequality**

This causal compatibility inequality is violated by the data

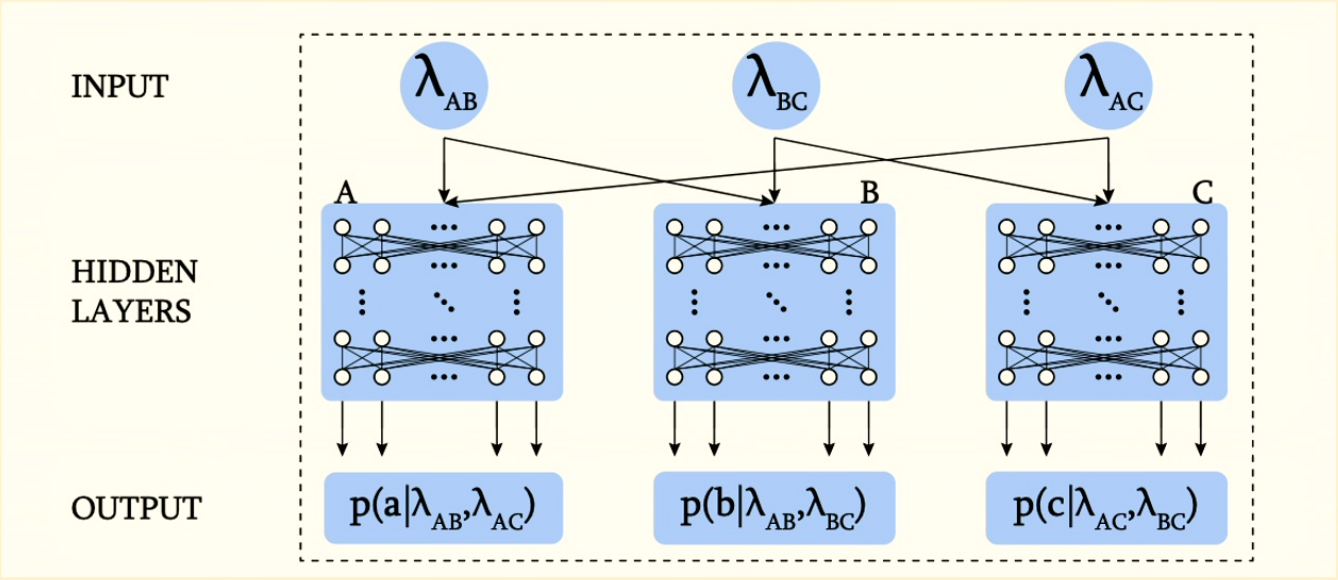
$$V_{\text{exp}} = -0.02436 \pm 0.00016$$

Polino et al., Nature Communications 14, 909 (2023)



Encode causal structure in the topology of a neural network

Krivachy, Cai, Cavalcanti, Tavakoli, Gisin, Brunner, Nature Quant Inf 6 (2020).



Applications for Quantum Technology

The formalism and conceptual scheme of causal inference resolved various puzzles of statistics (e.g., Simpson's paradox, Berkson's paradox)

The lesson:
We must unscramble the omelette of inference and causation both conceptually and in the formalism

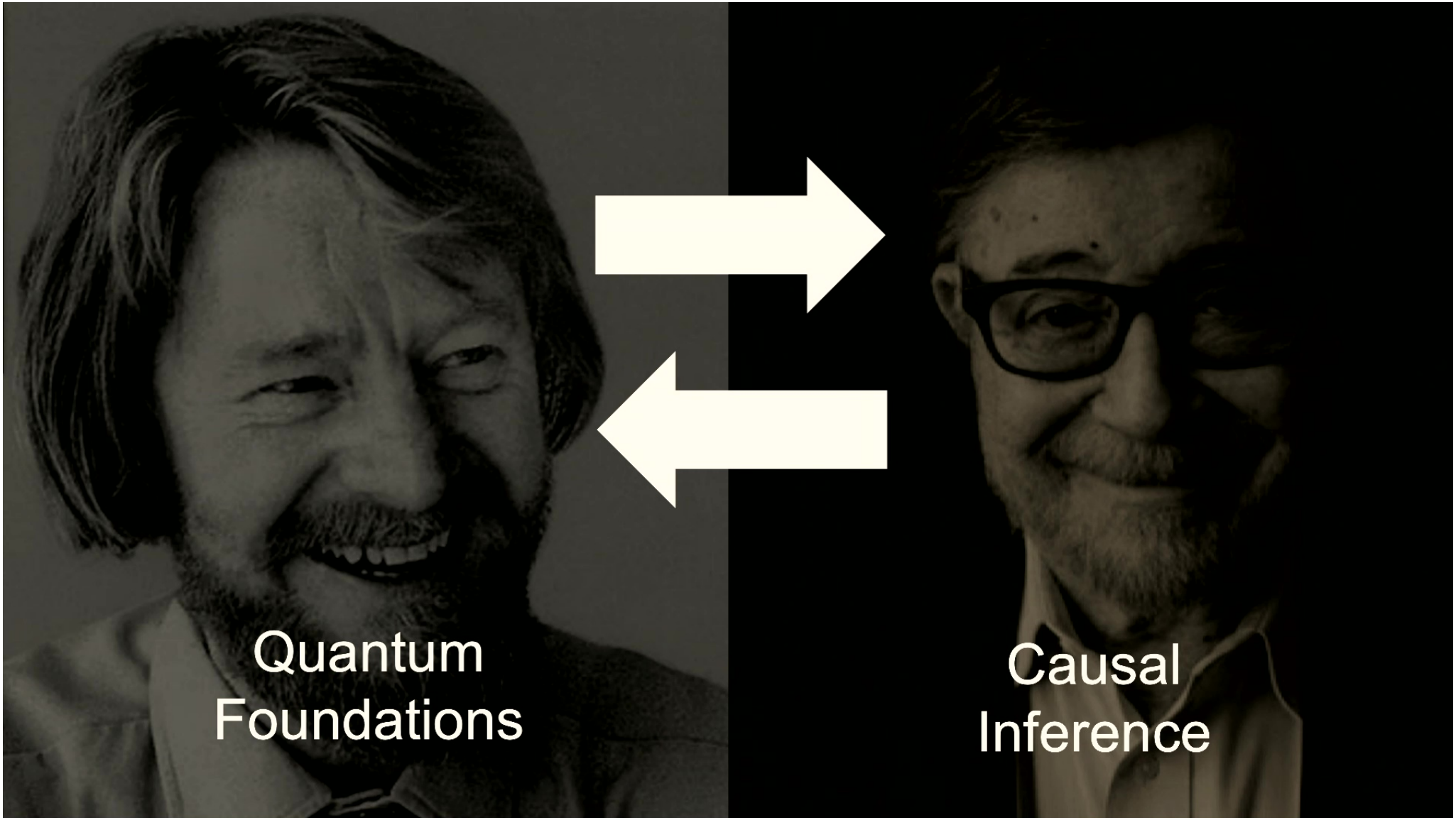
But what hope do we have of succeeding in the quantum context if we do not understand how to do so in the classical context?

$$\frac{\text{Quantum Causation and Inference}}{\text{Classical Causation and Inference}} \approx \frac{\text{Relativistic Notions of Space and Time}}{\text{PreRelativistic Notions of Space and Time}}$$

See: Schmid, Selby, RWS, arXiv:2009.03297

QUANTUM THEORY

RELATIVITY


Quantum
Foundations
Causal
Inference


Thanks!