Title: Giant gravitons in Dp-brane holography

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Abstract:

We consider half BPS operators in maximally supersymmetric Yang Mills (SYM) in p+1 dimensions. These operators satisfy trace relations that are identical to those discussed in the p=3 case (N = 4 SYM). Nevertheless, the bulk explanation of these trace relations must differ from the p = 3 case as their holographic duals are not AdS spacetimes. We identify giant graviton solutions in the dual holographic backgrounds for -1 <= p <= 4. In the ’t Hooft limit, these giants are D(6-p) branes that wrap the internal sphere. We also follow the giants into the strong coupling region where they become other branes. Despite propagating in a non-AdS geometry, we find that the branes “feel” like they are in AdS. This is closely related to the emergent scaling symmetry present in these boundary theories.(based on https://arxiv.org/abs/2502.14249)

Giant gravitons in Dp-brane holography

Henry Lin, Stanford University

April 1, 2025

This talk is based on:
2502.14249 w/ **Gauri Batra**

see also:

[Lee & Stanford 2412.20769]

[Eleftheriou, Murthy, Rosselló 2501.13910]

[Biggs & Maldacena, 2303.09974]

't Hooft taught us that matrix theories contain strings.

The string coupling is $g_s \sim 1/N$.

String theories contain non-perturbative effects, e.g., $\sim e^{-N}$
D-branes with tension $\sim N$.

How to see this qualitatively? Trace relations.

As with 't Hooft's arguments, trace relations are generic in matrix theories. But trace relations have mostly been explored in AdS/CFT.

Today we will consider the bulk dual (giant gravitons) of these trace relations in super Yang mills in $p+1$ dimensions.

Partly motivated by new tech for studying the boundary theory in $p=0$ (BFSS) and $p=-1$ (IKKT) [..., HL, Zheng, Hartnoll, Liu, Komatsu, ...].

- Review of Dp-brane holography
- Giant gravitons in Dp-brane holography
- Matrix bootstrap for BFSS, future directions

Consider the effective field theory of Dp-branes, for $p \leq 3 \Rightarrow$ SU(N) SYM in $p+1$ dimensions.

$$g_{\mathrm{YM}}^2 \propto g_s \ell_s^{p-3}$$

The interaction is relevant for $p < 3$.
't Hooft coupling $\lambda = g_{\mathrm{YM}}^2 N \Rightarrow \tilde{\lambda} = g_{\mathrm{YM}}^2 N / \beta^{p-3}$.

Black brane solution

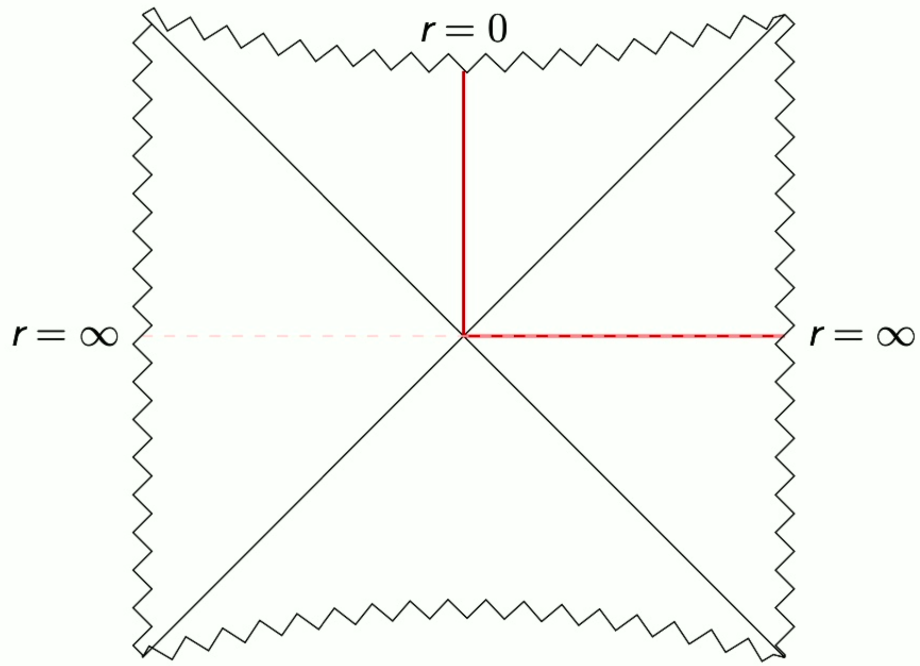
This solution is somewhat similar to the AdS black brane $\times S_{8-p}$

$$\frac{\mathrm{d}s^2}{\alpha'} = \left(\frac{z}{R_{\mathrm{AdS}}}\right)^{\frac{3-p}{5-p}} \left[R_{\mathrm{AdS}}^2 \left(\frac{h(z)\,\mathrm{d}\tau^2 + h^{-1}(z)\mathrm{d}z^2 + \mathrm{d}x_p^2}{z^2}\right) + \mathrm{d}\Omega_{8-p}^2\right],$$

$$h = 1 - \frac{z^d}{z_0^d}, \quad d = 1 + \frac{9-p}{5-p}, \quad R_{\mathrm{AdS}} = \frac{2}{5-p},$$

$$e^{-2\phi} = (d_p(2\pi)^{p-2}N)^2 \left(\frac{z}{R_{\mathrm{AdS}}}\right)^{\frac{7-p}{5-p}(p-3)},$$

$$A_{0\cdots p} = \sqrt{\alpha'}d_p(2\pi)^{p-2}N\left(\frac{z}{R_{\mathrm{AdS}}}\right)^{-2\frac{7-p}{5-p}}.$$


$r = 0$
$r = \infty$
$r = \infty$

Some features for $p = 0$:

$$\frac{\mathrm{d}s^2}{\alpha'} = \left(\frac{z}{R_{\mathrm{AdS}}}\right)^{\frac{3}{5}} \left[R^2_{\mathrm{AdS}}\left(\frac{h(z)\,\mathrm{d}\tau^2 + h^{-1}(z)\mathrm{d}z^2}{z^2}\right) + \mathrm{d}\Omega_8^2\right],$$

$$h = 1 - \frac{z^d}{z_0^d}, \quad d = 1 + \frac{9}{5},$$

$$e^{2\phi} \propto \frac{1}{N^2}\left(\frac{z}{R_{\mathrm{AdS}}}\right)^{\frac{21}{5}}.$$

- Sphere shrinks near boundary $z = 0$. When $z \sim 1$ curvature scale is of order $\sim \ell_s$.
- dilaton grows towards the horizon. SYM coupling is *relevant*.

Extrapolation to strong coupling

- $p = 0$ view D0s as gravitons in 11d $\Rightarrow$ boosted Schwarzschild black hole (homogeneous in the 11th dimension) $\Rightarrow$ BFSS conjecture
- $p = 2$, view the D2 branes as M2 branes, $AdS_4 \times S_7$ ABJM
- $p = 1$, S-duality relates D1 solution to F1s $\Rightarrow$ matrix string ($(R^8)^N/S_N$ CFT)
- $p = -1$ ...?

Relation to AdS

Fluctuations of the dilaton $\phi = \phi_{\text{sol}} + \chi$

$$
\begin{aligned}
I &\propto \int \mathrm{d}^{10}x\sqrt{g}e^{-2\phi_{\text{sol}}}(\nabla\chi)^2 \\
&= \int \mathrm{d}^{8-p}\Omega\,\mathrm{d}^{d-1}\vec{x}\,\mathrm{d}z\,\mathrm{d}\tau\,\sqrt{g_{\text{AdS}}}\left[(\nabla_{\mathrm{AdS}}\chi)^2 + m_k^2\chi^2\right]
\end{aligned}
$$

Using $m_k = k(k+7-p) \Rightarrow$ fields in AdS_{d+1}:

$$
\langle \mathcal{O}_\phi(x)\mathcal{O}_\phi(0)\rangle \sim \frac{1}{|x|^{2(\Delta-d-p-1)}}, \quad \Delta = R_{\mathrm{AdS}}(k+2)+2.
$$

This applies to SUGRA modes.

The GKP dictionary

Consider DBI action in the presence of a dilaton wave. Repeat decoupling argument. DBI action for Dp-branes:

$$I_{DBI} \sim \int \mathrm{d}^{p+1}x\, e^{-\phi(x,X)} F^2(x) + \cdots$$

Boundary operators schematically of the form

$$S_{\mathsf{SYM}} \to S_{\mathsf{SYM}} + \mathcal{N} \sum_j \frac{1}{k!} \int \mathrm{d}^{p+1}x\, \partial_{I_1} \cdots \partial_{I_k} \phi\, \mathrm{Tr}\Big(F^2_{\mu\nu} X^{(I_1} \cdots X^{I_k)}\Big).$$

This is a super-descendant of the 1/2 BPS operator:

$$\mathrm{Tr}\Big(F^2_{\mu\nu} X^{(I_1} \cdots X^{I_k)}\Big) \sim QQQQ\, \mathrm{Tr}\, X^{(I_1} \cdots X^{I_{k+2})}$$

We learn that the dimensions of the $\frac{1}{2}$ BPS operator:

$$\Delta = R_{\mathrm{AdS}}(k+2) + 2 \Rightarrow \Delta_{\frac{1}{2}\mathsf{BPS}} = R_{\mathrm{AdS}} k$$

Reproduce the relation we got from the gravitons:

$$\Delta_{\frac{1}{2}\text{BPS}} = R_{\text{AdS}} k$$

We can find a probe D$(6-p)$-brane solution, rotating on the S_{8-p} with

$$\Delta_{\frac{1}{2}\text{BPS}} = R_{\text{AdS}} k, \quad k \leq N$$

To explain the significance of $k \leq N$, let's first go back to the boundary analysis.

Giant gravitons

Consider multi-trace operators $\mathrm{Tr}\, Z^{k_1}\, \mathrm{Tr}\, Z^{k_2} \cdots \mathrm{Tr}\, Z^{k_n}$.

Finite N trace relations, e.g., $\mathrm{Tr}\, Z^2 - \mathrm{Tr}\, Z\, \mathrm{Tr}\, Z = 0$ for $N = 1$. Trace relations kick in at $k \sim N$.

More precisely, $Z^{\otimes k}$ furnishes a representation of GL(N) on $V^{\otimes k}$. Multi-traces of Z are class functions $Z \to gZg^{-1}$. Orthogonal basis for class functions are the characters "Schur polynomials" $\chi_R(Z)$. [Corley, Jevicki, Ramgoolam]

trace relations $\leftrightarrow$ anti-symmetric irreps exist for $k \leq N$.

Baby giants

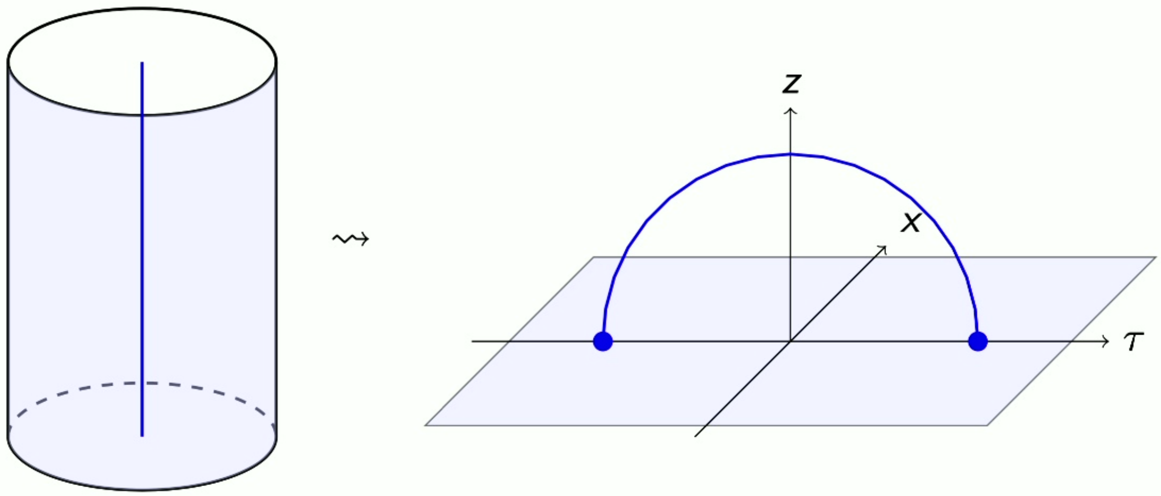
Baby version of the giant graviton expansion:

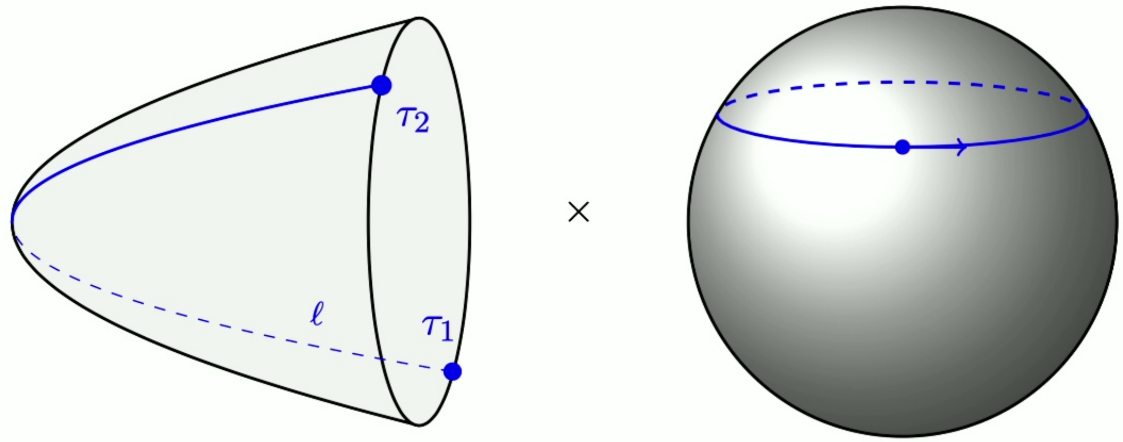
$$\sum_{\text{single column operators}} q^J = \frac{1}{1-q} - \frac{q^{N+1}}{1-q}$$

Second term looks related to D-branes $e^{-\#N}$. Note the minus sign [Lee & Stanford].

This discussion is p-independent. Relevant physics $\sim$ "matrices" not "CFT". What about the bulk?

Giant gravitons





This picture suppresses the spatial directions of the boundary theory and also an $S_{6-p} \subset S_{8-p}$ on which the D-brane is wrapped.

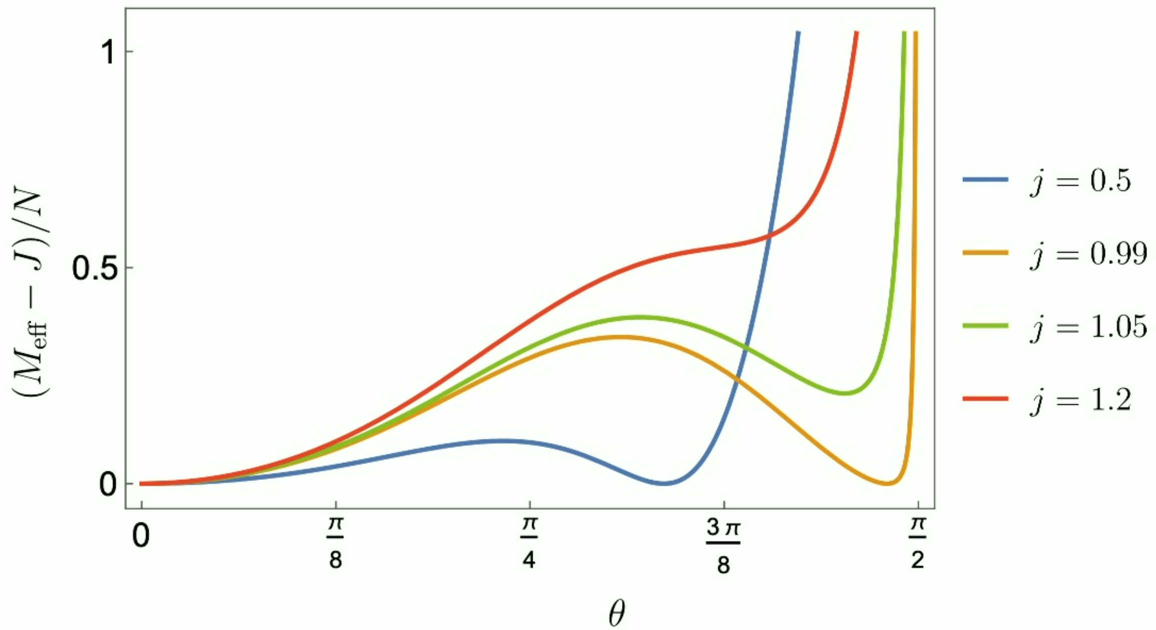
The giants feel AdS!

$$I = -g_s T_{6-p} \int_{\mathrm{D}(6-p)} (\mathrm{d}^{7-p} x \, e^{-\phi} \sqrt{-g_{\mathrm{D}(6-p)}} - A_{7-p})$$

$$\mathcal{L}/N = -\sin^{6-p}(\theta) \sqrt{R_{\mathrm{AdS}}^2 \left(\frac{h(z) - h^{-1}(z)\dot{z}^2 - \dot{\vec{x}}_p^2}{z^2} \right) - \cos^2\theta \dot{\phi}^2 - \dot{\theta}^2} + N \sin^{7-p}(\theta) \dot{\phi}.$$

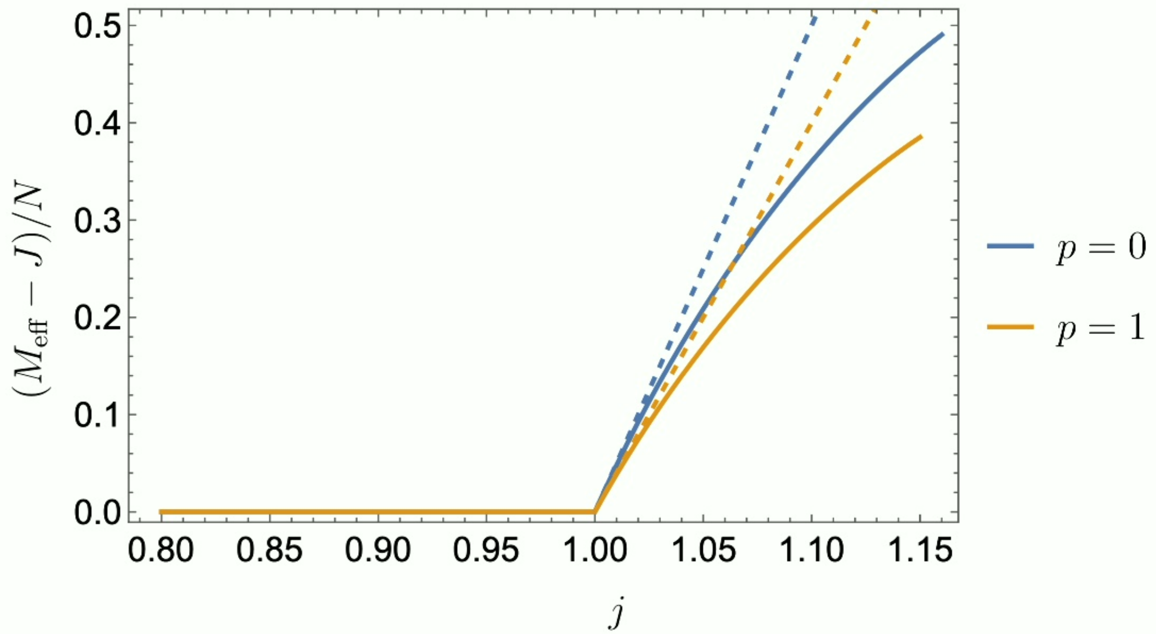
$\Rightarrow$ bulk explanation of why the trace relations are so uniform in p.

Bulk explanation of trace relations



Plot is for D0-brane/D6 giants. Look for stable solutions with R-charge $j \equiv J/N$.

Bulk explanation of trace relations



BPS only if $j = J/N \leq 1$.

$\Delta = R_{\rm AdS} M = \frac{2}{5-p} J$.

One can quantize certain fluctuations of the maximal giant. Following [Lee & Stanford], one can show this reproduces the second term in:

$$\sum_{\text{single column operators}} q^J = \frac{1}{1-q} - \frac{q^{N+1}}{1-q}$$

What happens to these branes when they enter the strong coupling region?

- D(-1)-brane/IKKT: D7 giant becomes an "NS7" or (p,q) brane
- D0-brane/BFSS: D6 giant becomes a KK monopole
- D1-brane/F1: D5 giant becomes an NS5 brane
- D2-brane/ABJM: D4 giant becomes M5 brane
- D3-brane/$\mathcal{N} = 4$: D3 is self-dual
- D4-brane/6D (2,0): D2 giant becomes M2 brane

In the $p = -1, 1, 2, 3, 4$ cases we checked that the brane still feels like it is in AdS.

$p > 4$ holography is weird and we are confused.

- $p = 5$: little string theory
- $p = 6$, D6 branes, the giants are D0 branes on an S_2. This is a relativistic version of the Haldane problem.

$$H = \sqrt{m^2 + J^2 - q^2} = \sqrt{\mathrm{j}(\mathrm{j}+1)}$$
$$\mathrm{j} = \frac{1}{2}N, \frac{1}{2}N + 1, \cdots ,$$

BPS condition. No operators with $J/N \leq 1/2$.

Future directions

- Dual giants/global "AdS"
- LLM
- BMN/Polarized IKKT
- Identifying the NS5 branes in the Matrix string
- Matrix bootstrap approach to giants?