Title: Natural Proof Checking and AI

Speakers: Peter Koepke

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Abstract:

From the start of AI, mathematical theorem proving has been an important challenge and technique. We sketch the topics of Automated and Interactive Theorem Proving and present the checking of naturally readable mathematical texts in the Naproche proof system. This involves translations between informal, semi-formal and formal mathematical languages and shows great potential for the use of new AI techniques.

ℕatural Proof Checking + AI

by Peter Koepke

Mathematical Institute and Hausdorff Center for Mathematics, University of Bonn

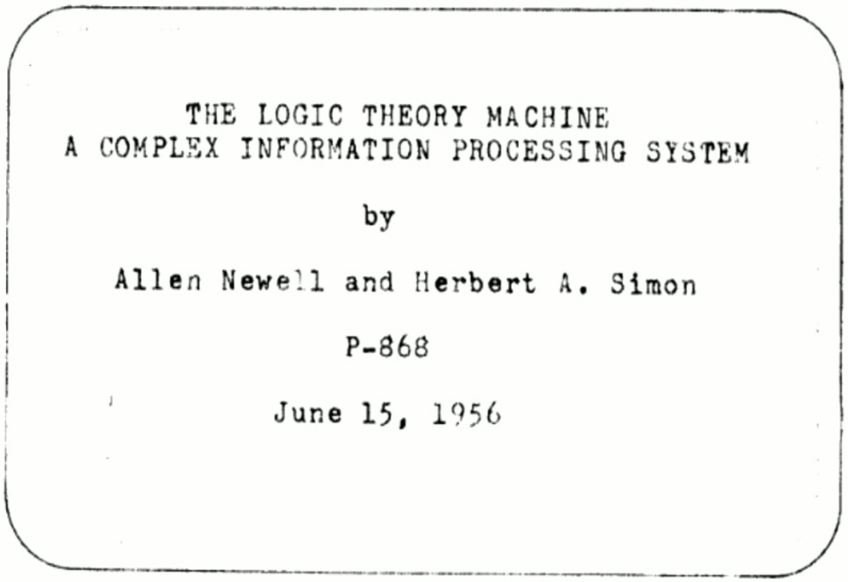
Abstract

From the start of AI, mathematical theorem proving has been an important challenge and technique. We sketch the topics of Automated and Interactive Theorem Proving and present the checking of naturally readable mathematical texts in the ℕaproche proof system. This involves translations between informal, semi-formal and formal mathematical languages and holds great potential for the deployment of new AI techniques.

UNIVERSITÄT BONN

hcm HAUSDORFF CENTER FOR MATHEMATICS

Dartmouth Workshop 1956: The Logic Theorist by Allan Newell and Herbert Simon



THE LOGIC THEORY MACHINE
A COMPLEX INFORMATION PROCESSING SYSTEM

by

Allen Newell and Herbert A. Simon

P-868

June 15, 1956

Simon Newell

Automated Theorem Prover (ATP)

Proving propositional theorems from A.N.Whitehead and B.Russell, *Principia Mathematica*

Searching through an infinite search space using heuristics

Proving Proposition 2.01 of *Principia Mathematica*

100 MATHEMATICAL LOGIC

$*2\cdot01.\ \vdash : p \supset \sim p \,.\supset .\, \sim p$

$$(p \rightarrow \neg p) \rightarrow \neg p$$

$$\downarrow$$

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$$\downarrow$$

$$(p \vee p) \rightarrow p$$
$$(\neg p \vee \neg p) \rightarrow \neg p$$
$$(p \rightarrow \neg p) \rightarrow \neg p$$

$\left[\text{Taut}\ \frac{\sim p}{p}\right] \vdash : \sim p \vee \sim p \,.\supset .\, \sim p$ (1)

$[(1).(*1\cdot01)]\ \vdash : p \supset \sim p \,.\supset .\, \sim p$

Translate Original to formal language: $(p \rightarrow \neg p) \rightarrow \neg p$

Start search at proof goal

Try proof rules with results similar to goal

Application of definition of $\rightarrow$

Substitution of p by $\neg p$

Reduction to axioms

Logic Theorist found the Whitehead-Russell proof

William McCune, 1996: Automatically proving Robbins Conjecture with EQP

Premises: ... + Robbins equation

$\ldots, \neg(\neg(a \vee b) \vee \neg(a \vee \neg b)) = a$

Goal: Huntington equation

$\neg(\neg a \vee b) \vee \neg(\neg a \vee \neg b) = a$

$\downarrow$

EQP Equational Prover

$\downarrow$

Success!

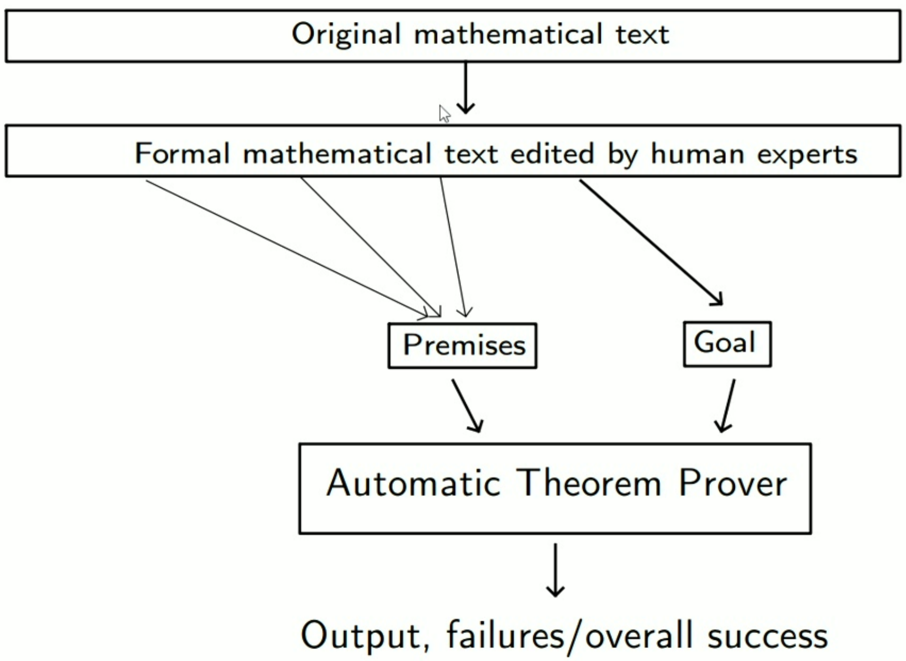
William McCune

Success! after 8 days on an RS/6000 processor with 30 MBytes

Every Robbins Algebra is a Boolean Algebra

EQP is the predecessor of modern ATPs like E and Vampire

Interactive Theorem Proving (ITP)

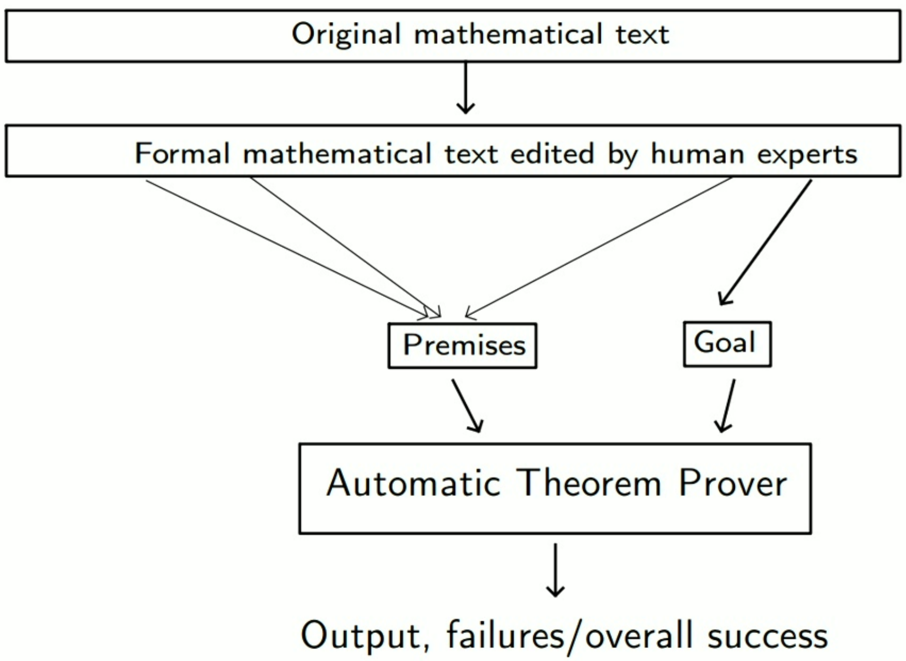


Combining expert insights and experiences with ATPs: divide final goal into smaller local steps that can be solved by computer

Steps will be translated into Premises and Goals

ITP systems:
Automath, Mizar, Isabelle, Rocq (previously Coq), HOL ..., Lean, ..., ℕaproche

Interactive Theorem Proving (ITP)



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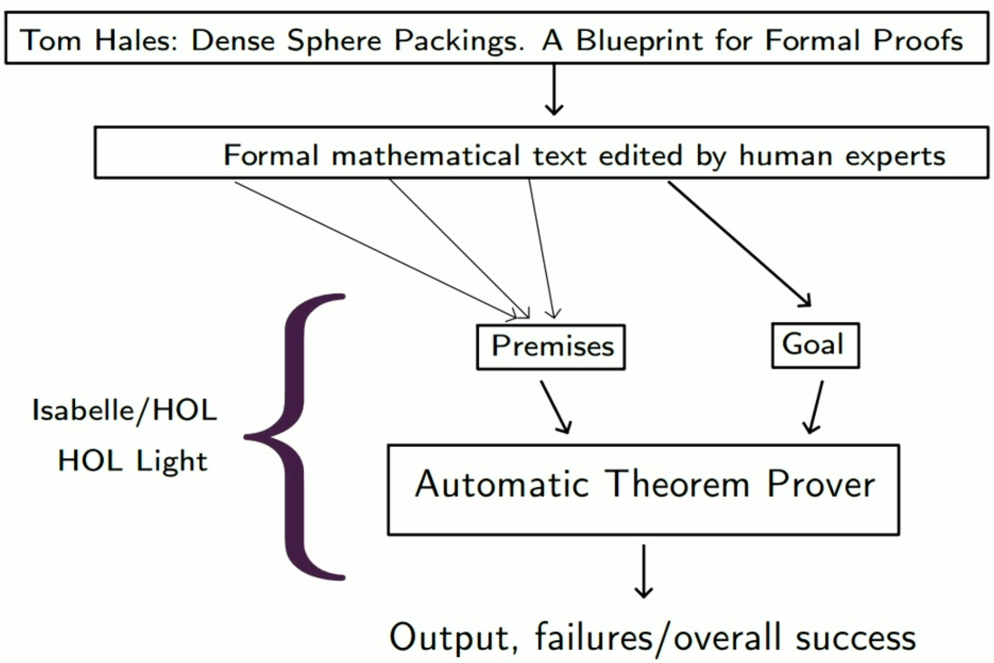
A Formal Proof of the Kepler Conjecture in Isabelle/HOL and HOL Light



Sphere packing

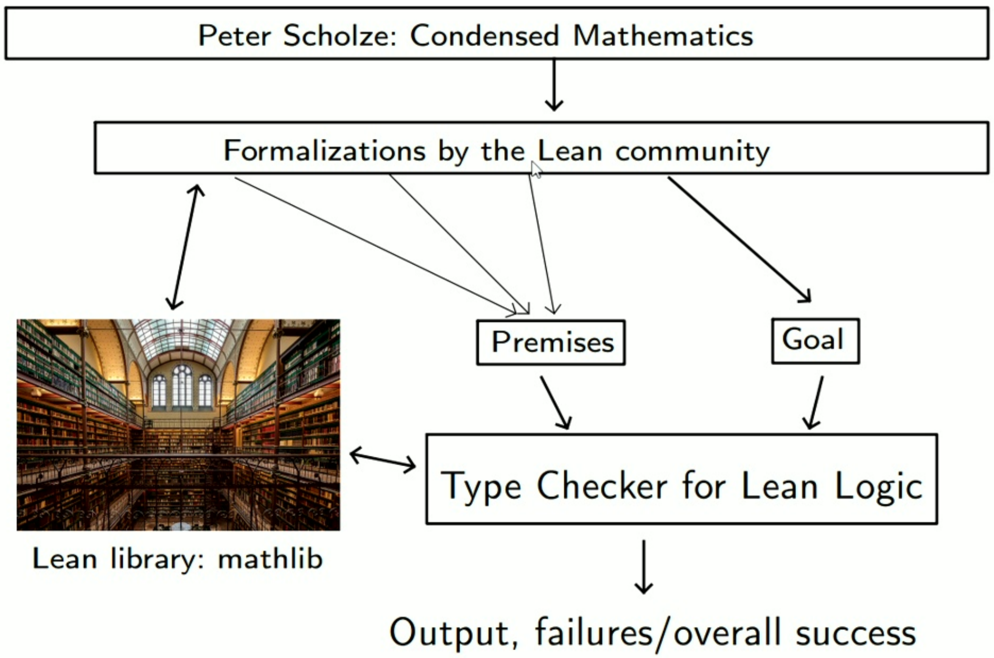
Thomas Hales

Flyspeck project 2003 - 2014

21 collaborators

Combining formal proof in Isabelle/HOL with formal proofs in HOL Light

Formalizing and Supporting Cutting Edge Research with Lean



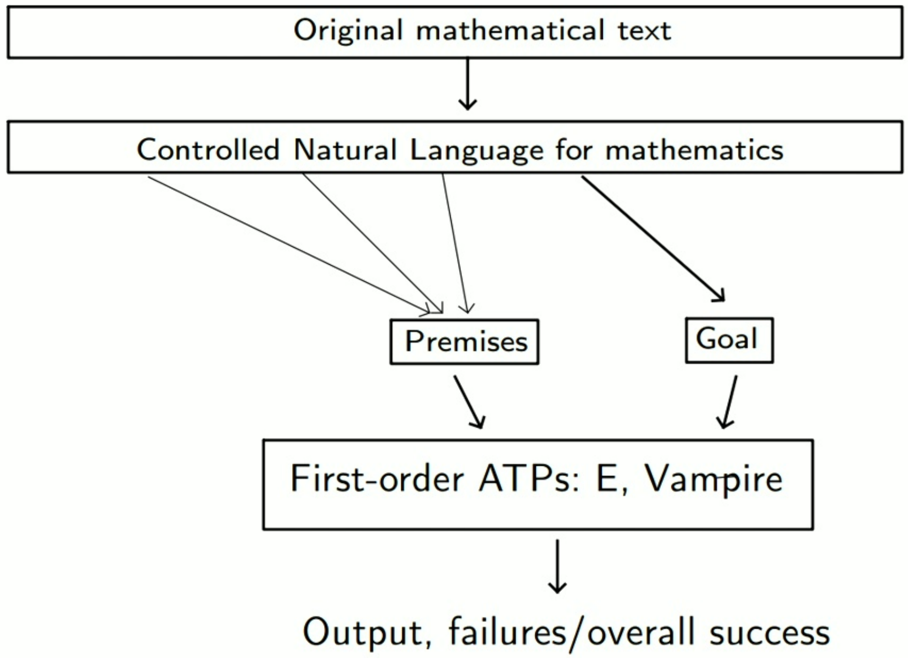
LEAN

Programming Language and Theorem Prover

Developed by Leonardo de Moura and Sebastian Ullrich

Intensely maintained mathlib library:
101061 Definitions
195721 Theorems
556 Contributors
(March 2025)

Natural Proof Checking, $\mathbb{N}$aproche



Naturalness of proof texts

Controlled Natural Language (CNL) defined by a formal phrase structure grammar

Readable input in LaTeX format

Strong ATP to allow human-like proof steps

Example formalizations to demonstrate feasibility of natural language in proof systems

$\mathbb{N}$aproche is available at `https://naproche.github.io/` or as a component of Isabelle 2024

$\mathbb{N}$aproche Excerpts

Some complex analysis:

Proposition. (Maximum modulus principle) *Assume f is a holomorphic function and the domain of f is a region. If f has a local maximal point then f is constant.*

Proof. Let z be a local maximal point of f. Take ε such that $B_\varepsilon(z)$ is a subset of $\mathrm{dom}(f)$ and $|f(w)| \leq |f(z)|$ for every element w of $B_\varepsilon(z)$.

Let us show that f is constant on $B_\varepsilon(z)$.

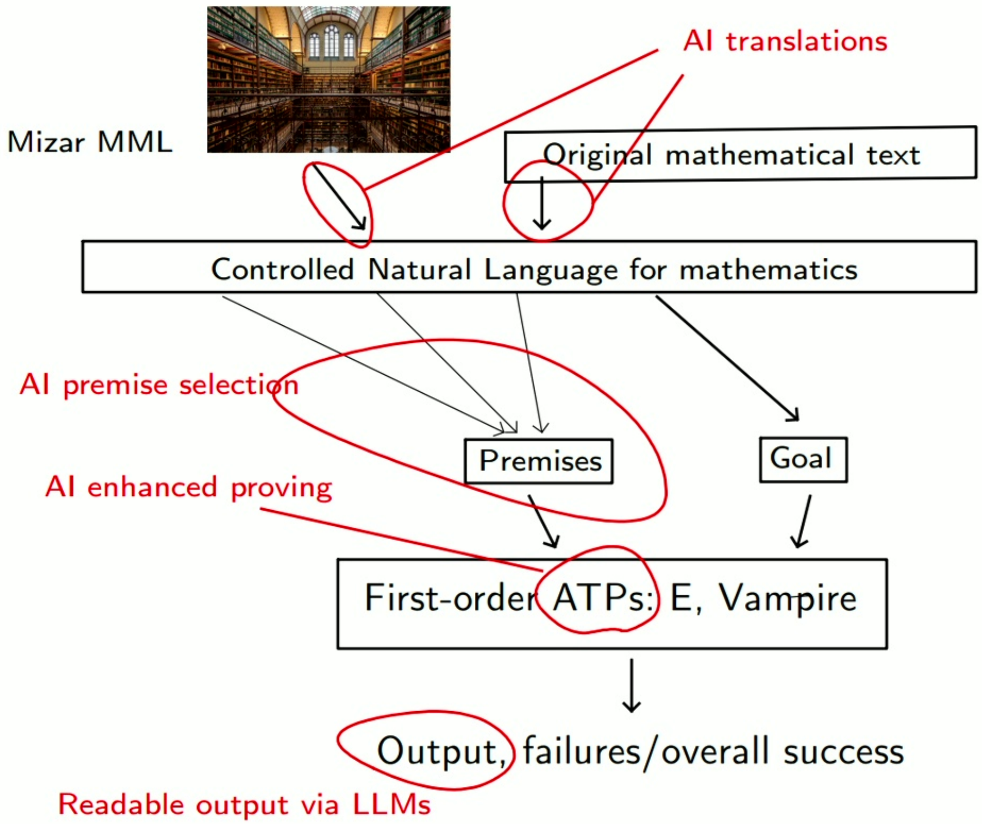
Proof. Assume the contrary. ... □

From a $\mathbb{N}$aproche formalization of perfectoid rings:

Definition. *R is perfectoid iff R is complete and uniform and there exists a pseudouniformizer ϖ of R such that $\varpi^{p,R} | p^{[R]}$ in R^o within R and*

$$\Phi^R \colon R^o / \varpi \cong R^o / \varpi^{p,R}.$$

Natural Proof Checking and Artificial Intelligence



Enhancing symbolic AI with neural AI

Translations with LLMs

Selection of premises with machine learning

Natural paraphrasings of formal statements and proofs by LLMs

Translating the Mizar Mathematical Library MML into the Controlled Natural Language

Thank you!

https://naproche.github.io/

https://isabelle.in.tum.de/website-Isabelle2024/