

Title: Recurrent neural networks for Rydberg atom arrays

Speakers: Mohamed Hibat Allah

Collection/Series: Special Seminars

Subject: Other

Date: May 02, 2025 - 9:30 AM

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Abstract:

Rydberg atom arrays have emerged as powerful quantum simulators, capable of preparing strongly correlated phases of matter that are potentially challenging to access with classical computational methods. A major focus has been on realizing these arrays on frustrated geometries, aiming to stabilize exotic many-body states like spin liquids. In this talk, I will show how two-dimensional recurrent neural network (RNN) wave functions can be used to study the ground states of Rydberg atom arrays on the kagome lattice. For Hamiltonians previously investigated in this geometry, I will demonstrate that the RNN finds no evidence for exotic spin liquid phases or emergent glassiness. In particular, I will argue that signals of glassy behavior, such as a nonzero Edwards-Anderson order parameter seen in quantum Monte Carlo (QMC) studies, may arise from artifacts related to long autocorrelation times. These results highlight the potential of language model-inspired approaches, like RNNs, for advancing the study of frustrated quantum systems and Rydberg atom physics more broadly.

arXiv paper: <https://arxiv.org/pdf/2405.20384>



Recurrent Neural Networks (RNNs) for Rydberg atom arrays

Mohamed Hibat-Allah

May 2nd, 2025

M.H., E. Merali, G. Torlai, R. Melko, J. Carrasquilla, arXiv: 2405.20384

Simulating quantum systems with classical computers?

Wave function:

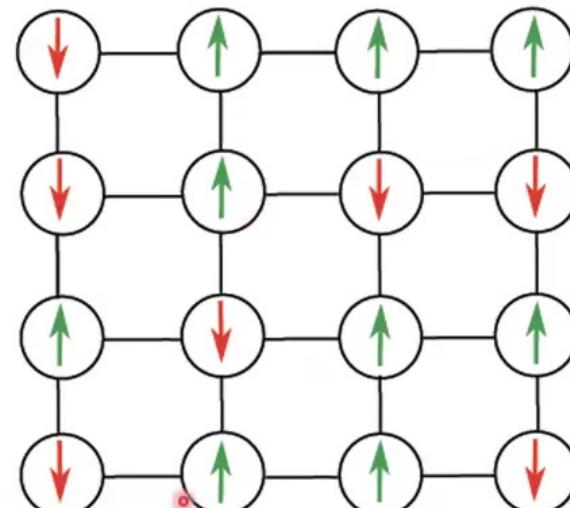
$$\Psi = (\Psi_1, \Psi_2, \dots, \Psi_{2^N}) \in \mathbb{C}^{2^N}$$

The module squared of the wave function defines a probability distribution:

$$|\Psi_1|^2 + |\Psi_2|^2 + \dots + |\Psi_{2^N}|^2 = 1$$

For N = 266:

$$2^{266} \approx 10^{80} \sim \text{number of atoms in our known universe}$$



Simulating quantum systems with classical computers?

Simulating Physics with Computers

Richard P. Feynman

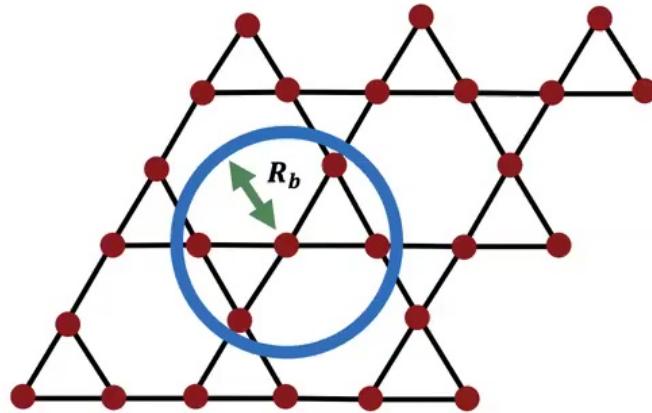
Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

"...if you want to make a simulation of nature, you'd better make it quantum mechanical,..."

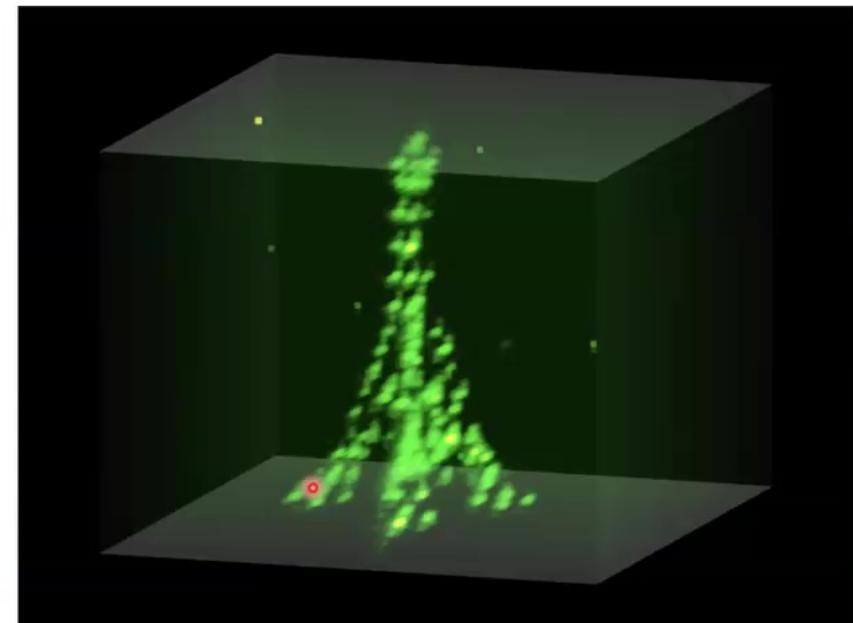


Quantum Simulator: Rydberg atom arrays



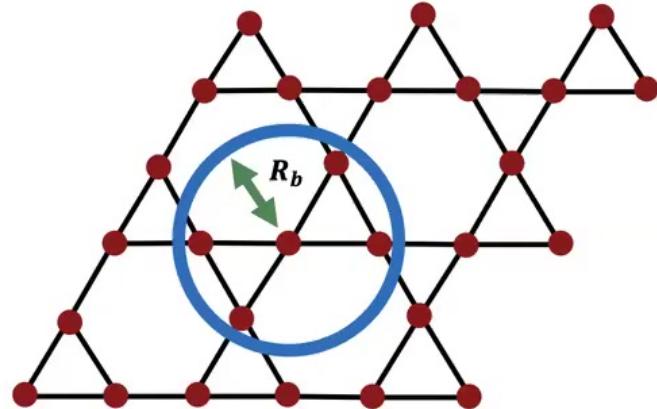
Programmable quantum simulator:

- Artificially simulate phases of matter in the lab.
- Realizable in the lab by manipulating Rydberg atoms.
- Can be implemented on different geometries with tunable parameters.



Credit: Thierry Lahaye/CNRS, 2018

Rydberg atom arrays



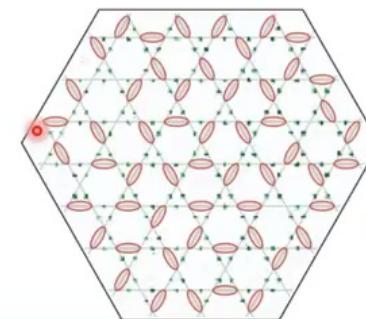
Programmable quantum simulator:

- Artificially simulate phases of matter in the lab.
- Realizable in the lab by manipulating Rydberg atoms.
- Can be implemented on different geometries with tunable parameters.

TOPOLOGICAL MATTER

Probing topological spin liquids on a programmable quantum simulator

G. Semeghini¹, H. Levine¹, A. Keesling^{1,2}, S. Ebadi¹, T. T. Wang¹, D. Bluvstein¹, R. Verresen¹, H. Pichler^{3,4}, M. Kalinowski¹, R. Samajdar¹, A. Omran^{1,2}, S. Sachdev^{1,5}, A. Vishwanath^{1*}, M. Greiner^{1*}, V. Vuletić^{6*}, M. D. Lukin^{1*}



Realization of “Topological order” in Rydberg atom arrays
(2021)

Goal of this talk

Showcase that we can use Recurrent Neural Networks (RNNs) to
understand the physics of Rydberg atoms arrays on Kagome lattice

Goal of this talk

Density Matrix Renormalization Group: Spin liquid phase (topological order).



R. Samajdar et al., PNAS 2021.

Quantum Monte Carlo: Spin glass phase.



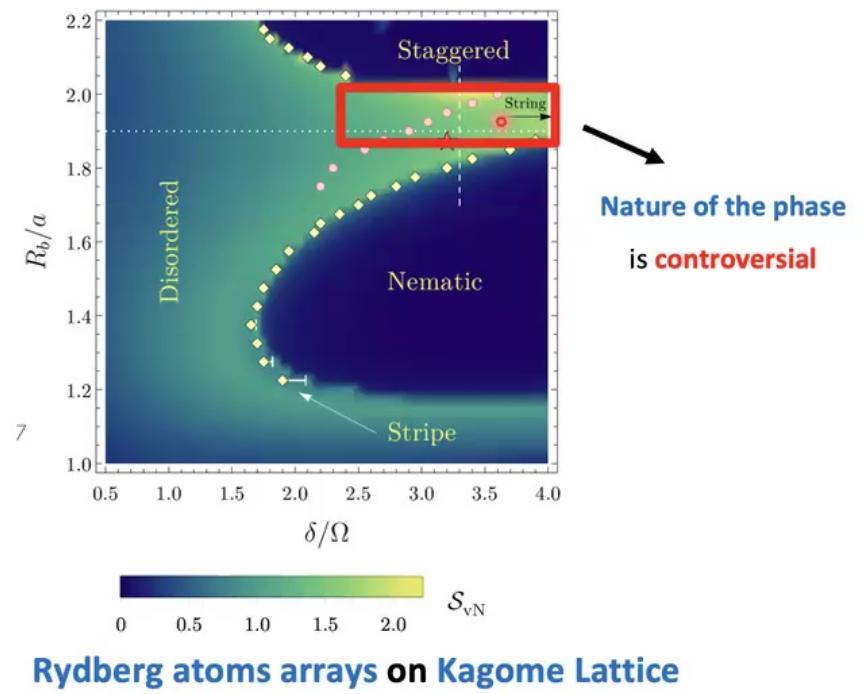
Z. Yan et al., PRL 2023.

2D Recurrent Neural Networks (RNNs): Paramagnetic phase.

Quantum Monte Carlo (fine-tuned simulations): Paramagnetic phase.

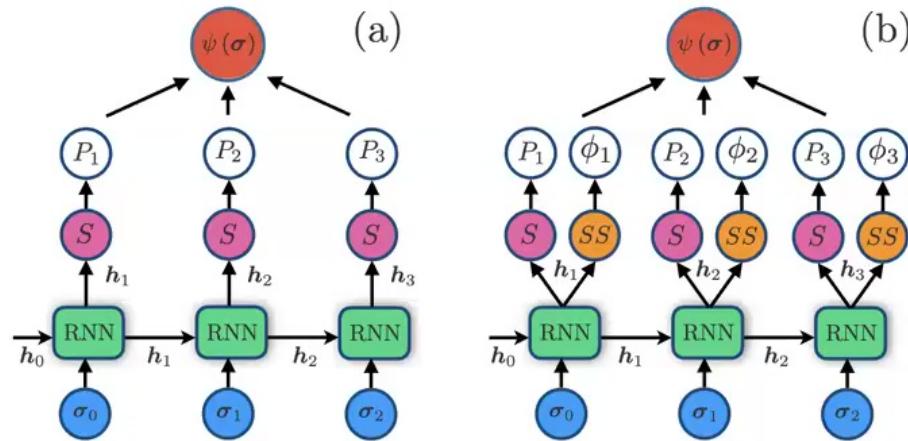
M.H. E. Merali, G. Torlai, R. Melko, J. Carrasquilla, arXiv: 2405.20384

Very promising example where RNNs offer a powerful complementary numerical method in quantum matter.



Outline

- I. Recurrent Neural network (RNN) wave functions
- II. Variational annealing with RNNs.
- III. Investigating topological order with RNNs.
- IV. RNNs for Rydberg atoms arrays on Kagome Lattice.



I. Recurrent Neural Network Wave Functions

[M.H, M. Ganahl, L. Hayward, R. Melko, J. Carrasquilla, Recurrent Neural Network Wave Functions, Physical Review Research, Jun 2020.](#)

[M.H, R. Melko, J. Carrasquilla, Supplementing RNN wave functions with symmetry and annealing to improve accuracy, ML for Physical Sciences Workshop, NeurIPS, Dec 2021.](#)

[M.H, R. Melko, J. Carrasquilla, Investigating topological order using Recurrent Neural Networks, Physical Reviews B, Aug 2023.](#)

Ground state problem

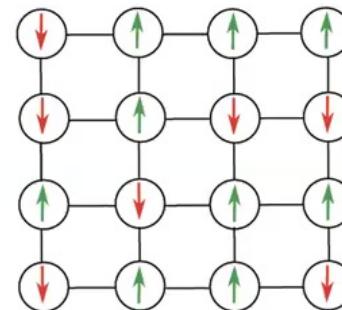
$$H\Psi_G = E_G \Psi_G$$

Hamiltonian
(matrix with size $2^N \times 2^N$)

Ground State Wave function
(complex-valued vector with size 2^N)

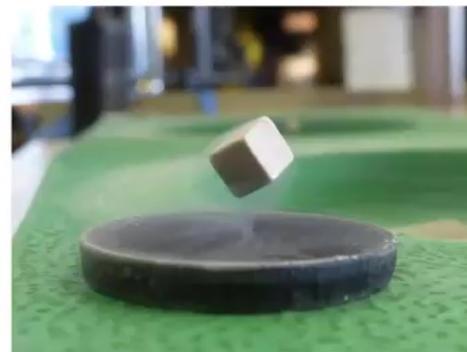
The lowest energy of the system
(Lowest eigenvalue)

Interactions are fully described by
the Hamiltonian : H

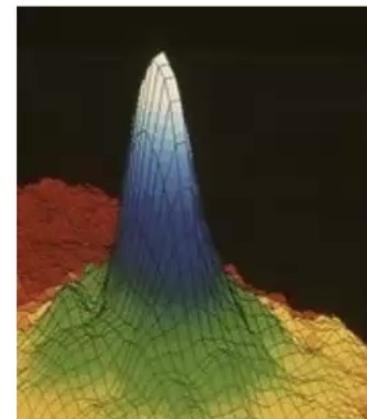


Why do we need ground states?

- To understand quantum phenomena (superconductivity, Bose-Einstein condensate,...).



Credit: Phys.org



Credit: © SPL

Exact diagonalization

•

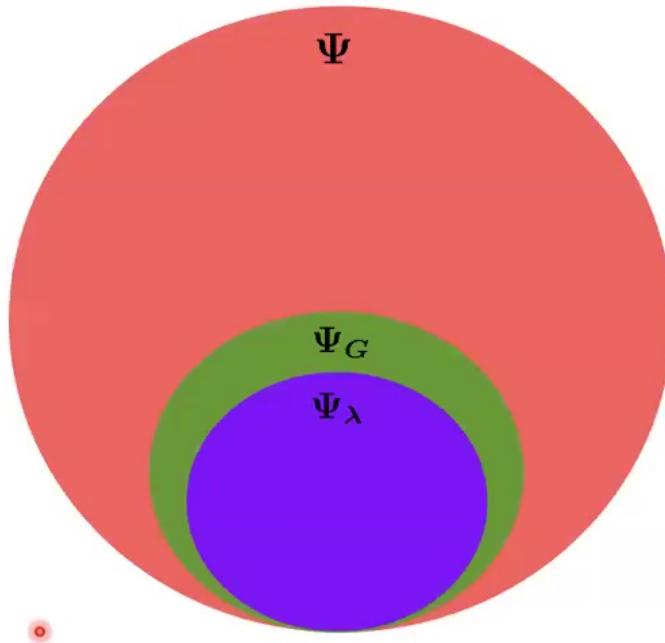
$$\mathbf{H} = \begin{pmatrix} * & \cdots & * \\ \vdots & \ddots & \vdots \\ * & \cdots & * \end{pmatrix} \xrightarrow[2^N]{\text{Diagonalize}} \begin{pmatrix} \mathbf{E}_G & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & * \end{pmatrix}$$

Exact diagonalization

$$\mathbf{H} = \begin{pmatrix} * & \cdots & * \\ \vdots & \ddots & \vdots \\ * & \cdots & * \end{pmatrix} \xrightarrow[2^N]{\text{Diagonalize}} \begin{pmatrix} \mathbf{E}_G & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & * \end{pmatrix}$$

- Computational complexity of diagonalization: $\sim \mathcal{O}(2^{cN})$
- Example: 2^{110} FLOPS is about 3 times the age of the known universe on a modern GPU (10¹⁵ FLOPS per second)

Variational Principle



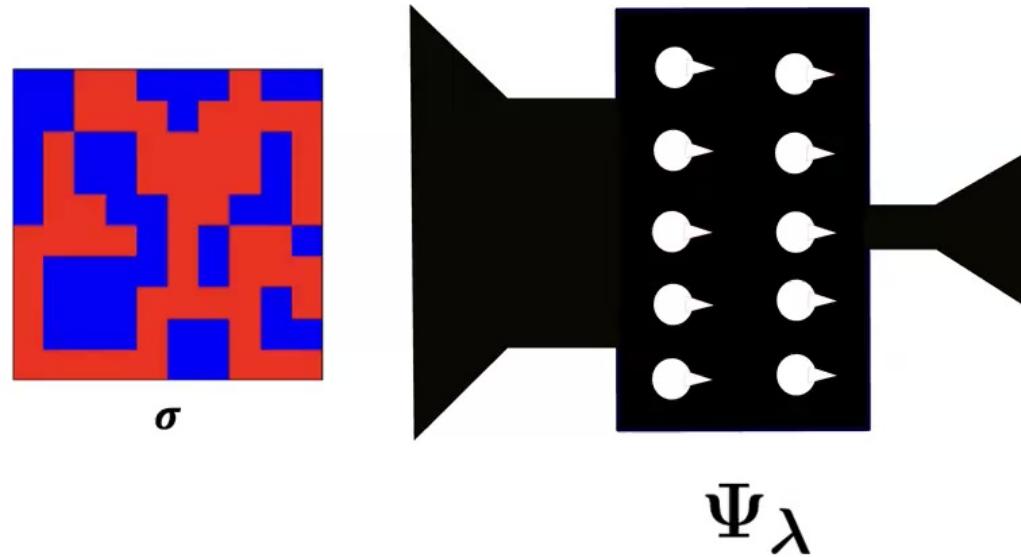
$$\begin{aligned} H\Psi_G &= E_G \Psi_G \\ \downarrow & \\ E_G &= \min_{\|\Psi\|_2=1} \Psi^\dagger H \Psi \\ \downarrow & \\ E_G &\lesssim \min_{\lambda} \boxed{\Psi_\lambda^\dagger H \Psi_\lambda} \end{aligned}$$

Variational energy
=

Cost function

Ψ_λ needs to be expressive enough!

Neural network quantum states



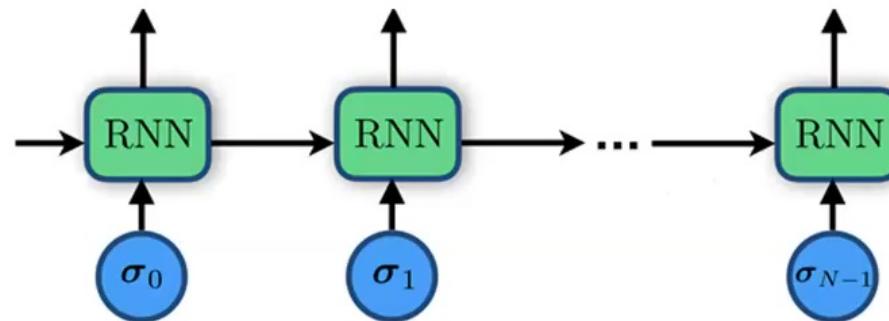
Carleo and Troyer, Science, 2017

RNNs for Rydberg atom arrays



15

Recurrent Neural Networks (RNNs)



An RNN is a Universal Turing Machine approximator: can be seen as a general-purpose computer that can perform any classical computation.

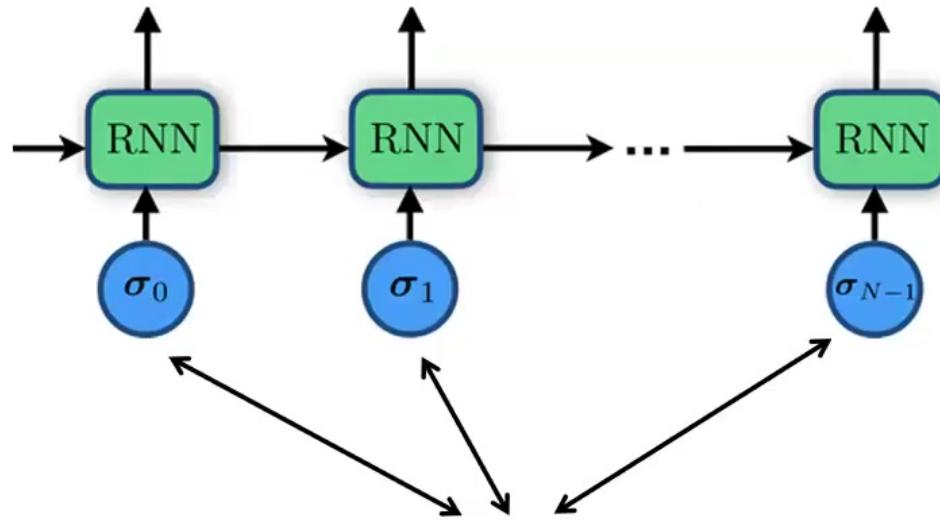
Very powerful at generating sequential data:

- Speech recognition, machine translation,...

Siegelmann and Sontag, ACM, 1982

Chung and Siegelmann, NeurIPS 2021

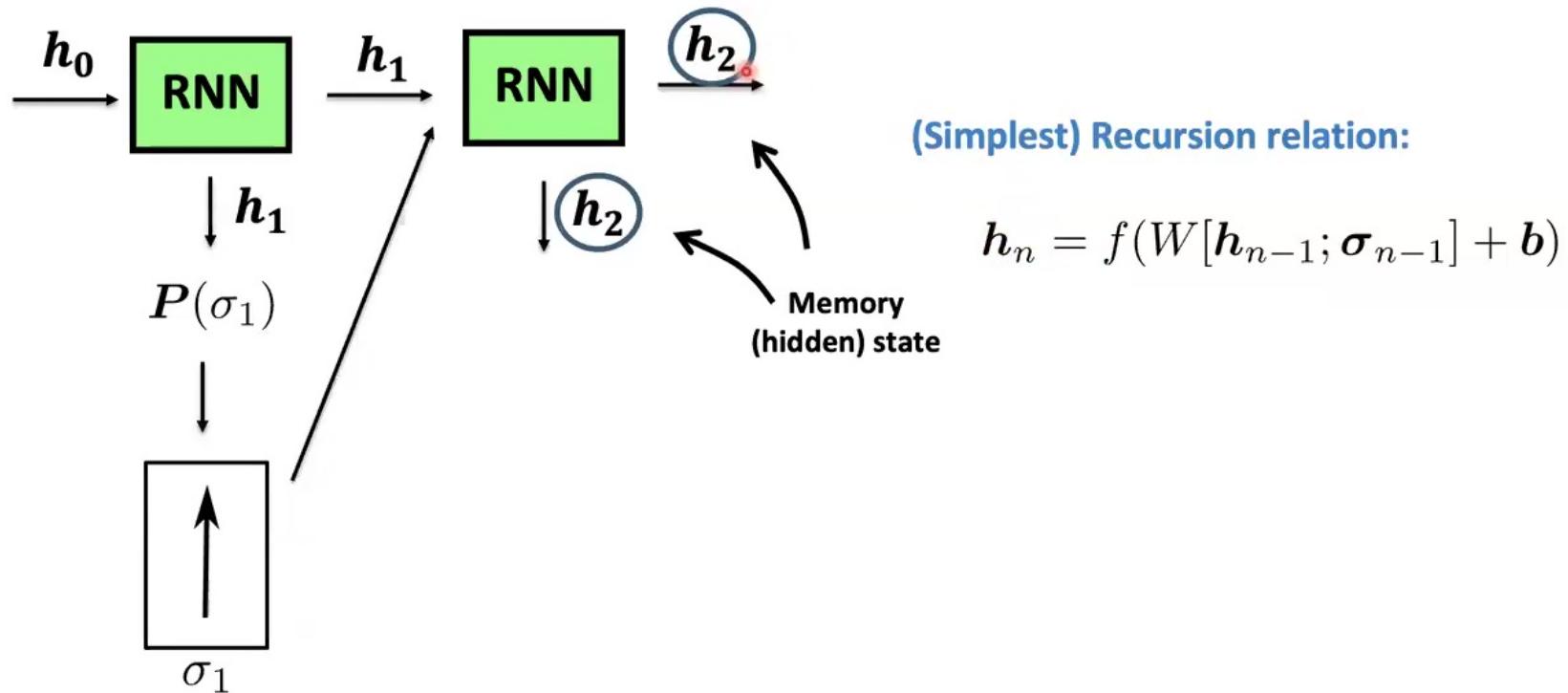
RNNs can be used in many-body Physics



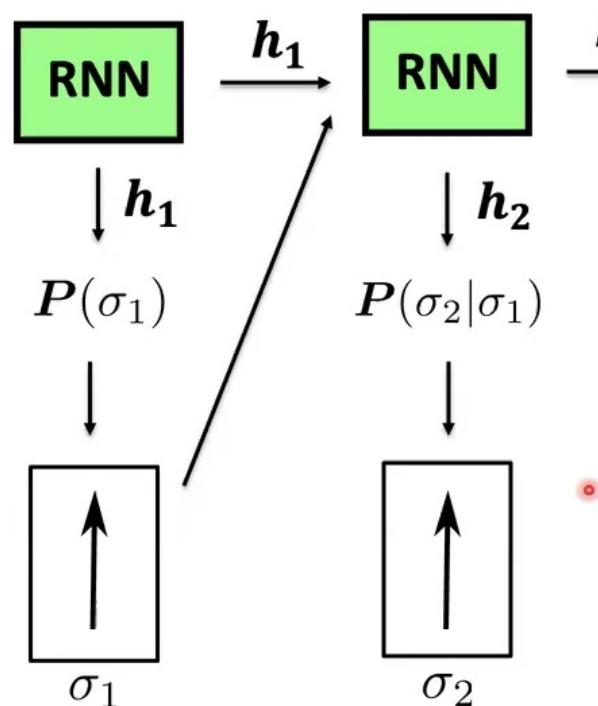
“Spins”, “occupation number” or “excitation” instead of “Words”

• “Artificial intelligence for advanced functional materials: Exploring current and future directions”, Journal of Physics: Materials, 2025

Autoregressive sampling



Autoregressive sampling



(Simplest) Recursion relation:

$$\mathbf{h}_n = f(W[\mathbf{h}_{n-1}; \boldsymbol{\sigma}_{n-1}] + \mathbf{b})$$

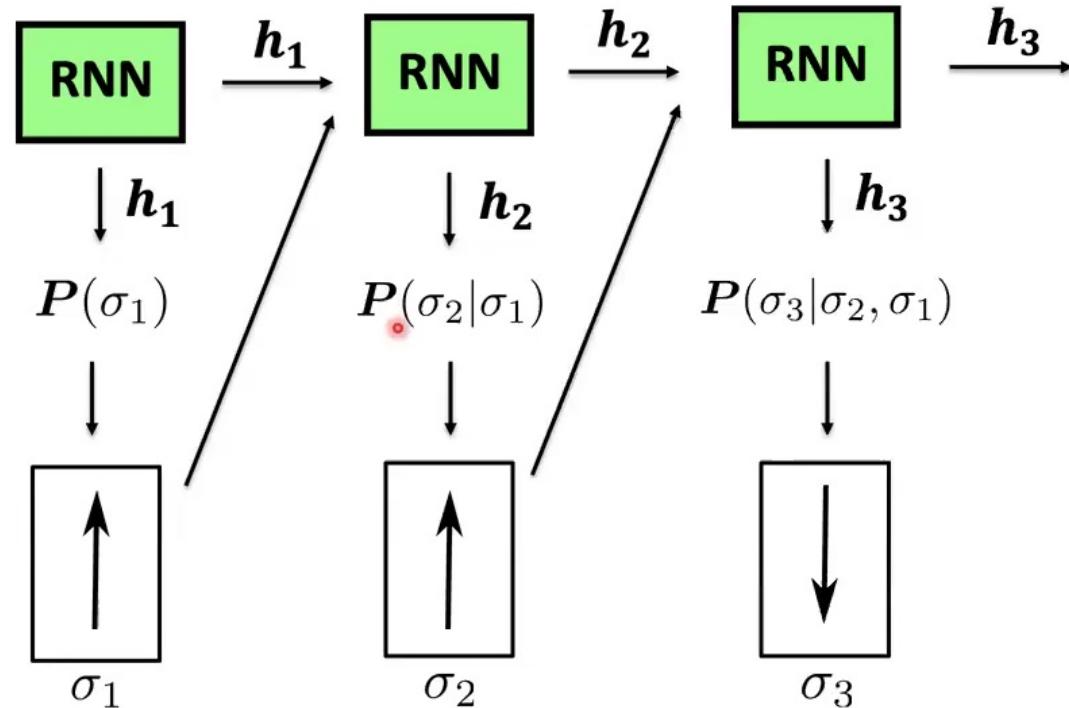
Conditional probability:

$$P(\sigma_i|\sigma_{<i}) = \text{Softmax}(U\mathbf{h}_n + \mathbf{c}) \cdot \boldsymbol{\sigma}_n$$

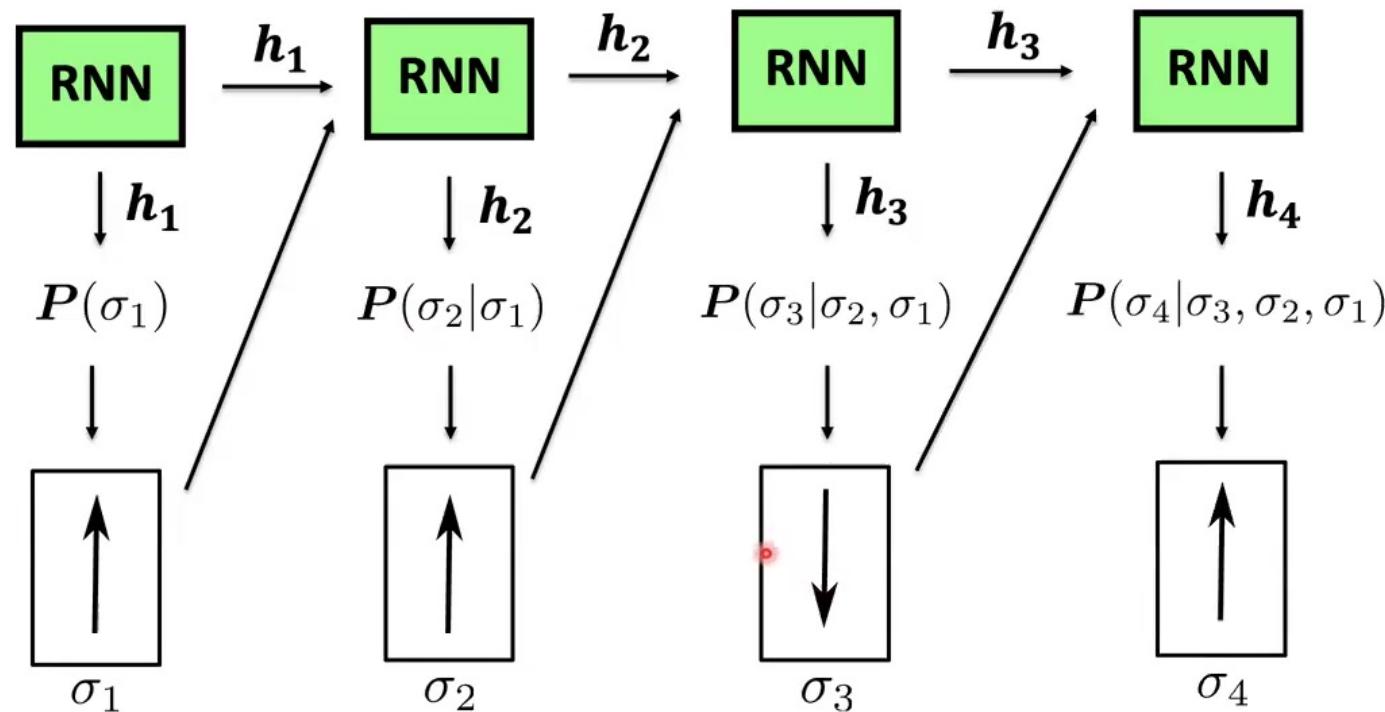
W , U , b and c are the parameters of the RNN.

In this work, we use gated recurrent units (GRUs).

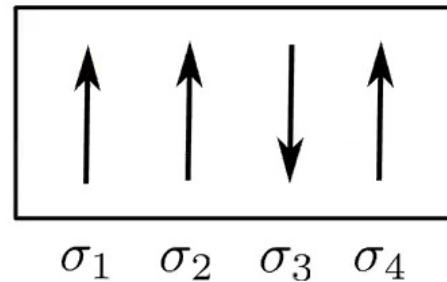
Autoregressive sampling



Autoregressive sampling



RNN wave functions

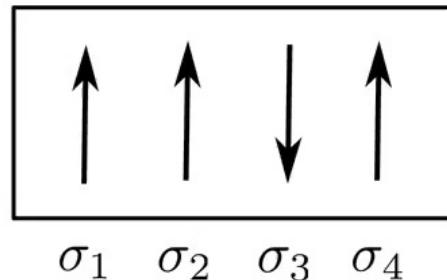


$$|\Psi_{\text{RNN}}(\sigma_1, \sigma_2, \sigma_3, \sigma_4)|^2 = P(\sigma_1)P(\sigma_2|\sigma_1)P(\sigma_3|\sigma_2, \sigma_1)P(\sigma_4|\sigma_3, \sigma_2, \sigma_1)$$



[M.H, M. Ganahl, L. Hayward, R. Melko, J. Carrasquilla, Recurrent Neural Network Wave Functions, PRResearch, 2020.](#)

Autoregressive sampling



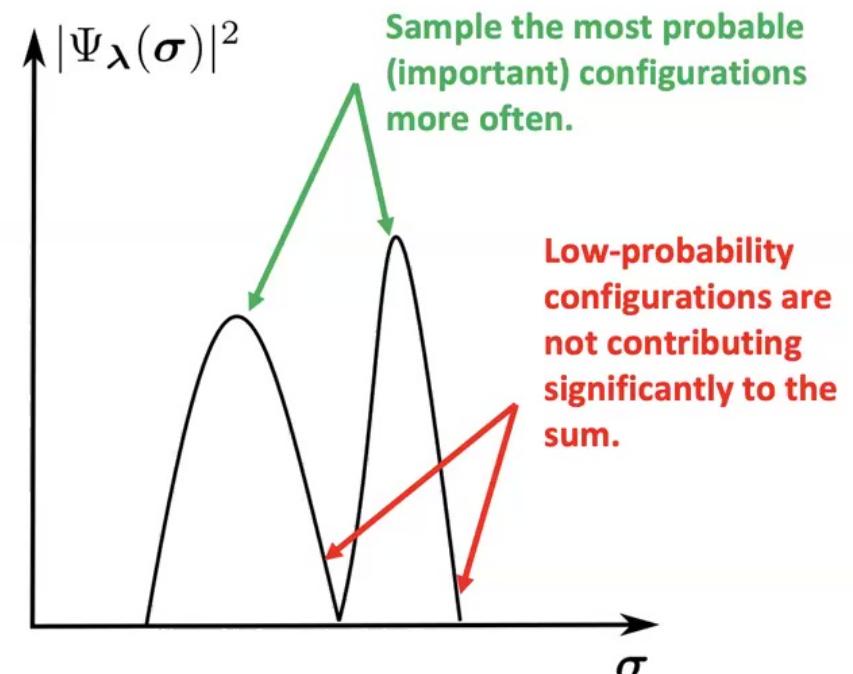
$$|\Psi_{\text{RNN}}(\sigma_1, \sigma_2, \sigma_3, \sigma_4)|^2 = \mathbf{P}(\sigma_1)\mathbf{P}(\sigma_2|\sigma_1)\mathbf{P}(\sigma_3|\sigma_2, \sigma_1)\mathbf{P}(\sigma_4|\sigma_3, \sigma_2, \sigma_1)$$

The samples are **independents** and can be generated in **parallel** $\{\sigma^{(j)}\}_{i=1}^{N_s}$

Importance Sampling (Variational Monte Carlo)

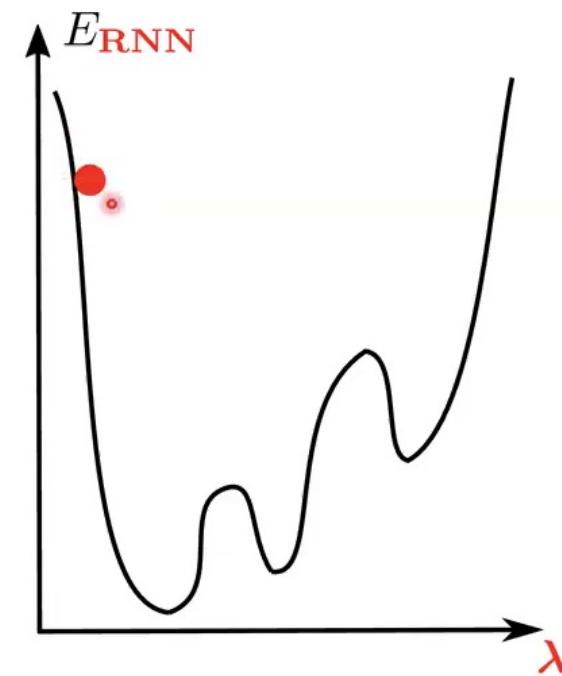
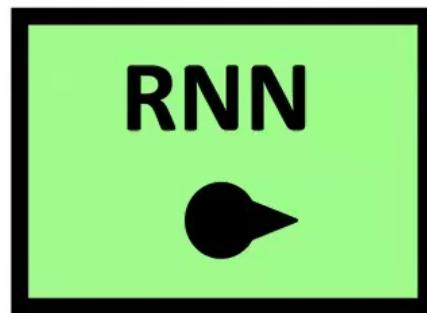
$$\begin{aligned} E_{\lambda} &= \Psi_{\lambda}^{\dagger} H \Psi_{\lambda} \\ &= \sum_{\sigma} |\Psi_{\lambda}(\sigma)|^2 E_{\text{loc}}(\sigma) \\ &\approx \frac{1}{N_s} \sum_{\sigma \sim |\Psi_{\lambda}(\sigma)|^2} E_{\text{loc}}(\sigma) \end{aligned}$$

No need for training data.
The data comes from the model itself!

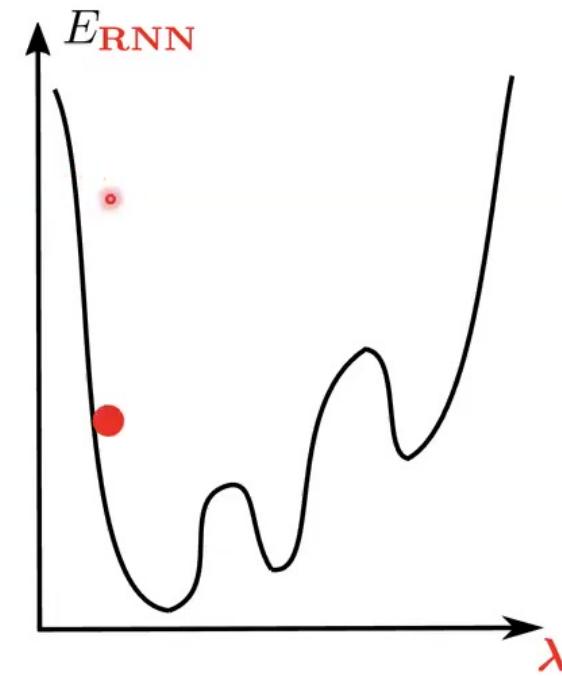


Becca and Sorella, Quantum Monte Carlo Approaches for Correlated Systems, Cambridge, 2017.

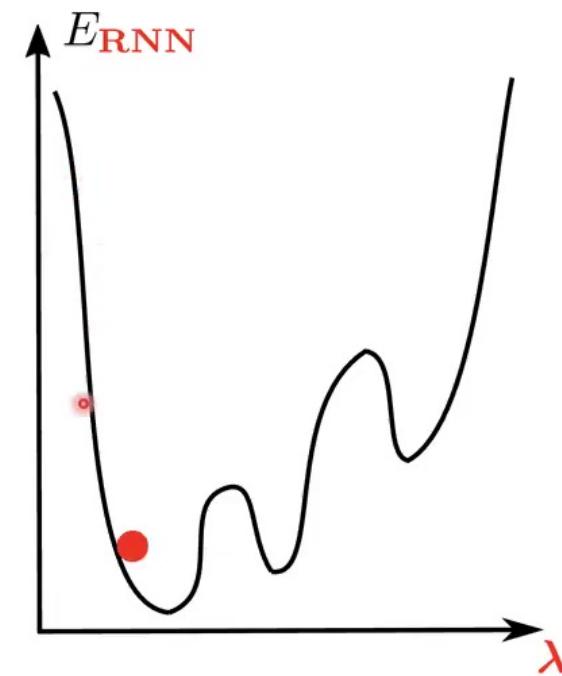
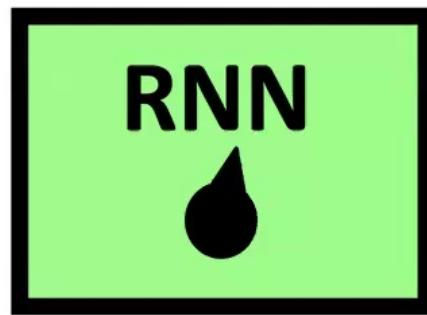
RNN wave functions optimization



RNN wave functions optimization



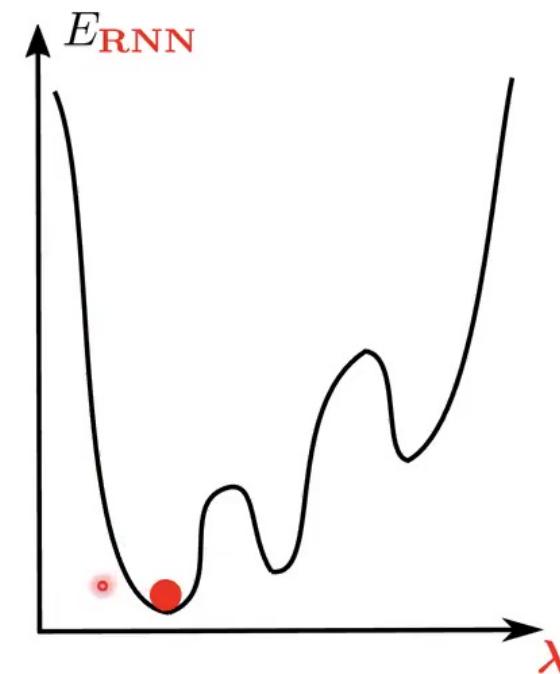
RNN wave functions optimization



RNN wave functions optimization



M.H. M. Ganahl, L. Hayward, R. Melko, J. Carrasquilla,
Recurrent Neural Network Wave Functions,
PRResearch, 2020.



An approximation
of the ground state
is found!

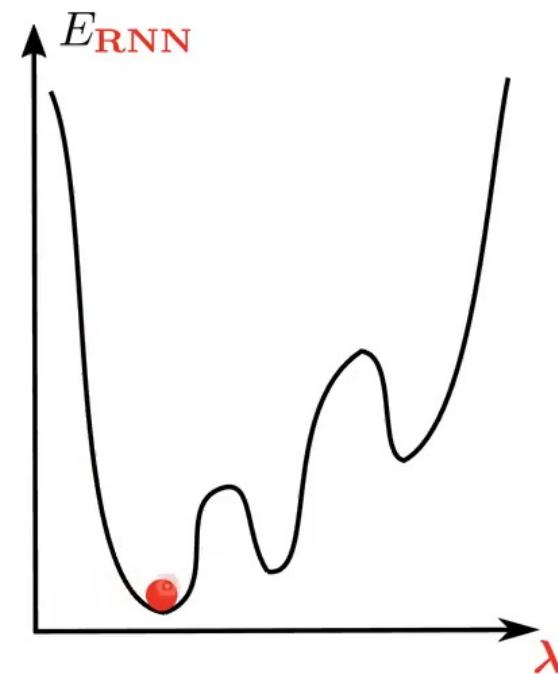
$$\Psi_{\text{RNN}} \approx \Psi_G$$

$$E_{\text{RNN}} \approx E_G$$

RNN wave functions optimization



M.H. M. Ganahl, L. Hayward, R. Melko, J. Carrasquilla,
Recurrent Neural Network Wave Functions,
PRResearch, 2020.

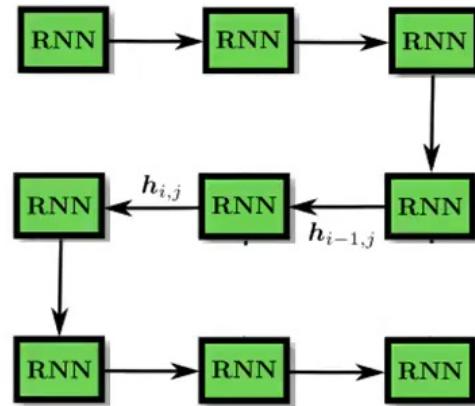


An approximation
of the ground state
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$$\Psi_{\text{RNN}} \approx \Psi_G$$

$$E_{\text{RNN}} \approx E_G$$

1D recurrent neural networks

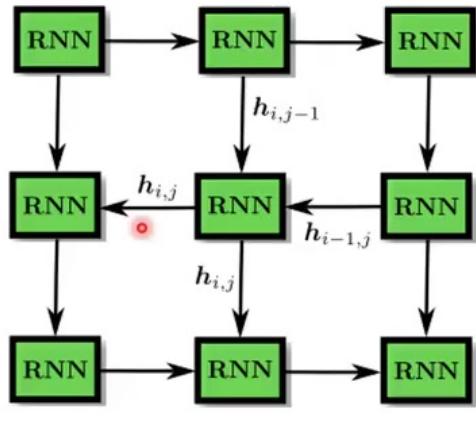


1D recursion relation:

$$\mathbf{h}_n = f(W[\mathbf{h}_{n-1}; \sigma_{n-1}] + \mathbf{b})$$



2D recurrent neural networks



1D recursion relation:

$$h_n = f(W[h_{n-1}; \sigma_{n-1}] + b)$$

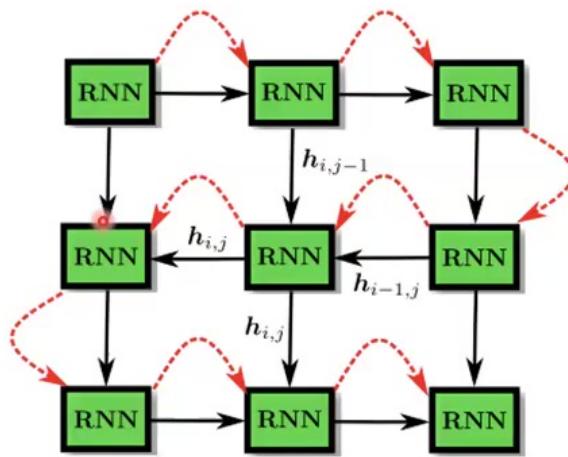
2D recursion relation:

$$h_{i,j} = f\left(W^{(h)}[h_{i-1,j}; \sigma_{i-1,j}] + W^{(v)}[h_{i,j-1}; \sigma_{i,j-1}] + b\right)$$

M.H. M. Ganahl, L. Hayward, R. Melko, J. Carrasquilla, RNN Wave functions, PRResearch, 2020.

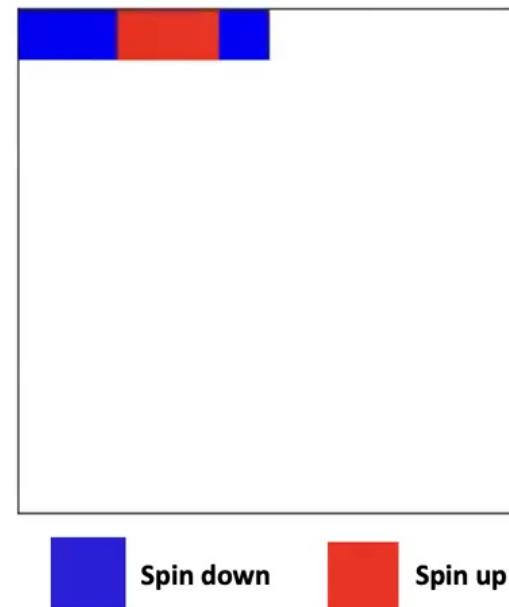
A. Graves et al., Multi-dimensional recurrent neural networks, 2007.

Zigzag Autoregressive Sampling

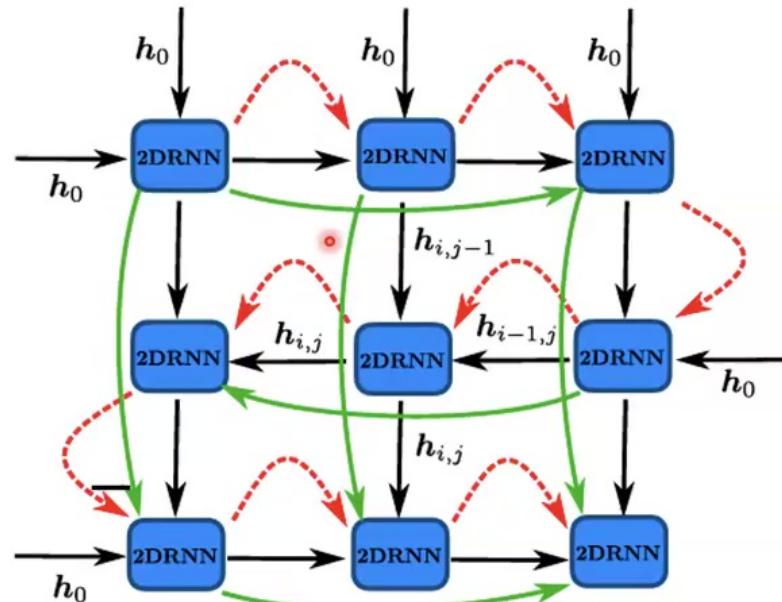


M.H. M. Ganahl, L. Hayward, R. Melko, J. Carrasquilla, RNN Wave functions, PRResearch, 2020.

A. Graves et al., Multi-dimensional recurrent neural networks, 2007.



2D periodic recurrent neural networks

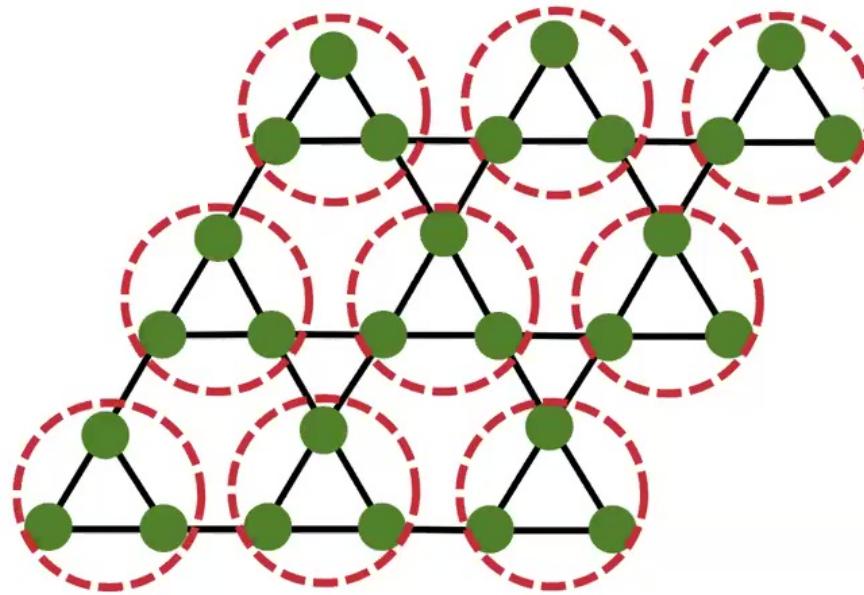


$$\mathbf{h}_{i,j} = f\left(W[\text{Neighbours}(\mathbf{h}_{i,j}); \text{Neighbours}(\boldsymbol{\sigma}_{i,j})] + \mathbf{b}\right)$$

M.H, R. Melko, J. Carrasquilla, Physical Reviews B, (2023)

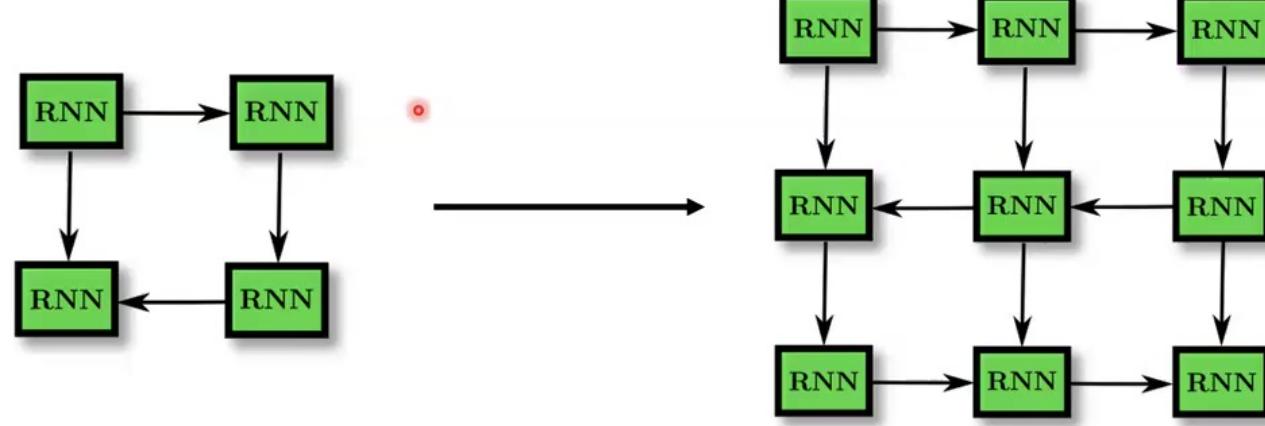
D. Luo et al, Physical Review Research 5, 013216 (2023)

Kagome to Square mapping



M.H., R. Melko, J. Carrasquilla, Physical Reviews B, (2023)

Iterative retraining of RNNs



C. Roth, arXiv:2003.06228, 2020

M.H., R. Melko, J. Carrasquilla, NeurIPS, 2021

M.H., R. Melko, J. Carrasquilla, Physical Reviews B, 2023

S. Moss, R. Wiersema, M.H., J. Carrasquilla, R. Melko, arXiv:2502.17144, 2025



Heating



Slow Cooling



Strong Metal

Credit: FUJITSU Digital Annealer

II. Variational annealing with RNNs

M.H., E. Inack, R. Wiersema, R. Melko, J. Carrasquilla, Variational Neural Annealing, [Nature Machine Intelligence](#), Oct 2021.

Variational Annealing with RNNs

$$F_{RNN}(T) = \langle \hat{H} \rangle_{RNN} - TS_{RNN}$$

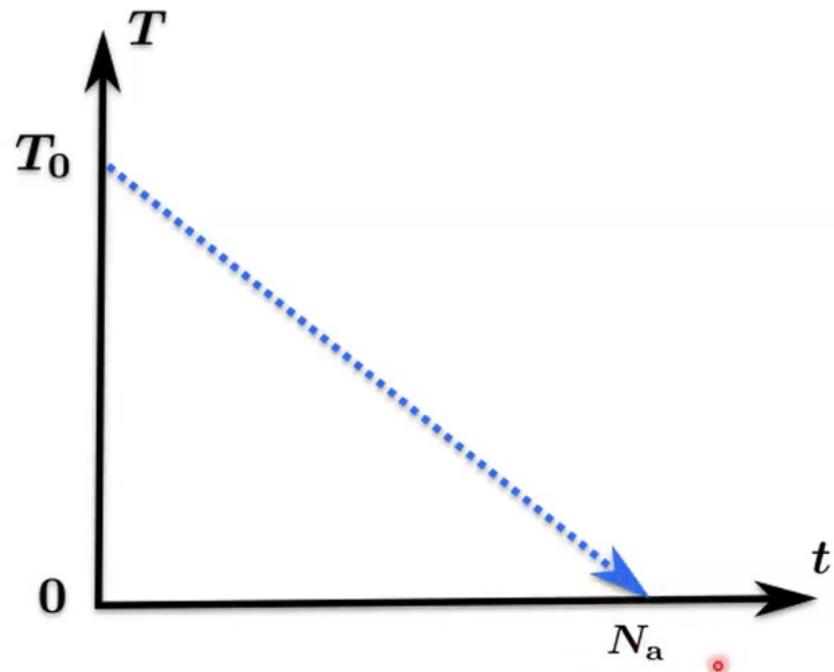
“Free energy” = Cost function

Shannon entropy term (thermal-like fluctuations)

C. Roth, arXiv:2003.06228, 2020

M.H, E. Inack, R. Wiersema, R. Melko, J. Carrasquilla, Nature Machine Intelligence, Oct 2021

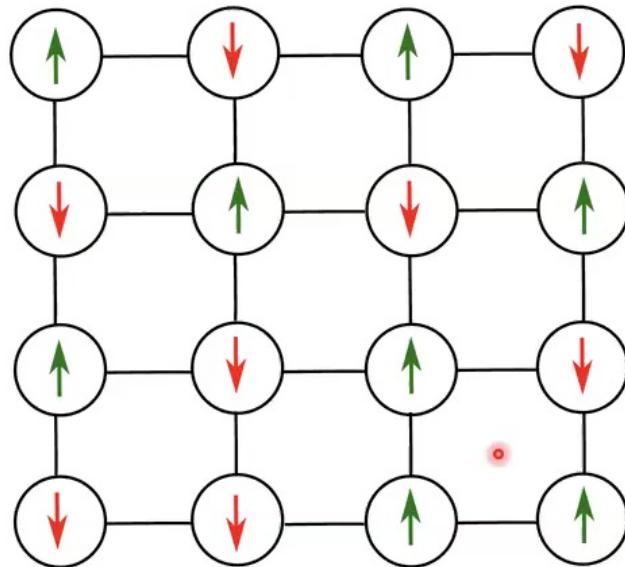
Annealing schedule



$$\mathbf{F}_{\text{RNN}}(\mathbf{T}) = \langle \hat{\mathbf{H}} \rangle_{\text{RNN}} - \mathbf{T} \mathbf{S}_{\text{RNN}}$$

M.H. E. Inack, R. Wiersema, R. Melko, J. Carrasquilla, [Nature Machine Intelligence, Oct 2021.](#)

Example: Spin Ising glass in 2D



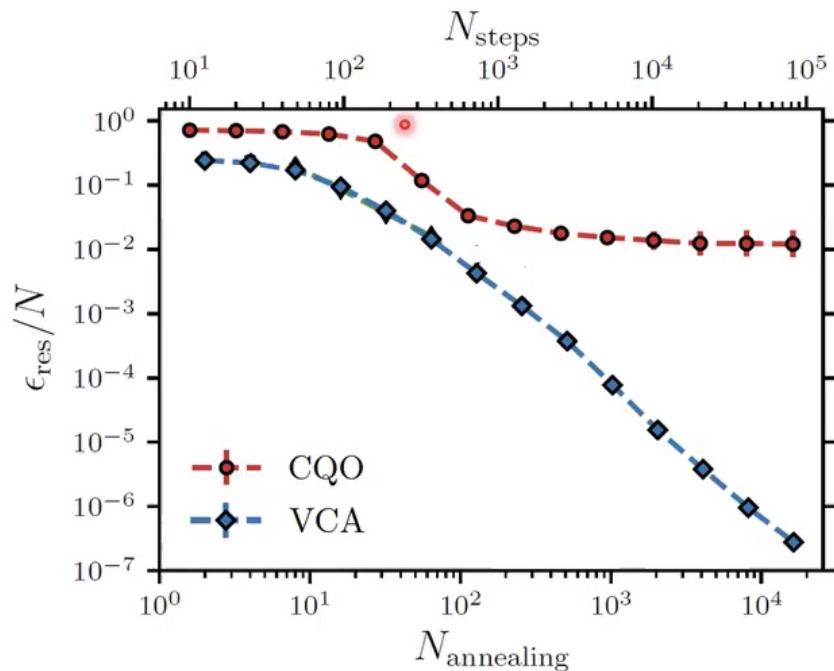
Edwards-Anderson model:

$$H_{EA} = \sum_{\langle i,j \rangle} J_{ij} \sigma_i \sigma_j$$

The couplings are drawn from **random uniform** distribution in the range [-1,1)

[M.H. E. Inack, R. Wiersema, R. Melko, J. Carrasquilla, Nature Machine Intelligence, Oct 2021.](#)

Example: Spin Ising glass in 2D ($N = 10 \times 10$ spins)



Edwards-Anderson model:

$$H_{\text{EA}} = \sum_{\langle i,j \rangle} J_{ij} \sigma_i \sigma_j$$

Classical-Quantum Optimization (CQO)

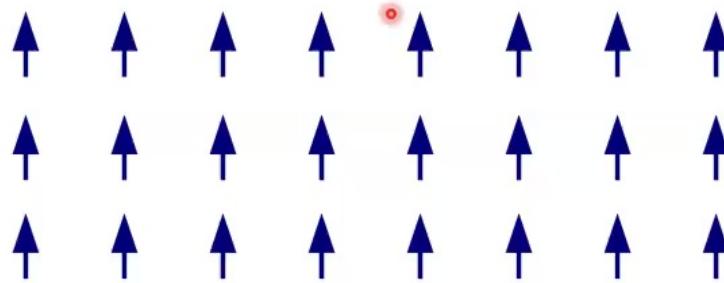
- $T = 0, B_x = 0$

Variational Classical Annealing (VCA)

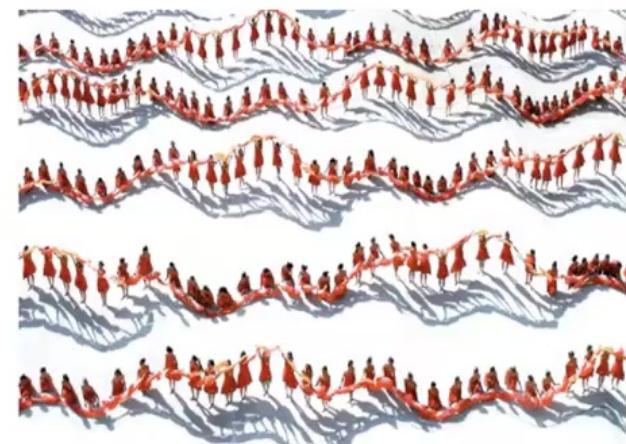
- $T(t) = T_0(1 - t/N_{\text{annealing}})$

M.H. E. Inack, R. Wiersema, R. Melko, J. Carrasquilla, Nature Machine Intelligence, Oct 2021.

III. Investigating topological order with RNNs



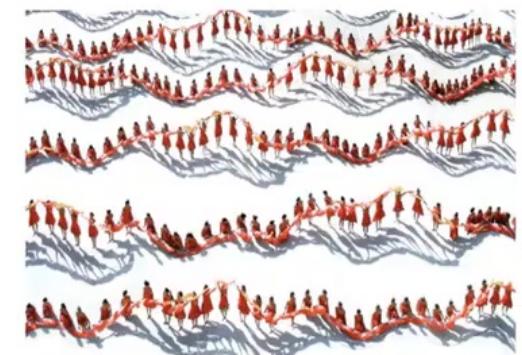
Ferromagnet (non topological – local)
(credit: Michael Schmidt)



Spin liquid (topological – nonlocal)
(credit: Xiao-Gang Wen)

Topological order

- Zero-temperature phase of quantum matter.
 - Quantum Hall Effect.
 - Fault-tolerant quantum computation.
- Can be characterized using 'Topological entanglement entropy'.



Spin liquid (credit: Xiao-Gang Wen)

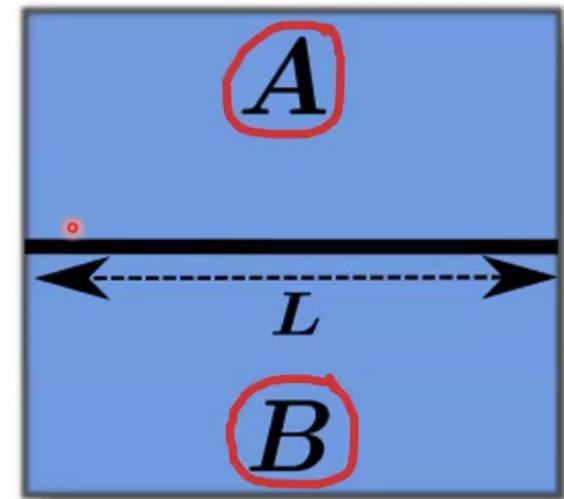
Topological entanglement entropy

$$S_2(A) = -\log(\text{Tr}(\rho_A^2))$$

Second Renyi entropy can be computed using RNN wave functions with the Swap trick.

M.H., M. Ganahl, L. Hayward, R. Melko, J. Carrasquilla, PRRResearch, 2020.

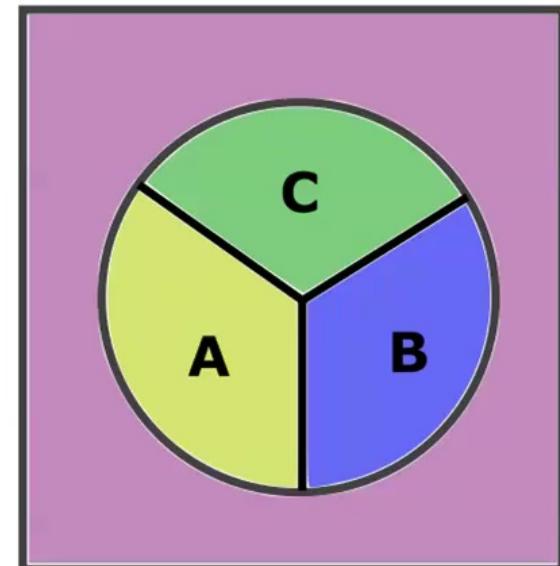
$$S_2(A) = aL - \gamma + \mathcal{O}(L^{-1})$$



Kitaev-Preskill construction

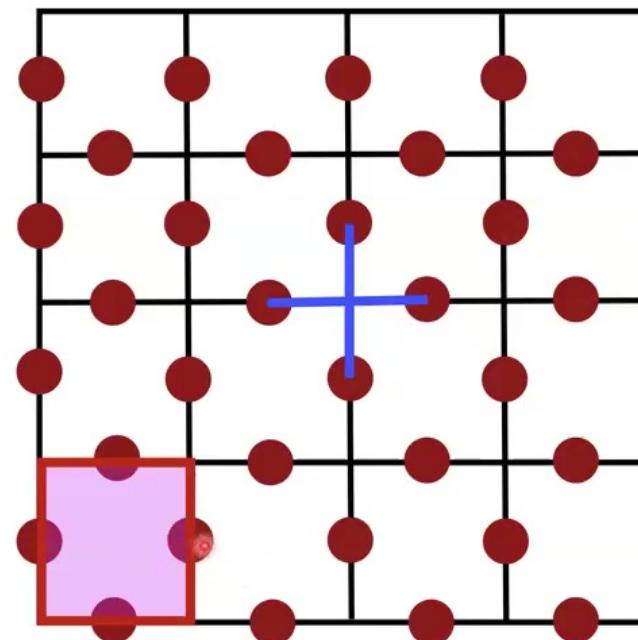
$$\begin{aligned}\gamma = & -S_A - S_B - S_C \\ & + S_{AB} + S_{AC} + S_{BC} \\ & - S_{ABC}\end{aligned}$$

Kitaev, Preskill, 2005



2D Toric Code

$$\hat{H} = - \sum_p \Pi_{i \in p} \hat{\sigma}_i^z - \sum_v \Pi_{i \in v} \hat{\sigma}_i^x$$



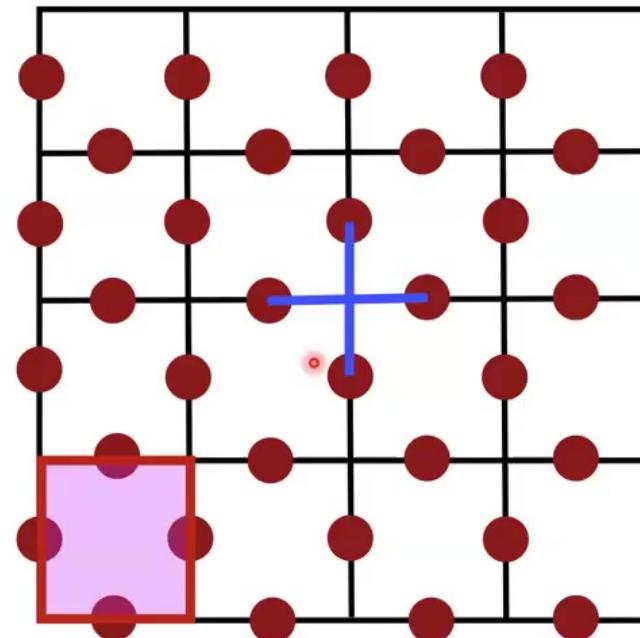
M.H, R. Melko, J. Carrasquilla, Physical Review B, Aug 2023

2D Toric Code

$$\hat{H} = - \sum_p \Pi_{i \in p} \hat{\sigma}_i^z - \sum_v \Pi_{i \in v} \hat{\sigma}_i^x$$

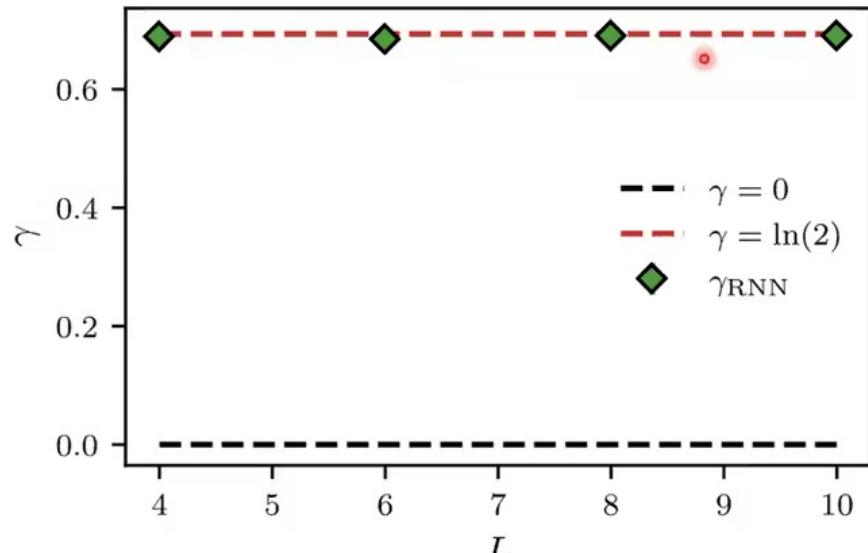
$$\gamma = \ln(2)$$

Z_2 topological order



M.H, R. Melko, J. Carrasquilla, Physical Review B, Aug 2023

2D Toric Code



TEE computed using Kitaev-Preskill construction

M.H., R. Melko, J. Carrasquilla, Physical Review B, Aug 2023

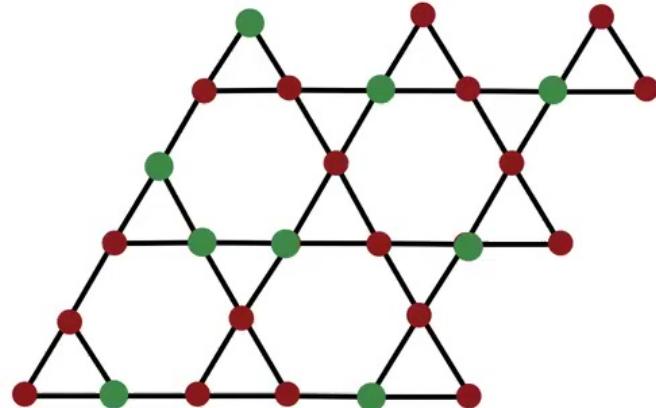
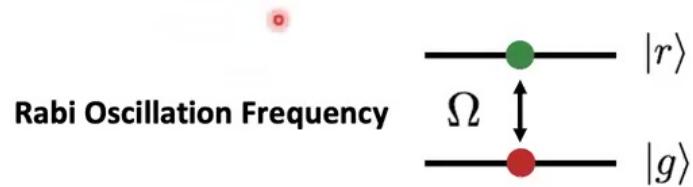
IV. RNN wave functions for Rydberg atoms on Kagome Lattice

M.H., E. Merali, G. Torlai, R. Melko, J. Carrasquilla, arXiv: 2405.20384

Rydberg atom arrays

$$\hat{H} = \sum_{i=1}^N \frac{\Omega}{2} \left(|g\rangle_i \langle r| + |r\rangle_i \langle g| \right) - \delta \sum_{i=1}^N |r\rangle_i \langle r| + \frac{1}{2} \sum_{(i,j)} V(\|\mathbf{x}_i - \mathbf{x}_j\|) |r\rangle_i \langle r| \otimes |r\rangle_j \langle r|.$$

Rabi Oscillation for each atom:

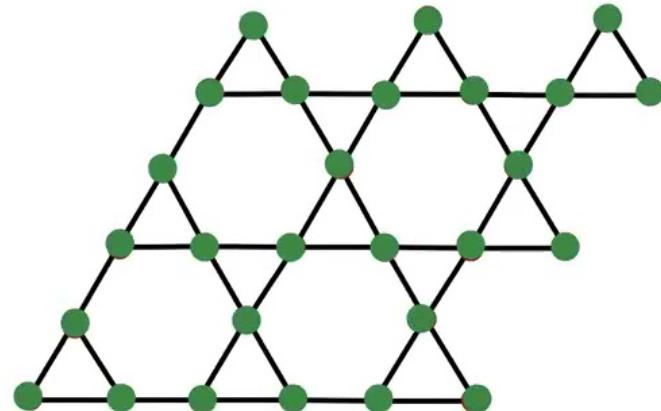


Rydberg atom arrays

$$\hat{H} = \sum_{i=1}^N \frac{\Omega}{2} \left(|g\rangle_i \langle r| + |r\rangle_i \langle g| \right) - \delta \sum_{i=1}^N |r\rangle_i \langle r| \\ + \frac{1}{2} \sum_{(i,j)} V(|\mathbf{x}_i - \mathbf{x}_j|) |r\rangle_i \langle r| \otimes |r\rangle_j \langle r|.$$

Laser detuning: δ

Encourage all Rydberg atoms to be in the
excited state level $|r\rangle$



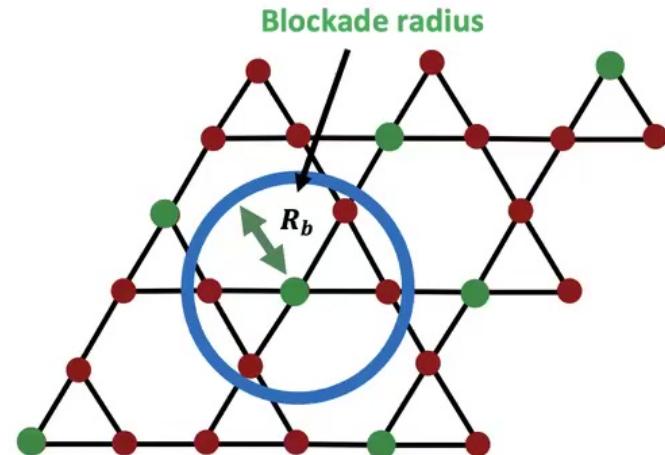
Rydberg atom arrays

$$\hat{H} = \sum_{i=1}^N \frac{\Omega}{2} \left(|g\rangle_i \langle r| + |r\rangle_i \langle g| \right) - \delta \sum_{i=1}^N |r\rangle_i \langle r| + \frac{1}{2} \sum_{(i,j)} V(\|\mathbf{x}_i - \mathbf{x}_j\|) |r\rangle_i \langle r| \otimes |r\rangle_j \langle r|.$$

Van-der-Waals interaction:

$$V(R) = \Omega \frac{R_b^6}{R^6}$$

A penalty for two atoms within the blockade
radius R_b to be both excited.



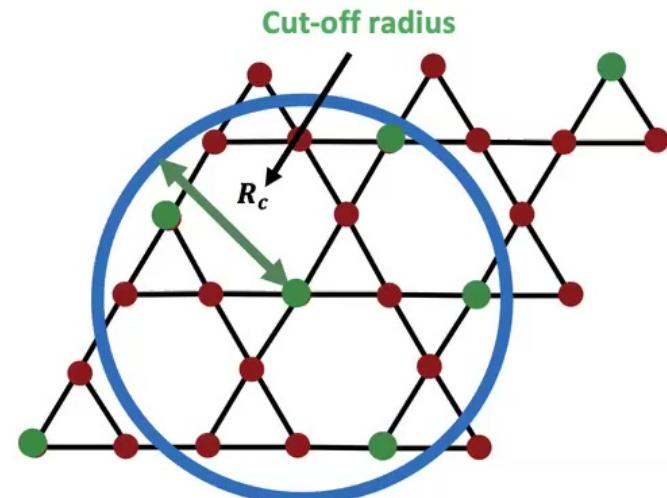
Rydberg atom arrays

$$\hat{H} = \sum_{i=1}^N \frac{\Omega}{2} \left(|g\rangle_i \langle r| + |r\rangle_i \langle g| \right) - \delta \sum_{i=1}^N |r\rangle_i \langle r| + \frac{1}{2} \sum_{(i,j)} V(\|\mathbf{x}_i - \mathbf{x}_j\|) |r\rangle_i \langle r| \otimes |r\rangle_j \langle r|.$$

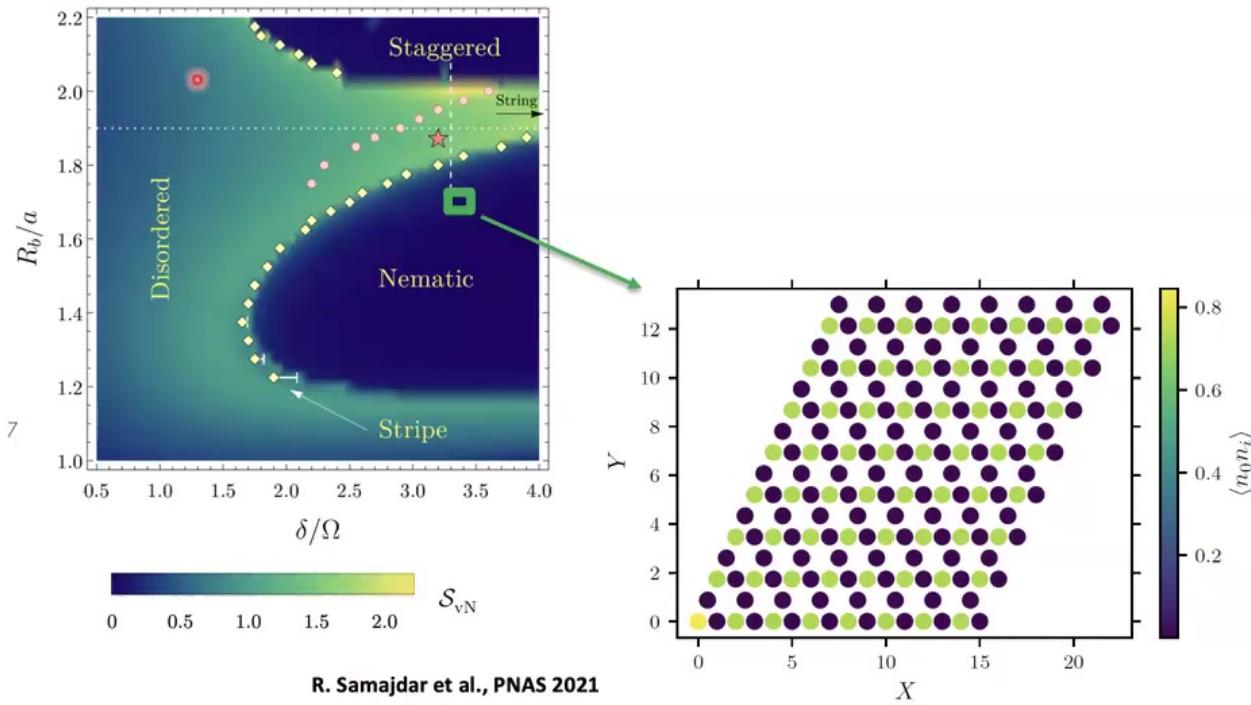
Cut-off radius

For $R > R_c$, interactions are set to zero.

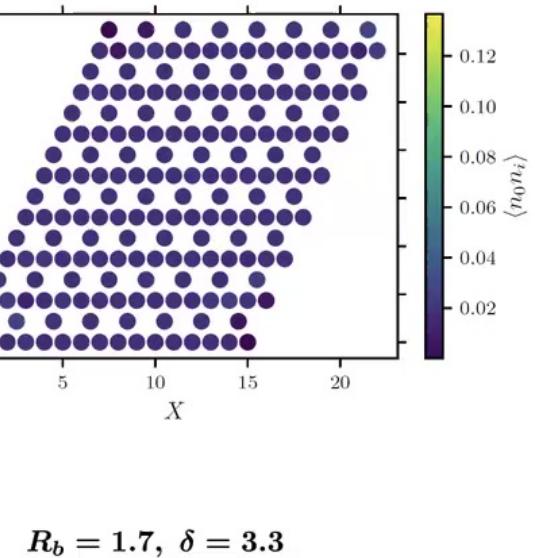
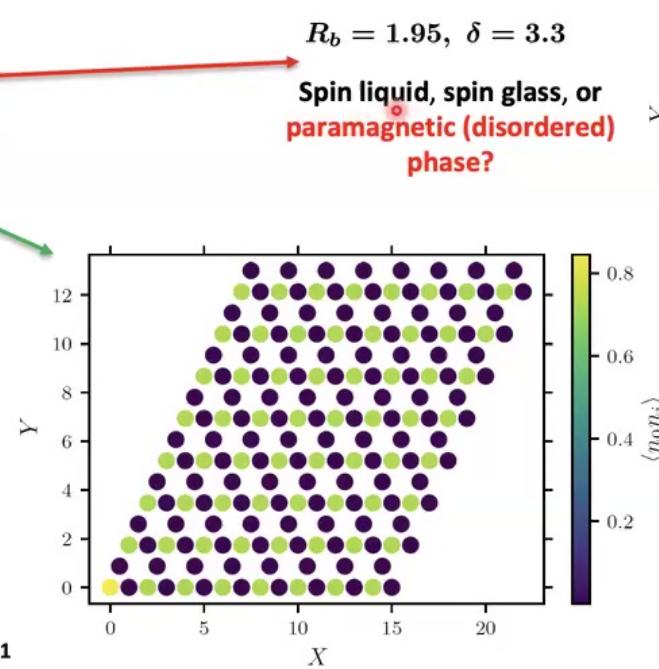
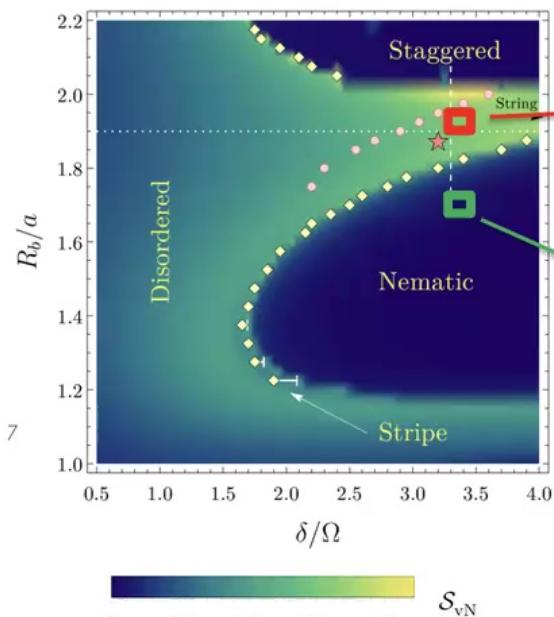
$$V(R) = \Omega \frac{R_b^6}{R^6}$$



Rydberg atom arrays on Kagome Lattice



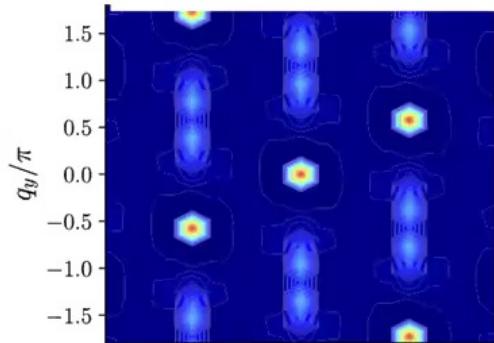
Rydberg atom arrays on Kagome Lattice



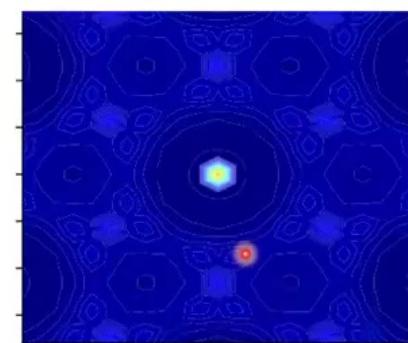
R. Samajdar et al., PNAS 2021

More annealing steps

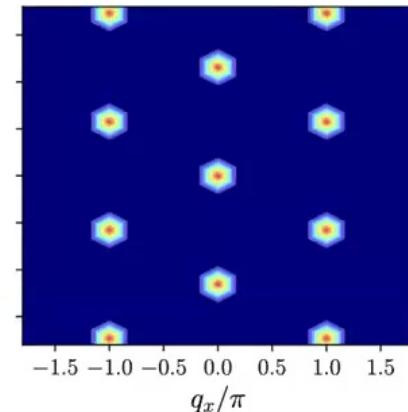
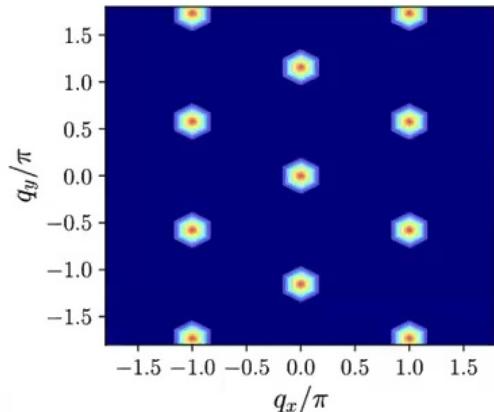
$N_a = 0$



$N_a \gg 1$



paramagnetic (disordered) phase



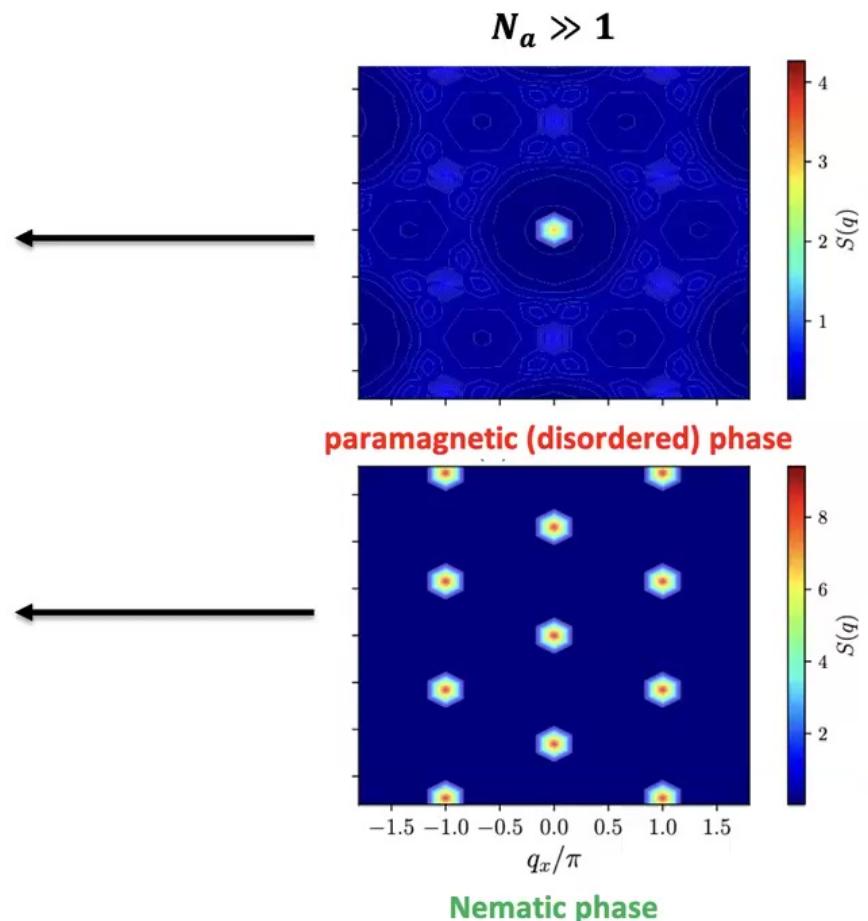
Nematic phase

M.H, E. Merali, G. Torlai, R. Melko, J. Carrasquilla, arXiv: 2405.20384

RNNs for Rydberg atom arrays

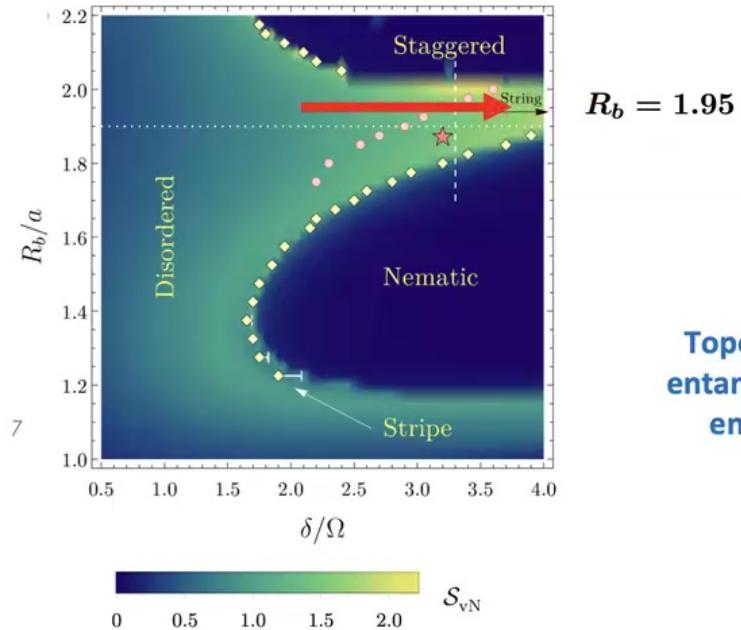
55

Difficult optimization landscape: annealing is helpful
to escape excited states (local minima) and converge
close to the ground state.



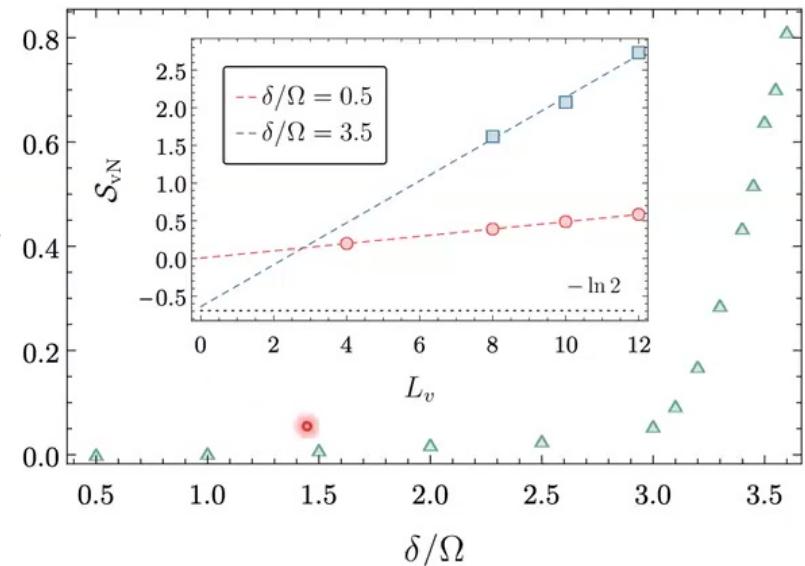
Easy optimization landscape: annealing is not needed

DMRG prediction of topological order



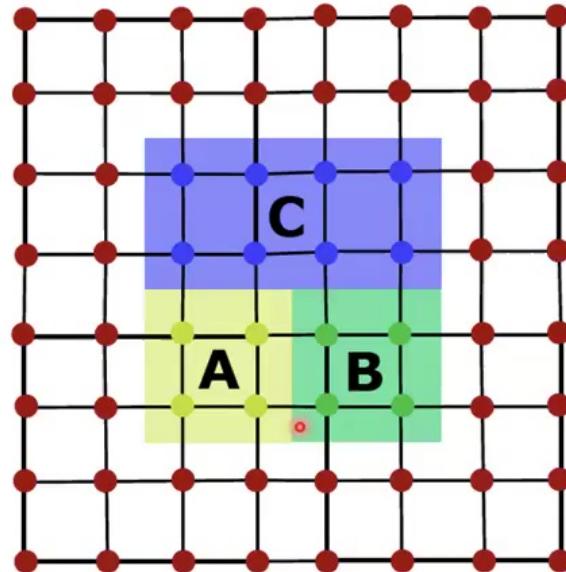
Topological
entanglement
entropy

$$\gamma$$



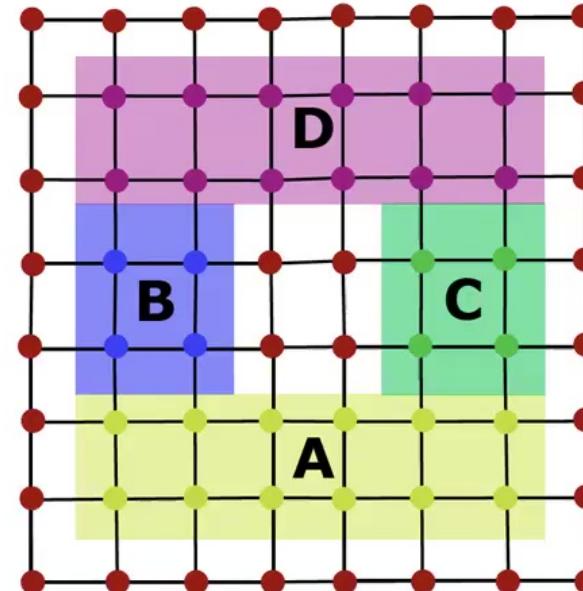
Samajdar et al., PNAS, 2021

TEE constructions



Kitaev-Preskill (KP) construction

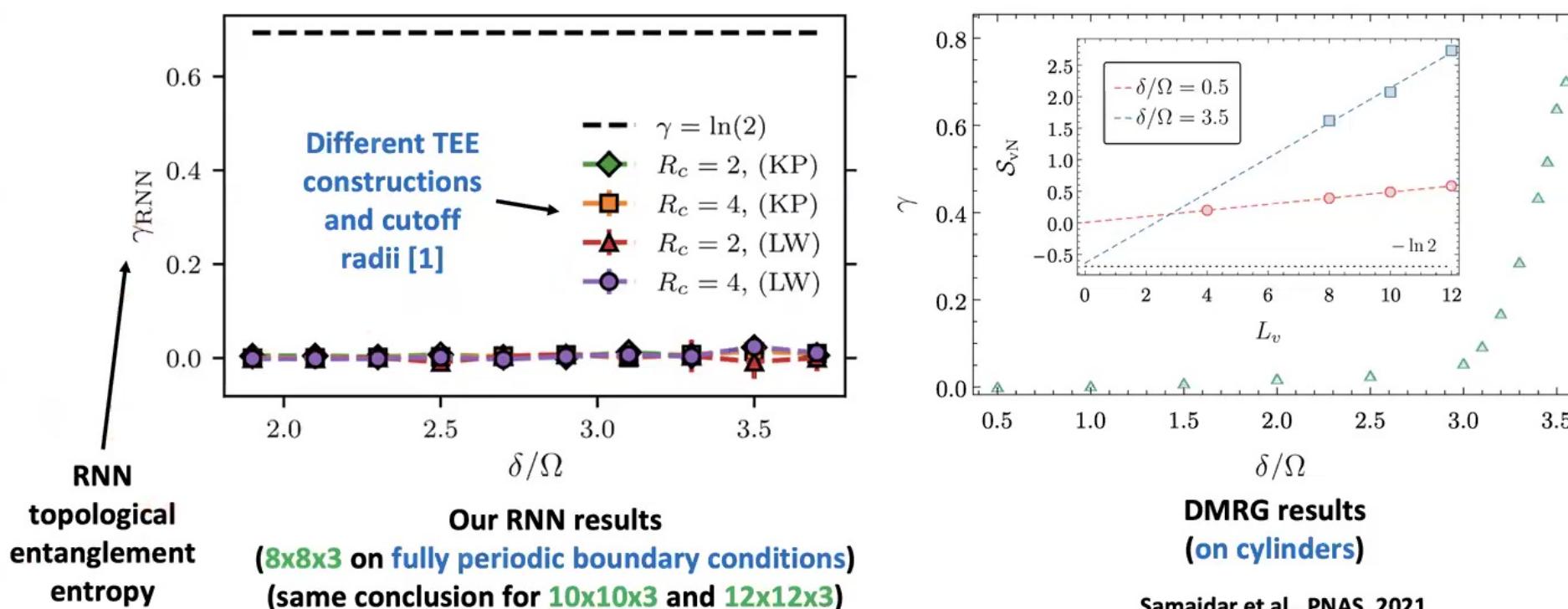
Kitaev, Preskill, 2005



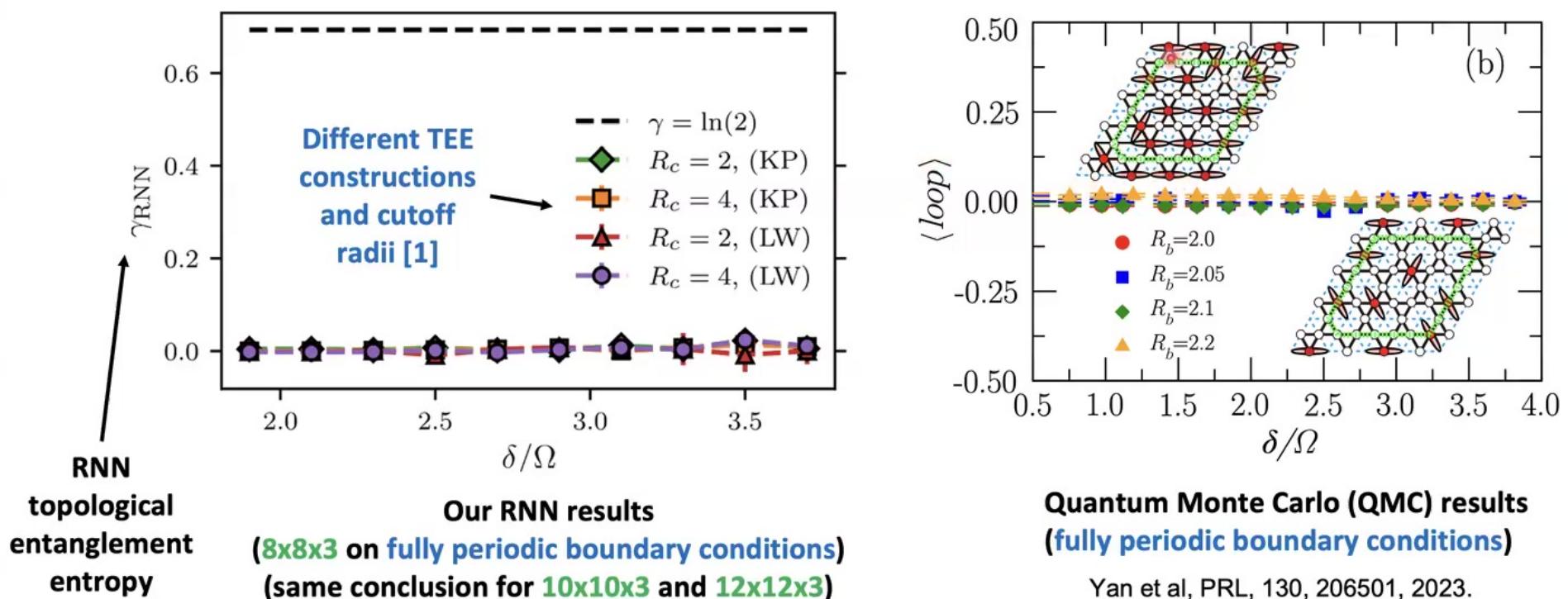
Levin-Wen (LW) construction

Levin, Wen, 2006

Absence of topological order



Absence of topological order



Spin-glass order?

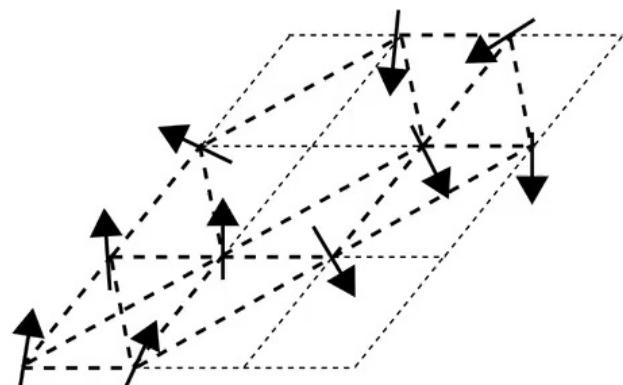
Edwards-Anderson order parameter:

$$q_{\text{EA}} = \frac{\sum_{i=1}^N \langle n_i - \rho \rangle^2}{N\rho(1-\rho)}$$

Average density

Close to 1: spin glass order.

Close to zero: no spin glass order.



Credit: Zureks, Wikipedia

Spin-glass order?

Edwards-Anderson order parameter:

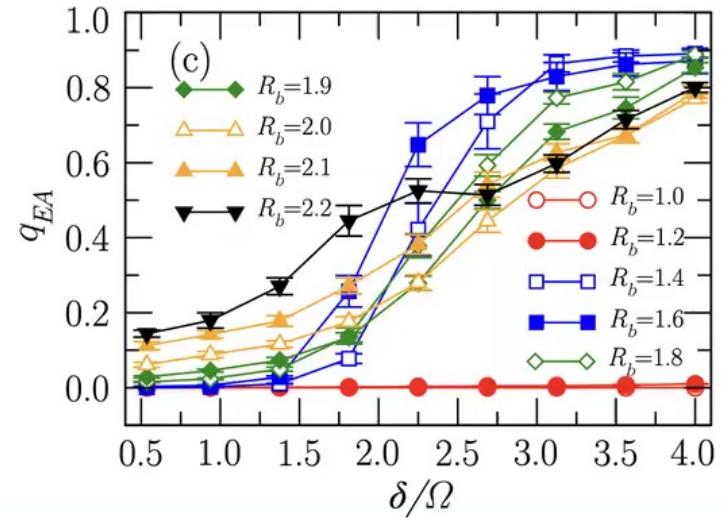
$$q_{EA} = \frac{\sum_{i=1}^N \langle n_i - \rho \rangle^2}{N\rho(1-\rho)}$$

Average density



Close to 1: spin glass order.

Close to zero: no spin glass order.

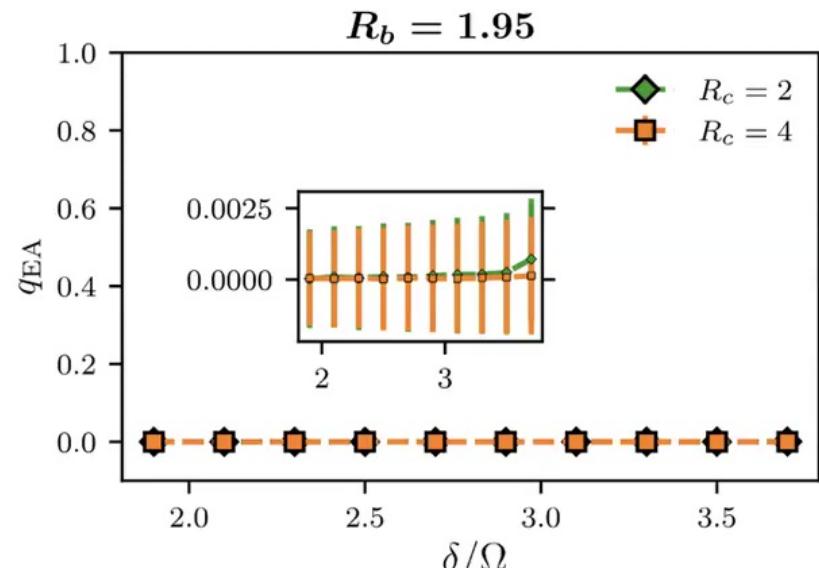


Quantum Monte Carlo (QMC) results

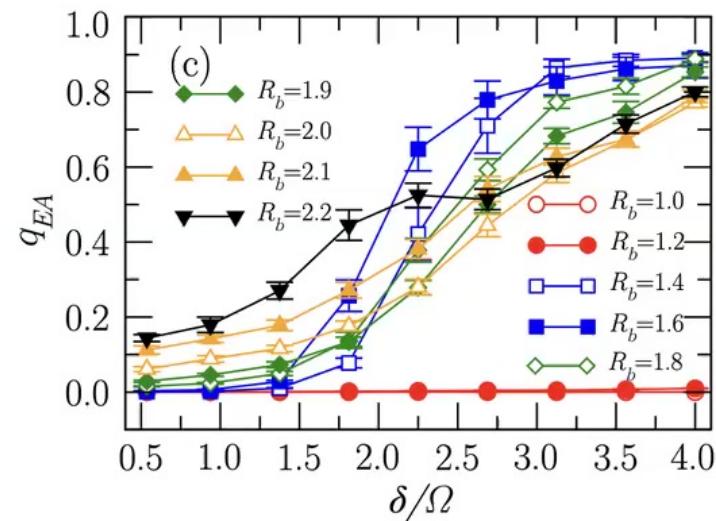
Yan et al, PRL, 130, 206501, 2023.

$$q_{EA} = \frac{\sum_{i=1}^N \langle n_i - \rho \rangle^2}{N\rho(1-\rho)}$$

Absence of spin-glass order



Our RNN results (8x8x3)
(perfect sampling: no autocorrelation)



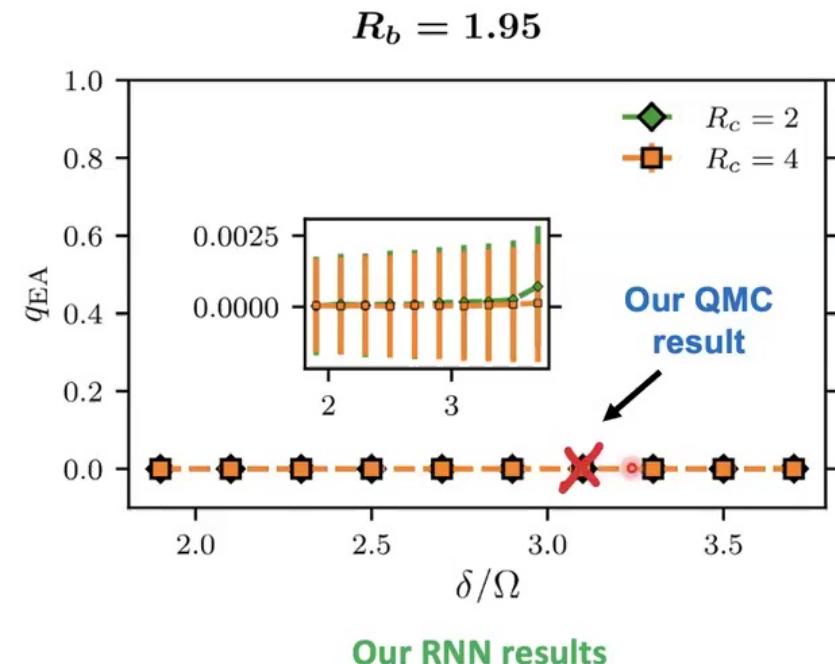
Quantum Monte Carlo (QMC) results
(approximate sampling in practice:
autocorrelation)

Yan et al, PRL, 130, 206501, 2023.

Absence of spin-glass order in our QMC calculations

We run QMC for a longer time compared to Ref. [1]

$$q_{\text{EA}}^{\text{QMC}} = 0.0000018(5) \text{ for } R_b = 1.95, \delta = 3.3$$

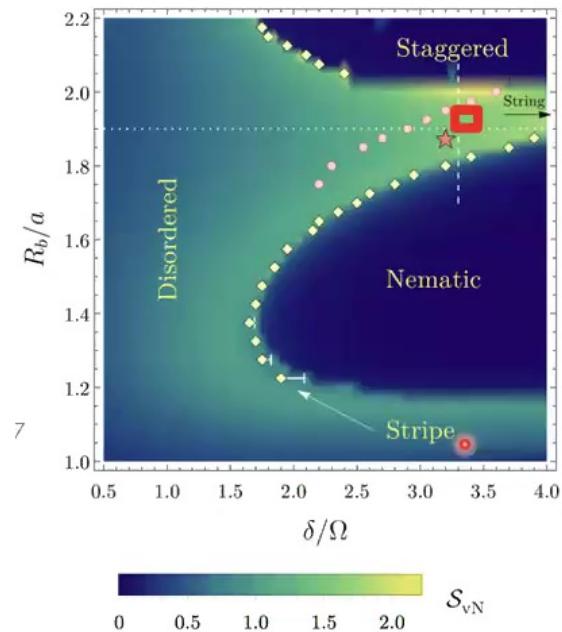


[1] Yan et al, PRL, 130, 206501, 2023.

M.H., E. Merali, G. Torlai, R. Melko, J. Carrasquilla, arXiv: 2405.20384

Final conclusion!

Both **RNNs** and **QMC** predict a **trivial paramagnetic (disordered) phase**



[M.H. E. Merali, G. Torlai, R. Melko, J. Carrasquilla, arXiv: 2405.20384](#)

Samajdar et al., PNAS, 2021

Time comparison: RNNs vs QMC

L	RNN: P100	RNN: A100	QMC (CPU)
6	26.567 h	3.460 h	48.751 h
8	4.074 h	1.537 h	70.422 h

M.H., E. Merali, G. Torlai, R. Melko, J. Carrasquilla, [arXiv: 2405.20384](#)

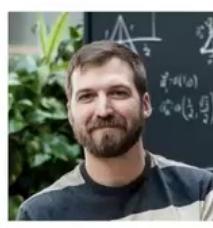
Takeaways

1. Our RNN variational calculations suggest the absence of topological and spin-glass orders in Rydberg atoms arrays on Kagome Lattice (backed up by QMC).
2. RNN wave functions and language-model based quantum states offer a powerful complementary numerical method in quantum matter.

Acknowledgments



Juan
Carrasquilla



Roger
Melko



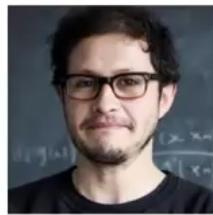
Giacomo
Torlai



Ejaz
Merali

M.H, E. Merali, G. Torlai, R. Melko, J. Carrasquilla, [arXiv: 2405.20384](#)

Thank you for your attention!



Juan
Carrasquilla



Roger
Melko



Lauren
Hayward



Estelle
Inack



Roeland
Wiersema



Martin
Ganahl



Shoummo
Khandoker



Jawaril
Abedin



Schuyler
Moss



Giacomo
Torlai



Ejaaz
Merali

Non-weight sharing

	2DRNN-NWS ($R_c = 2$)	2DRNN ($R_c = 2$)
E/N	-0.59447(2)	-0.59666(2)
q_{EA}	0.001416(6)	0.000205(1)
γ_{RNN}	-0.006(6)	-0.003(20)

Table V. A table comparing the 2DRNN results with weight sharing with the 2DRNN with no-weight sharing (2DRNN-NWS). Here use the system size $N = 8 \times 8 \times 3$ and $R_b = 1.95, \delta = 3.3$ for the comparison.



Variational Annealing with RNNs

$$F_{RNN}(T) = \langle \hat{H} \rangle_{RNN} - TS_{RNN}$$

Free energy = Cost function

Shannon entropy:

$$S_{RNN} = - \sum_{\sigma} P_{RNN}(\sigma) \log(P_{RNN}(\sigma)) = \langle -\log(P_{RNN}(\sigma)) \rangle_{RNN}$$

Expectation values
over RNN distribution

$$P_{RNN}$$

M.H, E. Inack, R. Wiersema, R. Melko, J. Carrasquilla, Variational Neural Annealing, *Nature Machine Intelligence*, Oct 2021.