Title: Lecture - Causal Inference, PHYS 777

Speakers: Robert Spekkens

Collection/Series: Causal Inference (Elective), PHYS 777, March 31 - May 2, 2025

Subject: Quantum Foundations

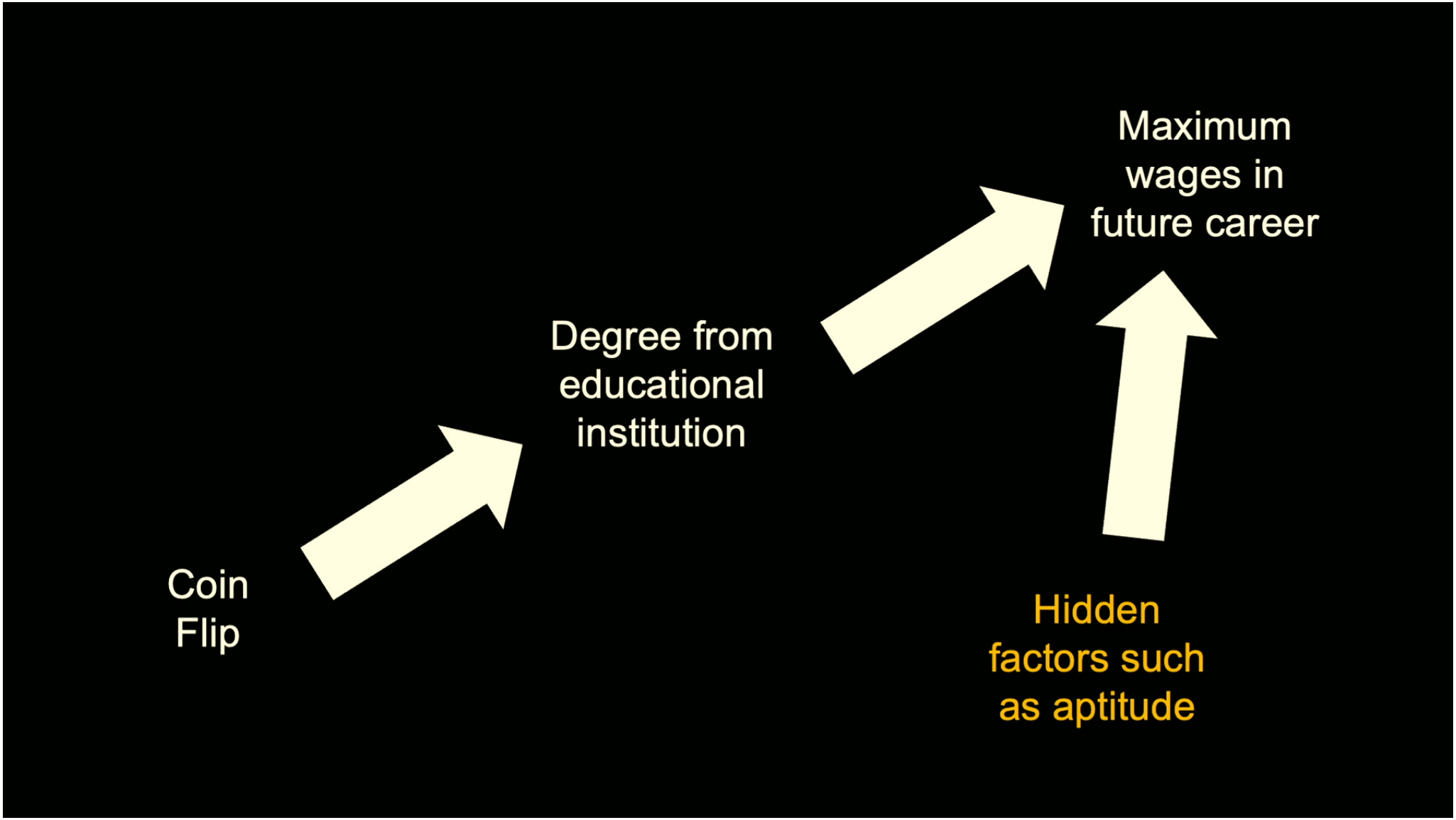
Date: April 04, 2025 - 11:30 AM

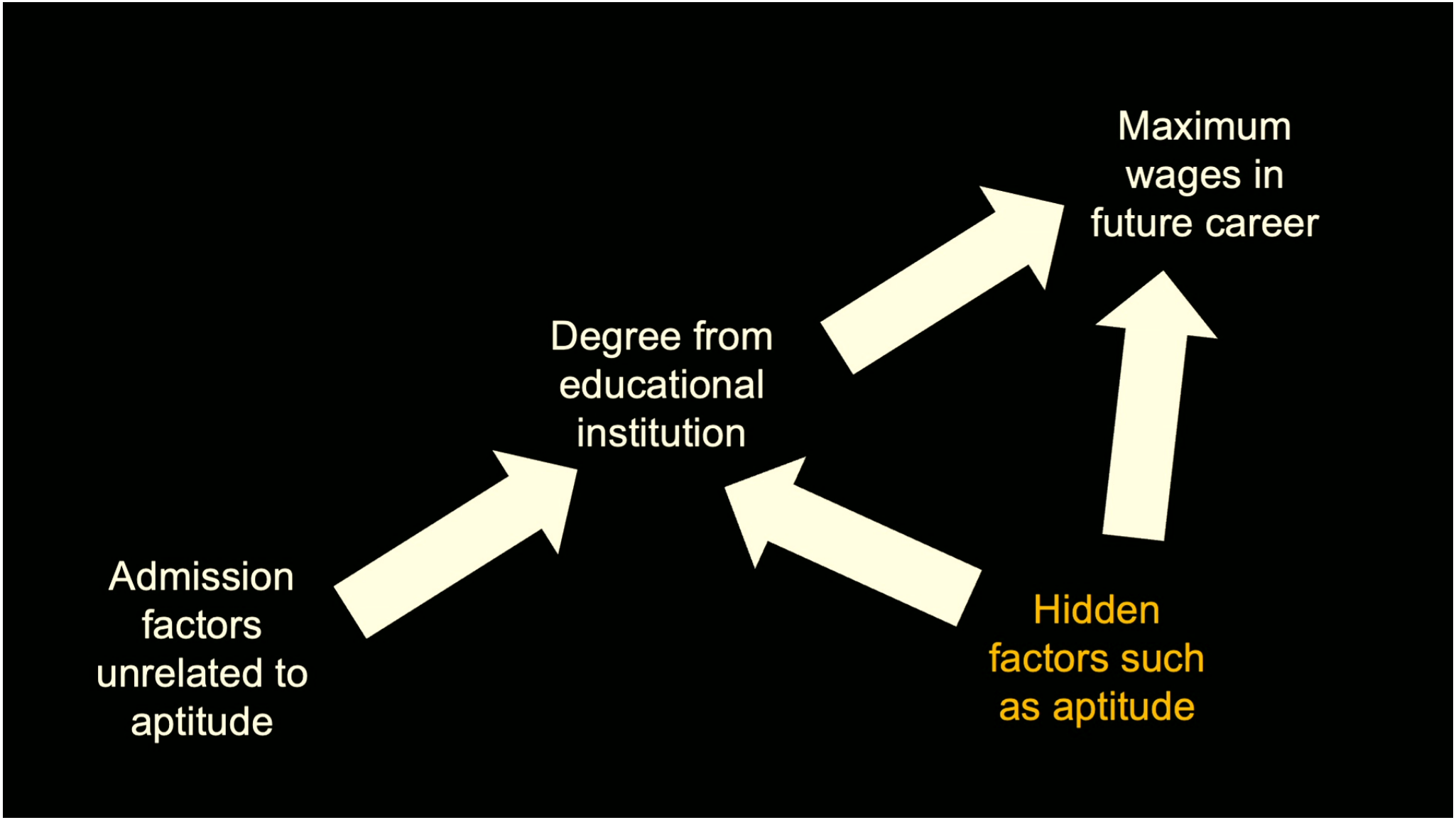
URL: https://pirsa.org/25040037

Maximum wages in future career above some threshold?

Degree from educational institution?

	Yes	No
Yes	79%	21%
No	43%	57%


Maximum wages in future career
Degree from educational institution
Coin Flip
Hidden factors such as aptitude


Maximum wages in future career
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Admission factors unrelated to aptitude
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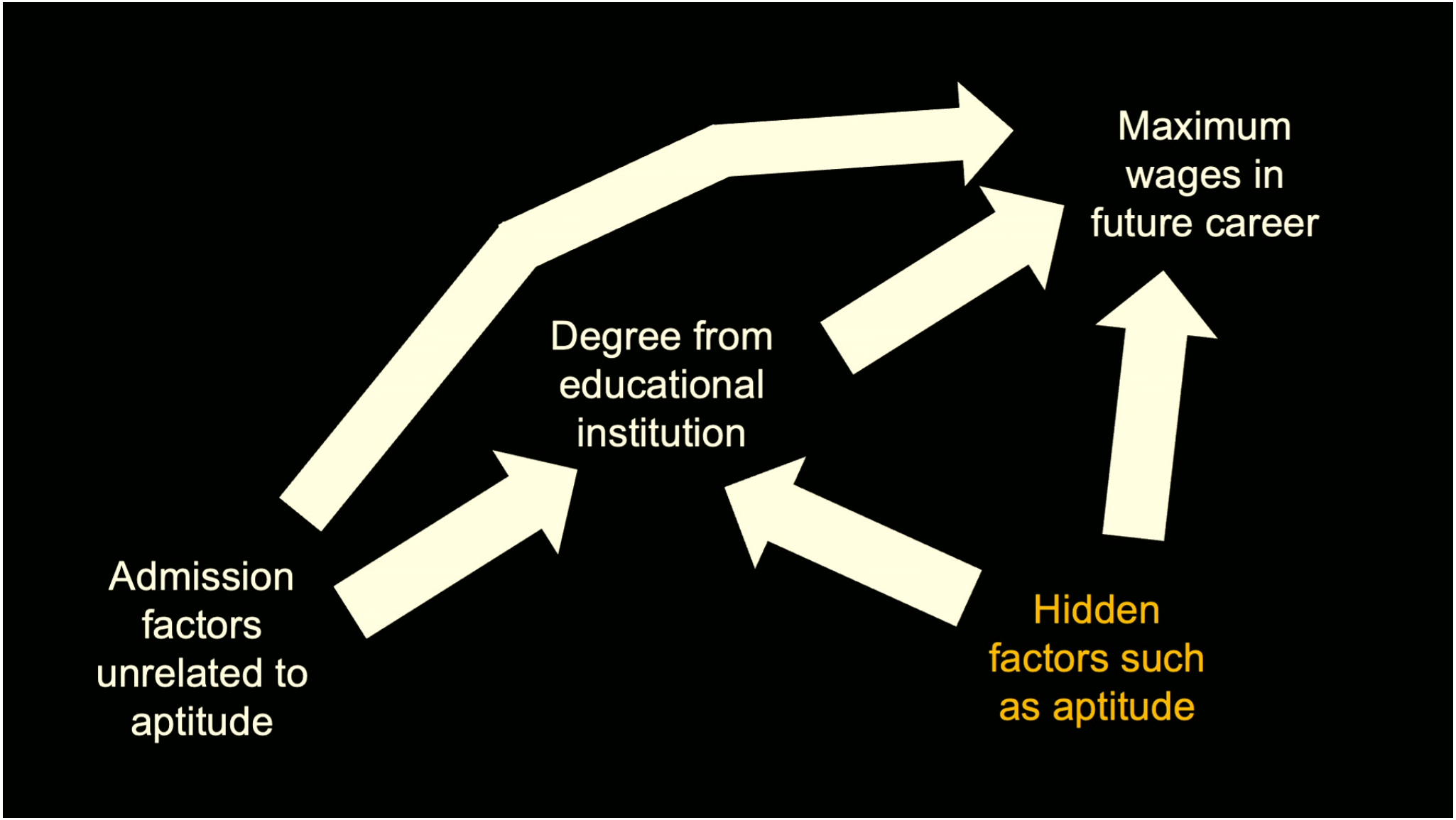
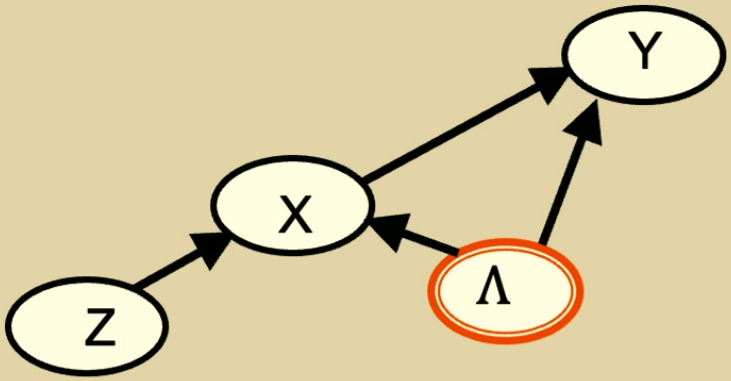

Maximum wages in future career
Degree from educational institution
Admission factors unrelated to aptitude
Hidden factors such as aptitude

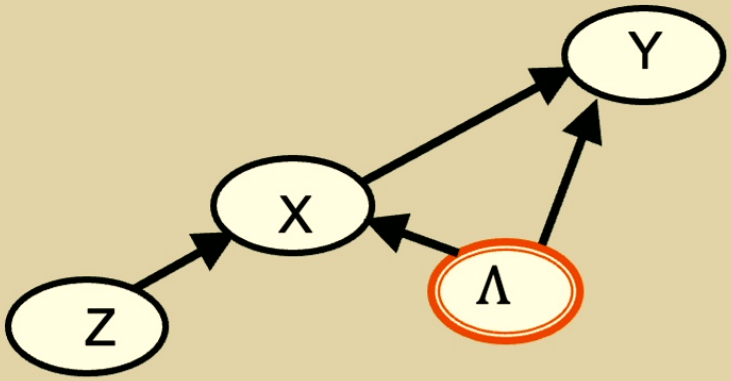
Causal structure



Parameters

$$P_{X|\Lambda Z}$$
$$P_{Y|\Lambda X}$$
$$P_{\Lambda}$$
$$P_{Z}$$

Causal structure



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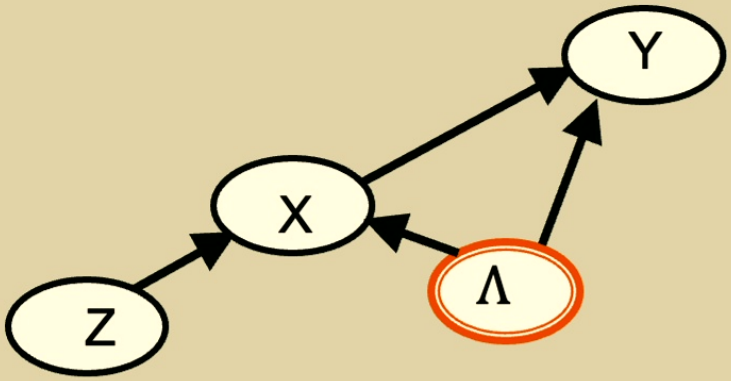
$$P_{Z}$$

$$P_{XYZ} = \sum_{\Lambda} P_{Y|X\Lambda} P_{X|Z\Lambda} P_{\Lambda} P_{Z}$$

A distribution is said to be compatible with a given causal structure if there are parameters that yield it

Causal structure



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$$P_{XYZ} = \sum_{\Lambda} P_{Y|X\Lambda} P_{X|Z\Lambda} P_{\Lambda} P_{Z}$$

Example of causal compatibility constraint:

$$P_{XY|Z}(00|0) + P_{XY|Z}(01|1) \leq 1$$

Pearl, 1993

The evidence

Z=0

	Y=0	Y=1
X=0	0.79	0.21
X=1	0.43	0.57

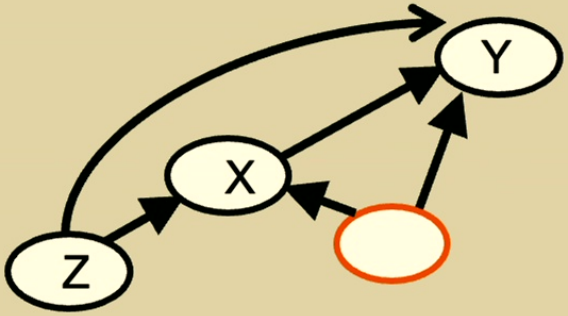
Z=1

	Y=0	Y=1
X=0	0.59	0.41
X=1	0.39	0.61

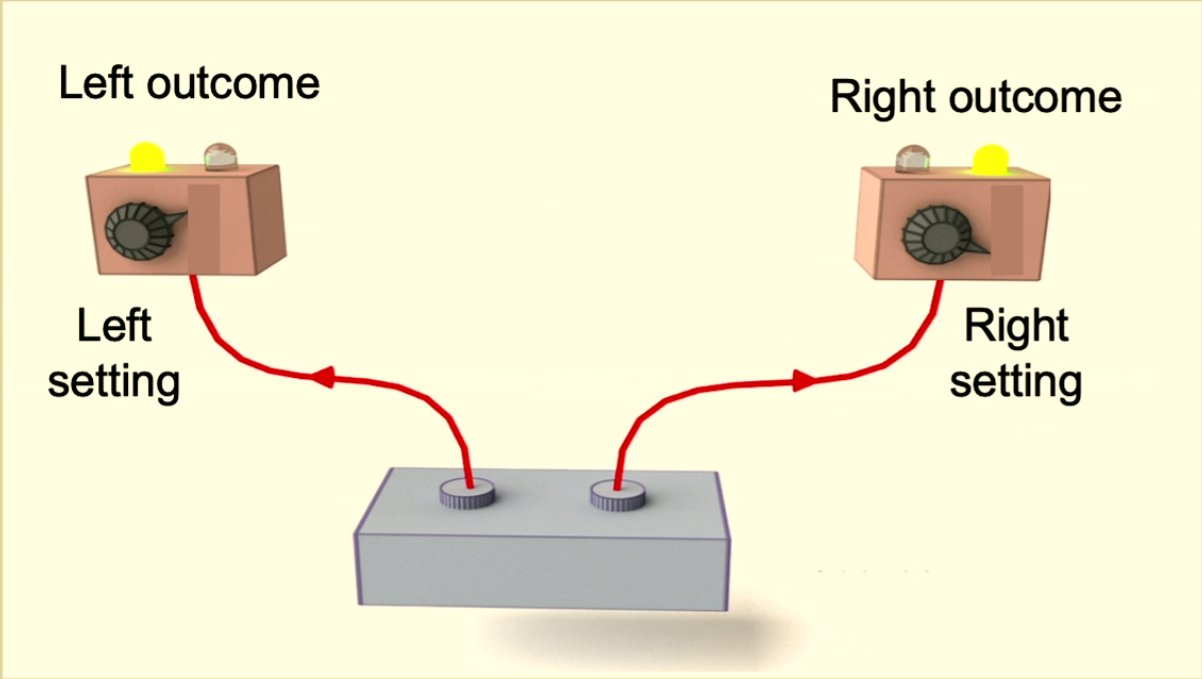
Violates the instrumental inequality!

The hypotheses

Implies a constraint:
the instrumental inequality



Bell’s theorem


Left outcome
Right outcome
Left
setting
Right
setting

Left setting and Right setting	Left outcome and Right outcome			
	0 and 0	0 and 1	1 and 0	1 and 1
0 and 0	43%	7%	7%	43%
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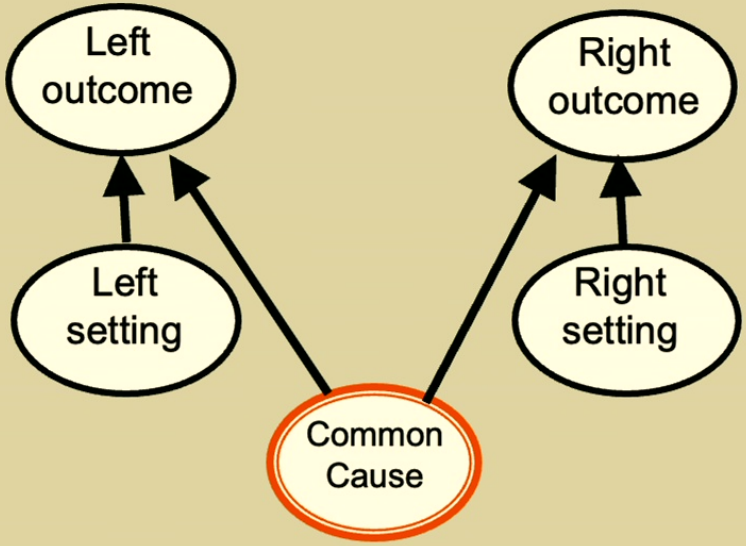
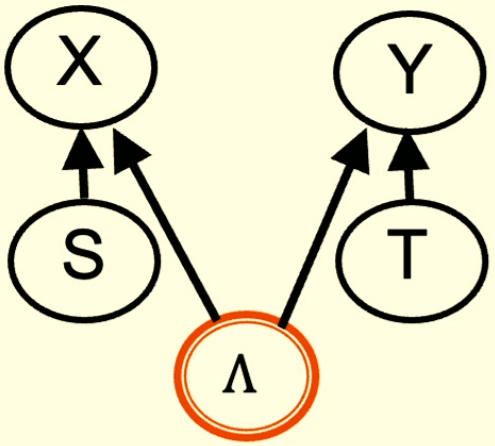
Violates Bell inequalities
(up to Tsirelson bound)

The natural hypothesis



Implies Bell inequalities

Causal structure



Parameters

$$P_{X|S\Lambda}$$
$$P_{Y|T\Lambda}$$
$$P_{\Lambda}$$

$$P_{XY|ST} = \sum_{\Lambda} P_{Y|T\Lambda} P_{X|S\Lambda} P_{\Lambda}$$

Examples of causal compatiblity constraints:

$$P_{X|ST} = P_{X|S}$$
$$P_{Y|ST} = P_{Y|T}$$

$$\tfrac{1}{4}\sum_{x=y} P_{XY|ST}(xy|00) + \tfrac{1}{4}\sum_{x=y} P_{XY|ST}(xy|01)$$
$$\tfrac{1}{4}\sum_{x=y} P_{XY|ST}(xy|10) + \tfrac{1}{4}\sum_{x\neq y} P_{XY|ST}(xy|11) \leq \tfrac{3}{4}$$

Clauser, Horne, Shimony and Holte, Phys. Rev. Lett.23, 880 (1967)

The evidence

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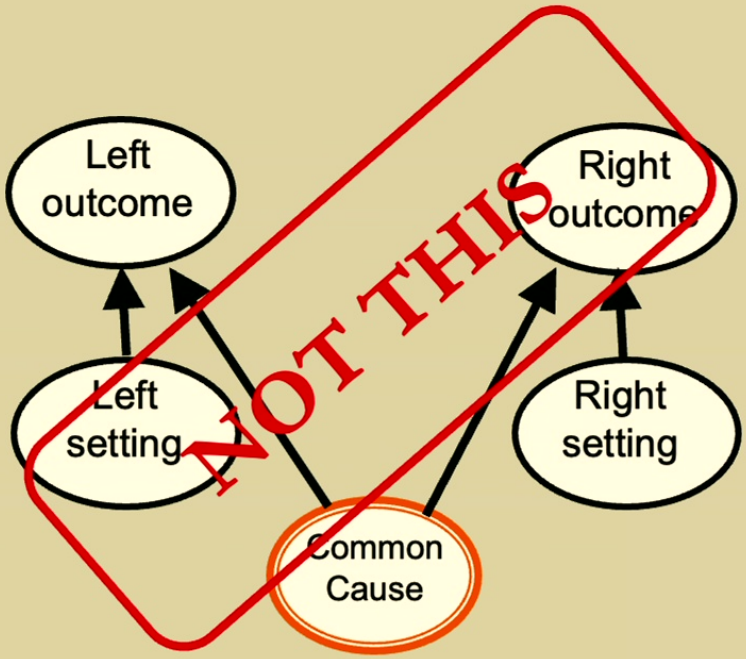
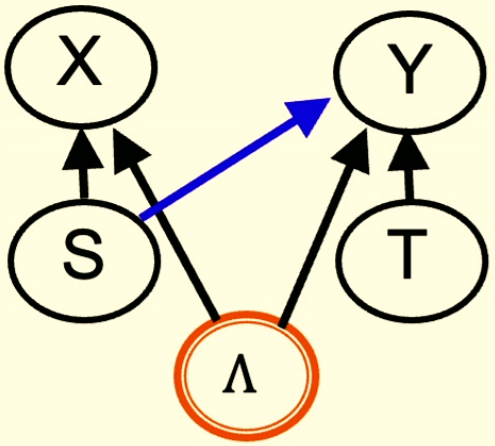


Violates the
Bell Inequalities!

Implies a constraint:
Bell Inequalities

Incompatible

Causal structure



Parameters

$P_{X|S\Lambda}$

$P_{Y|ST\Lambda}$

P_{Λ}

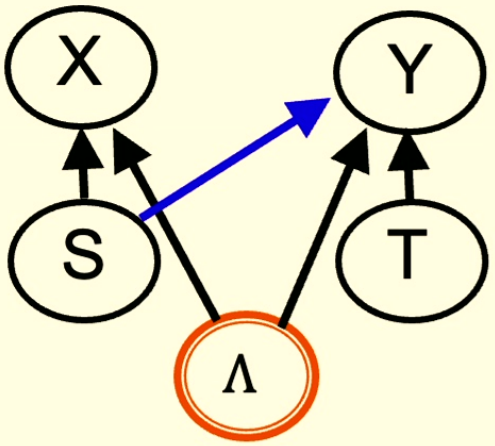
$$P_{XY|ST} = \sum_{\Lambda} P_{Y|ST\Lambda} P_{X|S\Lambda} P_{\Lambda}$$

Causal compatibility constraints:

$P_{X|ST} = P_{X|S}$

But the data *also* satisfies $P_{Y|ST} = P_{Y|T}$

Causal structure



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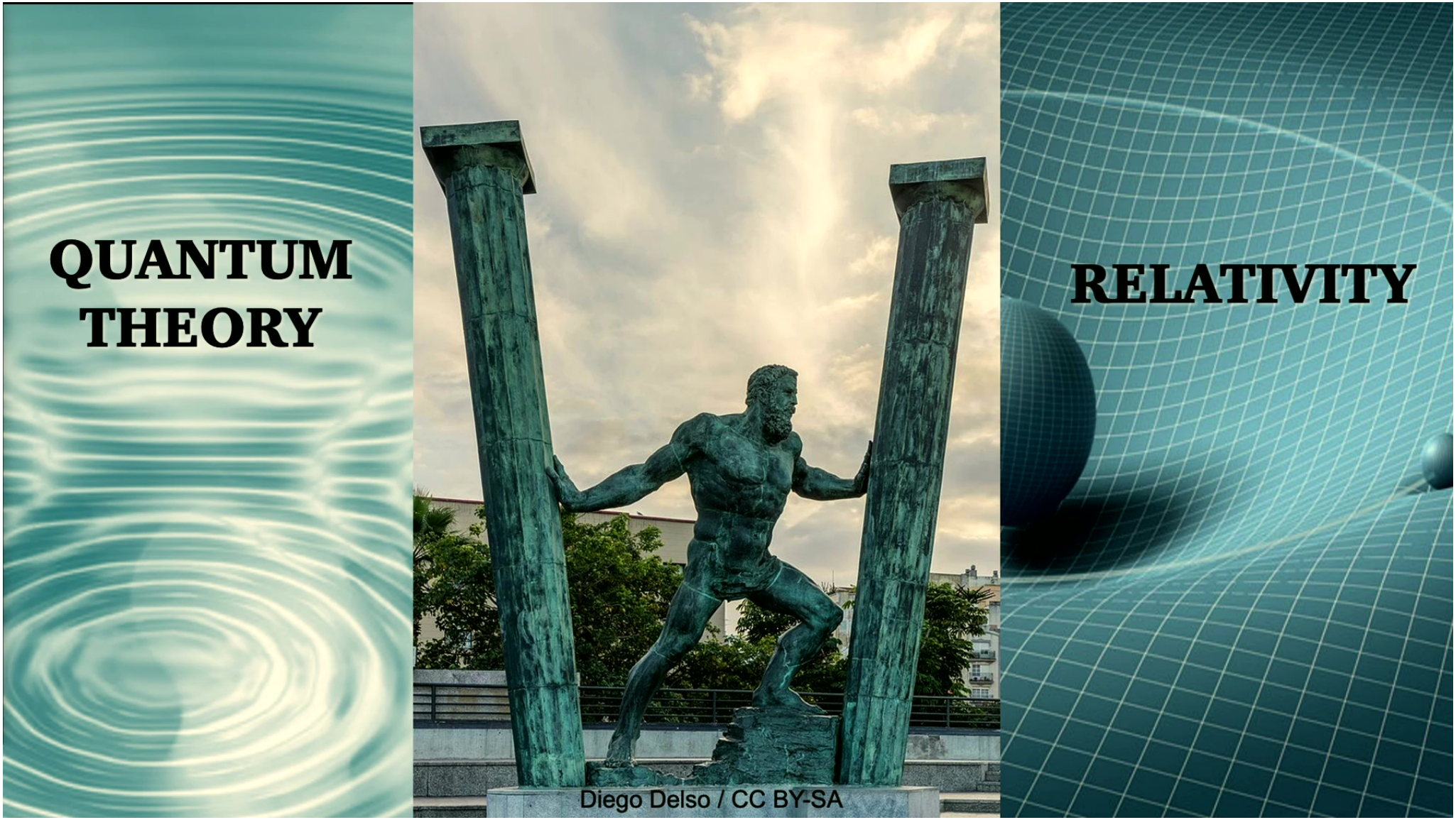
Reproducing this requires fine-tuning

Wood and RWS, New J. Phys. 17, 033002 (2015)


QUANTUM
THEORY
RELATIVITY
Diego Delso / CC BY-SA

The evidence

Left setting and Right setting	Left outcome and Right outcome: 0 and 0	0 and 1	1 and 0	1 and 1
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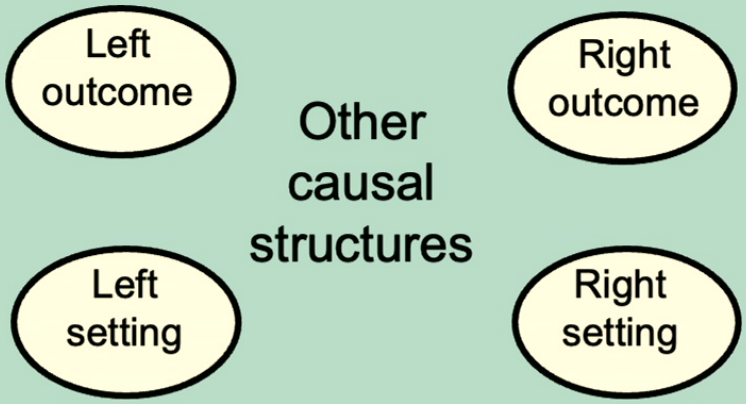
The usual suspects



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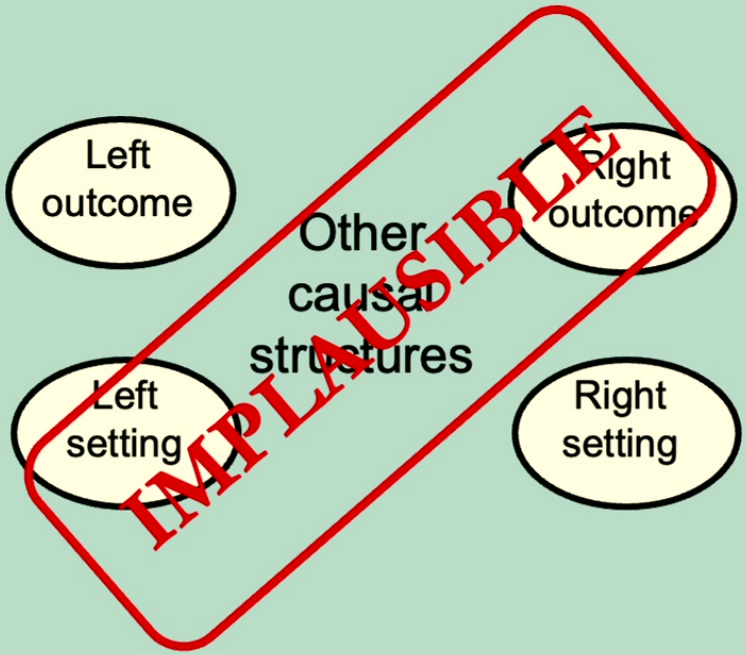

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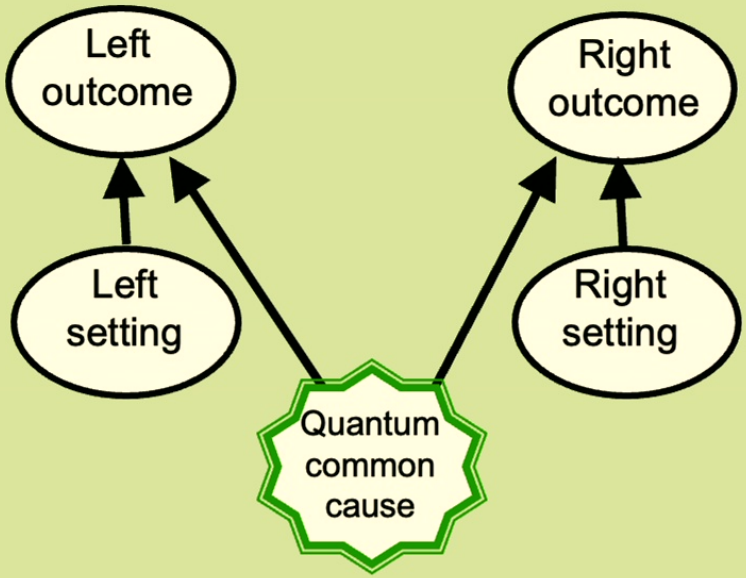
A new possibility



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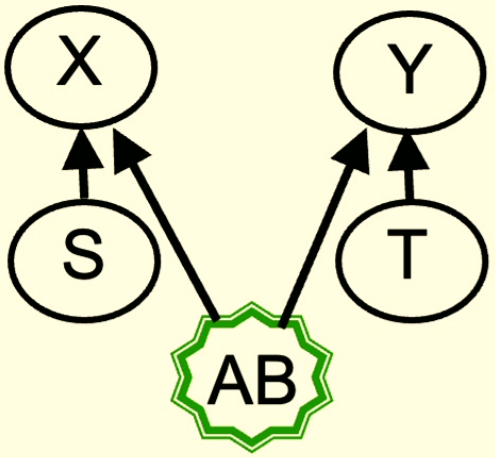

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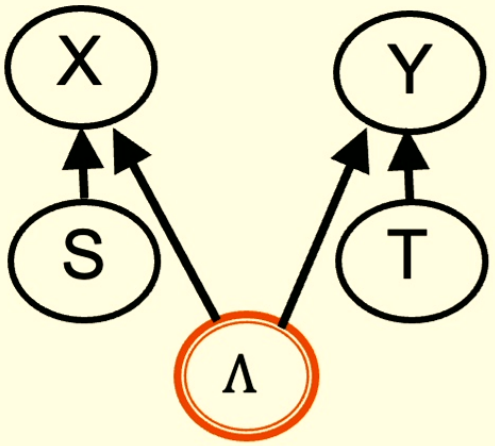
Tsirelson, Lett. Math. Phys. 4, 93 (1980)

Causal structure



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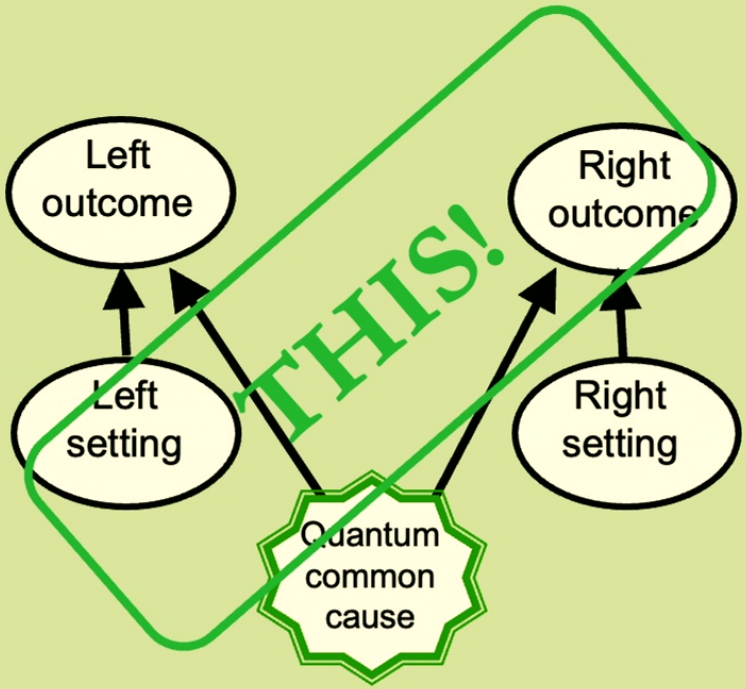
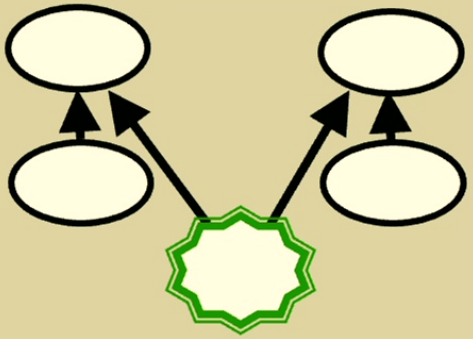


Compatible

Violation of Bell inequalities

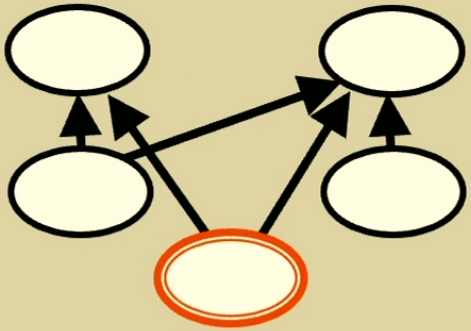
Witnessing quantumness



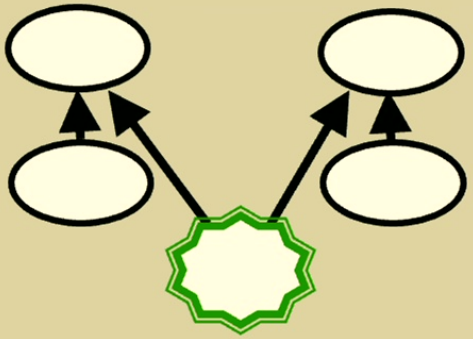
Violation of Bell inequalities

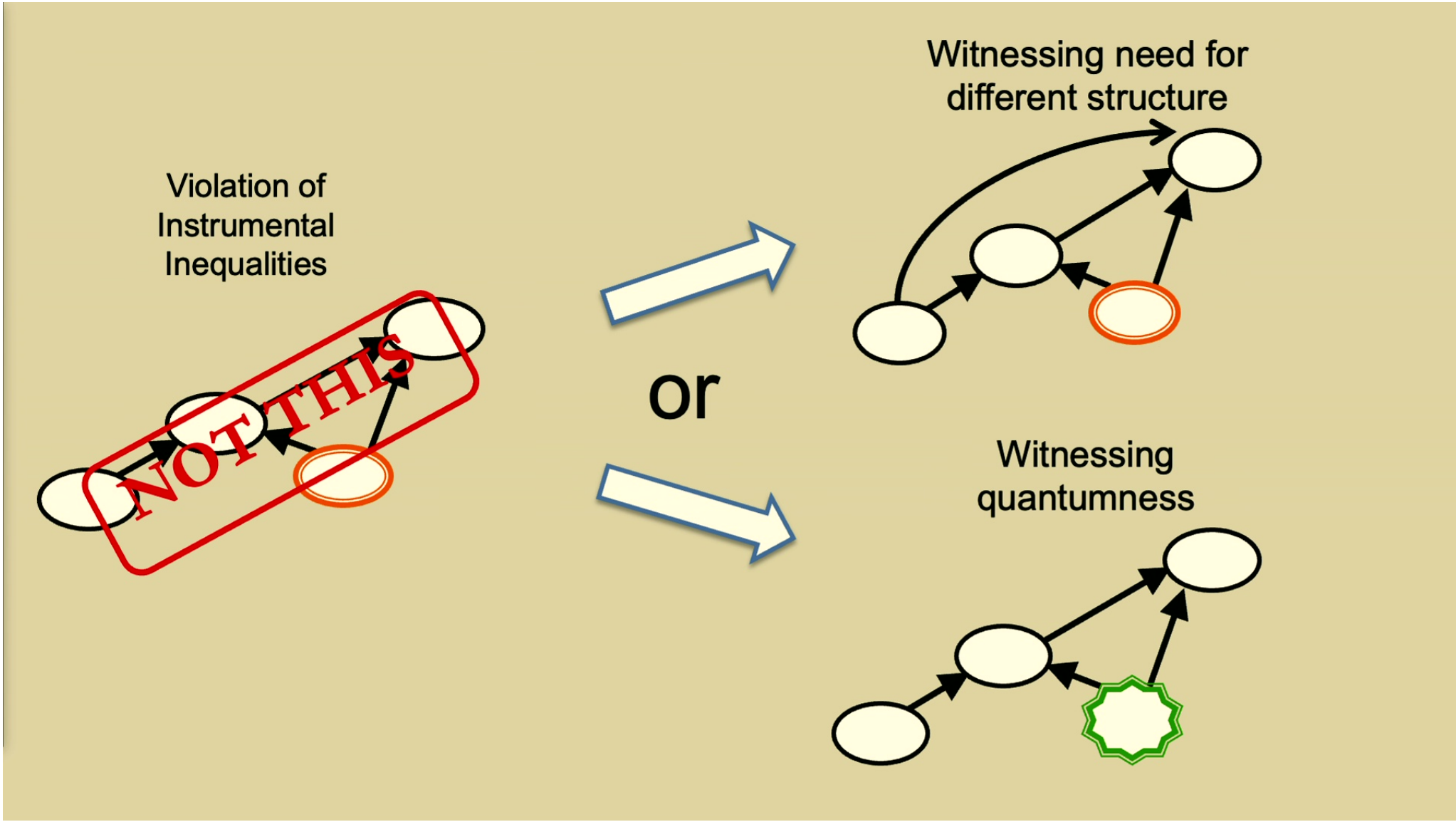
or

Witnessing need for different structure

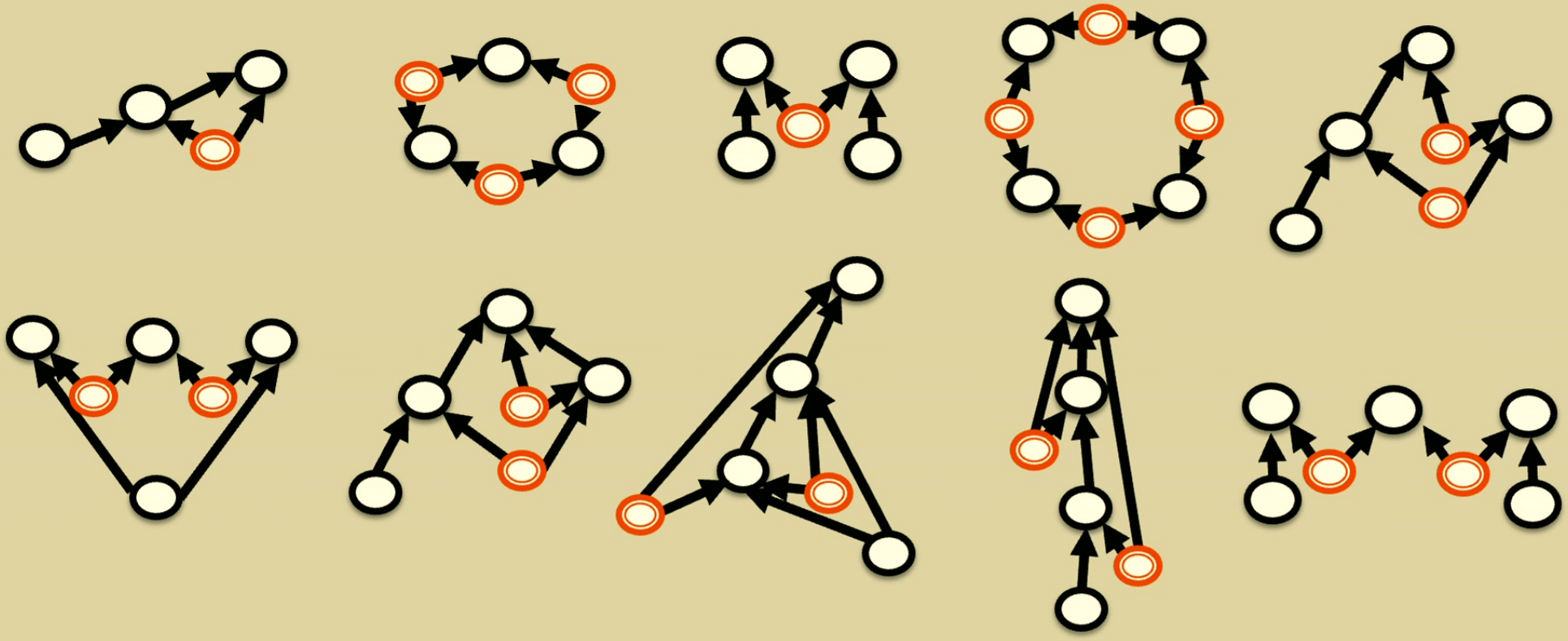


Witnessing quantumness




Violation of Instrumental Inequalities
NOT THIS
or
Witnessing need for different structure
Witnessing quantumness

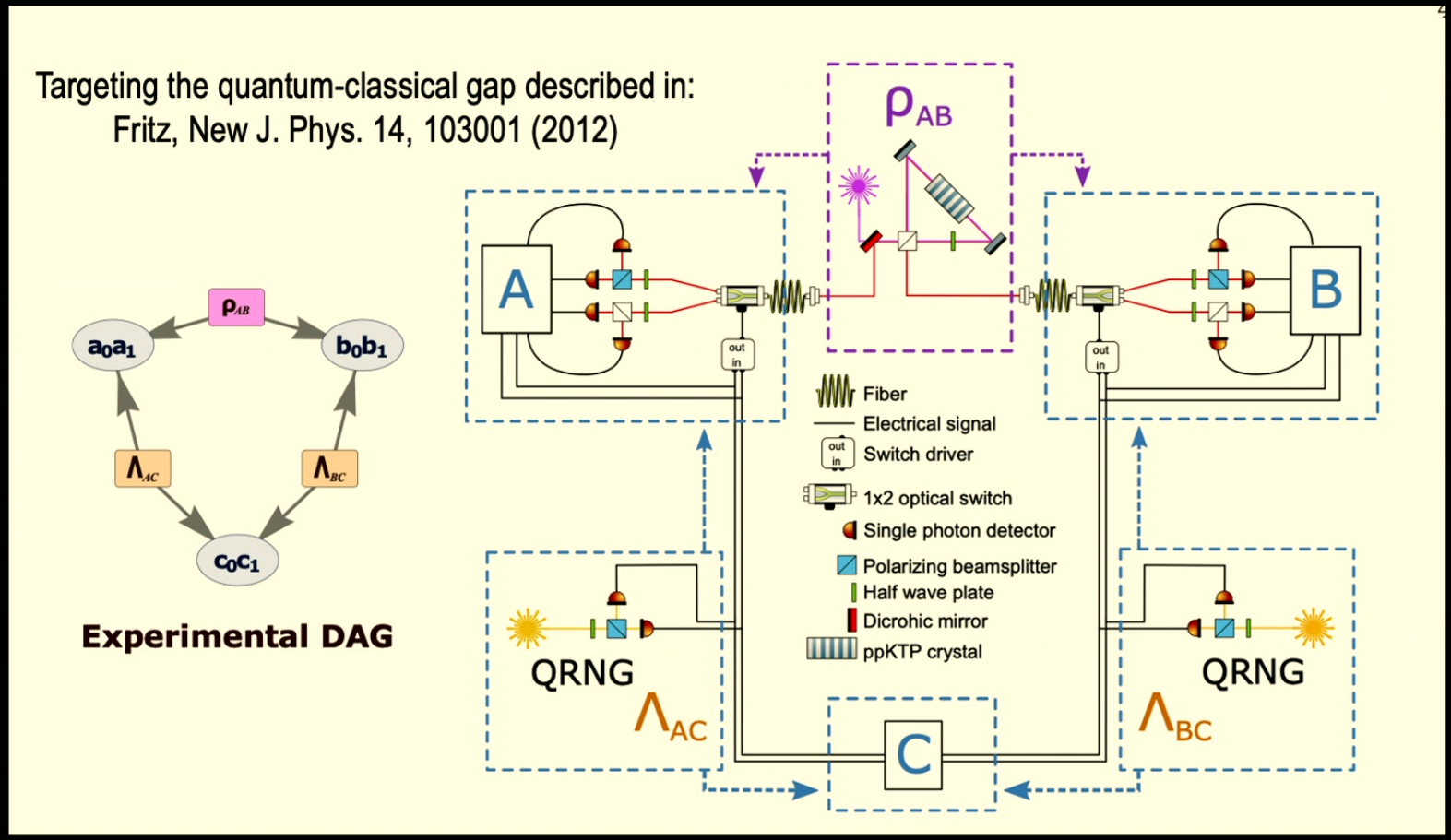
Some causal structures that admit of inequality constraints



They are ubiquitous: Ansanelli, Wolfe, RWS, arXiv:2502.07891

Collaboration with experimental group of Fabio Sciarrino

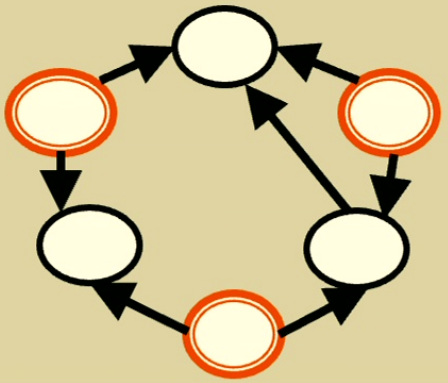
Polino et al., Nature Communications 14, 909 (2023)



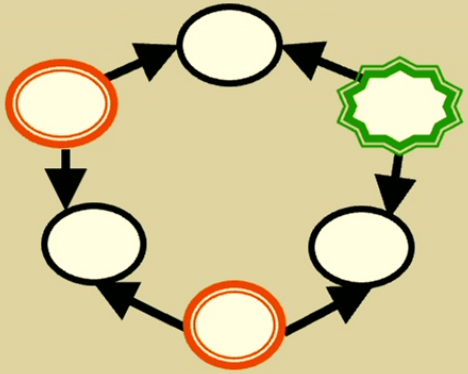
Violation of certain causal compatibility inequalities

or

Witnessing need for different structure

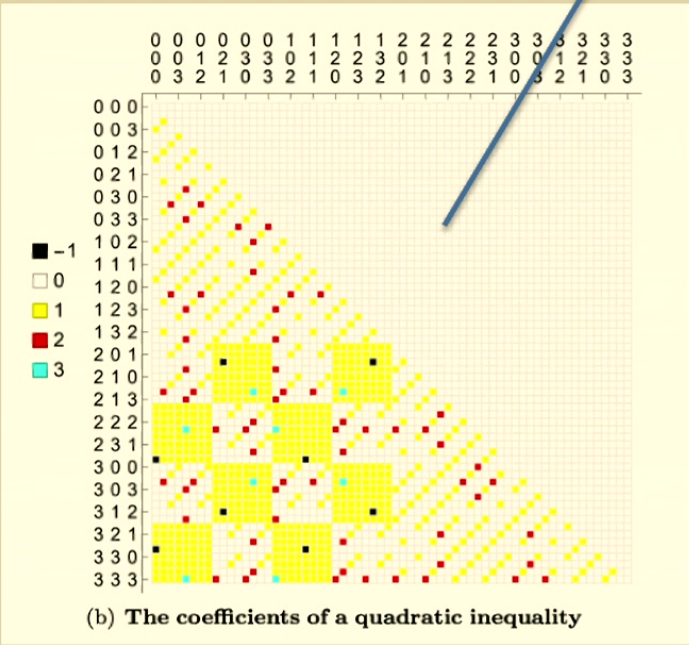


Witnessing quantumness



Data-seeded inflation technique

$$V := \sum_{a,b,c,a',b',c'} y_{abca'b'c'} P_{ABC}(abc) P_{ABC}(a'b'c') \geq 0$$

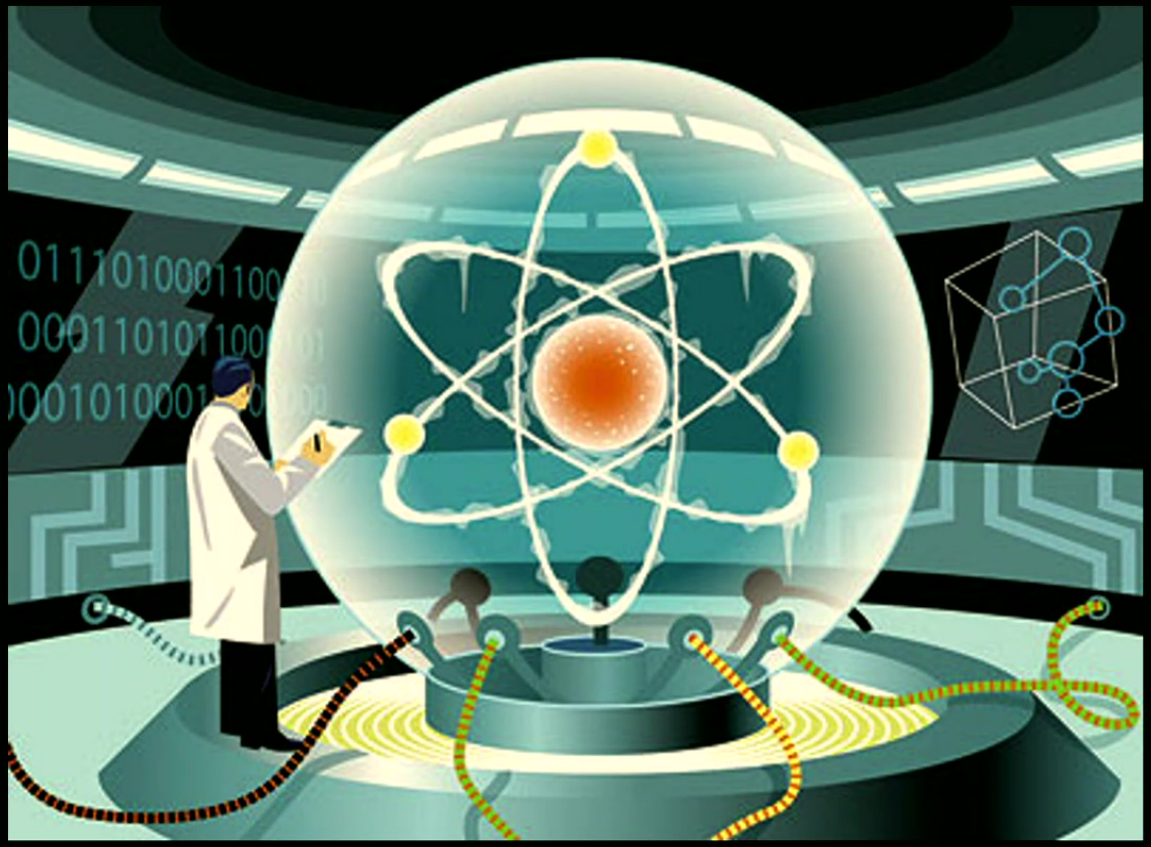


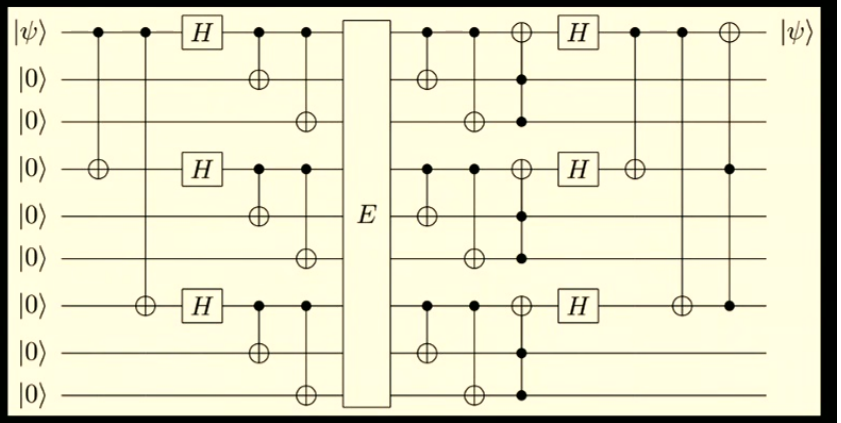
(b) The coefficients of a quadratic inequality

This causal compatibility inequality is violated by the data

$$V_{\text{exp}} = -0.02436 \pm 0.00016$$

Polino et al., Nature Communications 14, 909 (2023)




$|\psi\rangle$
$|0\rangle$
$|0\rangle$
$|0\rangle$
$|0\rangle$
$|0\rangle$
$|0\rangle$
$|0\rangle$
$|0\rangle$
H
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H
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E
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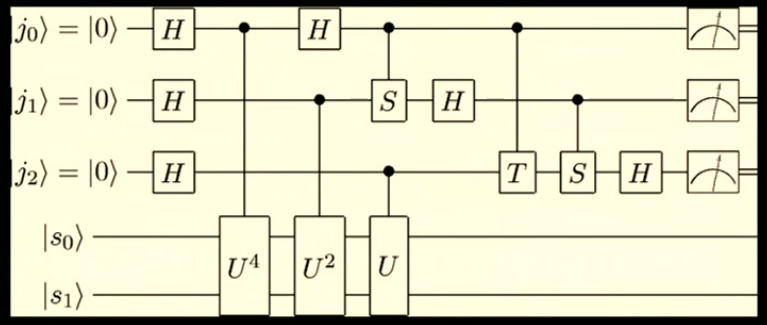
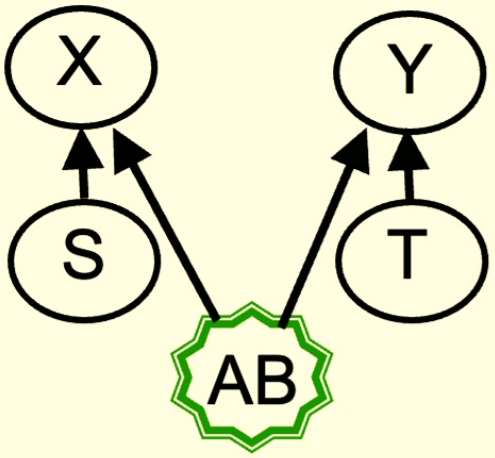
H
H
$|\psi\rangle$


$|j_0\rangle = |0\rangle$
$|j_1\rangle = |0\rangle$
$|j_2\rangle = |0\rangle$
$|s_0\rangle$
$|s_1\rangle$
H
H
H
U^4
U^2
U
S
H
T
S
H

$$\frac{\text{Quantum Causation and Inference}}{\text{Classical Causation and Inference}} \approx \frac{\text{Relativistic Notions of Space and Time}}{\text{PreRelativistic Notions of Space and Time}}$$

How to define causation

Causal structure



Parameters

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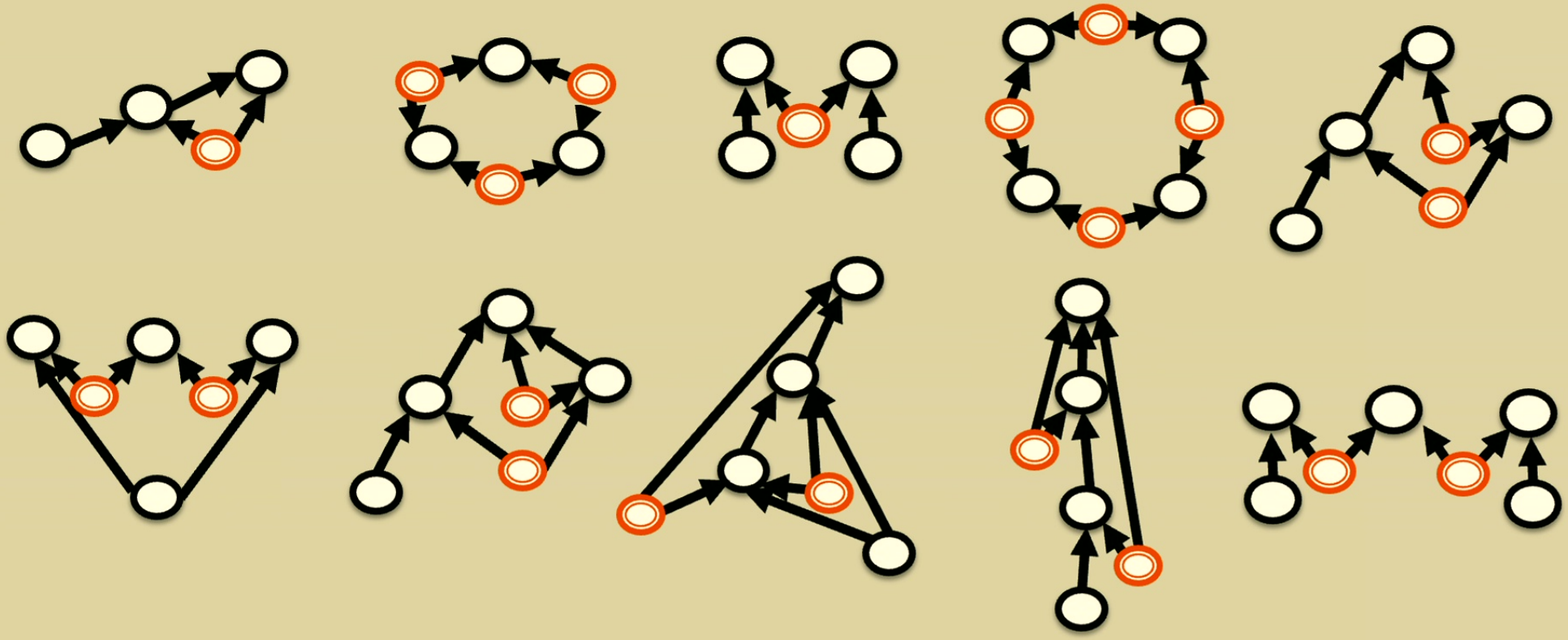
Tsirelson, Lett. Math. Phys. 4, 93 (1980)

Some causal structures that admit of inequality constraints



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