Title: Lecture - AdS/CFT, PHYS 777

Speakers: David Kubiznak

Collection/Series: AdS/CFT (Elective), PHYS 777, March 31 - May 2, 2025

Subject: Quantum Fields and Strings, Quantum Gravity

Date: April 30, 2025 - 9:00 AM

URL: https://pirsa.org/25040035

HOLOGRAPHIC SUPERCONDUCTORS

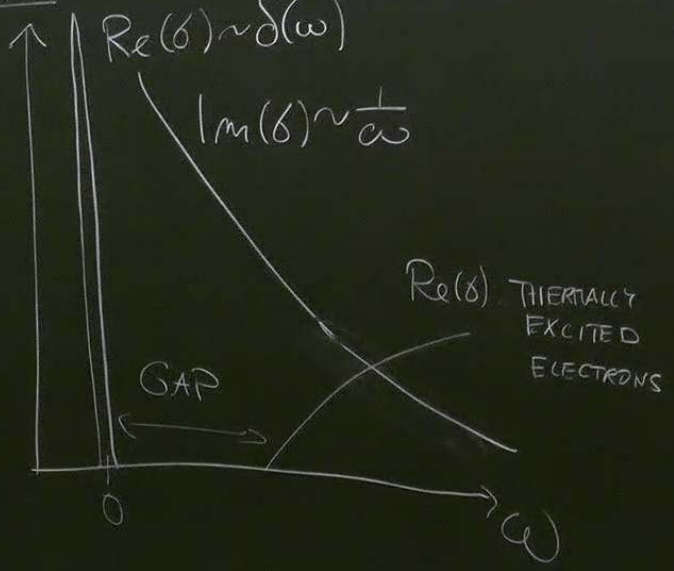
a) NORMAL SUPERCONDUCTORS:

2 BASIC FEATURES:

- DIVERGING DC CONDUCTIVITY σ (OHM'S LAW: $J = \sigma E$)
- MEISSNER EFFECT (MAGNETIC FIELD EXPELLED FROM SUPERCONDUCTOR - $B = 0$)

IN FACT:

IN FACT:

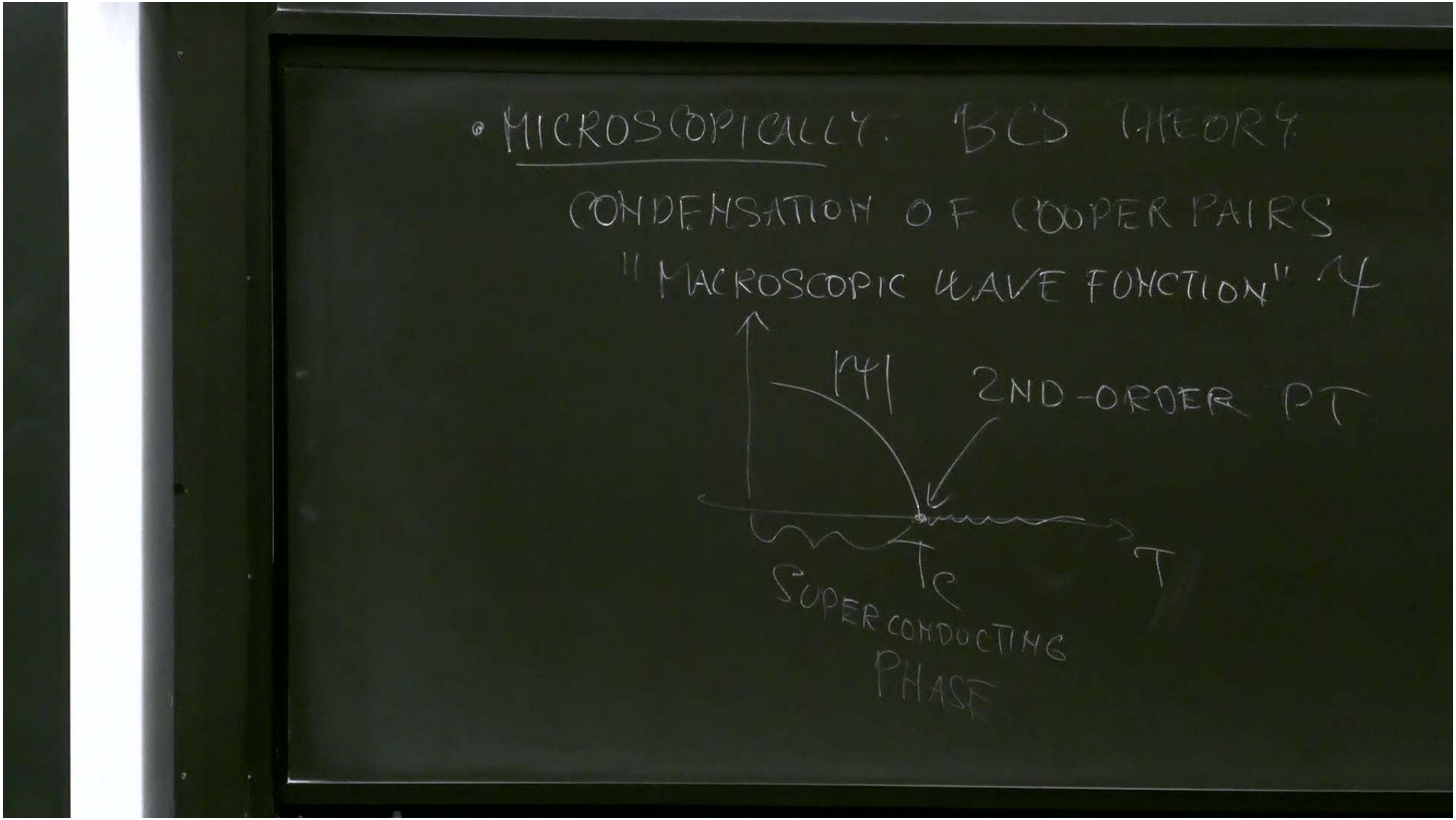


NOTE: REAL & IMAGINARY PARTS ARE RELATED BY KRAMERS-KRONIG RELATIONS

$$\mathrm{Im}(\sigma) = -\frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\mathrm{Re}(\sigma(\omega'))}{\omega' - \omega} d\omega'$$

$\Rightarrow$ $\frac{1}{\omega}$ POLE IN $\mathrm{Im}(\sigma)$ $\leftrightarrow$ $\delta(\omega)$ FOR $\mathrm{Re}(\sigma)$

EASIER TO IDENTIFY NUMERICALLY.


• MICROSCOPICALLY: BCS THEORY
CONDENSATION OF COOPER PAIRS
"MACROSCOPIC WAVE FONCTION" ψ
|ψ|
2ND-ORDER PT
T
T_c
SUPERCONDUCTING
PHASE

- PHENOMENOLOGICAL DESCRIPTION: LANDAU-GINSBURG THEORY

$$\mathcal{L} = \frac{\hbar^2}{2m^*} |D_i \psi|^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{1}{4} F_{ij}^2 \qquad D_i = \partial_i - i e^* \frac{A_i}{\hbar}, \quad m^* = 2m_e, \quad e^* = 2e$$

- WRITE $\psi = |\psi(x)| e^{i \frac{e^* \phi(x)}{\hbar}}$

$$\mathcal{L} = \frac{\hbar^2}{2m^*} (\partial_i |\psi|)^2 + \frac{e^{*2}}{2m^*} |\psi|^2 (\partial_i \phi - A_i)^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{1}{4} F_{ij}^2 \qquad (*)$$

SPEC: 1) $A_i = 0$

$$\mathcal{L} = \frac{\hbar^2}{2m^*}|D_i\psi|^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{1}{4}F_{ij}^2 \qquad D_i = \partial_i - i\frac{e^* A_i}{\hbar}, \quad m^* = 2m_e, \quad e^* = 2e$$

- WRITE $\psi = |\psi(x)|\, e^{i\frac{e^*\phi(x)}{\hbar}}$

$$\mathcal{L} = \frac{\hbar^2}{2m^*}(\partial_i|\psi|)^2 + \frac{e^{*2}}{2m^*}|\psi|^2(\partial_i\phi - A_i)^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{1}{4}F_{ij}^2 \qquad (*)$$

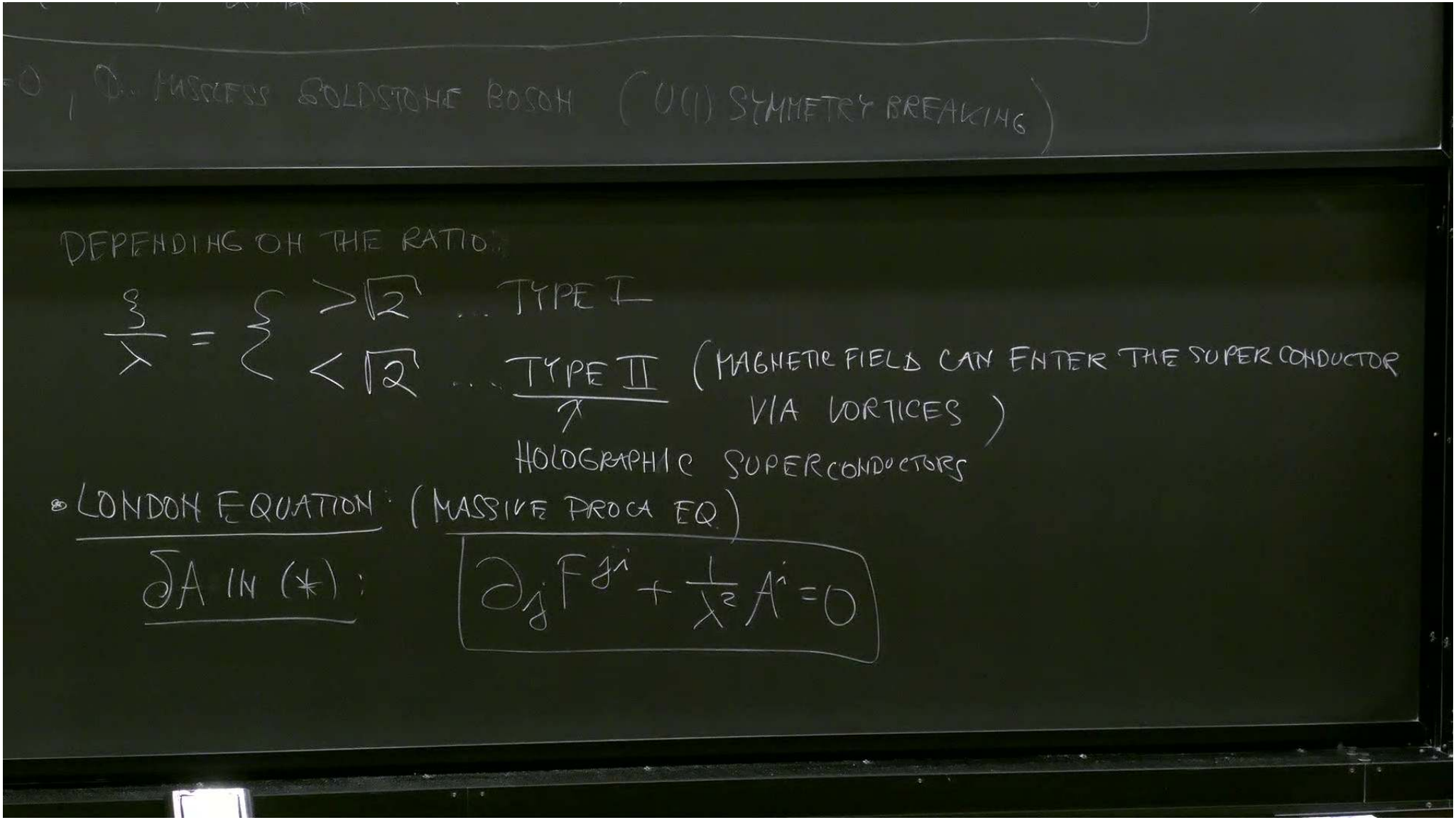
SPEC: 1) $A_i = 0$, ϕ MASSLESS GOLDSTONE BOSON (U(1) SYMMETRY BREAKING)

ii) $A_1 \neq 0$. ϕ CAN BE ABSORBED BY GAUGE TRANSF. TO A_1

$\rightarrow A_1$ IS MASSIVE (HIGGS MECHANISM)

$$\frac{1}{m_A^2} \sim \frac{m_*}{e_*^2 |\psi|^2} \sim \frac{a}{b} \sim \lambda^2 \quad \ldots \text{PENETRATION LENGTH}$$

$$\frac{1}{m_\psi^2} \sim \frac{\hbar^2}{2 m_* a} \sim \xi^2 \quad \ldots \text{CORRELATION LENGTH}$$

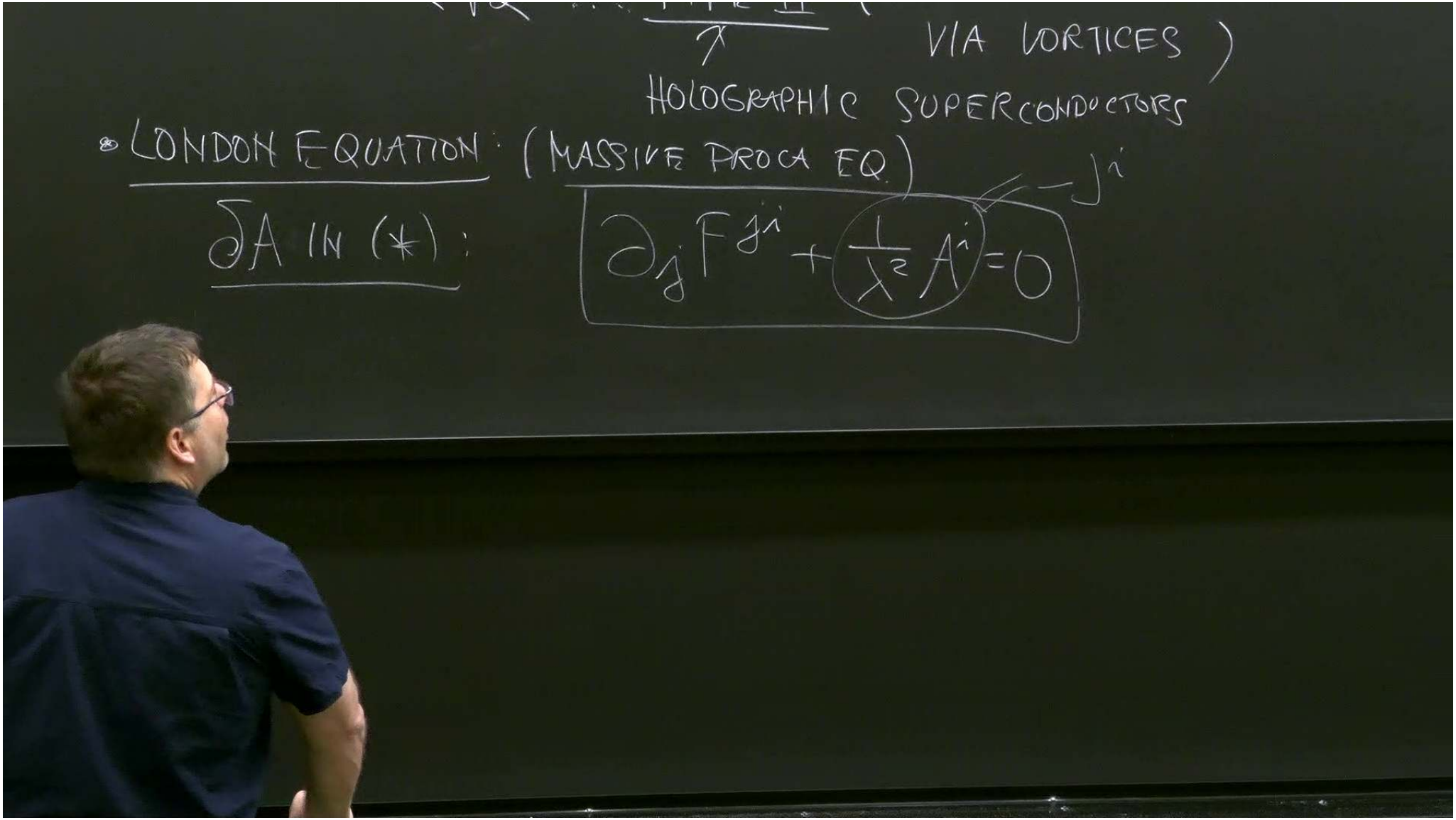

, 2.. MASSLESS GOLDSTONE BOSON (U(1) SYMMETRY BREAKING)
DEPENDING ON THE RATIO:
$\frac{\xi}{\lambda} = \begin{cases} > \sqrt{2} & \ldots \text{TYPE I} \\ < \sqrt{2} & \ldots \underline{\text{TYPE II}} \end{cases}$ (MAGNETIC FIELD CAN ENTER THE SUPER CONDUCTOR VIA VORTICES)
HOLOGRAPHIC SUPERCONDUCTORS
• LONDON EQUATION: (MASSIVE PROCA EQ.)
∂A IN (*): $\partial_j F^{ji} + \frac{1}{\lambda^2} A^i = 0$

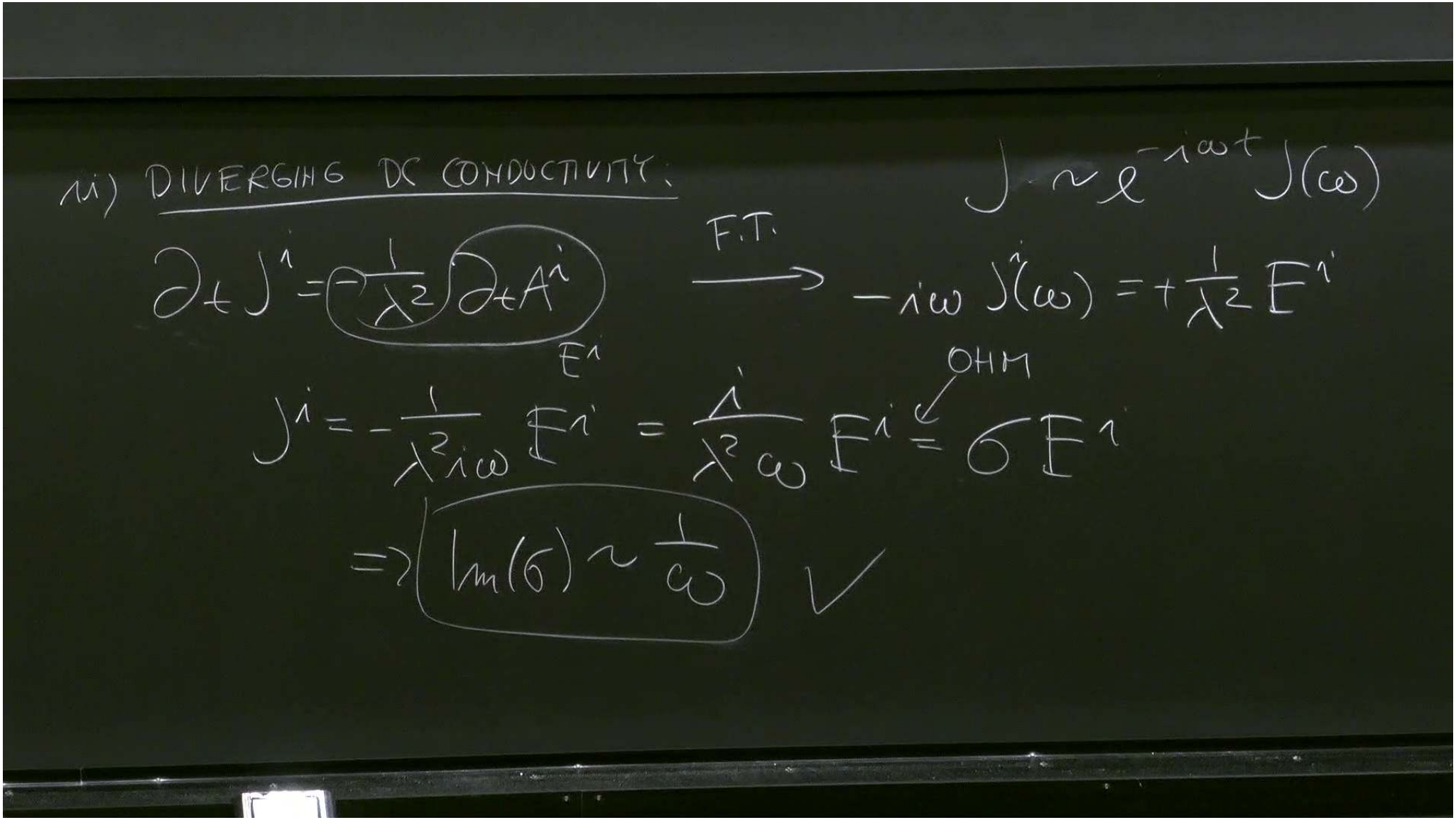
IMMEDIATE CONSEQUENCES:

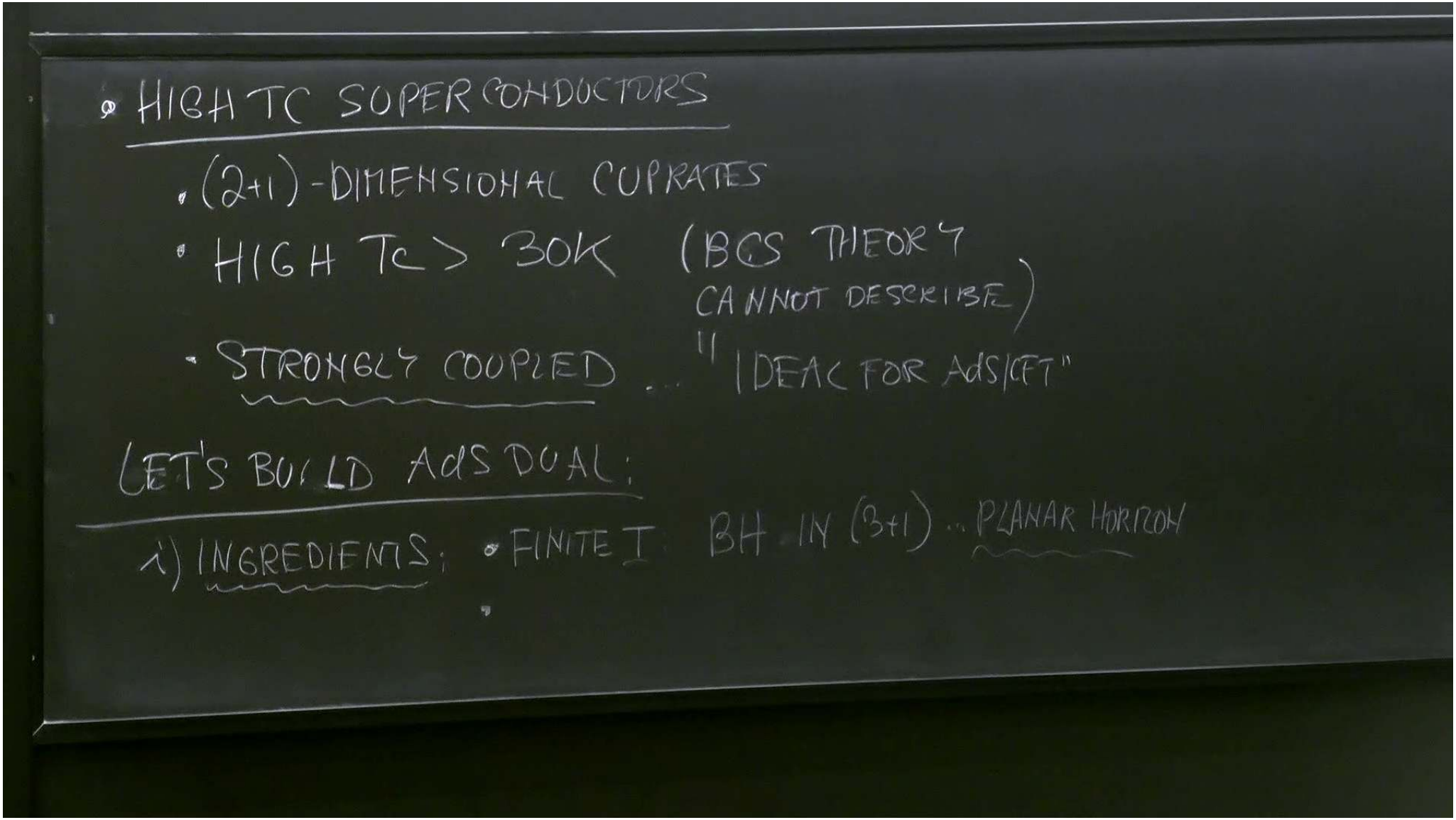
i) MEISSNER EFFECT: $A_i = (0, A_y(x), 0)$

$$\partial_i F^{iy} = \partial_x F^{xy} = -\partial_x^2 A_y = -\frac{1}{\lambda^2} A_y$$

$$B_z = F_{xy} = \partial_x A_y = B_0 e^{-x/\lambda}$$


VIA VORTICES)
HOLOGRAPHIC SUPERCONDUCTORS
LONDON EQUATION : (MASSIVE PROCA EQ.)
$-j^i$
∂A IN (*) :
$\partial_j F^{ji} + \frac{1}{\lambda^2} A^i = 0$

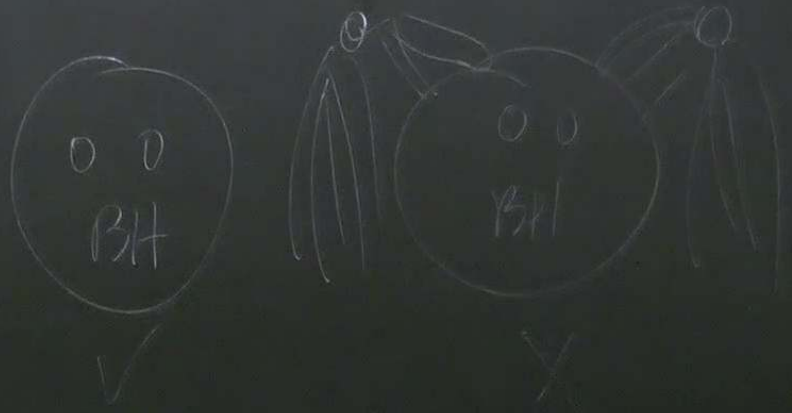

ii) DIVERGING DC CONDUCTIVITY:
$J \sim e^{-i\omega t} J(\omega)$
$\partial_t J^i = -\frac{1}{\lambda^2} \partial_t A^i$
F.T.
$-i\omega J^i(\omega) = +\frac{1}{\lambda^2} E^i$
E^i
OHM
$J^i = -\frac{1}{\lambda^2 i\omega} E^i = \frac{i}{\lambda^2 \omega} E^i = \sigma E^i$
$\Rightarrow \mathrm{Im}(\sigma) \sim \frac{1}{\omega}$ ✓


• HIGH TC SUPERCONDUCTORS
• (2+1)-DIMENSIONAL CUPRATES
• HIGH Tc > 30K (BCS THEORY CANNOT DESCRIBE)
• STRONGLY COUPLED ... "IDEAL FOR AdS/CFT"
LET'S BUILD AdS DUAL:
i) INGREDIENTS: • FINITE T BH IN (3+1) .. PLANAR HORIZON

SEEK BH → BH 2ND-ORDER PT.

∴ CONDENSATION OF COMPLEX SCALAR FIELD IN THE BULK?

WISDOM: IMPOSSIBLE AS WE HAVE NO HAIR THEOREMS



NOT TRUE FOR AdS BHS (OBSER)

ENTROPY

• WE ALSO NEED A FIELD IN THE BULK TO STUDY CONDUCTIVITY ...

H^3-THEORY

$$S = \int d^4x \sqrt{g} \left(R - 2\Lambda - \frac{1}{4} F_{MN}^2 - |D_M \psi|^2 - m^2 |\psi|^2 \right)$$

$$D_M = \nabla_M - ieA_M$$

"HAIRY BH" FOR $T < T_c$

"NO HAIR BH" FOR $T > T_c$

- SIMPLEST: TEST FIELD APPROX

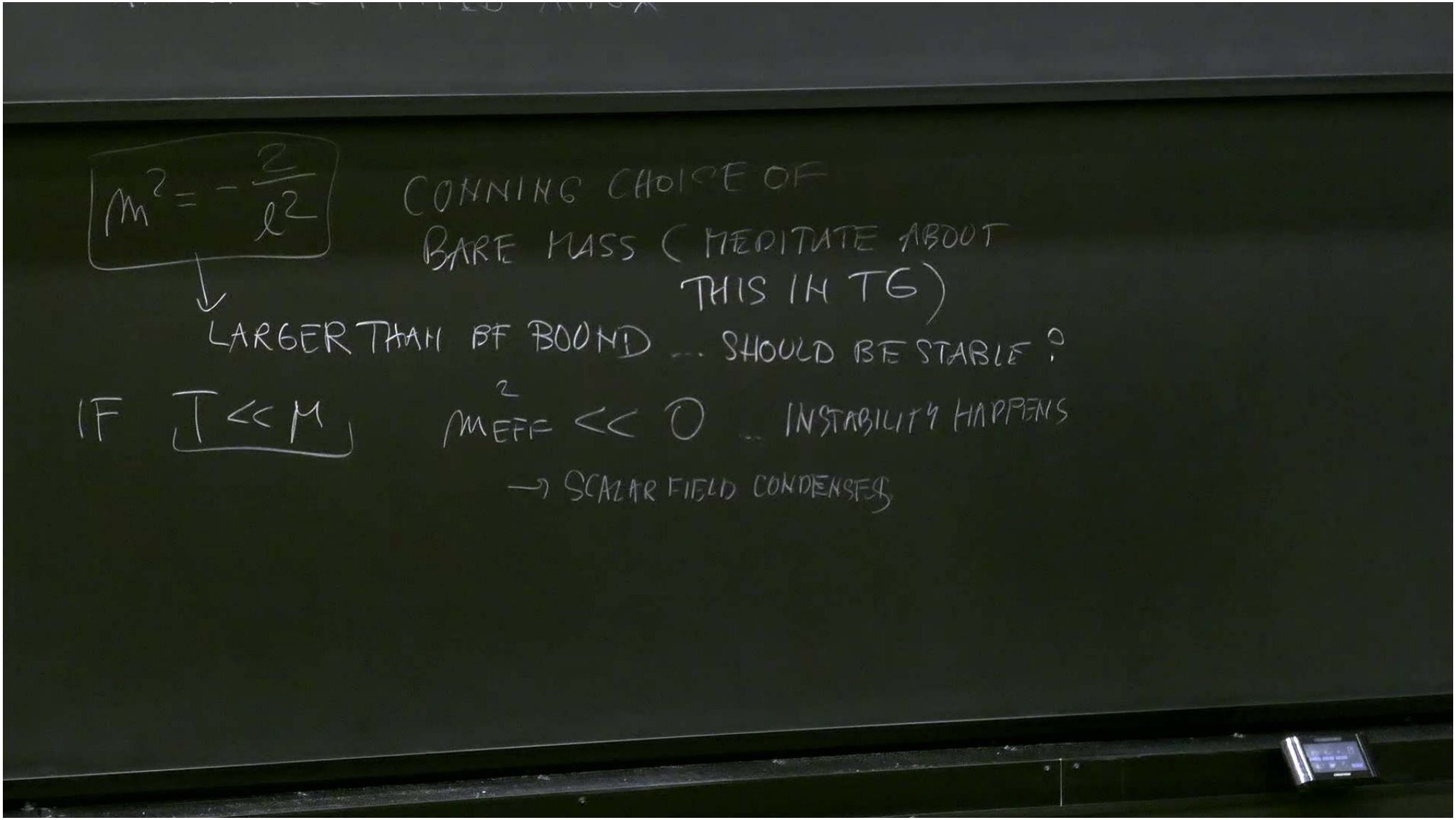
$$ds^2 = \left(\frac{r_0}{\ell}\right)^4 \frac{1}{u^2}\left(-f\,dt^2 + dx^2 + dy^2\right) + \ell^2 \frac{du^2}{f u^2}, \quad f = 1 - u^3, \quad u \in (0, 1]$$

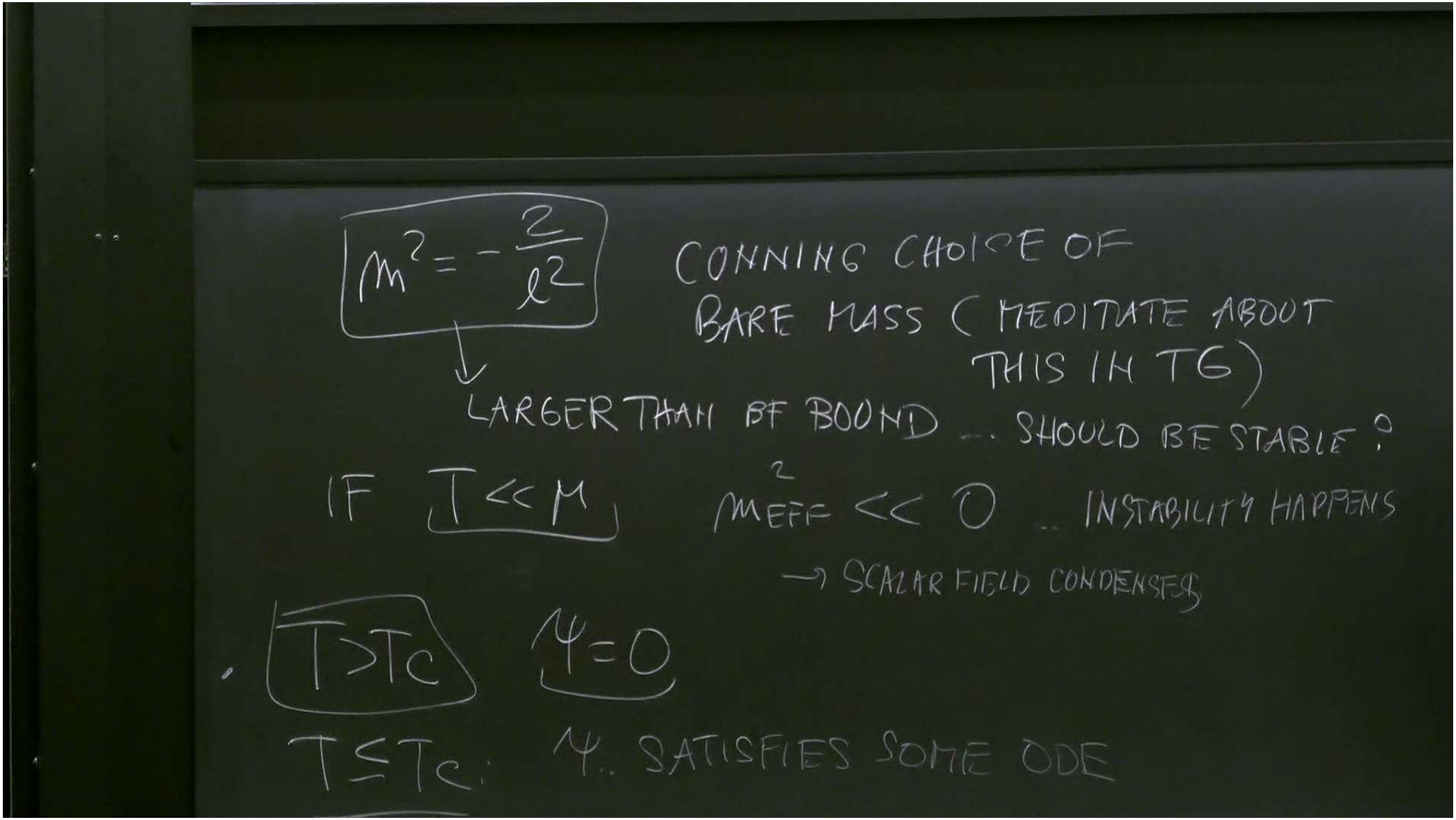
BOUNDARY ↑ HORIZON ↑

GIVES $\boxed{T = \frac{3 r_0}{4\pi \ell^2}}$

ii) PHASE TRANSITION: $\boxed{A = \mu(1-u)\,dt}$

NOTE: $m^2_{\text{EFF}} = m^2 + g^{MN} A_M A_N = m^2 - \left(\frac{\mu}{T}\right)^2 f(u)$ ↑ 0


$m^2 = -\frac{2}{\ell^2}$
CONNING CHOICE OF
BARE MASS (MEDITATE ABOUT
THIS IN TG)
LARGER THAN BF BOUND ... SHOULD BE STABLE?
IF $T \ll M$
$m_{EFF}^2 \ll 0$ ... INSTABILITY HAPPENS
→ SCALAR FIELD CONDENSES


$m^2 = -\frac{2}{\ell^2}$
CONNING CHOICE OF BARE MASS (MEDITATE ABOUT THIS IN TG)
LARGER THAN BF BOUND ... SHOULD BE STABLE?
IF $T \ll M$
$m_{EFF}^2 \ll 0$ .. INSTABILITY HAPPENS
→ SCALAR FIELD CONDENSES
$T > T_c$
$\Psi = 0$
$T \leq T_c$: Ψ.. SATISFIES SOME ODE

NEAR $u=0$ (BOUNDARY)

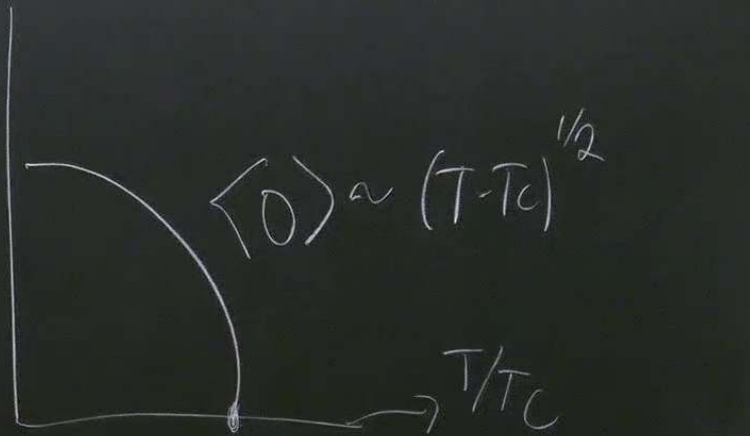
$\psi \sim \psi_0 u^{\Delta_-} + \psi_+ u^{\Delta_+}$

$\Delta_+ = 2$

ψ_0 ↑ SOURCE $=0$

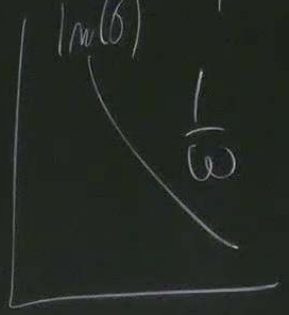
(SPONTANEOUS SYMMETRY BREAKING)

ψ_+ ↑ $\langle O \rangle$

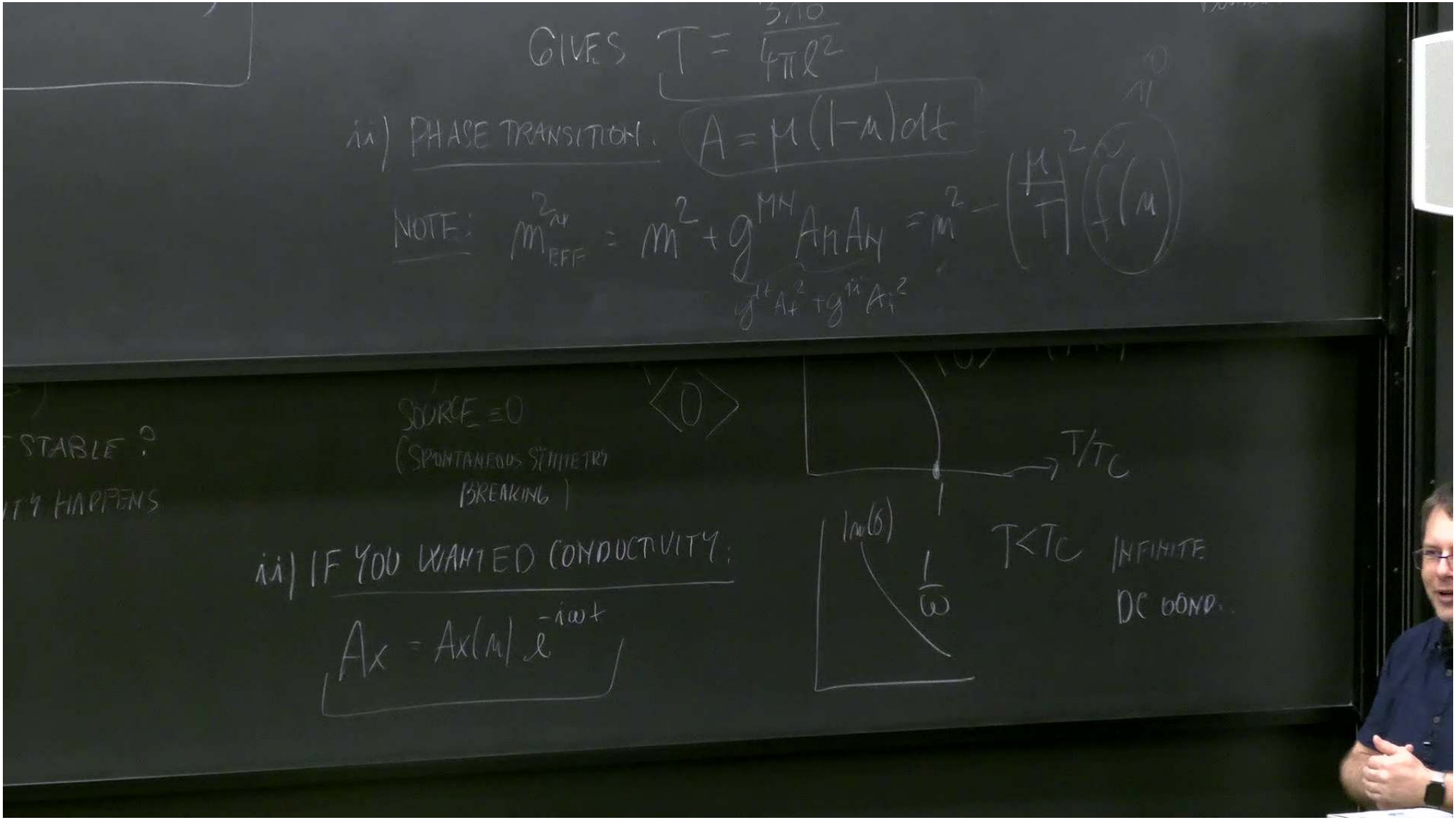


ii) IF YOU WANTED CONDUCTIVITY:

$A_x = A_x(u)\, e^{-i\omega t}$



$T<T_c$ INFINITE DC COND.


GIVES $T = \frac{3/10}{4\pi R^2}$
ii) PHASE TRANSITION. $A = \mu(1-m)dt$
NOTE: $m^{2}_{A\mu\,EFF} = m^2 + g^{MN}A_M A_N = m^2 - \left(\frac{\mu}{T}\right)^2 f(m)$
$g^{11}A_t^2 + g^{ii}A_i^2$
STABLE ?
IT HAPPENS
SOURCE = 0
(SPONTANEOUS SYMMETRY BREAKING)
$\langle O \rangle$
T/Tc
Im(G)
$\frac{1}{\omega}$
T<Tc INFINITE DC COND.
ii) IF YOU WANTED CONDUCTIVITY:
$A_x = A_x(u)\, e^{-i\omega t}$