Title: Lecture - AdS/CFT, PHYS 777

Speakers: David Kubiznak

Collection/Series: AdS/CFT (Elective), PHYS 777, March 31 - May 2, 2025

Subject: Quantum Fields and Strings, Quantum Gravity

Date: April 30, 2025 - 9:00 AM

URL: https://pirsa.org/25040035

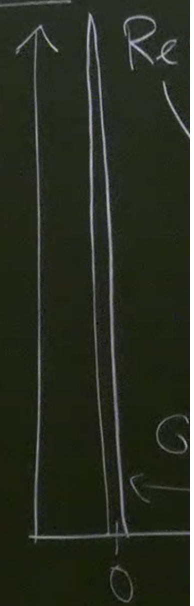
HOLOGRAPHIC SUPERCONDUCTORS

a) NORMAL SUPERCONDUCTORS:

2 BASIC FEATURES

- DIVERGING DC CONDUCTIVITY σ
  (OHM'S LAW: $J = \sigma E$)
- MEISSNER EFFECT (MAGNETIC FIELD EXPELLED FROM SUPERCONDUCTOR - $B=0$)

IN FACT:



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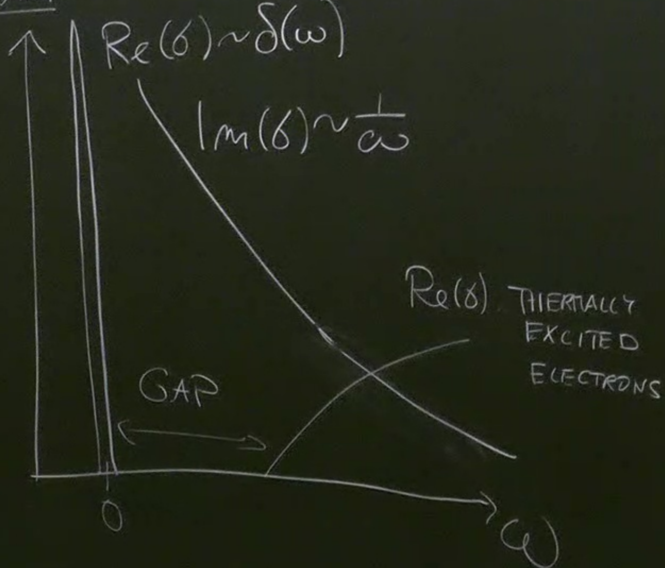


NOTE: REAL & IMAGINARY PARTS ARE RELATED BY KRAMERS-KRONIG RELATIONS

$$\boxed{\mathrm{Im}(\sigma) = -\frac{1}{\pi} P\int_{-\infty}^{\infty} \frac{\mathrm{Re}(\sigma(\omega'))}{\omega'-\omega}\, d\omega'}$$

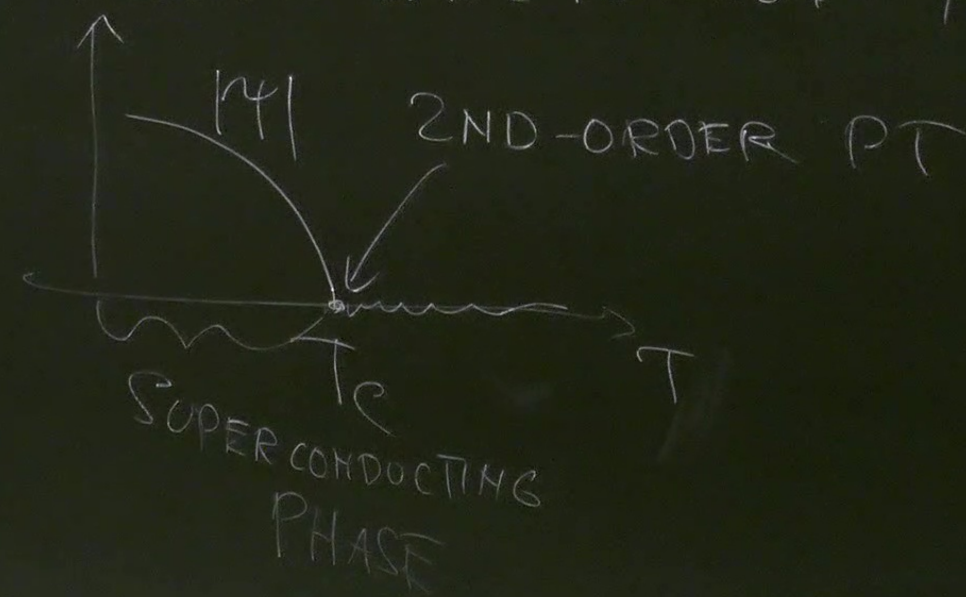
$\Rightarrow$ $\frac{1}{\omega}$ POLE IN $\mathrm{Im}(\sigma)$ $\leftrightarrow$ $\delta(\omega)$ FOR $\mathrm{Re}(\sigma)$

EASIER TO IDENTIFY NUMERICALLY

- <u>MICROSCOPICALLY</u>: BCS THEORY

CONDENSATION OF COOPER PAIRS

"MACROSCOPIC WAVE FUNCTION" Ψ



• PHENOMENOLOGICAL DESCRIPTION: LANDAU-GINSBURG THEORY

$$\boxed{\mathcal{L} = \frac{\hbar^2}{2m_*}|D_i\psi|^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{1}{4}F_{ij}^2}$$

$$D_i = \partial_i - i e_* \frac{A_i}{\hbar}, \quad m_* = 2m_e, \quad e_* = 2e$$

• WRITE $\psi = |\psi(x)|\, e^{i e_* \frac{\phi(x)}{\hbar}}$

$$\boxed{\mathcal{L} = \frac{\hbar^2}{2m_*}\left(\partial_i|\psi|\right)^2 + \frac{e_*^2}{2m_*}|\psi|^2\left(\partial_i\phi - A_i\right)^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{1}{4}F_{ij}^2} \qquad (*)$$

SPEC: i) $A_i = 0$

• THE PHENOMENOLOGICAL [illegible] GINZBURG-LANDAU [illegible]

$$\boxed{\mathcal{L} = \frac{\hbar^2}{2m_*}|D_i\psi|^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{1}{4}F_{ij}^2} \qquad D_i = \partial_i - i e_* \frac{A_i}{\hbar}, \quad m_* = 2m_e, \quad e_* = 2e$$

• WRITE $\psi = |\psi(x)|\, e^{i \frac{e_* \phi(x)}{\hbar}}$

$$\boxed{\mathcal{L} = \frac{\hbar^2}{2m_*}\left(\partial_i|\psi|\right)^2 + \frac{e_*^2}{2m_*}|\psi|^2\left(\partial_i\phi - A_i\right)^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{1}{4}F_{ij}^2} \qquad (*)$$

SPEC: i) $A_i = 0$, ϕ: MASSLESS GOLDSTONE BOSON (U(1) SYMMETRY BREAKING)

ii) $\underline{A_i \neq 0}$. ϕ CAN BE ABSORBED BY GAUGE TRANSF. TO $\underline{A_i}$

$\rightarrow$ A_i IS MASSIVE (HIGGS MECHANISM)

$$\frac{1}{m_A^2} \sim \frac{m_*}{e_*^2 \underbrace{|\psi|^2}_{=\frac{a}{b}}} \sim \lambda^2 \quad \ldots$$

PENETRATION LENGTH

$$\frac{1}{m_\psi^2} \sim \frac{\hbar^2}{2 m_* a} \sim \xi^2 \quad \ldots$$

CORRELATION LENGTH

=0 , Φ.. MASSLESS GOLDSTONE BOSON (U(1) SYMMETRY BREAKING)

DEPENDING ON THE RATIO:

$$\frac{\xi}{\lambda} = \begin{cases} > \sqrt{2} & \dots \text{TYPE I} \\ < \sqrt{2} & \dots \underline{\text{TYPE II}} \end{cases}$$

(MAGNETIC FIELD CAN ENTER THE SUPERCONDUCTOR VIA VORTICES)

HOLOGRAPHIC SUPERCONDUCTORS

* <u>LONDON EQUATION</u>: (<u>MASSIVE PROCA EQ.</u>)

<u>δA IN $(*)$</u>:

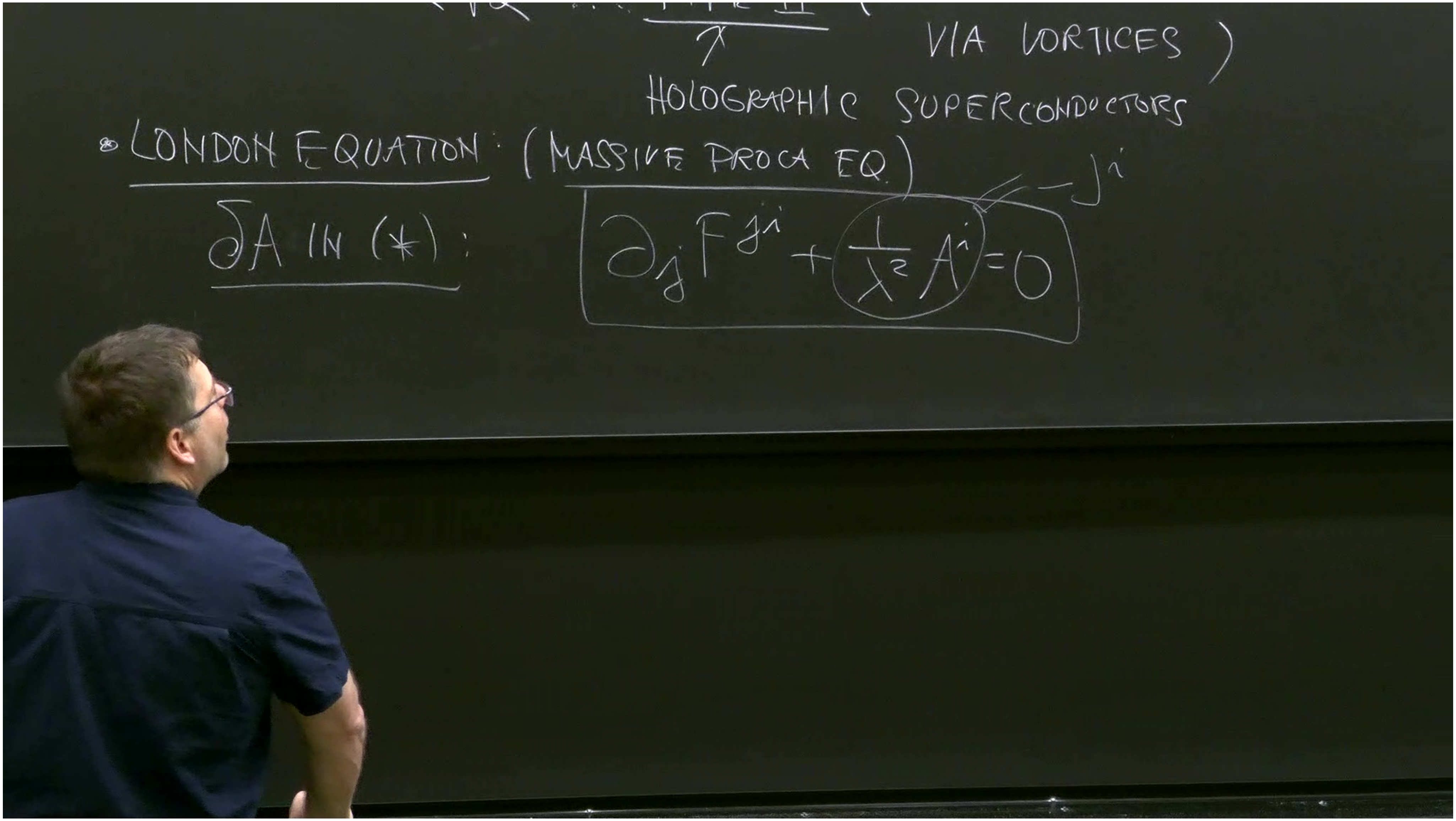
$$\boxed{\partial_j F^{ji} + \frac{1}{\lambda^2} A^i = 0}$$

IMMEDIATE CONSEQUENCES:

i) MEISSNER EFFECT: $A_i = (0, A_y(x), 0)$

$$\partial_i F^{iy} = \partial_x F^{xy} = -\partial_x^2 A_y = -\frac{1}{\lambda^2} A_y$$

$$B_z = F_{xy} = \partial_x A_y = B_0 e^{-x/\lambda}$$


VIA VORTICES)
HOLOGRAPHIC SUPERCONDUCTORS
⊛ LONDON EQUATION: (MASSIVE PROCA EQ)
∂A IN (*):
$\boxed{\partial_j F^{ji} + \frac{1}{\lambda^2} A^i = 0}$
j^i

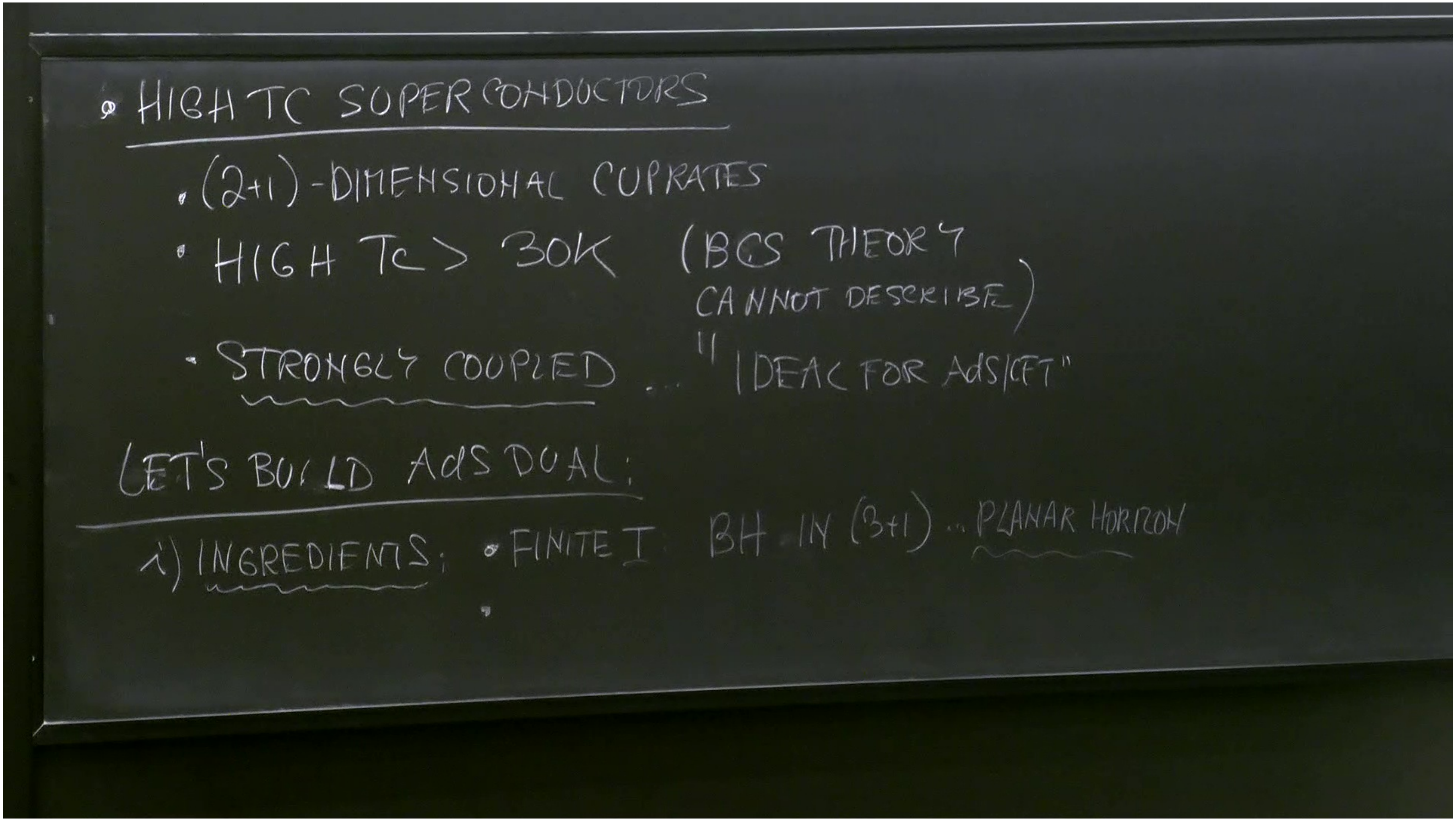
ii) DIVERGING DC CONDUCTIVITY:

$$J \sim e^{-i\omega t} J(\omega)$$

$$\partial_t J^i = \underbrace{-\frac{1}{\lambda^2}\partial_t A^i}_{E^i} \xrightarrow{\text{F.T.}} -i\omega\, J^i(\omega) = +\frac{1}{\lambda^2} E^i$$

$$J^i = -\frac{1}{\lambda^2 i\omega} E^i = \frac{i}{\lambda^2 \omega} E^i \overset{\text{OHM}}{=} \sigma E^i$$

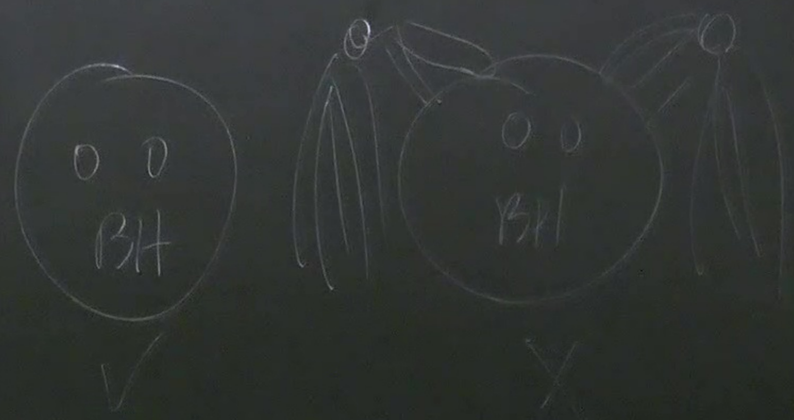
$$\Rightarrow \boxed{\mathrm{Im}(\sigma) \sim \frac{1}{\omega}} \quad \checkmark$$


HIGH TC SUPERCONDUCTORS
(2+1)-DIMENSIONAL CUPRATES
HIGH Tc > 30K (BCS THEORY CANNOT DESCRIBE)
STRONGLY COUPLED ... "IDEAL FOR AdS/CFT"
LET'S BUILD AdS DUAL:
i) INGREDIENTS: FINITE T BH IN (3+1) ... PLANAR HORIZON

SEEK BH → BH 2ND-ORDER PT.

. CONDENSATION OF COMPLEX SCALAR FIELD IN THE BULK:

WISDOM: IMPOSSIBLE AS WE HAVE NO HAIR THEOREMS



NOT TRUE FOR AdS BHs
(GUBSER)

ENTROPY

· WE ALSO NEED A_μ FIELD IN THE BULK TO STUDY CONDUCTIVITY ...

H^3-THEORY

$$\boxed{\begin{gathered}S = \int d^4x \sqrt{g} \left(R - 2\Lambda - \tfrac{1}{4} F_{MN}^2 - |D_M \psi|^2 - m^2 |\psi|^2 \right) \\ D_M = \partial_M - ie A_M\end{gathered}}$$

"HAIRY BH" FOR $T<T_c$

"NO HAIR BH" FOR $T>T_c$

- SIMPLEST: TEST FIELD APPROX

$$ds^2 = \left(\frac{r_0}{\ell}\right)^4 \frac{1}{u^2}\left(-f\,dt^2 + dx^2 + dy^2\right) + \ell^2 \frac{du^2}{f u^2}, \quad f = 1-u^3, \quad u \in (0,1)$$

BOUNDARY HORIZON

GIVES $\boxed{T = \frac{3 r_0}{4\pi \ell^2}}$

ii) PHASE TRANSITION: $\boxed{A = \mu(1-u)\,dt}$

NOTE: $m^2_{\text{EFF}} = m^2 + g^{MN} A_M A_N = m^2 - \left(\frac{\mu}{T}\right)^2 \tilde{f}(u)$

$\boxed{m^2 = -\frac{2}{\ell^2}}$ CONNING CHOISE OF BARE MASS (MEDITATE ABOUT THIS IN TG)

↓

LARGER THAN BF BOOND ... SHOULD BE STABLE ?

IF $\boxed{T \ll M}$ $m^2_{EFF} \ll 0$ .. INSTABILITY HAPPENS

→ SCALAR FIELD CONDENSES

$$\boxed{m^2 = -\frac{2}{\ell^2}}$$

CONNING CHOICE OF BARE MASS (MEDITATE ABOUT THIS IN TG)

↓ LARGER THAN BF BOUND ... SHOULD BE STABLE ?

IF $\underline{T \ll \mu}$ $m^2_{EFF} \ll 0$ ... INSTABILITY HAPPENS

→ SCALAR FIELD CONDENSES

$\boxed{T > T_c}$ $\underline{\psi = 0}$

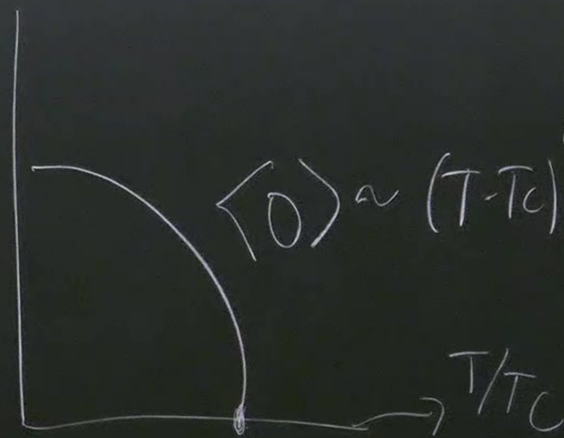
$T \leq T_c$: ψ.. SATISFIES SOME ODE

NEAR $u=0$ (BOUNDARY)

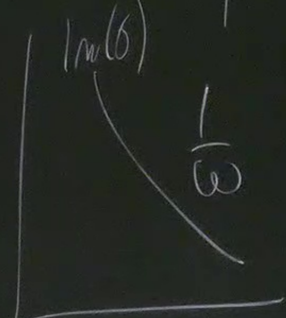
$$\psi \sim \psi_0 \, u^{\Delta_-} + \psi_+ \, u^{\Delta_+}$$

ψ_0: SOURCE $\equiv 0$ (SPONTANEOUS SYMMETRY BREAKING)

ψ_+: $\langle O \rangle$



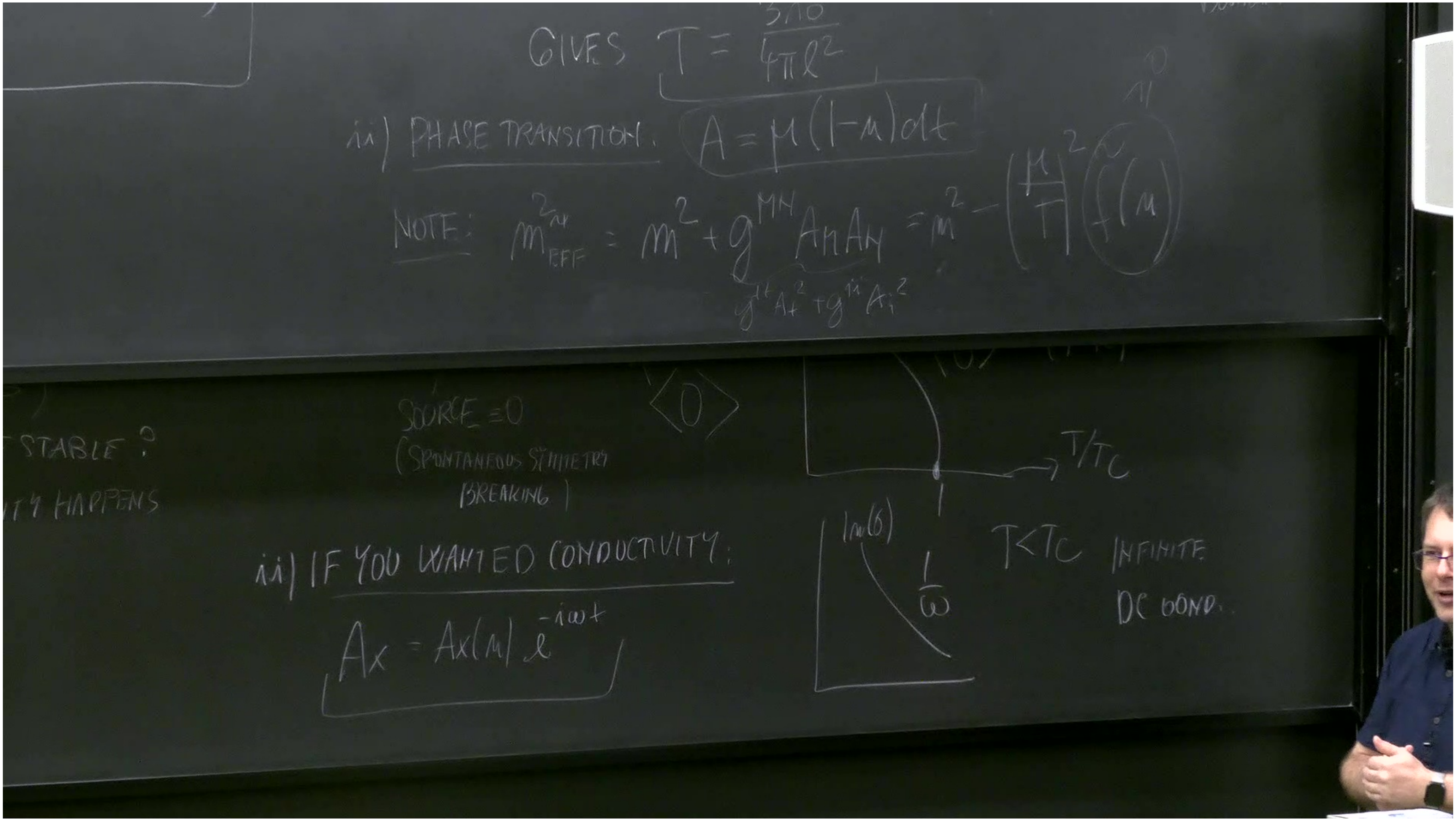
$\langle O \rangle \sim (T-T_c)^{1/2}$



$T<T_c$ INFINITE DC COND.

ii) IF YOU WANTED CONDUCTIVITY:

$$\boxed{A_x = A_x(u)\, e^{-i\omega t}}$$


GIVES T = 3r0 / 4πℓ²
ii) PHASE TRANSITION: A = μ(1−u)dt
NOTE: m²EFF = m² + g^MN AM AN = m² − (μ/T)² f(u)
g^tt At² + g^ii Ai²
STABLE ?
TY HAPPENS
SOURCE = 0
(SPONTANEOUS SYMMETRY BREAKING)
⟨O⟩
T/Tc
iii) IF YOU WANTED CONDUCTIVITY:
Ax = Ax(u) e^(−iωt)
Im(σ)
1/ω
T<Tc INFINITE DC COND.