Title: Lecture - AdS/CFT, PHYS 777

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Collection/Series: AdS/CFT (Elective), PHYS 777, March 31 - May 2, 2025

Subject: Quantum Fields and Strings, Quantum Gravity

Date: April 28, 2025 - 9:00 AM

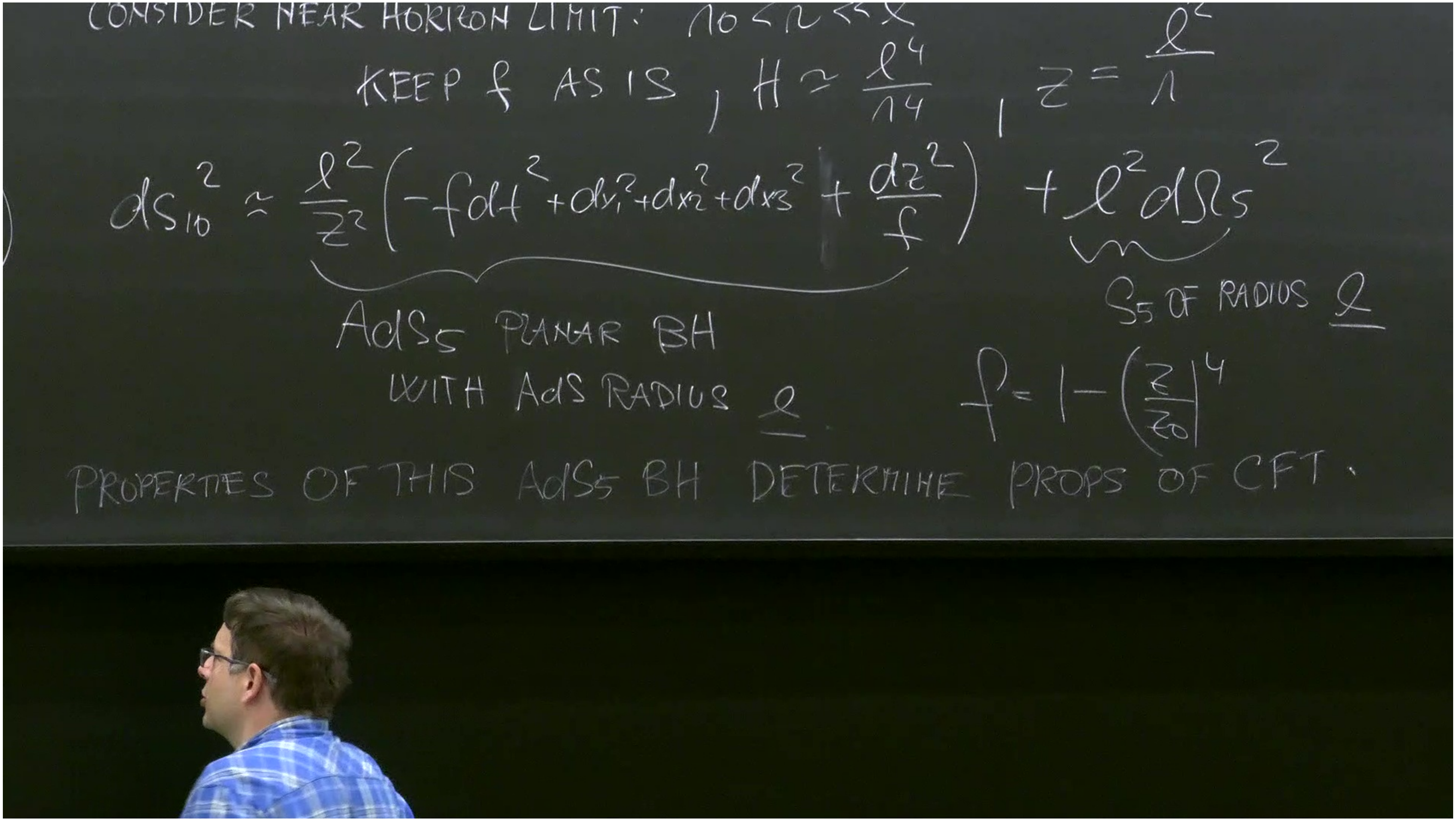
URL: https://pirsa.org/25040034

CFT AT FINITE TEMPERATURE

RECALL: NON-EXTREMAL BLACK BRANE IN IIB SUGRA:

$$ds_{10}^2 = H^{-1/2}\left(-f\,dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + H^{1/2}\left(\frac{dr^2}{f} + r^2 d\Omega_5^2\right)$$

$$\underbrace{H = 1 + \frac{\ell^4}{r^4}}_{r=0 \text{ INNER HORIZON}}, \quad \underbrace{f = 1 - \frac{r_0^4}{r^4}}_{\text{HORIZON AT } r_+ = r_0}$$


CONSIDER NEAR HORIZON LIMIT: $r_0 < r \ll \ell$
KEEP f AS IS, $H \simeq \frac{\ell^4}{r^4}$, $z = \frac{\ell^2}{r}$
$ds_{10}^2 \simeq \underbrace{\frac{\ell^2}{z^2}\left(-f dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + \frac{dz^2}{f}\right)}_{\text{AdS}_5 \text{ PLANAR BH WITH AdS RADIUS } \ell} + \underbrace{\ell^2 d\Omega_5^2}_{S_5 \text{ OF RADIUS } \ell}$
$f = 1 - \left(\frac{z}{z_0}\right)^4$
PROPERTIES OF THIS AdS5 BH DETERMINE PROPS OF CFT.

AT FINITE TEMPERATURE

UNLESS

$$\boxed{\boxed{T = T_H} = \frac{1}{\pi z_0}}$$

BEWARE: $x^\mu \to \lambda x^\mu$, $z \to \lambda z$, $z_0 \to \lambda z_0$

SYMMETRY

BUT: $T \to \frac{T}{\lambda}$ (ALL TEMPERATURES EXCEPT $T=0$ ARE "EQUIVALENT")

WITH AdS RADIUS ℓ. $\quad f = 1 - \left(\frac{z}{z_0}\right)$

PROPERTIES OF THIS AdS_5 BH DETERMINE PROPS OF CFT.

UNLESS ANOTHER SCALE IS PRESENT WE CANNOT HAVE PHASE TRANSITIONS

(HAWKING-PAGE HAPPENS FOR SPHERICAL BHS)

EXTRA SCALE.

$\boxed{\text{STATIONARY BH} \Longleftrightarrow \text{EQUILIBRIUM}}$

WE WANT SLIGHTLY PERTURB THIS TO STUDY

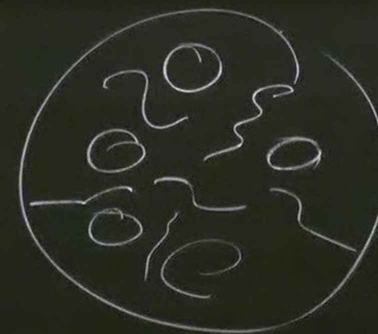
INTERESTING PROPS. OF THE SYSTEM (η ... VISCOSITY)

QUIVALENT")

EXCEPT $T=0$ ARE "EQUIVALENT")

PHYSICS OF QUARK-GLUON (QG) PLASMA

- STATE OF MATTER IN EARLY UNIVERSE
  (10^{13} K, 10^{-6} s)
- STRONGLY COUPLED (NOT A CFT)
- RECREATED AT LHC ... IN HYDRODYNAMICS REGIME



SHEAR VISCOSITY

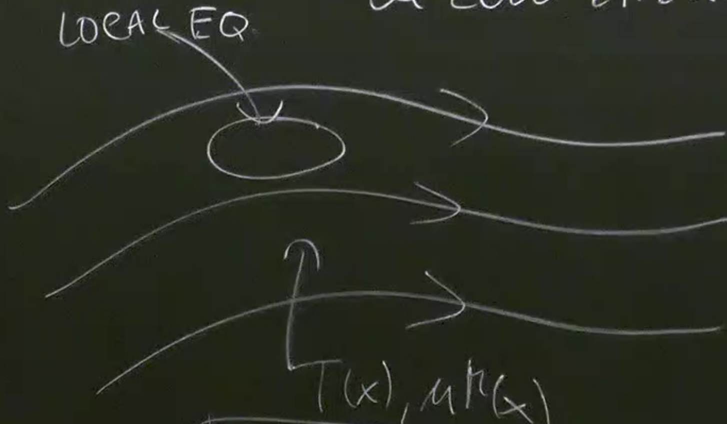
$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi} \times (1, 2.5)}$$ MEASURED

ENTROPY DENS

• MUCH SMALLER THAN TYPICAL MATTER ($\propto 10^3$ TIMES BIGGER)

• HYDRODYNAMICS $\equiv$ EFT DESCRIBING LONG RANGE ($\vec{q} \to 0$) & LOW ENERGY ($\omega \to 0$) FLUCTUATIONS

ME



"$\lambda_{FLUCTUATIONS} \gg \lambda_{MFP}$"

DYNAMICS: $\boxed{\nabla_\mu T^{\mu\nu} = 0}$

$\boxed{\nabla_\mu j^\mu = 0}$ $\quad \vec{j} \propto \alpha \vec{\nabla} T$

D-EOM FOR $\frac{D(D+1)}{2}$ UNKNOWNS

INSTEAD CHOOSE D-INDEP VARIABLES: $T(x)$, $u^{\mu}(x)$ $(u^2=-1)$

↑TEMP ↑VELOCITY

EXPRESS

$$T^{\mu\nu} = T^{\mu\nu}\left(\rho(T), P(T), u^{\mu}, + \text{COEFFICIENTS}\right)$$

CONSTITUTIVE RELATIONS

η, ζ

"GOD GIVEN"

... EXTERNA INPUT

INSTEAD CHOOSE D-INDEP VARIABLES: $T(x)$, $u^{\mu}(x)$ $\quad (u^2=-1)$

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↑ CONSTITUTIVE RELATIONS

↑ η, ζ

"GOD GIVEN"

... EXTERNA INPUT

Ex: $\nabla T \sim \dfrac{\Delta T}{\Delta X} \sim \dfrac{\lambda_{MFP}}{\lambda_{FLUCT}}$

($u^2=-1$)

...ITY

...FFICIENTS)

ξ

"...GIVEN"

...NA INPUT

• IN PRACTISE WE USE DERIVATIVE EXPANSION:

$$T^{\mu\nu} = \underbrace{(\rho+P)\,u^{\mu}u^{\nu} + P g^{\mu\nu}}_{\text{0TH-ORDER}} + \underbrace{\Pi^{\mu\nu}}_{\substack{\text{1ST-ORDER}\\ \text{(GAUGE DEPENDENT)}}} + O\left(\frac{\lambda_{mfp}}{\lambda_{fluct}}\right)^2 + \dots$$

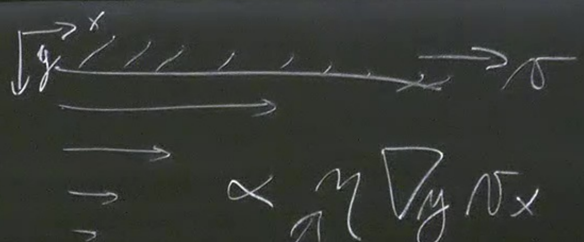
• IN LANDAU'S GAUGE: $T^{\mu}{}_{\nu}\,u^{\nu} = -\rho\,u^{\mu}$

$$\boxed{\Pi^{\mu\nu} = -2\eta\,\underset{\uparrow\,\text{TRACELESS}}{\sigma^{\mu\nu}} - 2\xi\,\theta\,P^{\mu\nu}}$$

SHEAR VISCOSITY

BULK VISCOSITY. ($\xi=0$ FOR CFT)

RECALL:



$\propto \eta \nabla_y v_x$

MEANING OF SHEAR VISCOSITY

$$\sigma^{\alpha\beta} = P^{\alpha\gamma} P^{\beta\delta} \left(\nabla_{(\gamma} u_{\delta)} - \frac{1}{d-1} P_{\gamma\delta} \theta \right)$$

$$\theta = \nabla_\alpha u^\alpha \, , \quad P^{\mu\nu} = u^\mu u^\nu + g^{\mu\nu}$$

WE WANT TO CALCULATE η/s:

STEP 1: USE HYDRO DESCRIPTION (MACROSCOPIC) (T5)

(1) $\boxed{\delta T^{xy}(\omega) = -i\omega\, \eta\, \underbrace{\overset{\circ}{h}_{xy}(\omega)}_{}}$ FICTICIOUS PERT OF MINK. METRIC.

$) - \frac{1}{d-1} P \eta \sigma \theta)$

$\mu_\mu{}^\nu + \eta^{\mu\nu}$

$T5)$

MINK. METRIC.

• STEP 2. TO RELATE THIS TO MICROSCOP. PHYSICS, WE USE LINEAR RESPONSE THEORY:

ASSUME SYSTEM IN EQUILIBRIUM DESCRIBED BY $\mathcal{S}_0$, AND PERTURB IT BY EXTERNAL SOURCE ϕ_0.

$$S \to S + \delta S, \quad \delta S = \int d^d x \; \mathcal{O}(x) \phi_0(x)$$

$$\Rightarrow \quad \delta\langle O(k)\rangle = -\underbrace{G_R^{00}}_{\text{RETARDED GREEN F.}}(k)\,\phi_0(k)$$

RESPONSE

SOURCE

FOR US:

$$\delta\langle \tau^{xy}\rangle = -G_R(k)\,\overset{\circ}{h}_{xy}(k) \qquad (2)$$

- COMPARE (1) & (2)

$$\boxed{\eta = \frac{1}{i\omega}\,G_R(k)}$$

KUBO FORMULA

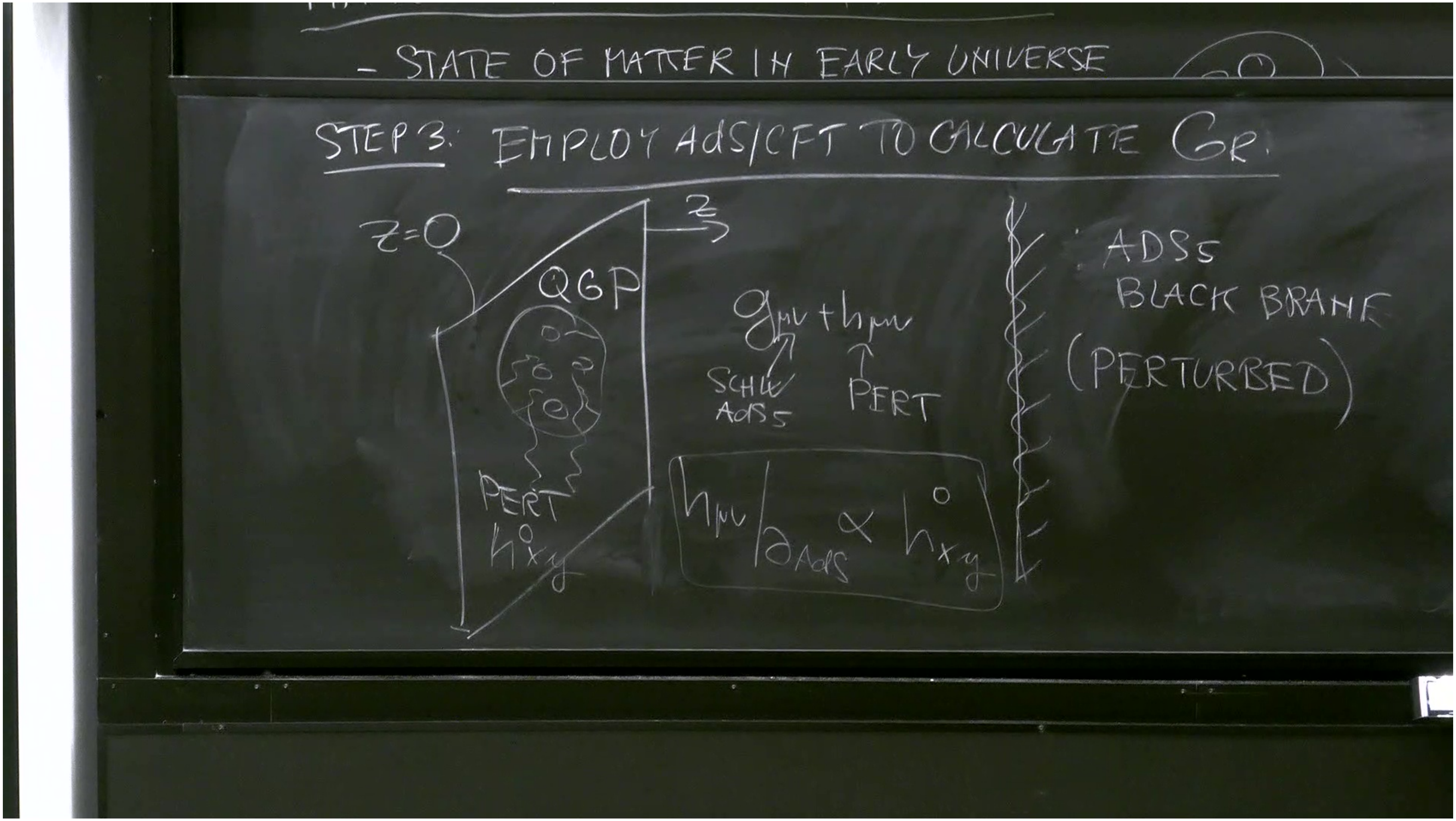
WHERE $G(k) \sim \int e^{ikx}\,\theta(t)\,\langle [\,T^{xy}(x), T^{xy}(0)\,] \rangle$

SO FAR NO AdS/CFT!

RCE

k) (2)

KUBO FORMULA


- STATE OF MATTER IN EARLY UNIVERSE
STEP 3: EMPLOY AdS/CFT TO CALCULATE G_R.
$z=0$
z
QGP
PERT
$\mathring{h}_{xy}$
$g_{\mu\nu} + h_{\mu\nu}$
SCHW AdS5
PERT
$\boxed{h_{\mu\nu}\big|_{\partial AdS} \propto \mathring{h}_{xy}}$
ADS5
BLACK BRANE
(PERTURBED)

WE WANT SLIGHTLY PERTURB THIS TO STUDY
INTERESTING PROPS. OF THE SYSTEM (

• RECIPE: i) SOLVE EOM FOR PERT $h_{\mu\nu}$ IN AdS5 SCHW.

ii) EXPRESS SOL IN TERMS OF BC's $\mathring{h}_{xy}$

iii) PLUG BACK TO THE ACTION $S[\mathring{h}_{xy}] = W[\mathring{h}_{xy}]$

$$\langle \tau^{xy}\tau^{xy}\rangle = -\frac{\delta^2 S[\mathring{h}_{xy}]}{\delta \mathring{h}_{xy}\,\delta \mathring{h}_{xy}} \longrightarrow G_R$$

iii) PLUG BACK TO THE ACTION $S[h^{0}_{xy}] = W[h^{0}_{xy}]$

$$\langle \eta^{xy}\eta^{xy}\rangle = -\frac{\delta^2 S[h^{0}_{xy}]}{\delta h^{0}_{xy}\,\delta h^{0}_{xy}} \longrightarrow G_R$$

$$\boxed{G(\omega) = \frac{\pi N_c^2 T^3}{8} i\omega}\qquad \frac{\pi N_c^2 T^3}{8} = \eta$$

UNIVER. LOWER BOUND

$$s = \frac{\pi^2}{2} N_c^2 T^3 \quad (T5) \Rightarrow \boxed{\frac{\eta}{s} = \frac{1}{4\pi}}$$

λ FLUCTUATIONS $\gg \lambda$ MFP

DYNAMICS: $\boxed{\nabla_\mu T^{\mu\nu} = 0}$

$T(x),\ u^\mu(x)$

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D-EOM FOR $\frac{D(D+1)}{2}$ UNKNOWNS