Title: Lecture - AdS/CFT, PHYS 777

Speakers: David Kubiznak

Collection/Series: AdS/CFT (Elective), PHYS 777, March 31 - May 2, 2025

Subject: Quantum Fields and Strings, Quantum Gravity

Date: April 10, 2025 - 9:00 AM

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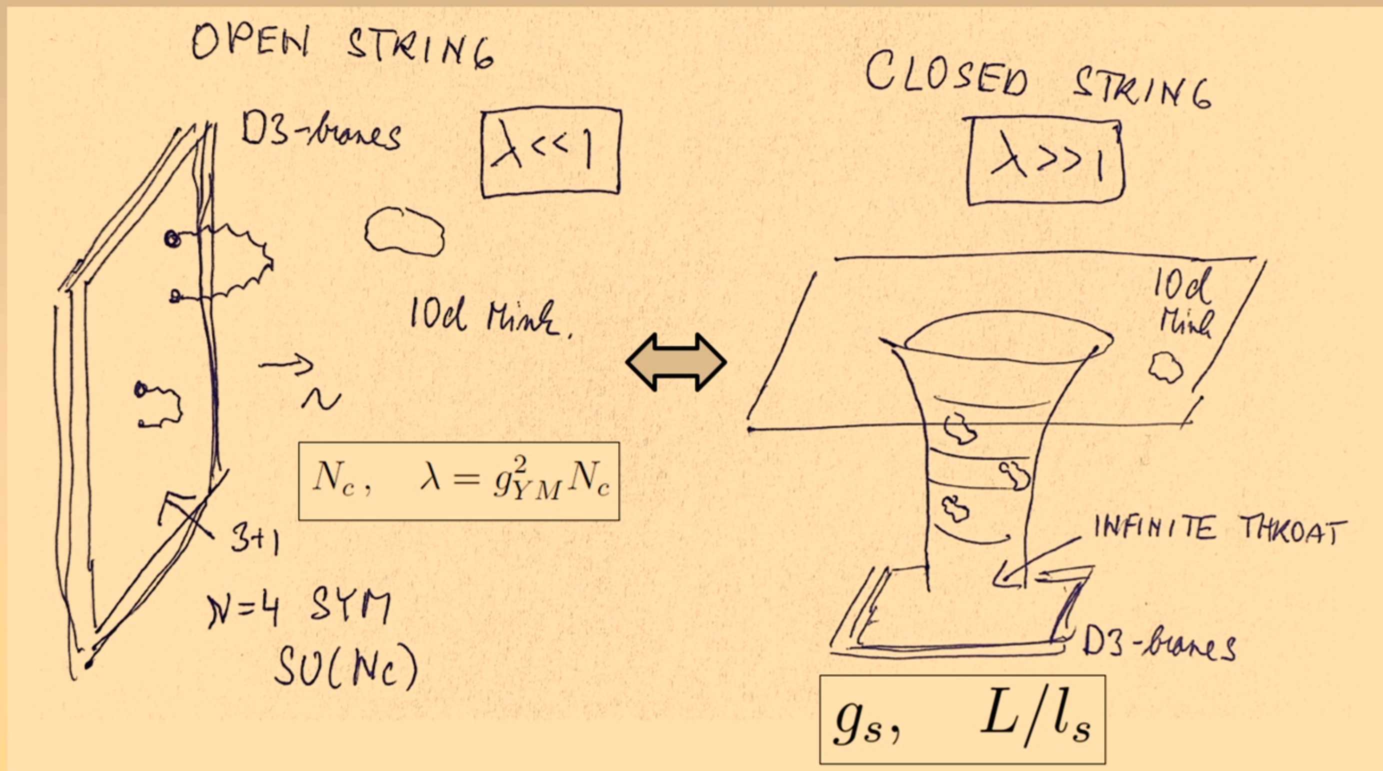
Central conjecture

Conjecture (Maldacena 1997). Type IIB superstring theory on $AdS_5 \times S^5$ is *dual* to $\mathcal{N} = 4$ $SU(N_c)$ SYM in $d = (3+1)$ dimensions.



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String theory — **Dual** — **QFT**

$$g_s, \quad L/l_s \quad \Longleftrightarrow \quad N_c\,, \quad \lambda = g^2_{YM} N_c$$

- A theory of **QG**
- A **QFT** without gravity

$$2\pi g_s = g^2_{YM} = \frac{\lambda}{N_c}\,, \qquad \frac{L^4}{l_s^4} = 4\pi g_s N_c = 2\lambda$$

Central conjecture

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Remarks:

- **Duality** means complete **equivalence!**
(No explicit change of variables is known. Conjecture supported by case by case evidence)

- **(In)sanity check:** match symmetries

AdS5xS5: SO(4,2) x SO(6)

SYM: SO(4,2) x SO(6)

CFT in 4d — rotations of 6 scalars Φ^i

- **Holographic:** SYM lives on the boundary of AdS_5

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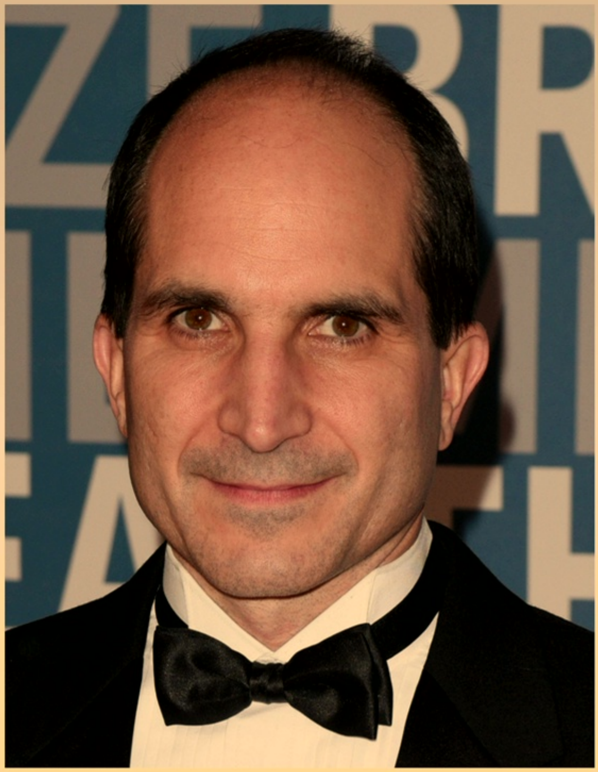


Useful weaker version

$$2\pi g_s = g_{YM}^2 = \frac{\lambda}{N_c}, \qquad \frac{L^4}{l_s^4} = 4\pi g_s N_c = 2\lambda$$

Classical (super)gravity as a limit of ST:

- Strings almost point-like – standard geometry valid

$$L \gg l_s \quad \Rightarrow \quad \lambda \gg 1$$

- Quantum corrections (loops) are small $1 \gg g_s \sim \frac{\lambda}{N_c}$

Weak (weakly interacting) gravity is dual to strongly coupled QFT

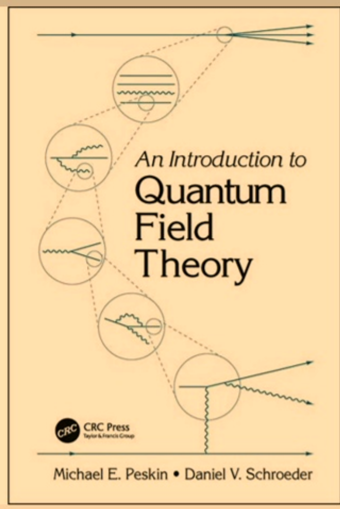
$$1 \ll \lambda \ll N_c$$

(**strong-weak duality**)

Amazing tool to study strongly coupled QFTs (by simple gravitational calculations).

What is conformal field theory?

= special QFT with bigger spacetime symmetry



Invariant under conformal transformations:

$$x^\mu \;\to\; x'^\mu(x): \quad \eta_{\mu\nu} \;\to\; \Omega^2(x)\eta_{\mu\nu}$$

spec. $t \to \lambda t\,, \quad \vec{x} \to \lambda \vec{x}$

$$\phi(x) \;\to\; \phi'(x') = \lambda^{-\Delta}\phi(x)$$

scaling dimension

Theorem: Conformal (superconformal symmetry is the largest one for non-trivial (interacting) QFTs.

What is conformal field theory?

- **Examples:**

$\beta(g) = 0$

???

1) Maxwell equations: $\partial_\mu F^{\mu\nu} = 0$
2) Massless Dirac equation: $\gamma^\mu \partial_\mu \psi = 0$
3) classical $\lambda\phi^4$ in $D=4$: $\Box\phi = \frac{\lambda}{3!}\phi^3$
4) classical YM: $D_\mu F^{\mu\nu} = 0$

- **Correlations:**

$$\langle O(x) O(y) \rangle \propto \frac{1}{(x-y)^{2\Delta}}$$

- **Applications**:
  - phase transitions: $\{\alpha, \beta, \ldots\} \leftrightarrow \{\Delta\}$
  - string theory
  - mapping space of QFTs ("perturbations" of CFT)

What is AdS?

= maximally symmetric solution of EE with negative cosmological constant

Embedding perspective: "hyperboloid in $\mathbb{R}^{2,d-1}$"

$$\begin{aligned} ds^2 &= -(dY^{-1})^2 - (dY^0)^2 + (dY^1)^2 + \cdots + (dY^{d-1})^2 \equiv \eta^{2,d-1}_{AB} dY^A dY^B \,, \\ -\ell^2 &= -(Y^{-1})^2 - (Y^0)^2 + (Y^1)^2 + \cdots + (Y^{d-1})^2 = \eta^{2,d-1}_{AB} Y^A Y^B \,. \end{aligned}$$

- Eliminating Y^{-1} from the constraint we get:

$$g_{ab} = \eta^{1,d-1}_{ab} - \frac{Y_a Y_b}{\ell^2 + Y^a Y_a}$$

$$Y_a = \eta^{1,d-1}_{ab} Y^b$$

- Yields:

$$G_{ab} + \Lambda g_{ab} = 0$$

$$R_{abcd} = -\frac{1}{\ell^2}(g_{ac}g_{bd} - g_{ad}g_{bc})$$

$$\Lambda = -\frac{(d-1)(d-2)}{2\ell^2}$$

What is AdS?

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- Both invariant under:

$$\tilde{x}^A = \Lambda^A{}_B x^B \quad \text{where} \quad \eta^{2,d-1}_{AB} = \eta^{2,d-1}_{CD} \Lambda^C{}_A \Lambda^D{}_B \,.$$

$$\Lambda^A{}_B = \delta^A_B + \lambda^A{}_B \quad \Rightarrow \quad \lambda_{AB} = \eta^{2,d-1}_{AC} \lambda^C{}_B = -\lambda_{BA} \ldots \binom{d+1}{2} \text{ generators}$$

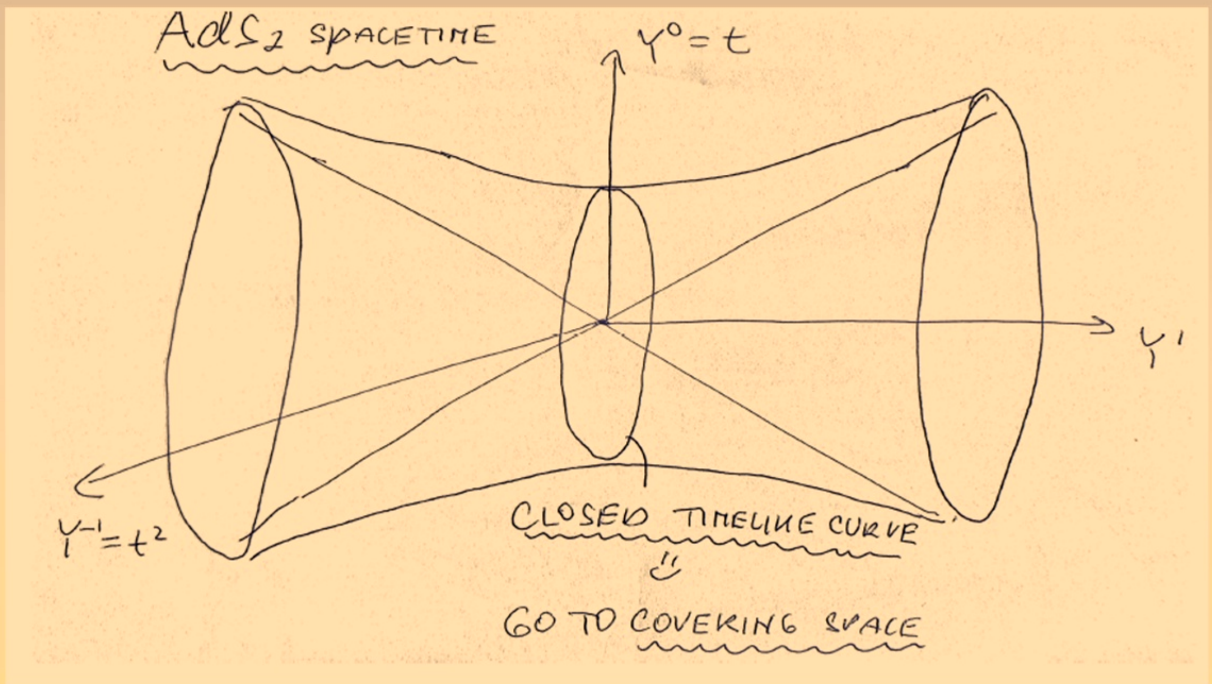
Maximal number of symmetries

- $\Lambda^A{}_B$ form a representation of O(d-1,2)

What is AdS?

Embedding perspective: "hyperboloid in $\mathbb{R}^{2,d-1}$"

$$
\begin{aligned}
ds^2 &= -(dY^{-1})^2 - (dY^0)^2 + (dY^1)^2 + \cdots + (dY^{d-1})^2 \equiv \eta^{2,d-1}_{AB} dY^A dY^B , \\
-\ell^2 &= -(Y^{-1})^2 - (Y^0)^2 + (Y^1)^2 + \cdots + (Y^{d-1})^2 = \eta^{2,d-1}_{AB} Y^A Y^B .
\end{aligned}
$$



AdS global coordinates

- Single out Y^{-1} and Y^{0} and "move" the constraint

$$Y^{-1} = \ell \cosh \tilde{\rho} \cos \tilde{t}\,, \quad Y^{0} = \ell \cosh \tilde{\rho} \sin \tilde{t}\,, \quad Y^{i} = \ell \sinh \tilde{\rho} \Omega_i$$

$$\Omega_i^2 \; = \; 1$$ (angular coordinates on the sphere)

$$ds^2 = \ell^2 \Big(- \cosh^2 \tilde{\rho} d\tilde{t}^2 + d\tilde{\rho}^2 + \sinh^2 \tilde{\rho} d\Omega_{d-2}^2 \Big)$$

- Let's go to the **universal covering space**:

$$\tilde{t} \sim \tilde{t} + 2\pi \quad \Longrightarrow \quad \tilde{t} \in (-\infty, \infty)$$

… no longer have closed timelike curves (CTCs).

AdS global coordinates

- Can further compactify "radial coordinate": $\tan\theta = \sinh\tilde{\rho}$,

$$ds^2 = \frac{\ell^2}{\cos^2\theta}\left(-d\tilde{t}^2 + d\theta^2 + \sin^2\theta d\Omega_{d-2}^2\right)$$

$$\theta \in [0, \pi/2)$$

… we have
(non Euclidean) **cylinder**

- **Conformal boundary** at $\theta = \pi/2$

$$ds^2|_{\partial\Omega} = -d\tilde{t}^2 + d\Omega_{d-2}^2$$

… **Einstein static Universe**: $\mathbb{R} \times S^{d-2}$

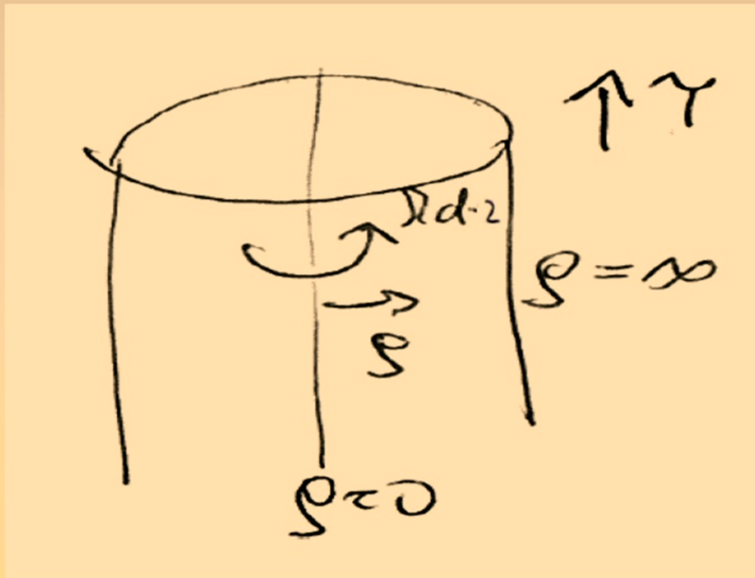
This is where the field theory lives!

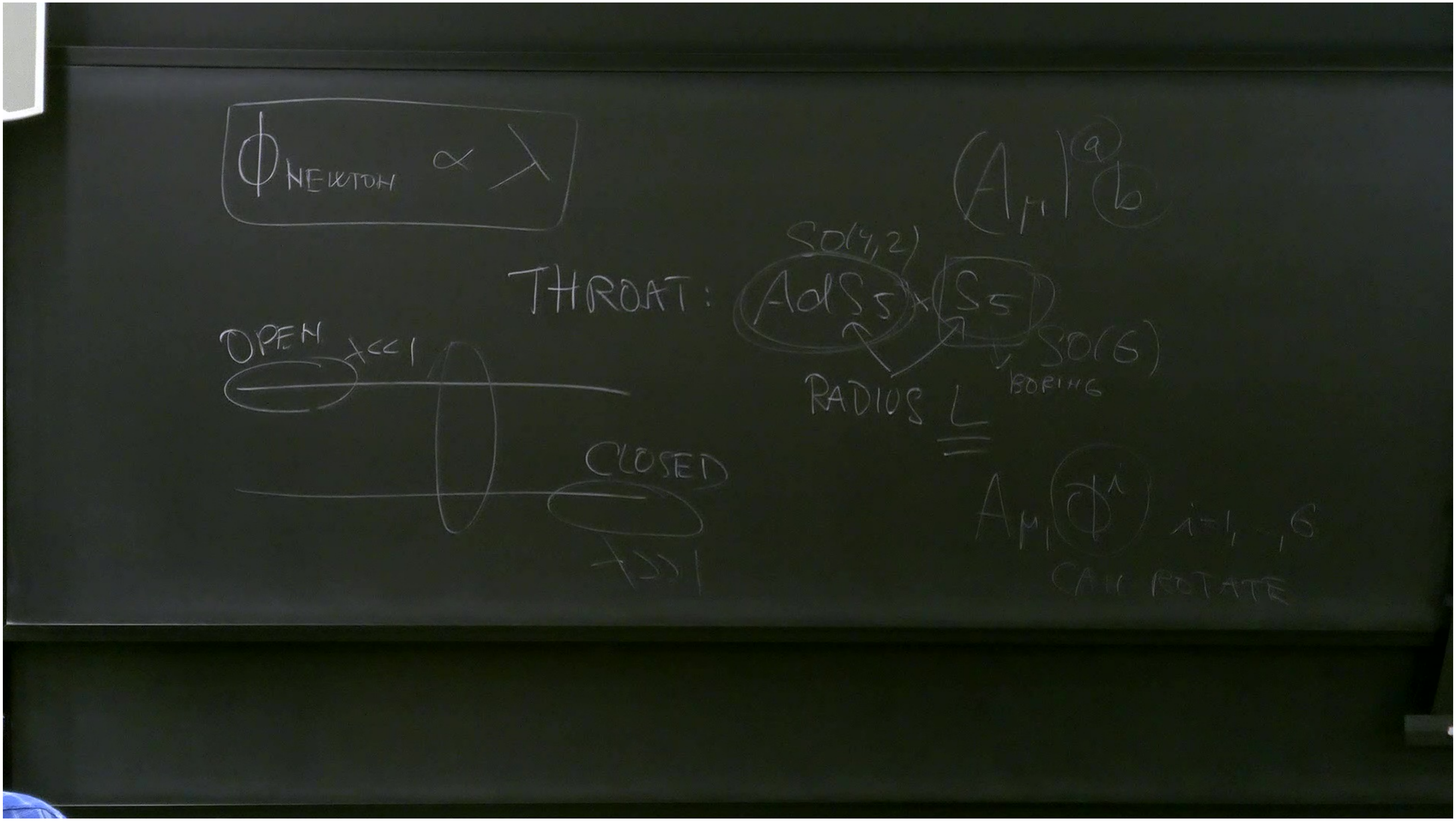
AdS global coordinates

- Instead of compactifying: $\rho = \ell \sinh \tilde{\rho}\,, \quad \tau = \ell \tilde{t}\,.$

$$ds^2 = -f d\tau^2 + \frac{d\rho^2}{f} + \rho^2 d\Omega^2_{d-2}\,, \quad f = 1 + \frac{\rho^2}{\ell^2}\,.$$

… "**usual global** (BH) coordinates"




ϕ NEWTON ∝ λ
THROAT: AdS5 × S5
SO(4,2)
SO(6)
BORING
RADIUS L
OPEN
λ<<1
CLOSED
λ>>1
CAN ROTATE

NOTE ALSO: "STANDARD QG CRITERION"

QG EFFECTS CAN BE NEGLECTED IF

$$\frac{L}{\ell_p} \gg 1$$

$$\frac{L^4}{\ell_p^4} \sim \frac{\ell_s^4 \lambda}{G_{10}^{1/2}} \sim \frac{g_s N_c \ell_s^4}{g_s \ell_s^4} \sim N_c \gg 1$$

NOTE ALSO "STANDARD QG CRITERION"

QG EFFECTS CAN BE NEGLECTED IF

$$\frac{L}{l_p} \gg 1$$

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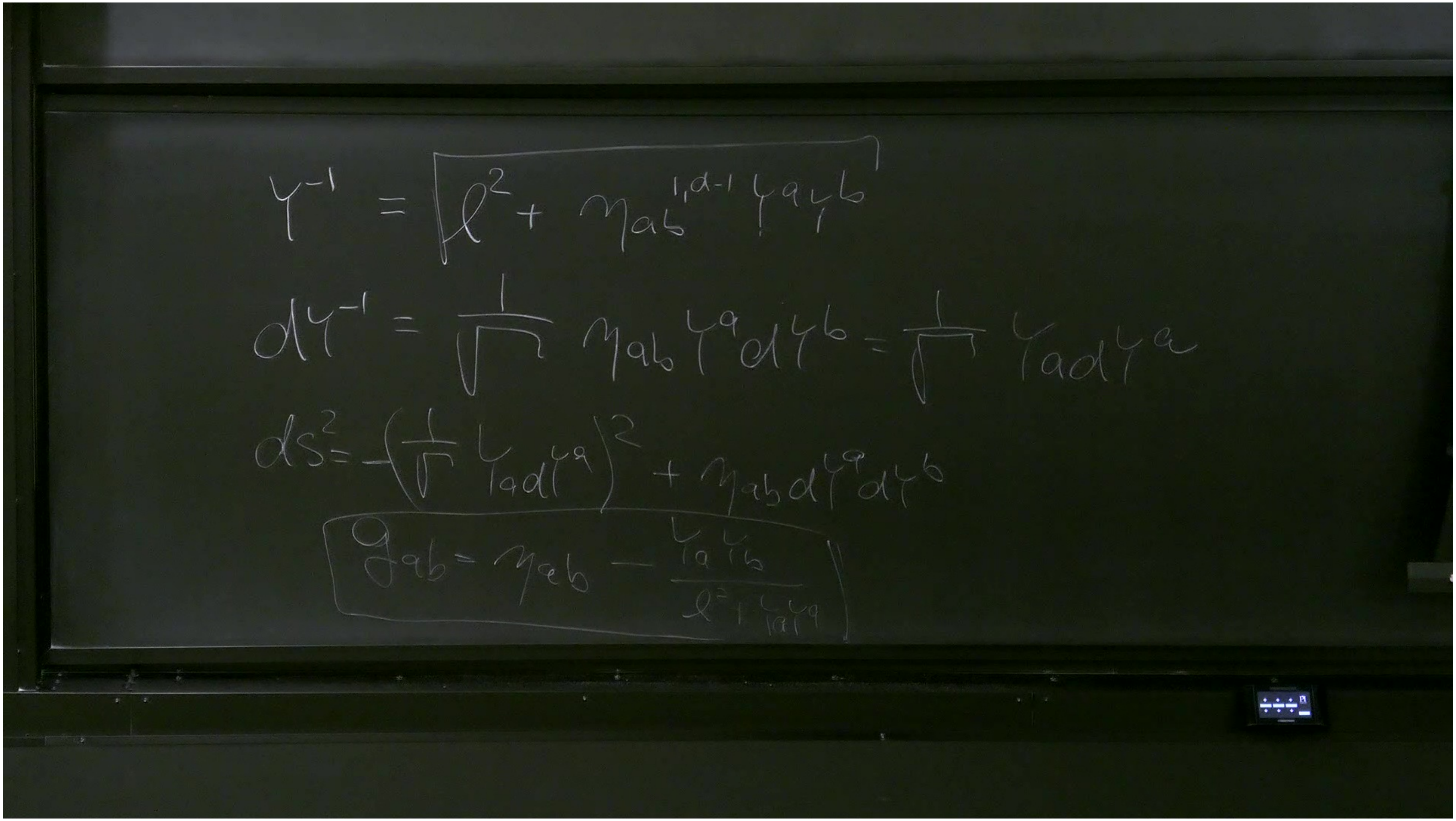
$Y^{-1} = \sqrt{\ell^2 + \eta_{ab}^{1,d-1} Y^a Y^b}$
$dY^{-1} = \frac{1}{\sqrt{\;}} \eta_{ab} Y^a dY^b = \frac{1}{\sqrt{\;}} Y_a dY^a$
$ds^2 = -\left(\frac{1}{\sqrt{\;}} Y_a dY^a\right)^2 + \eta_{ab} dY^a dY^b$
$g_{ab} = \eta_{ab} - \frac{Y_a Y_b}{\ell^2 + Y_a Y^a}$