Title: Lecture - AdS/CFT, PHYS 777

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Collection/Series: AdS/CFT (Elective), PHYS 777, March 31 - May 2, 2025

Subject: Quantum Fields and Strings, Quantum Gravity

Date: April 03, 2025 - 9:00 AM

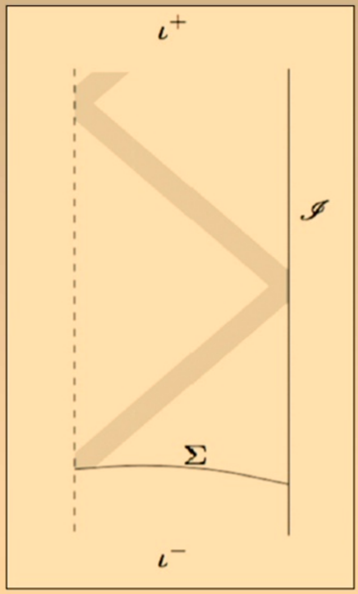
URL: https://pirsa.org/25040026

I) Black Holes in Anti de Sitter (AdS)

Global AdS4: a few basic facts

- **Anti de Sitter (AdS) space** = maximally symmetric solution of EE with negative Lambda:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0 \qquad \Lambda = -\frac{3}{\ell^2}$$



Schwarzschild-AdS black hole

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega_k^2 \qquad f = k - \frac{2m}{r} + \frac{r^2}{\ell^2}$$

- It is an **Einstein space** (vacuum with Lambda solution)

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0 \quad \Lambda = -\frac{3}{\ell^2}$$

- Choices of $k = 0, \pm 1$ correspond to various **horizon topologies** (with $d\Omega_k^2$ the corresponding metric).
- One can add, charge, or rotation – having charge-AdS, or Kerr-AdS black holes in 4 and higher dimensions.
- These have rather interesting properties.

Gravitational action in AdS

$$S_E = \frac{1}{16\pi}\int_M d^4x\sqrt{-g}\left[R + \frac{6}{\ell^2}\right] + \frac{1}{8\pi}\int_{\partial M} d^3x\sqrt{-h}\mathcal{K}$$
$$- \frac{1}{8\pi}\int_{\partial M} d^3x\sqrt{-h}\left[\frac{2}{\ell} + \frac{\ell}{2}\mathcal{R}(h)\right] ,$$

- The 2nd line are the <u>covariant</u> **AdS counterterms** (c.f. `vague' background subtraction in AF case)
- Variation yields:

$$\delta S_E = -\frac{1}{2}\int_{\partial\Omega} d^3x\sqrt{-h}\tau_{ab}\delta h^{ab}$$ + bulk EOM

here

$$8\pi\tau_{ab} = \mathcal{K}h_{ab} - \mathcal{K}_{ab} + \ell G_{ab}(h) - \frac{2}{\ell}h_{ab}$$

is up to trivial (infinite) scaling the **holographic stress tensor**

a) Hawking-Page transition

S.W. Hawking & D.N. Page, *Thermodynamics of black holes in anti-de-Sitter space*, Commun. Math. Phys. 87, 577 (1983).

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2 , \quad f = 1 - \frac{2M}{r} + \frac{r^2}{\ell^2} .$$

$$\begin{aligned} T &= \frac{1}{\beta} = \frac{\ell^2 + 3r_+^2}{4\pi\ell^2 r_+} , \\ G &= -\frac{1}{\beta}\log Z \approx \frac{S_E}{\beta} = \frac{r_+(\ell^2 - r_+^2)}{4\ell^2} \end{aligned}$$

..can plot this parametrically (**Homework 1**) to get:

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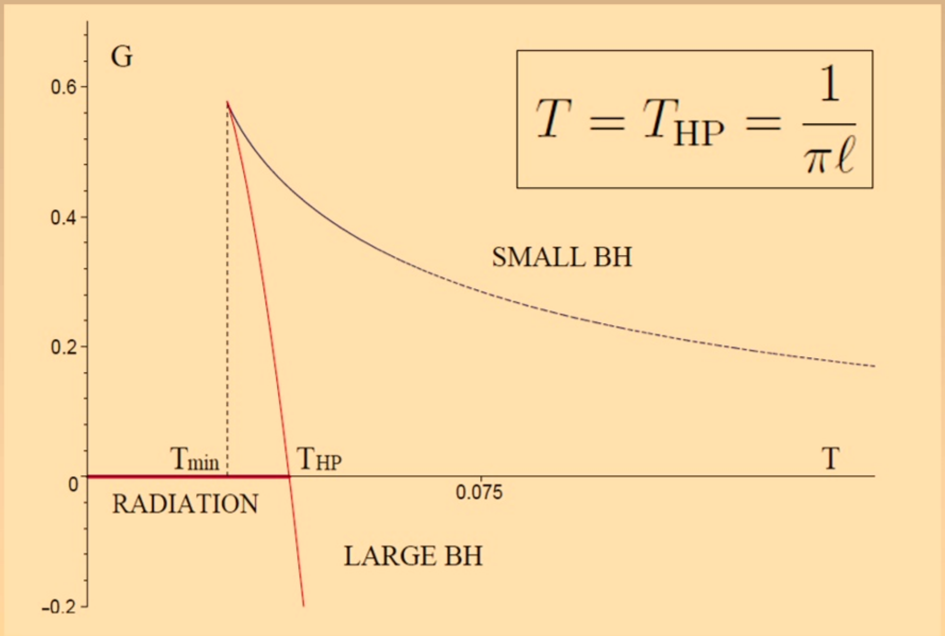


1st-order radiation/large black hole phase transition

(dual to **confinement/deconfinement** PT of QGP)

b) Charged AdS BHs: VdW criticality

- A. Chamblin, R. Emparan, C.V. Johnson and R.C. Myers, *Charged AdS black holes and catastrophic holography*, PRD60 (1999) 064018.

Van der Waals fluid

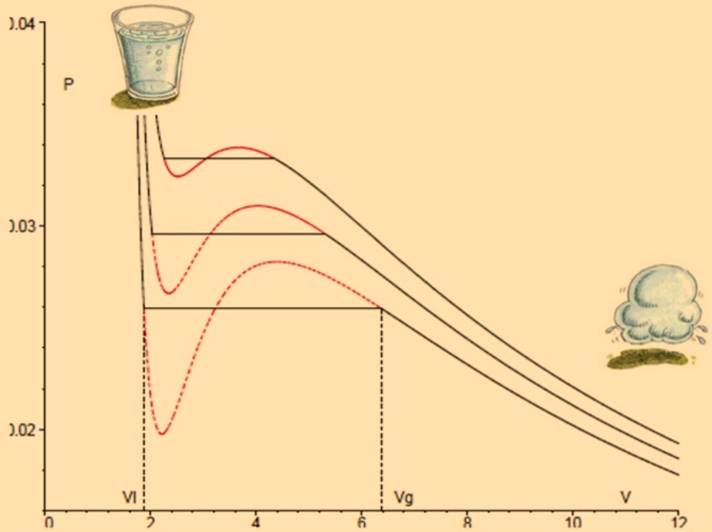


FIG. 2. Maxwell's equal area law. The 'oscillating' (dashed) part of the isotherm $T < T_c$ is replaced by an isobar, such that the areas above and below the isobar are equal one another.

$$\left(P + \frac{a}{v^2}\right)(v - b) = T$$

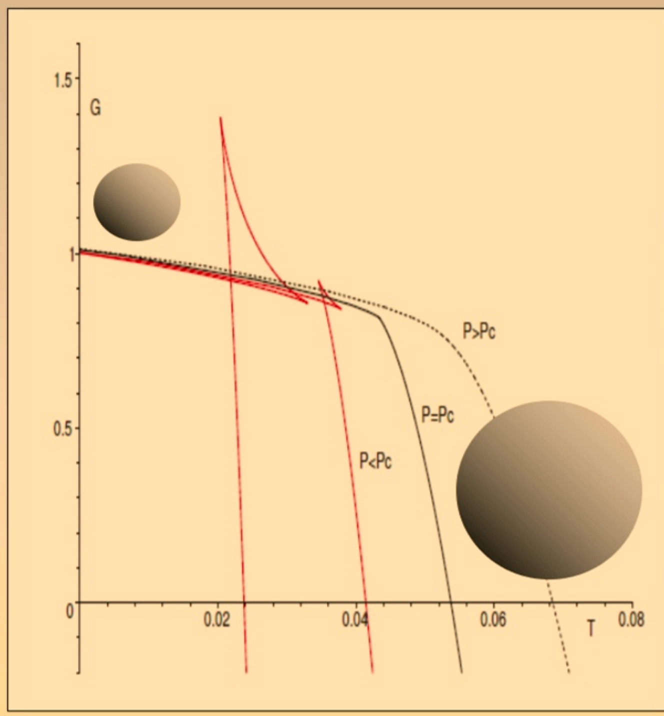
Parameter a measures the **attraction** between particles (a>0) and b corresponds to "**volume of fluid particles**".

Critical point:

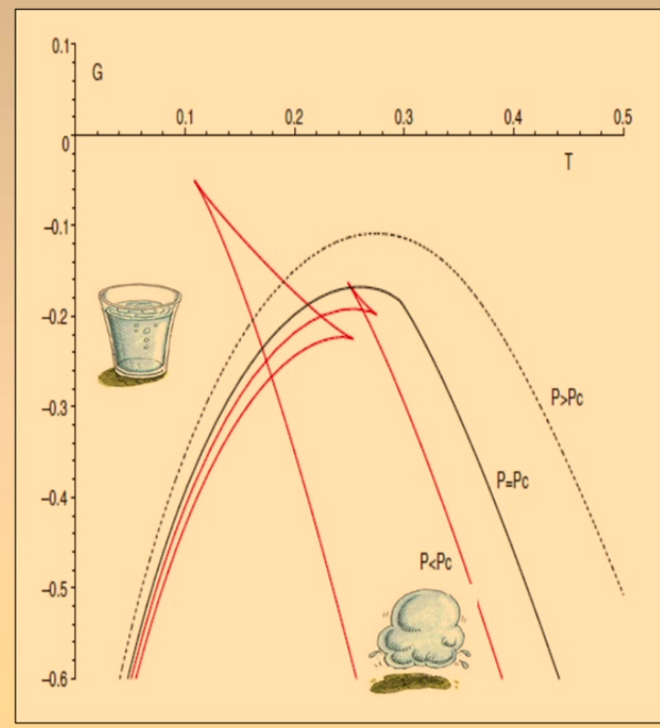
$$\rho_c = \frac{P_c v_c}{T_c} = \frac{3}{8}$$

Free energy: demonstrates standard swallow tail behavior

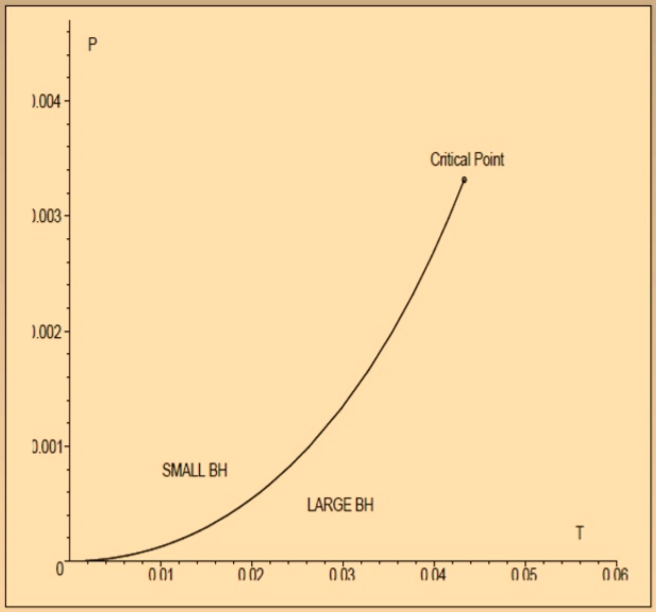
$$F = F(T,P,Q) = \frac{1}{4}\left(r_+ - \frac{8\pi}{3}Pr_+^3 + \frac{3Q^2}{r_+}\right)$$



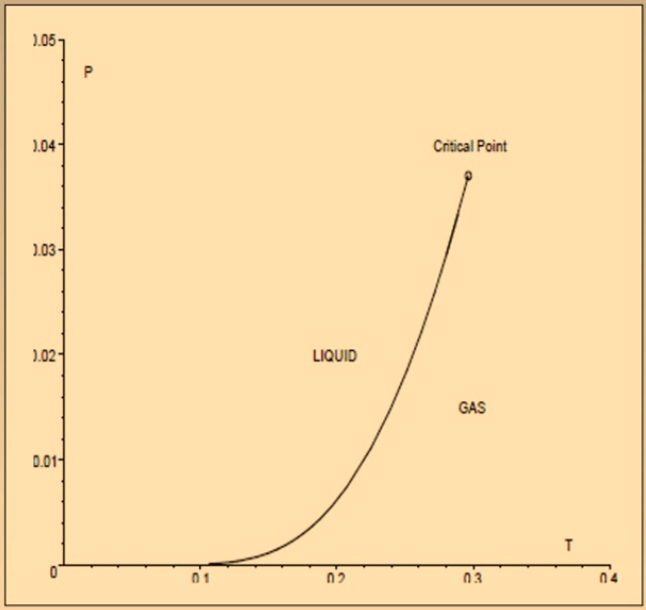
vs.



Phase diagrams: complete analogy



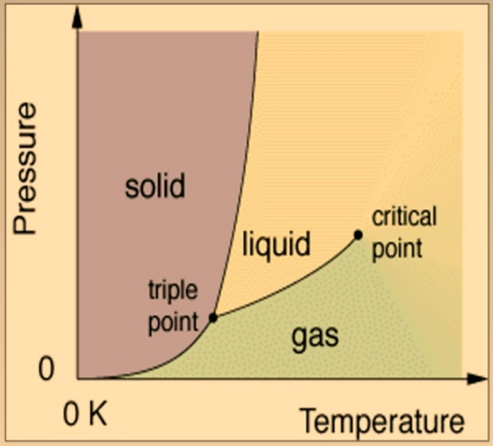
vs.

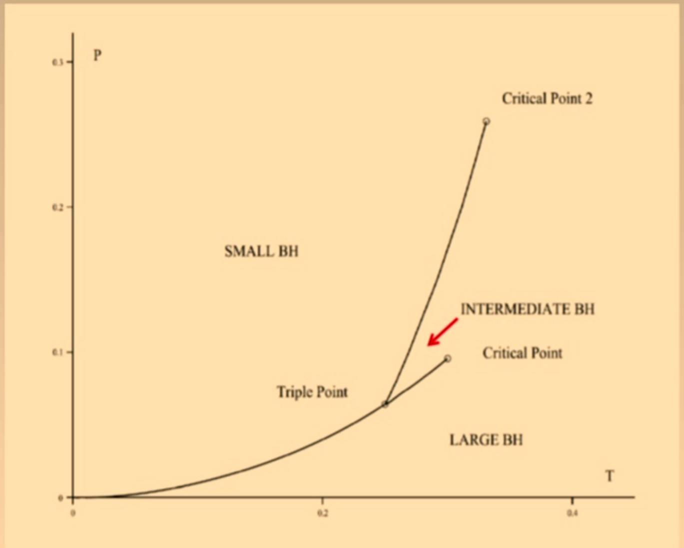


- Coexistence & critical point described by **Clausius-Clapeyron** and **Ehrenfest** equations
- **MFT critical exponents** $$\alpha = 0\,, \quad \beta = \frac{1}{2}\,, \quad \gamma = 1\,, \quad \delta = 3$$

More generally: black hole chemistry

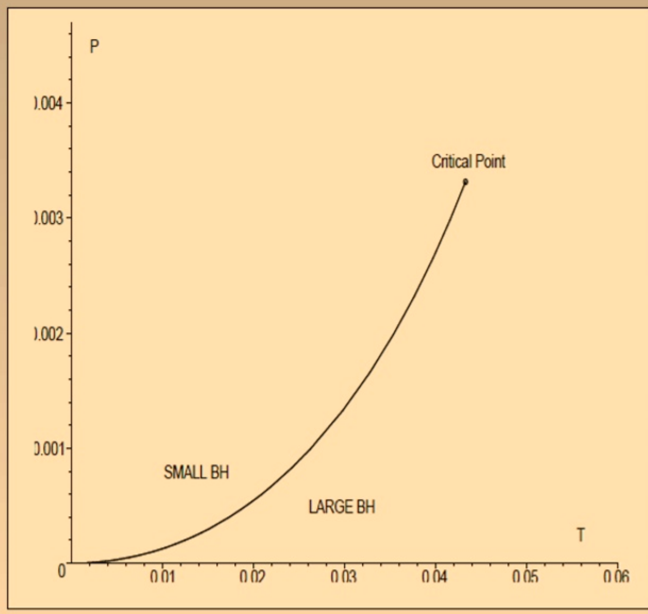
- **Triple point** and solid/liquid/gas analogue**:**



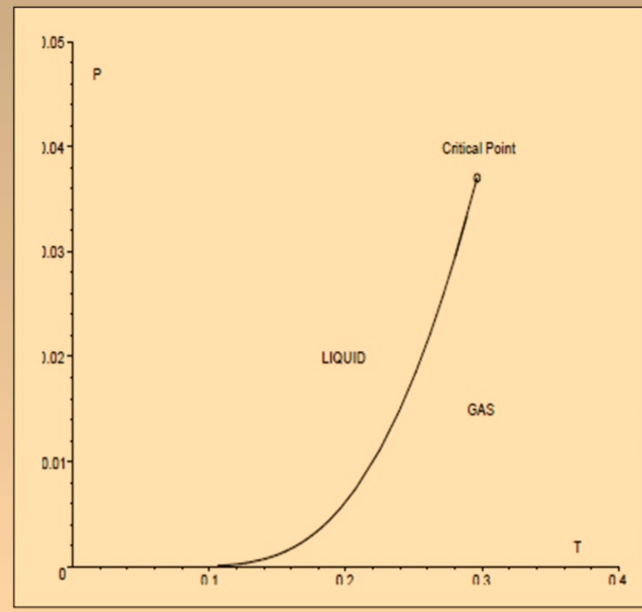


(can have n-tuple points)

Phase diagrams: complete analogy



vs.



- Coexistence & critical point described by **Clausius-Clapeyron** and **Ehrenfest** equations
- **MFT critical exponents**

$$\alpha = 0\,, \quad \beta = \frac{1}{2}\,, \quad \gamma = 1\,, \quad \delta = 3$$

Summary

1) **AdS black holes** are an interesting generalization of their asymptotically flat cousins.

2) **AdS action** covariantly constructed by **holographic renormalization**. It yields **holographic stress tensor,** from where CFT properties can be calculated.

3) AdS black holes feature very interesting **phase transitions: Hawking-Page,** Van der Waals, …

4) All these **phase transitions** have a **dual description** via the AdS/CFT duality. Can they be **observed** on the CFT side?

2) MOTIVATING AdS/CFT FROM ST

STRING THEORY = QUANTUM THEORY OF INTERACTING RELATIVISTIC STRINGS (OR HIGHER DIMEN. OBJECTS)

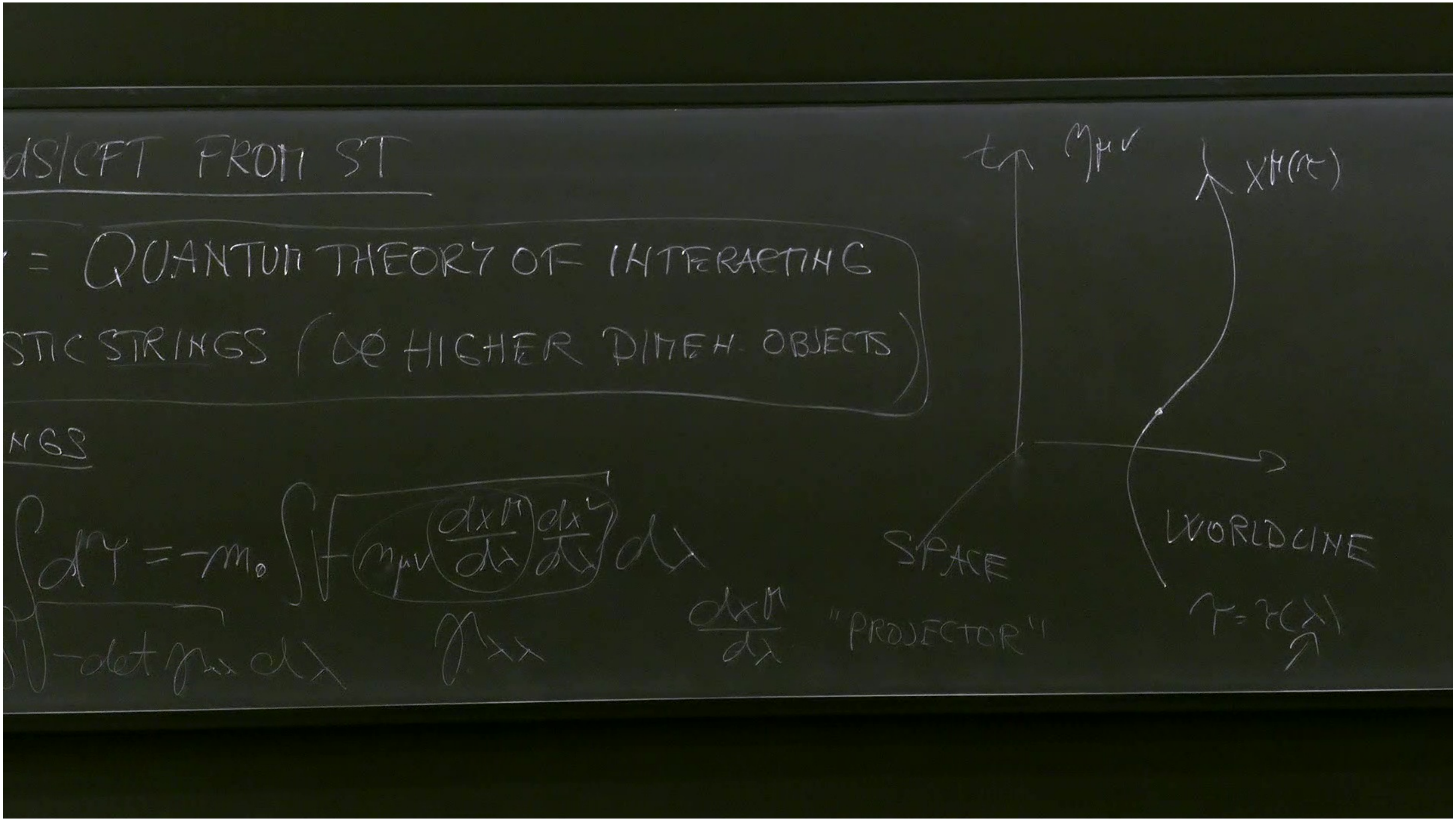
- CLASSICAL STRINGS

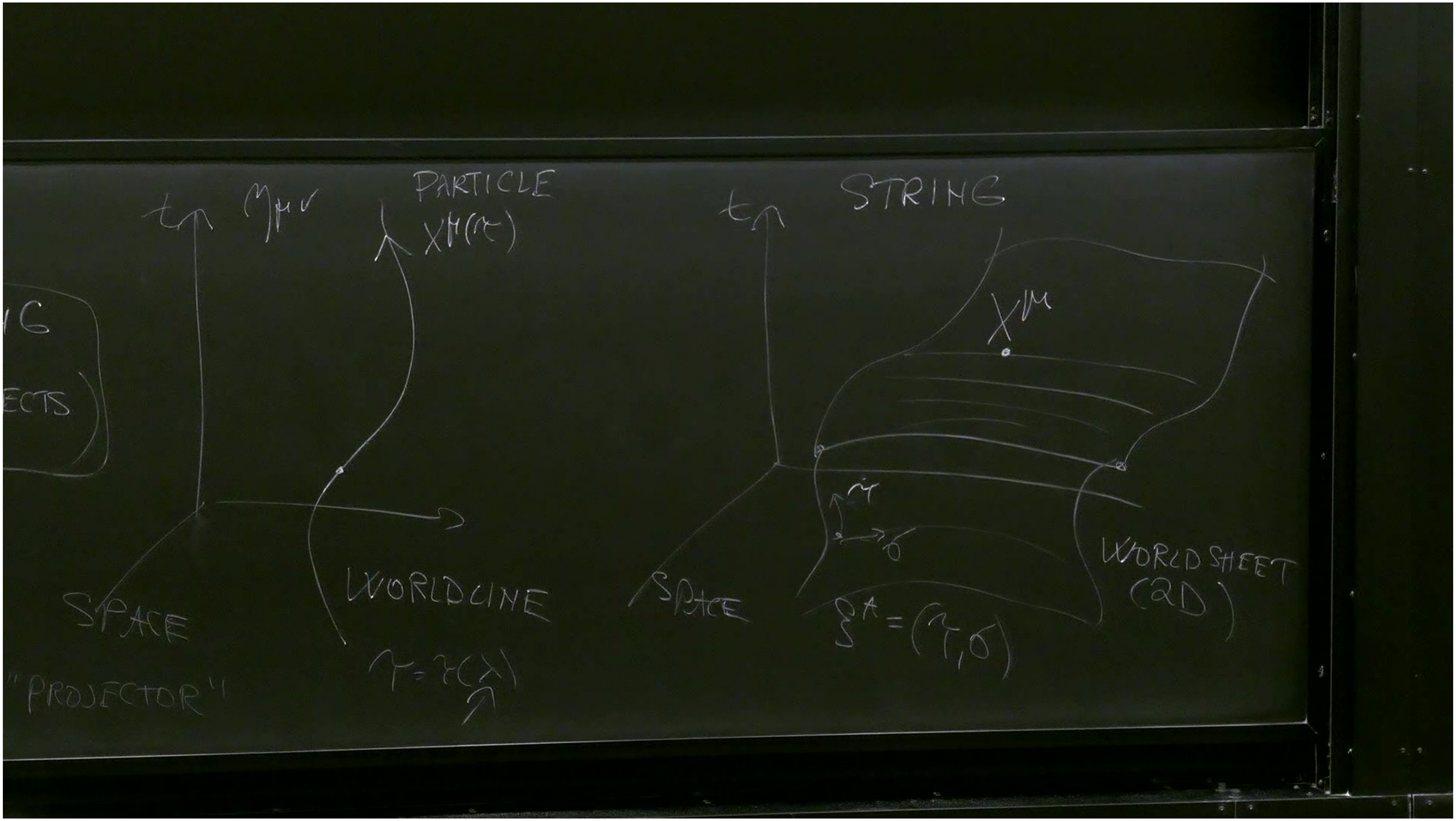
$$S_{pp}[X^\mu] = -m_0 \int d\tau = -m_0 \int \sqrt{-\eta_{\mu\nu} \frac{dX^\mu}{d\lambda} \frac{dX^\nu}{d\lambda}}\, d\lambda$$

$$\eta_{\mu\nu} \frac{dX^\mu}{d\lambda} \frac{dX^\nu}{d\lambda} = \gamma_{\lambda\lambda}$$

$$= -m_0 \int \sqrt{-\det \gamma_{\lambda\lambda}}\, d\lambda$$

SPACE


dS/CFT FROM ST
= QUANTUM THEORY OF INTERACTING
STIC STRINGS (HIGHER DIMEN. OBJECTS)
NGS
SPACE
WORLDLINE
"PROJECTOR"



PARTICLE
$\eta_{\mu\nu}$
t
$X^\mu(\tau)$
SPACE
WORLDLINE
$\tau = \tau(\lambda)$
"PROJECTOR"
ECTS
STRING
t
X^μ
SPACE
WORLDSHEET (2D)
$\xi^A = (\tau, \sigma)$

$$= -m_0 \int \sqrt{-\det \gamma_{\lambda\lambda}}\, d\lambda$$

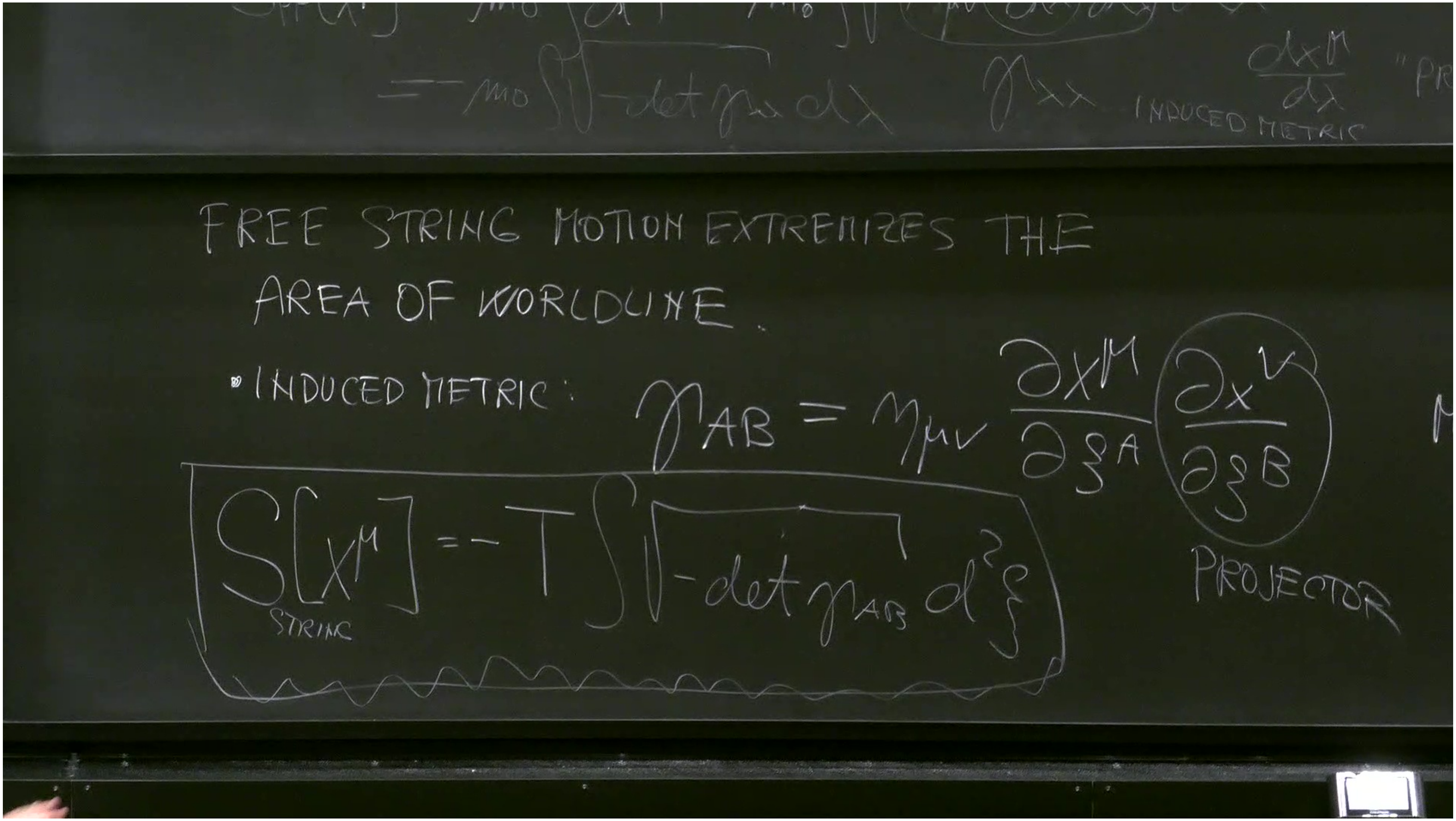
$\gamma_{\lambda\lambda}$ INDUCED METRIC $\qquad \frac{dx^\mu}{d\lambda}$ "PROJ

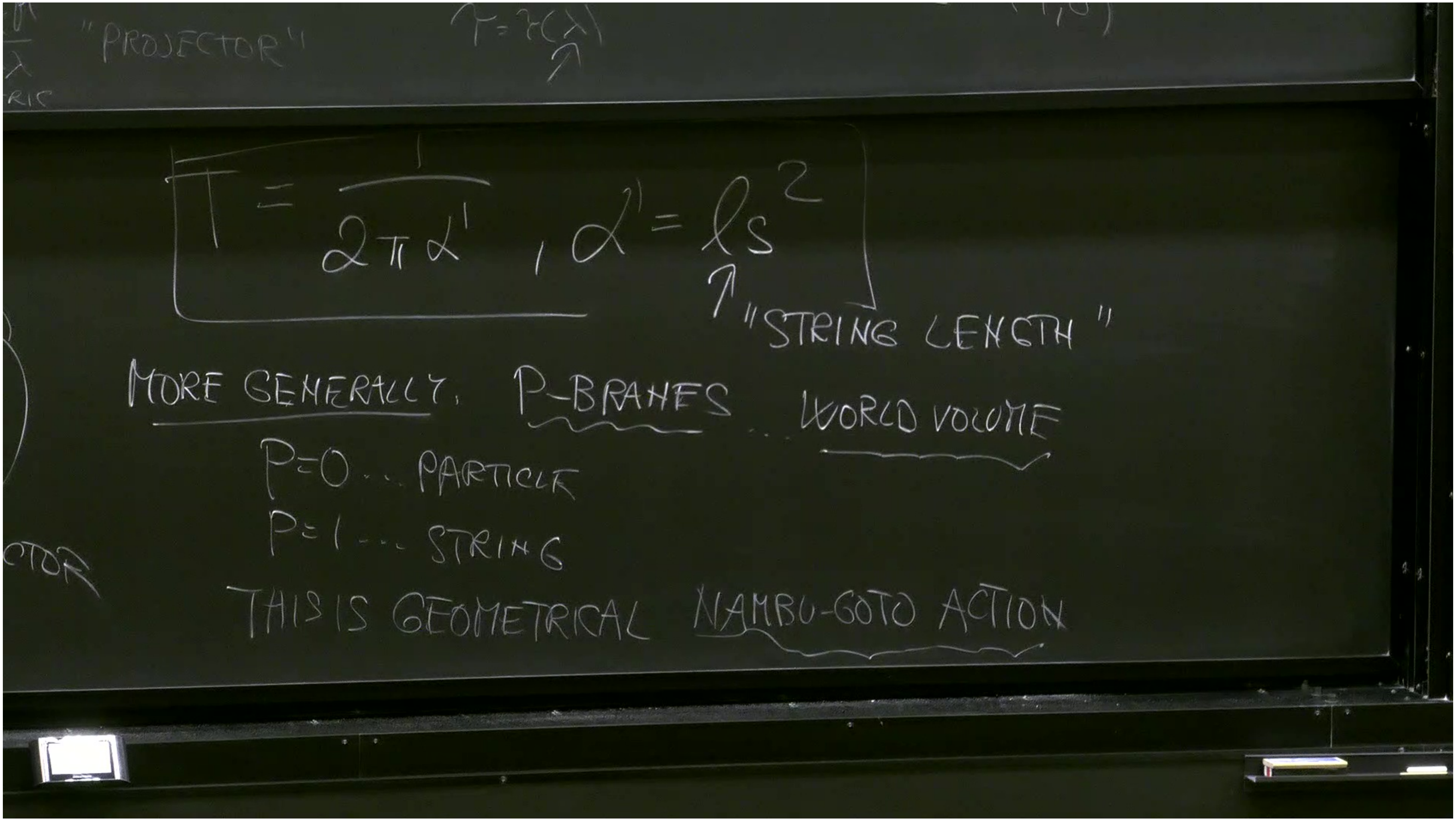
FREE STRING MOTION EXTREMIZES THE AREA OF WORLDLINE.

- INDUCED METRIC:

$$\gamma_{AB} = \eta_{\mu\nu} \frac{\partial x^\mu}{\partial \xi^A} \frac{\partial x^\nu}{\partial \xi^B}$$

PROJECTOR


INDUCED METRIC
"PR
FREE STRING MOTION EXTREMIZES THE
AREA OF WORLDLINE.
• INDUCED METRIC:
$\gamma_{AB} = \eta_{\mu\nu} \frac{\partial X^\mu}{\partial \xi^A} \frac{\partial x^\nu}{\partial \xi^B}$
PROJECTOR
$S_{STRING}[X^\mu] = -T \int \sqrt{-\det \gamma_{AB}}\, d^2\xi$


"PROJECTOR"
$\tau = \tau(\lambda)$
$T = \frac{1}{2\pi\alpha'}$, $\alpha' = l_s^2$
"STRING LENGTH"
MORE GENERALLY: P-BRANES ... WORLD VOLUME
P=0 ... PARTICLE
P=1 ... STRING
THIS IS GEOMETRICAL NAMBU-GOTO ACTION

• ALTERNATIVELY WE CAN CONSIDER POLYAKOV ACTION.

$$S_{\substack{POL\\PARTICLE}}[x^\mu, h] = \frac{1}{2}\int \left(\frac{1}{h}\eta_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} - m_0^2 h\right) d\lambda$$

h: LAGRANGE MULTIPLIER (NO DERIVATIVES OF h)

$$\delta h: \left(-\frac{1}{h^2}\gamma_{\lambda\lambda} - m_0^2\right)\delta h = 0$$

$$\Rightarrow h = \frac{1}{m_0}\sqrt{-\gamma_{\lambda\lambda}}$$

PLUG BACK $\Rightarrow$ RECOVER S_{PP}

- CLASSICALLY THE TWO ARE EQUIVALENT
- NICE & QUADRATIC
- CAN NOW TAKE $m_0 \to \emptyset$ & CAN ABSORB h INTO λ

$$S_{MASSLESS} = \frac{1}{2}\int \eta_{\mu\nu} \frac{dx^{\mu}}{d\lambda}\frac{dx^{\nu}}{d\lambda}\, d\lambda$$

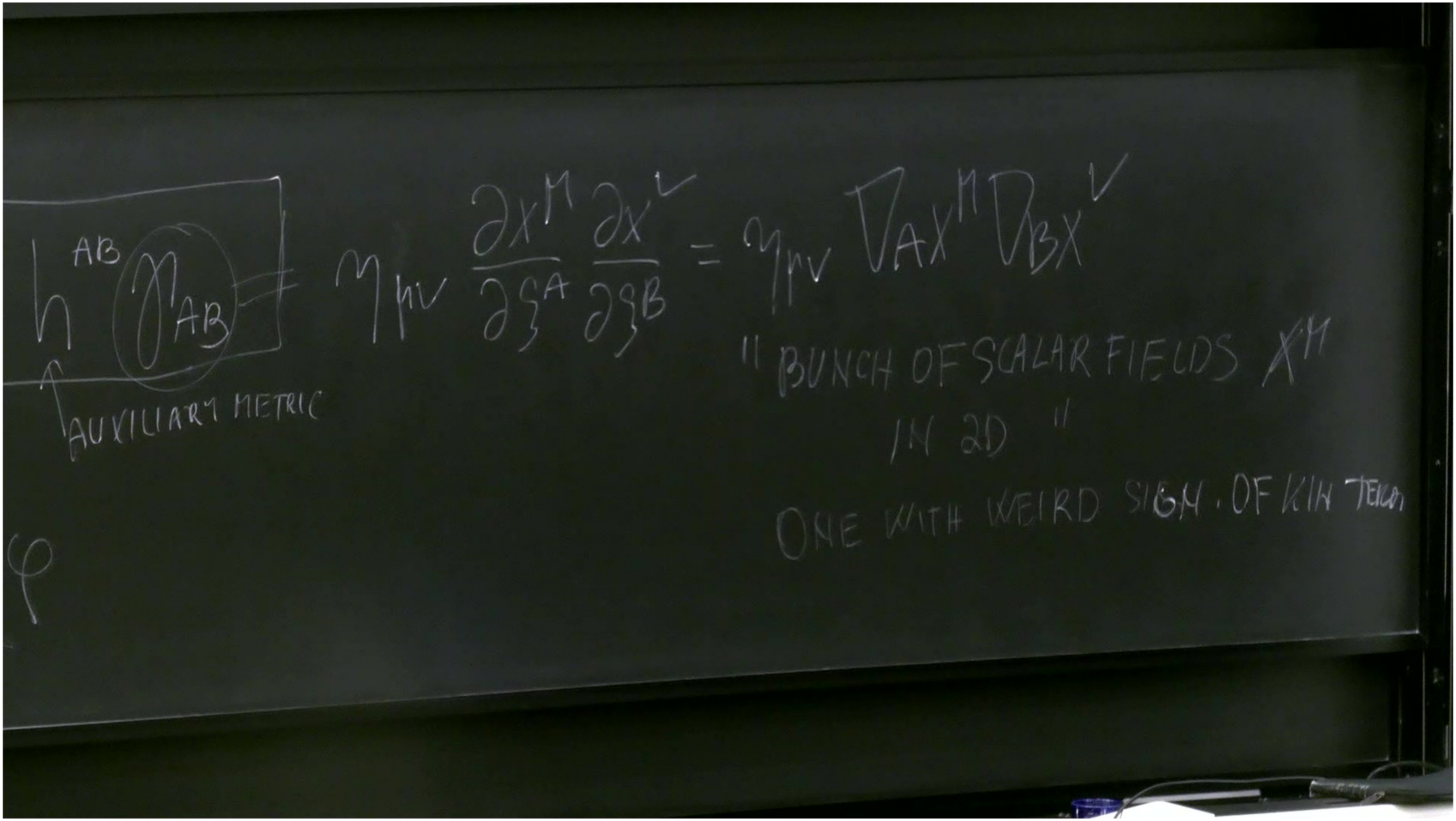
POLYAKOV FOR MASSLESS STRINGS.

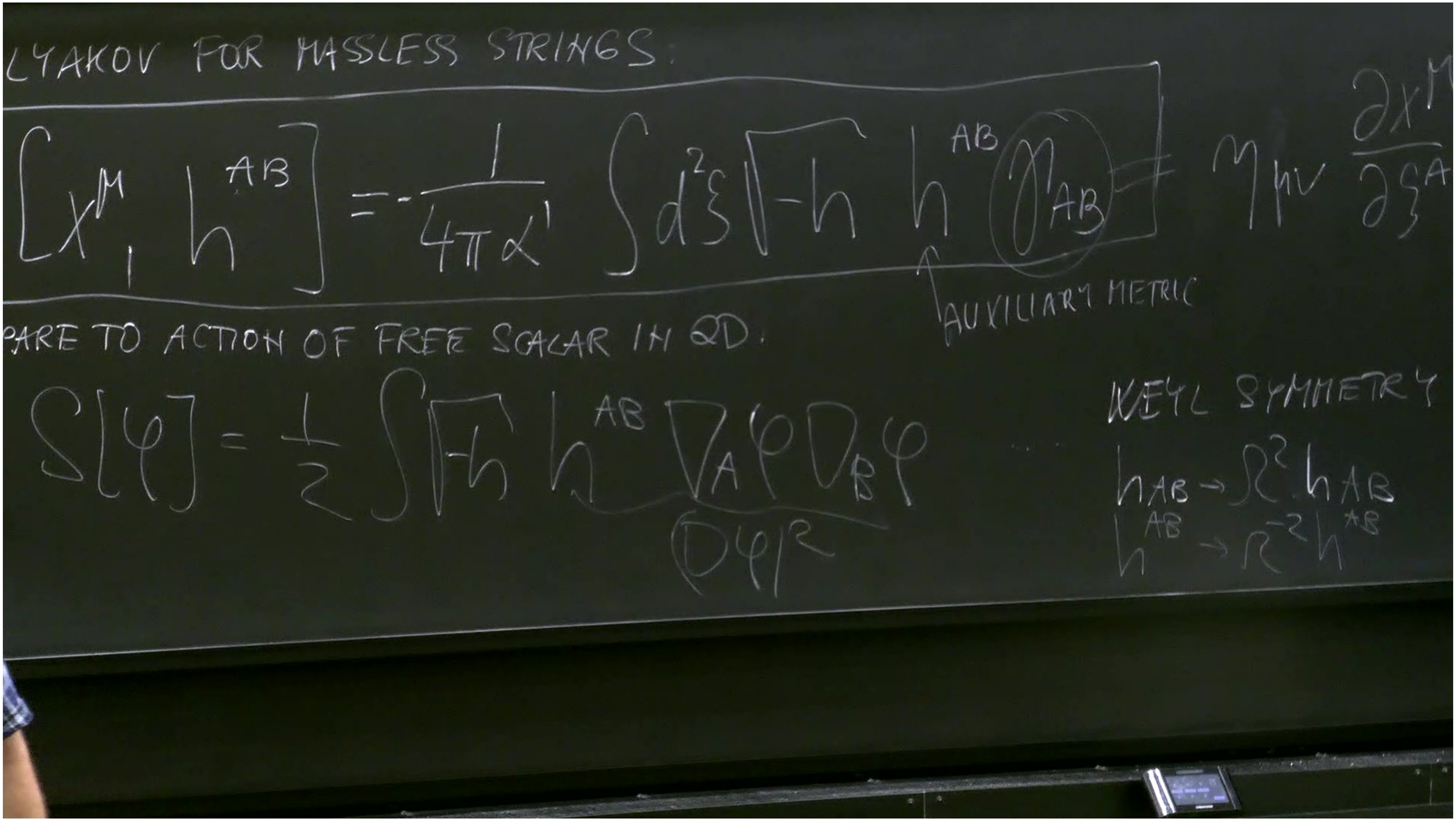
$$S[X^\mu, h^{AB}] = -\frac{1}{4\pi\alpha'}\int d^2\xi \sqrt{-h}\, h^{AB}\, \gamma_{AB}$$

↑ AUXILIARY METRIC

COMPARE TO ACTION OF FREE SCALAR IN 2D.

$$S[\varphi] = \frac{1}{2}\int \sqrt{-h}\, \underbrace{h^{AB}\nabla_A\varphi\nabla_B\varphi}_{(\nabla\varphi)^2}$$


$h^{AB} \gamma_{AB} \neq \eta_{\mu\nu} \frac{\partial x^\mu}{\partial \xi^A} \frac{\partial x^\nu}{\partial \xi^B} = \eta_{\mu\nu} \nabla_A X^\mu \nabla_B X^\nu$
AUXILIARY METRIC
"BUNCH OF SCALAR FIELDS X^μ
IN 2D"
ONE WITH WEIRD SIGN OF KIN TERM
φ


LYAKOV FOR MASSLESS STRINGS:
AUXILIARY METRIC
PARE TO ACTION OF FREE SCALAR IN 2D:
WEYL SYMMETRY

$$x^\mu \to x^\mu + \underline{\xi^\mu}$$

$$\eta_{\mu\nu} \to \Omega^2 \eta_{\mu\nu}$$