Title: Lecture - Cosmology, PHYS 621

Speakers: Neal Dalal

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Subject: Cosmology

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$$\Theta(\vec{k},\hat{p}) = -\Psi(k) + e^{i\vec{k}\cdot\hat{p}(\eta_r-\eta_0)}\left[\Theta_0(\vec{k},\eta_r) + \Psi(\vec{k},\eta_r) + 3\vec{\Theta}_1(\vec{k},\eta_r)\cdot\hat{p}\right] + \int_0^{\eta_r} d\eta\, e^{i\vec{k}\cdot\hat{p}(\eta-\eta_0)}\left(\dot{\Psi} - \dot{\Phi}\right)e^{-\tau}$$

$$\Theta(\vec{x},\hat{p}) = -\Psi(\vec{x}) + \left(\Theta_0(\vec{x}_s) + \Psi(\vec{x}_s)\right) + 3\vec{\Theta}_1(\vec{x}_s)\cdot\hat{p} + \int_0^{\eta_r} d\eta\left[\left(\dot{\Psi}(x_\ell) - \dot{\Phi}(x_\ell)\right)e^{-\tau}\right]$$

$$\Theta(\vec{k},\hat{p}) = -\Psi(k) + e^{i\vec{k}\cdot\hat{p}(\eta_r-\eta_0)}\left[\Theta_0(\vec{k},\eta_r) + \Psi(\vec{k},\eta_r) + 3\vec{\Theta}_1(\vec{k},\eta_r)\cdot\hat{p}\right] + \int_0^{\eta_r} d\eta\, e^{i\vec{k}\cdot\hat{p}(\eta-\eta_0)}\left(\dot{\Psi} - \dot{\Phi}\right)e^{-\tau}$$

in k space

$$\Theta(\vec{x},\hat{p}) = -\Psi(\vec{x}) + \left(\Theta_0(\vec{x}_s) + \Psi(\vec{x}_s)\right) + 3\vec{\Theta}_1(\vec{x}_s)\cdot\hat{p} + \int_0^{\eta_r} d\eta\left[\left(\dot{\Psi}(x_l) - \dot{\Phi}(x_l)\right)e^{-\tau}\right]$$

in real space

$$\Theta(\vec{x},\hat{p}) = \sum_{l,m} a_{lm}(\vec{x})\, Y_{lm}(\hat{p})$$

$$\langle a_{lm}$$

$$a_{lm}(x) = \int d^2\hat{p}\; Y^*_{lm}(\hat{p}) \int \frac{d^3k}{(2\pi)^3}\, e^{ik\cdot x}\, \Theta(\vec{k},\hat{p})$$

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} d^2\hat{p}\, d^2\hat{p}'\, e^{i(k-k')x} \langle \Theta(\vec{k},\hat{p})\, \Theta^*(\vec{k}',\hat{p}') \rangle$$

$$Y^*_{\ell m}(\hat{p})\, Y_{\ell' m'}(\hat{p}')$$

$$\Theta = \zeta_{init}(\vec{k}) \times T_\theta(\vec{k},\hat{p})$$

$$\langle \Theta(k,p)\, \Theta^*(k',p') \rangle = T(k,p)\, T^*(k',p') \langle \zeta_{init}(k)\, \zeta_{init}(k') \rangle$$

$$\langle \zeta_{init} \zeta^*_{init} \rangle = (2\pi)^3 \delta(k-k') P_\zeta$$

$$\langle \Theta(k,p)\, \Theta^*(k',p') \rangle = (2\pi)^3 \delta(k-k') P_\zeta(k)\, T(k,p)\, T^*(k',p')$$

$$\Theta(\vec{x}, \hat{p}) = \sum_{l,m} a_{lm}(\vec{x})\, Y_{lm}(\hat{p})$$

$$a_{lm}(x) = \int d^2\hat{p}\; Y^*_{lm}(\hat{p}) \int \frac{d^3k}{(2\pi)^3}\, e^{ik\cdot x}\, \Theta(\vec{k}, \hat{p})$$

$\langle a_{lm}\, a^*_{l'm'} \rangle$

$\Theta = \Sigma_{i}$

$\langle \Theta(k, p)\, \Theta$

$$\langle a_{\ell m}\, a^{*}_{\ell' m'} \rangle = \int \frac{d^3k}{(2\pi)^3}\, d^2\hat{p}\, d^2\hat{p}'\, P_{\zeta}(k)\, T(k,\hat{p})\, T^{*}(k,\hat{p}')\, Y^{*}_{\ell m}(\hat{p})\, Y_{\ell' m'}(\hat{p}')$$

$$\int d^2\hat{p}\, Y^{*}_{\ell m}(\hat{p})\, T(k,\hat{p})$$

$$\vec{\Theta}_1 = \frac{1}{a}\vec{\nabla} w_1 = \frac{i\vec{k}}{a} w_1$$

$$e^{i\vec{k}\cdot\hat{p}(\eta-\eta_0)} = 4\pi \sum_{l',m'} i^{l'} j_{l'}\left(k(\eta-\eta_0)\right) Y^*_{l'm'}(\hat{k})\, Y_{l'm'}(\hat{p})$$

$$\int d^2\hat{p}\; Y^*_{lm}(\hat{p})\, Y_{l'm'}(\hat{p}) = \delta_{ll'}\,\delta_{mm'}$$

$$\int d^2p\, Y^*_{\ell m}(\hat{p})\, T(\vec{k},\hat{p}) = 4\pi\, i^\ell\, Y^*_{\ell m}(\hat{k}) \Big[(\Theta_0 + \Psi)\, j_\ell(k(\eta_0 - \eta_r)) + \frac{3k}{(2\ell+1)a}\, \omega_1 \times \big[\ell\, j_{\ell-1}(k(\eta_0 - \eta_r))$$

$$
\Big[\left(\Theta_0 + \Psi\right) j_\ell\left(k(\eta_0 - \eta_r)\right)
$$

$$
+ \frac{3k}{(2\ell+1)a}\, \omega_1 \times \left[\ell\, j_{\ell-1}\left(k(\eta_0 - \eta_r)\right) - (\ell+1)\, j_{\ell+1}\left(k(\eta_0 - \eta_r)\right) \right]
$$

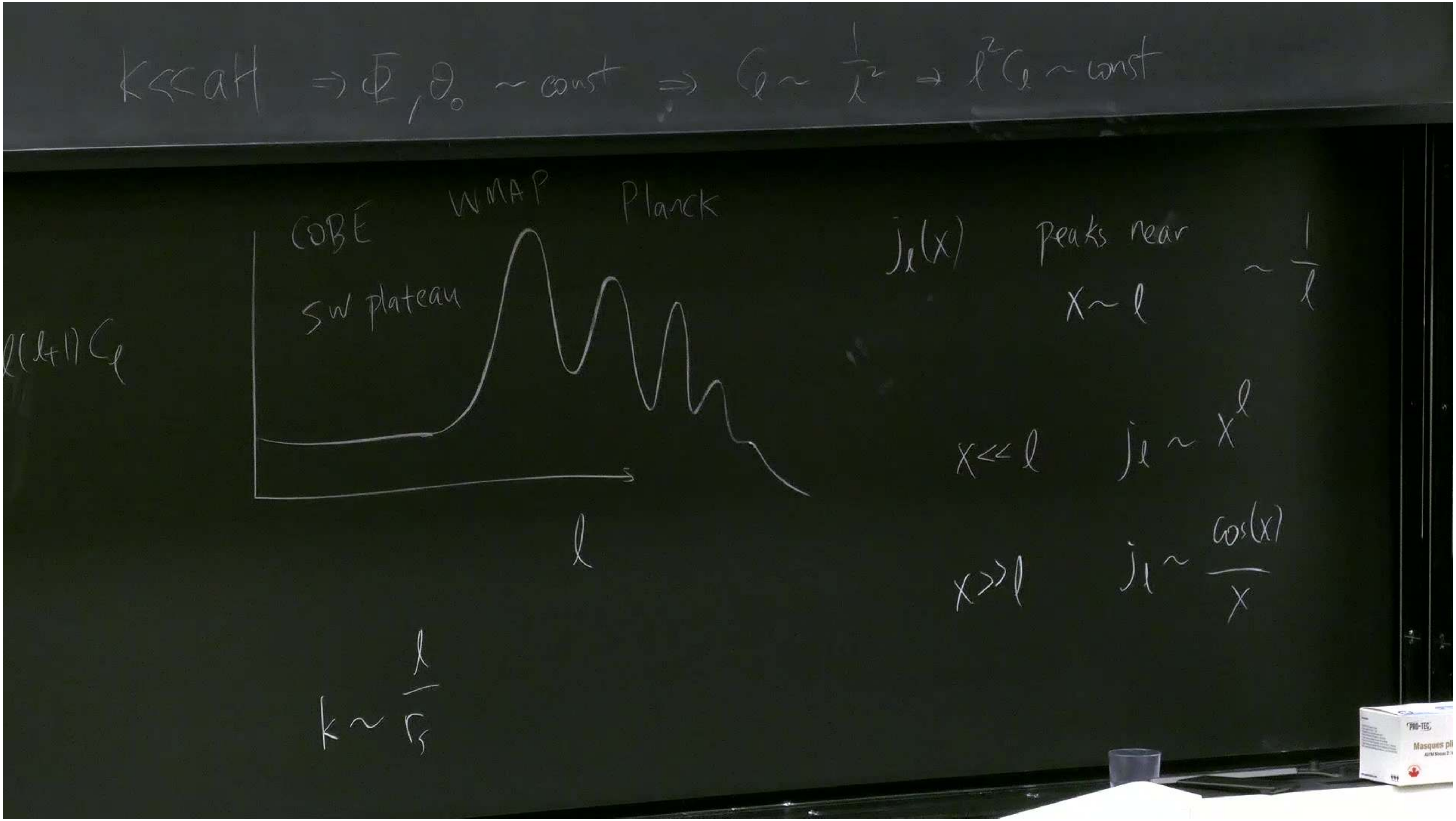
$$
+ \int d\eta \left(\dot{\Psi} - \dot{\Phi}\right) e^{-\tau}\, j_\ell\left(k(\eta - \eta_r)\right) \Big]
$$

$$\langle a_{\ell m}\, a^*_{\ell' m'} \rangle = \int \frac{d^3k}{(2\pi)^3} P_\zeta(k)\,(4\pi)^2\, Y^*_{\ell m}(\hat{k})\, Y_{\ell' m'}(\hat{k}) \left[\ \cdot\ \right]_{\ell m} \left[\ \ \right]_{\ell' m'}$$

$$\boxed{= \delta_{\ell\ell'}\delta_{mm'}\, \frac{2}{\pi} \int dk\, k^2\, P_\zeta(k) \left[\ \ \right]^2_{\ell m}}$$

Approximations

1. instant recombination
2. no reionization $(\tau = 0)$
3. no photon diffusion
4. no ν perts
5. no lensing
   SZ



$K \ll aH \Rightarrow \Phi, \theta_0 \sim \text{const} \Rightarrow C_\ell \sim \frac{1}{\ell^2} \rightarrow \ell^2 C_\ell \sim \text{const}$
COBE
WMAP
Planck
SW plateau
$\ell(\ell+1) C_\ell$
ℓ
$k \sim \frac{\ell}{r_s}$
$j_\ell(x)$ peaks near $x \sim \ell$
$\sim \frac{1}{\ell}$
$x \ll \ell \quad j_\ell \sim x^\ell$
$x \gg \ell \quad j_\ell \sim \frac{\cos(x)}{x}$
PRO-TEC