Title: Lecture - Cosmology, PHYS 621

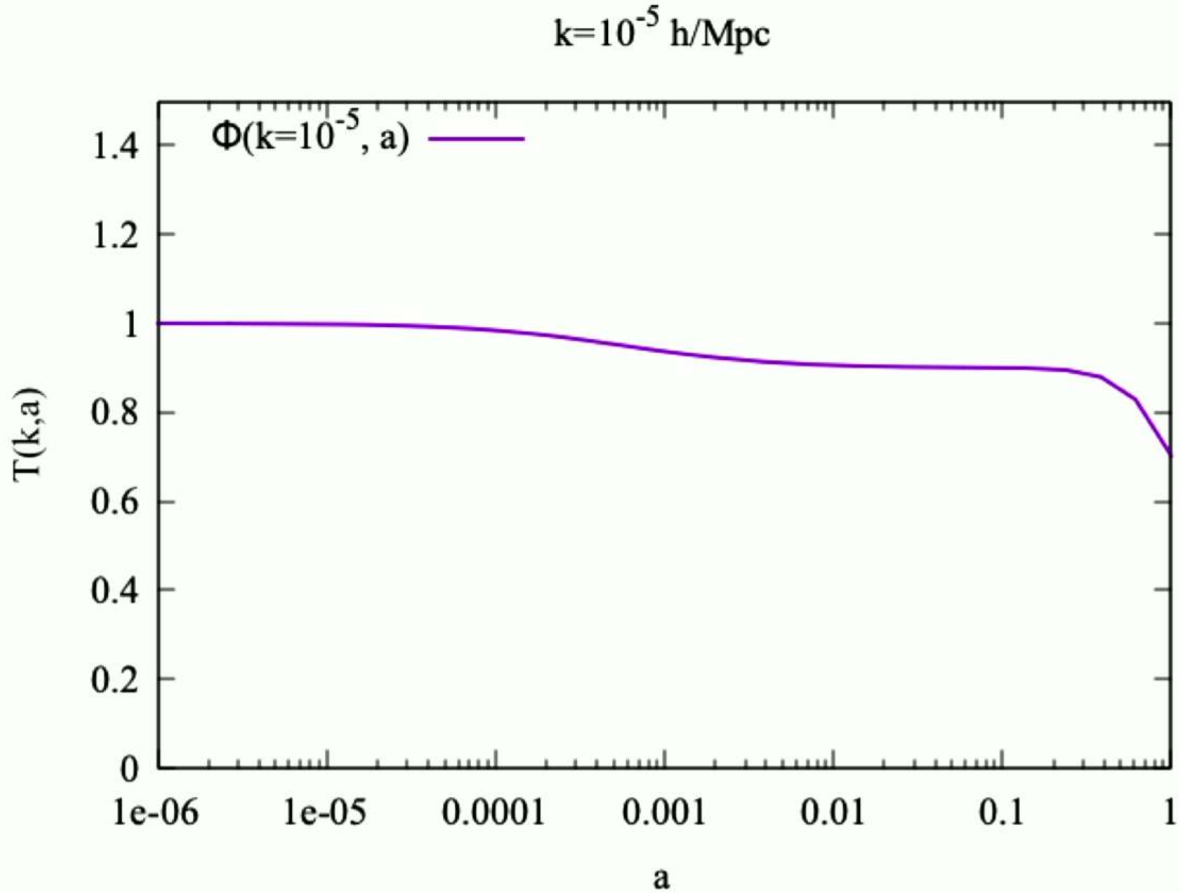
Speakers: Neal Dalal

Collection/Series: Cosmology (Elective), PHYS 621, March 31 - May 2, 2025

Subject: Cosmology

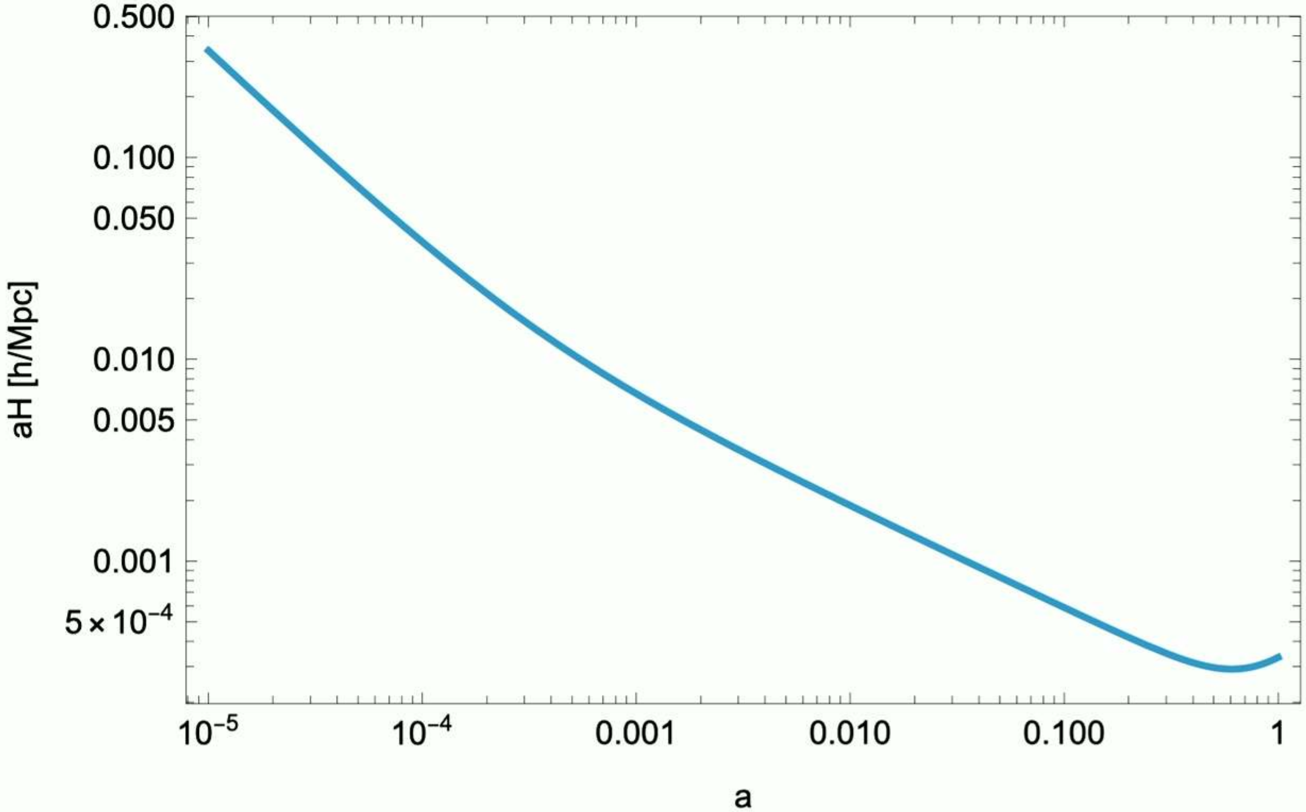
Date: April 28, 2025 - 2:00 PM

URL: https://pirsa.org/25040023



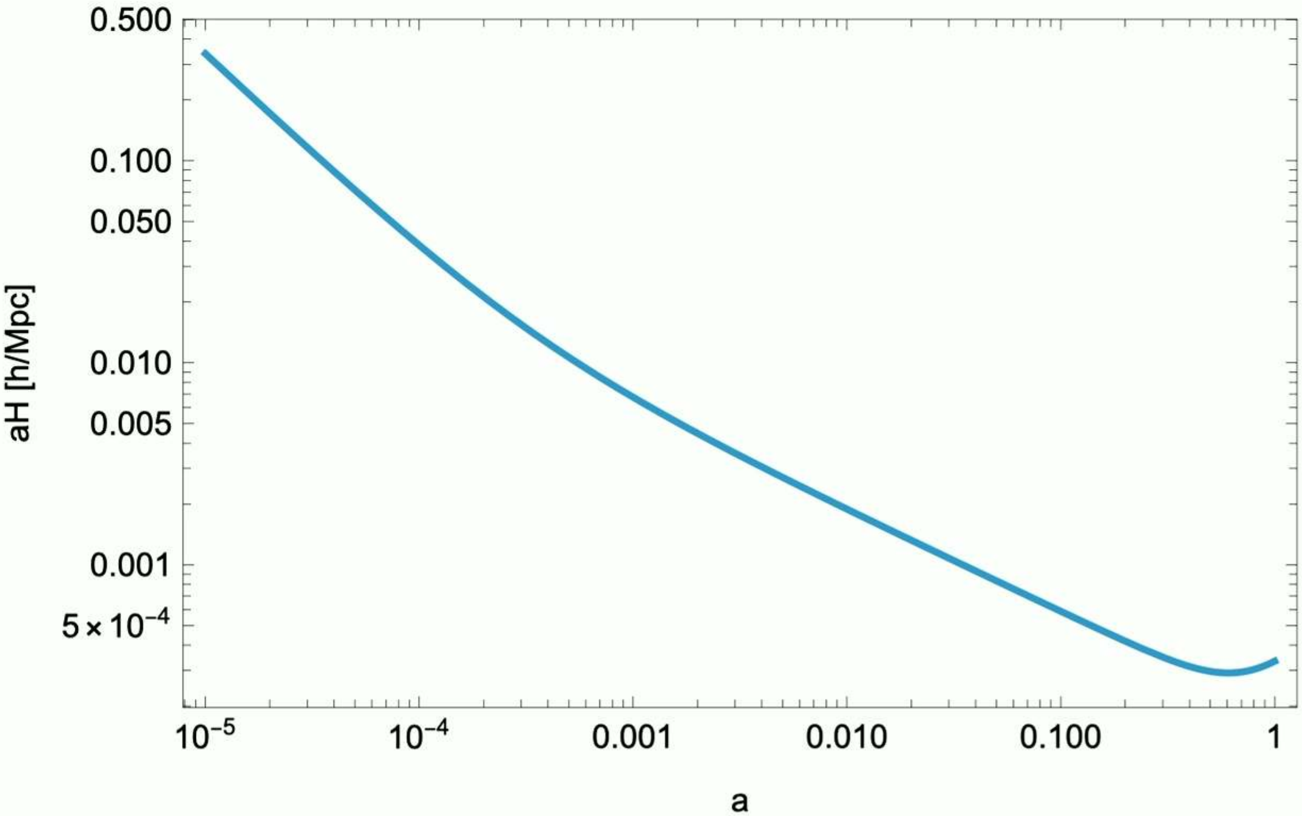
k=10^-5 h/Mpc
Φ(k=10^-5, a)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
T(k,a)
1e-06
1e-05
0.0001
0.001
0.01
0.1
1
a

Horizon entry when $k > aH$

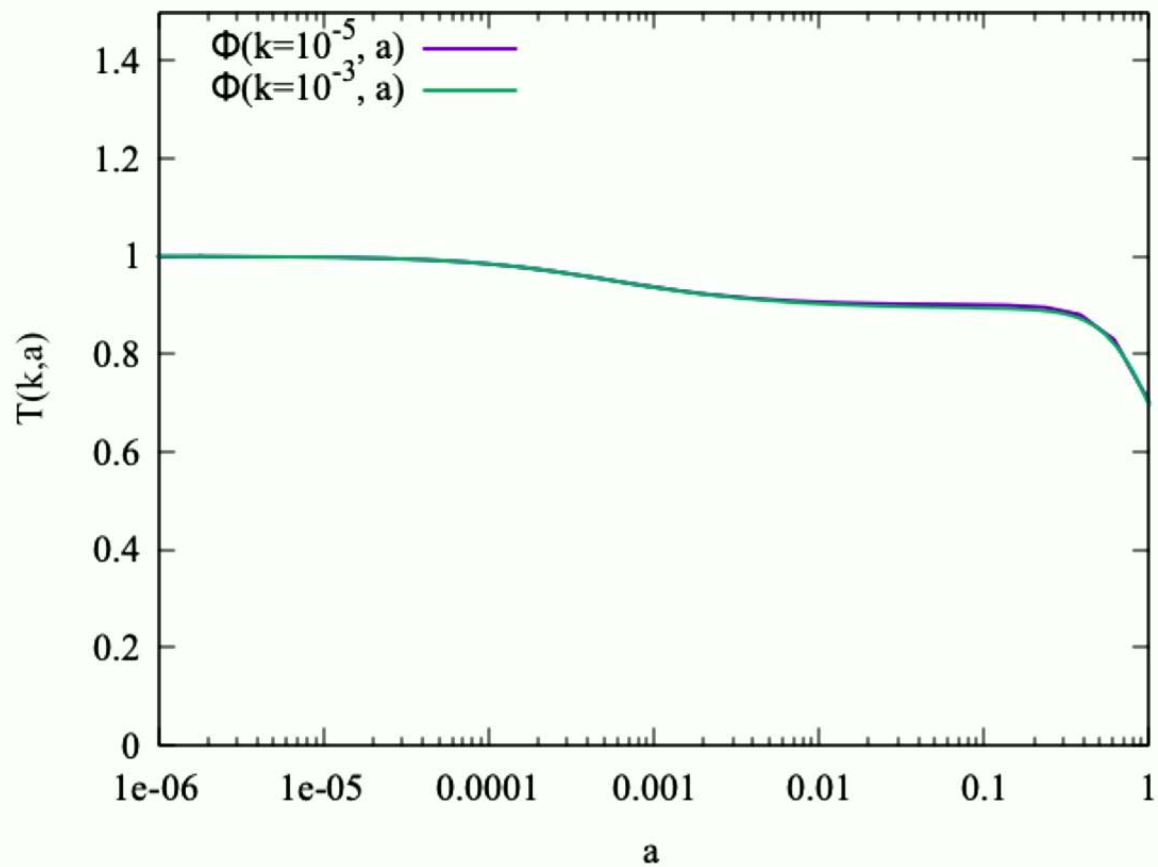


Horizon entry when $k > aH$

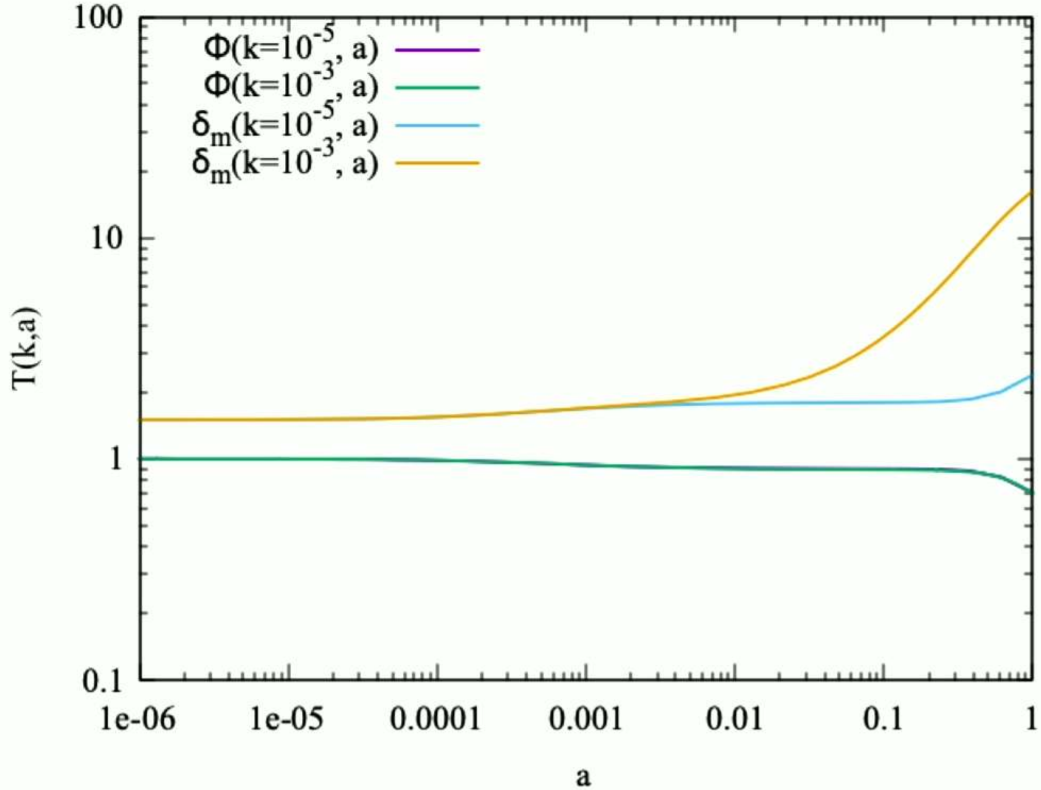
k

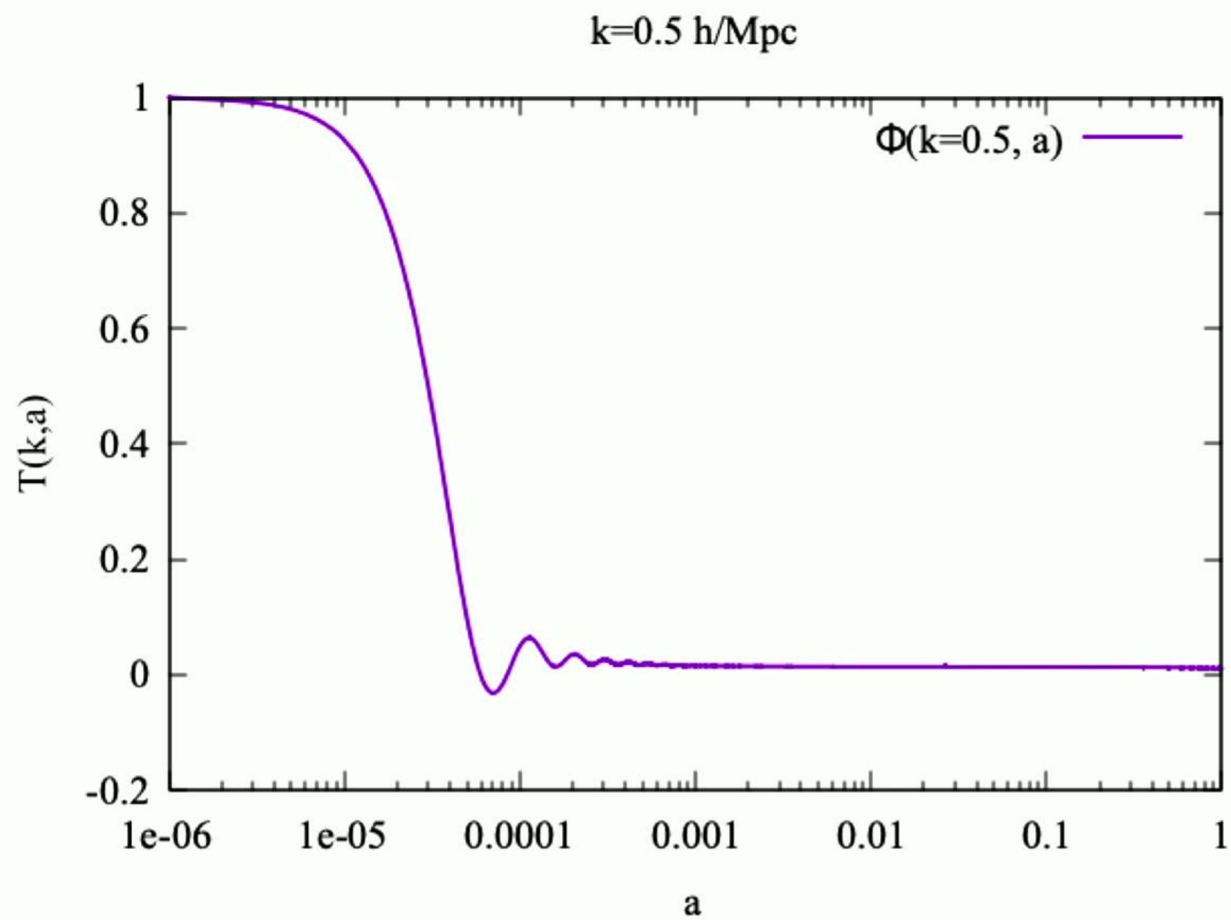


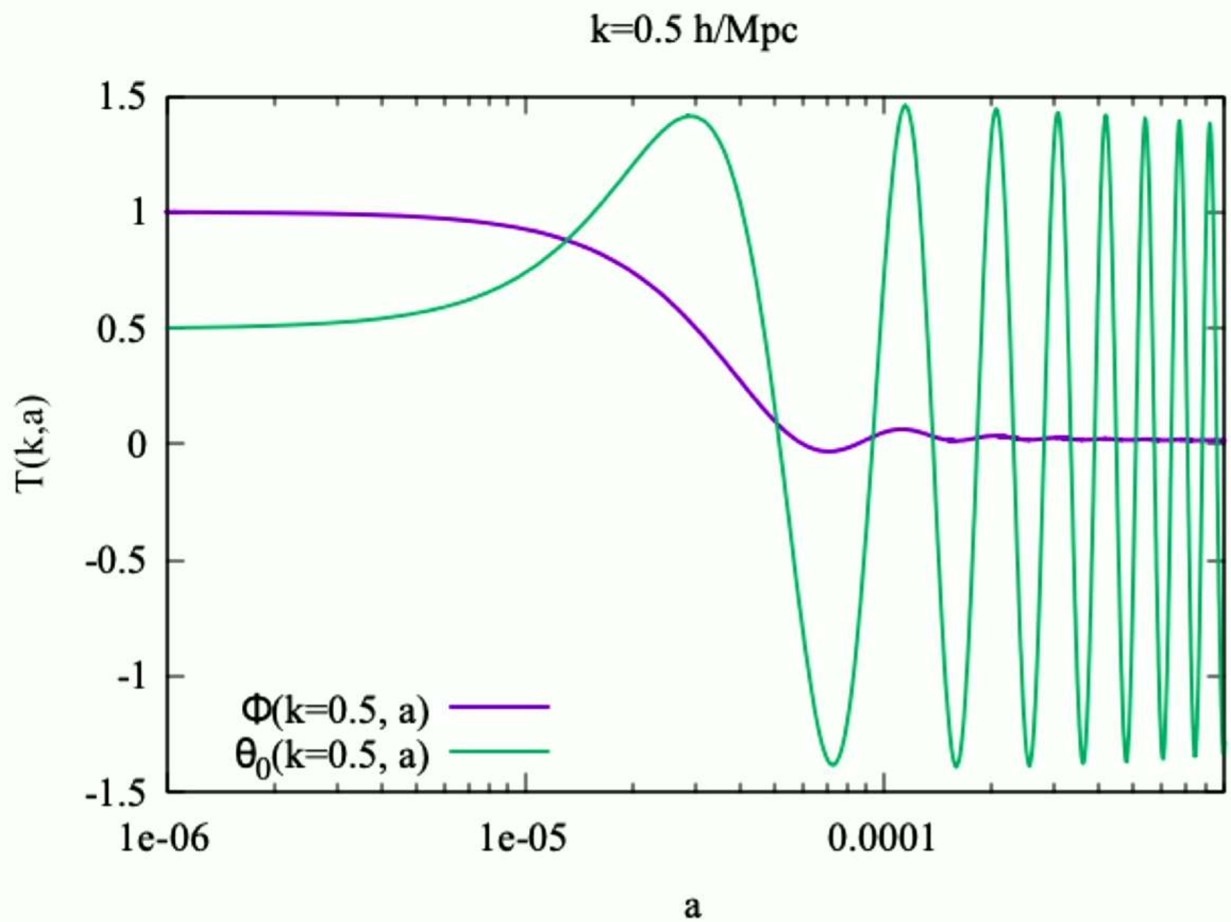
Time of horizon entry

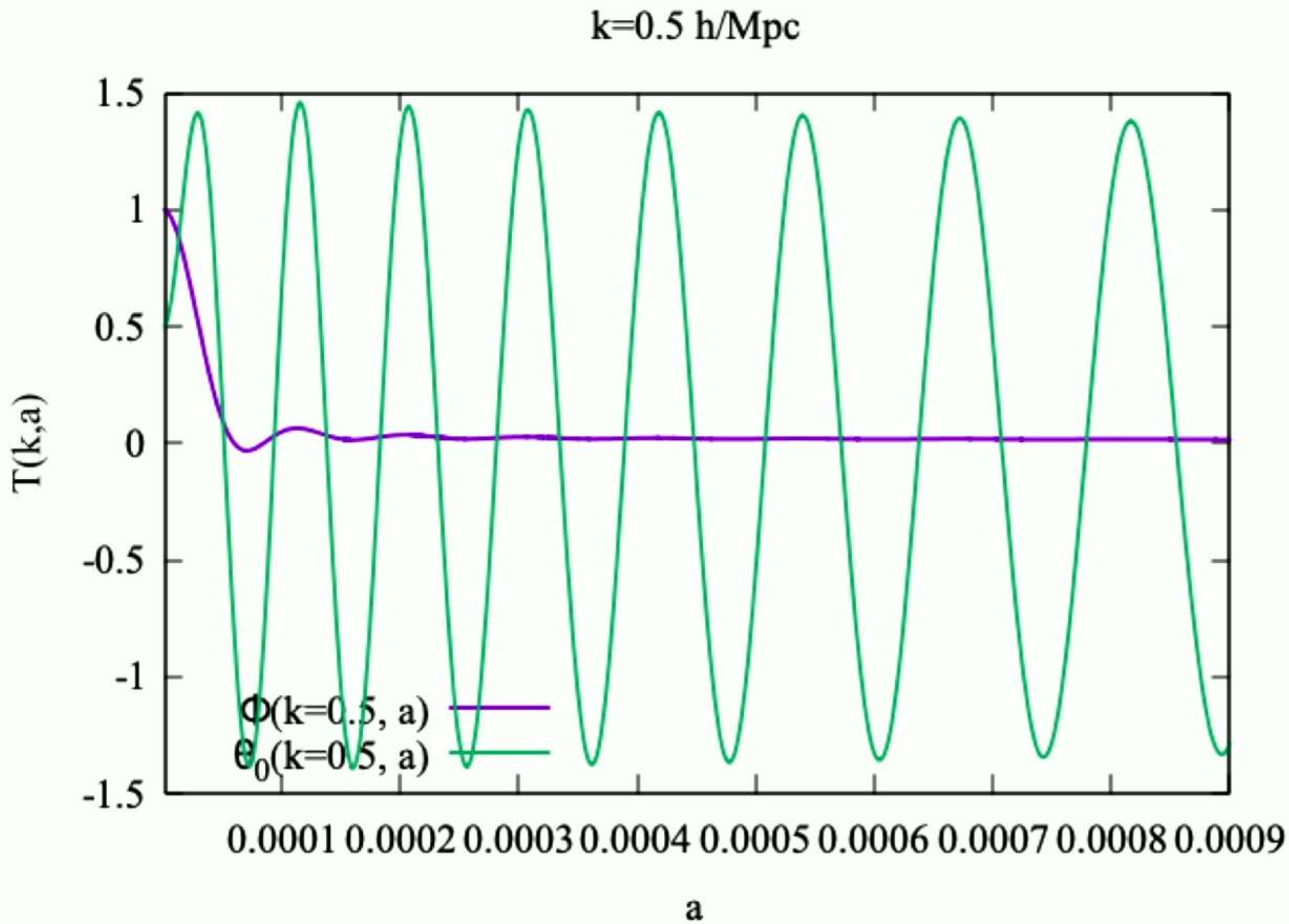


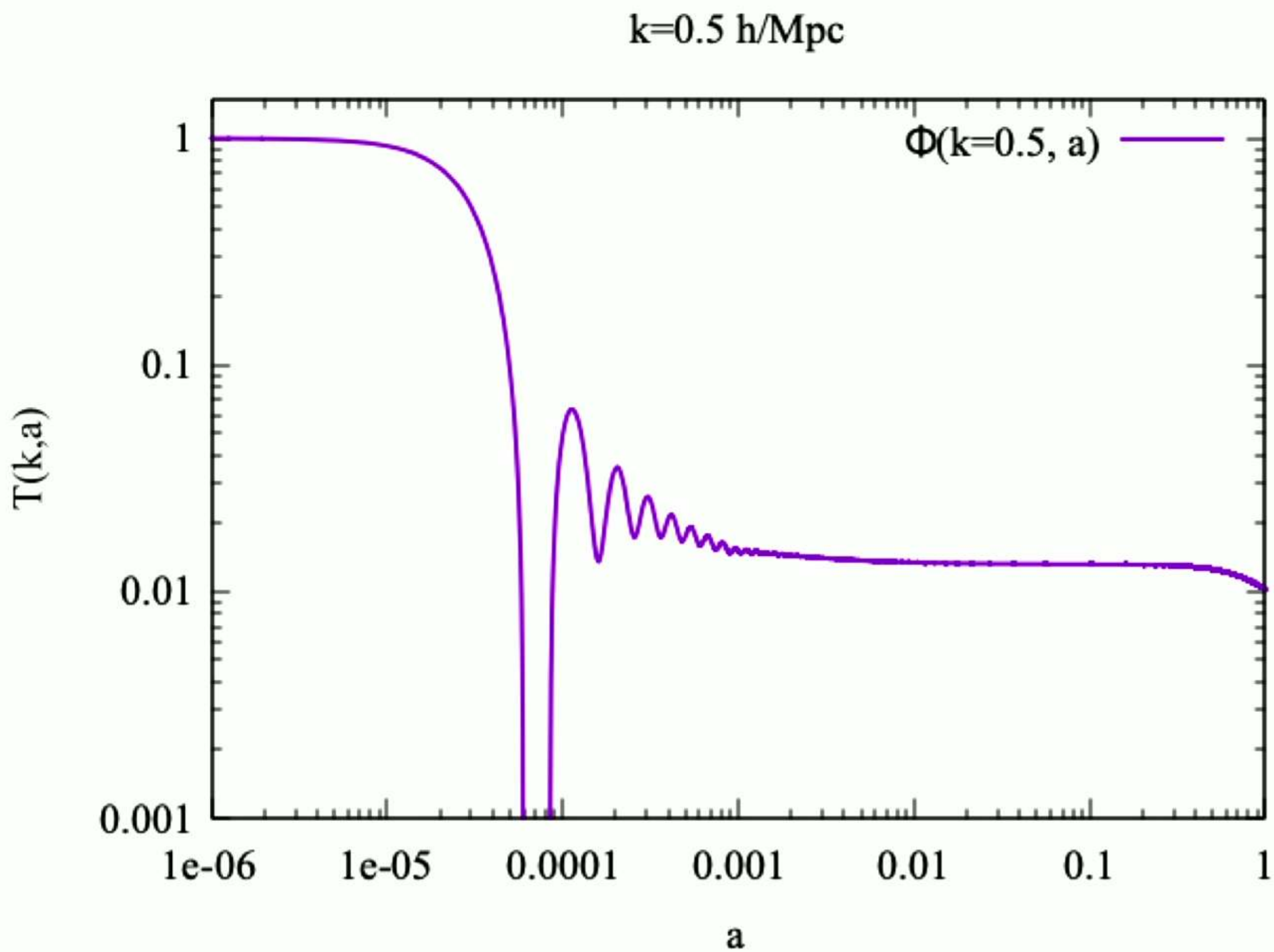
Φ(k=10^-5, a)
Φ(k=10^-3, a)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
T(k,a)
1e-06
1e-05
0.0001
0.001
0.01
0.1
1
a


Φ(k=10^-5, a)
Φ(k=10^-3, a)
δm(k=10^-5, a)
δm(k=10^-3, a)
100
10
1
0.1
T(k,a)
1e-06
1e-05
0.0001
0.001
0.01
0.1
1
a

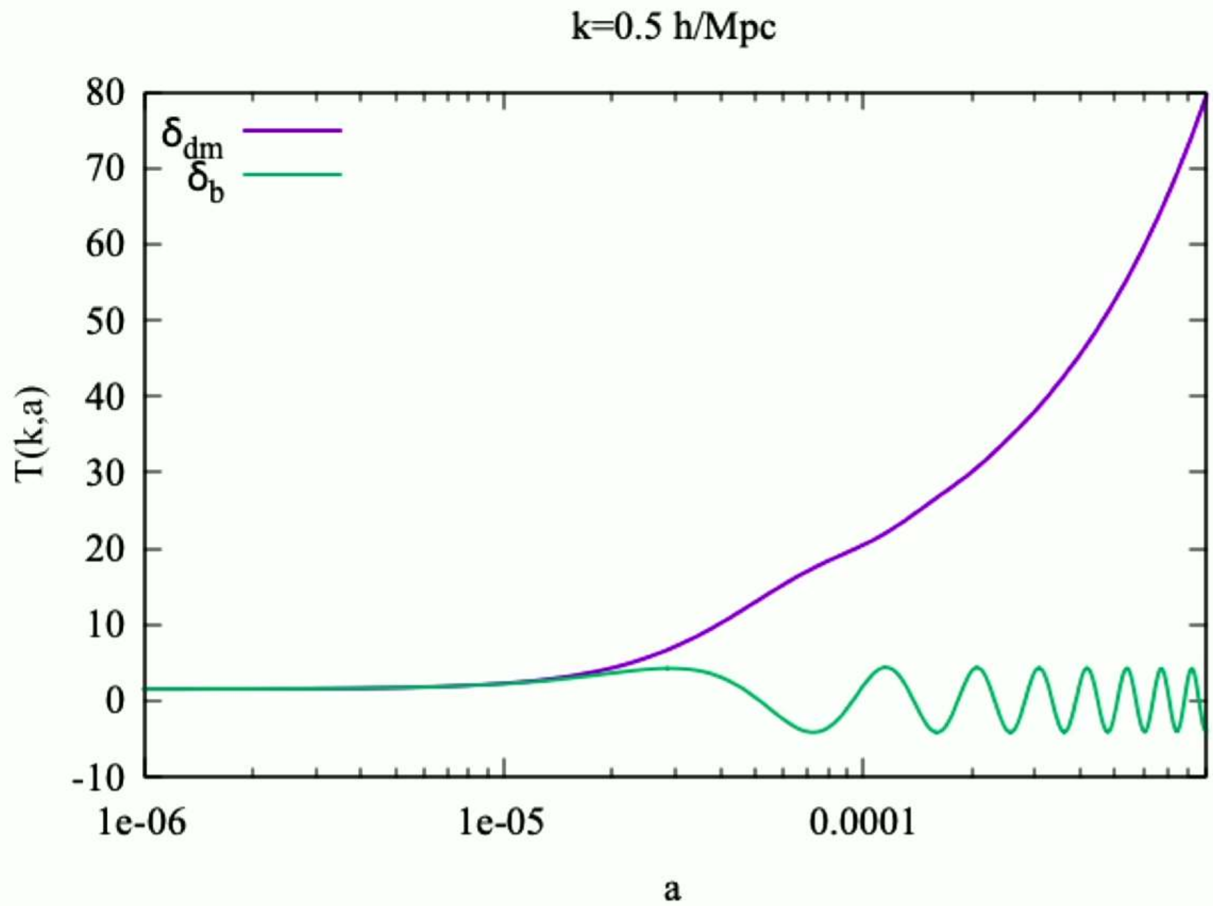

k=0.5 h/Mpc
Φ(k=0.5, a)
T(k,a)
1
0.8
0.6
0.4
0.2
0
-0.2
1e-06
1e-05
0.0001
0.001
0.01
0.1
1
a

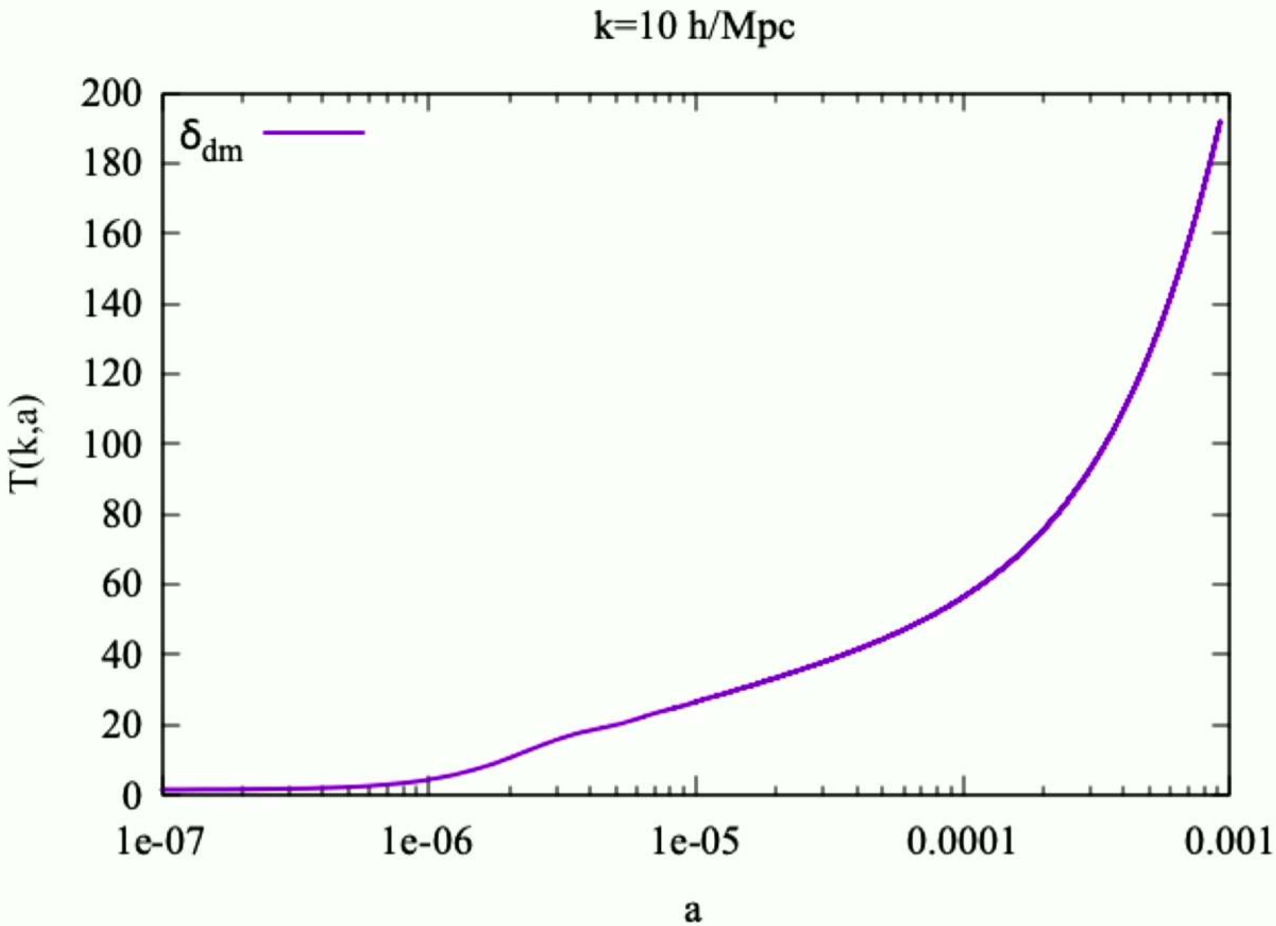

k=0.5 h/Mpc
1.5
1
0.5
0
-0.5
-1
-1.5
T(k,a)
Φ(k=0.5, a)
θ0(k=0.5, a)
1e-06
1e-05
0.0001
a

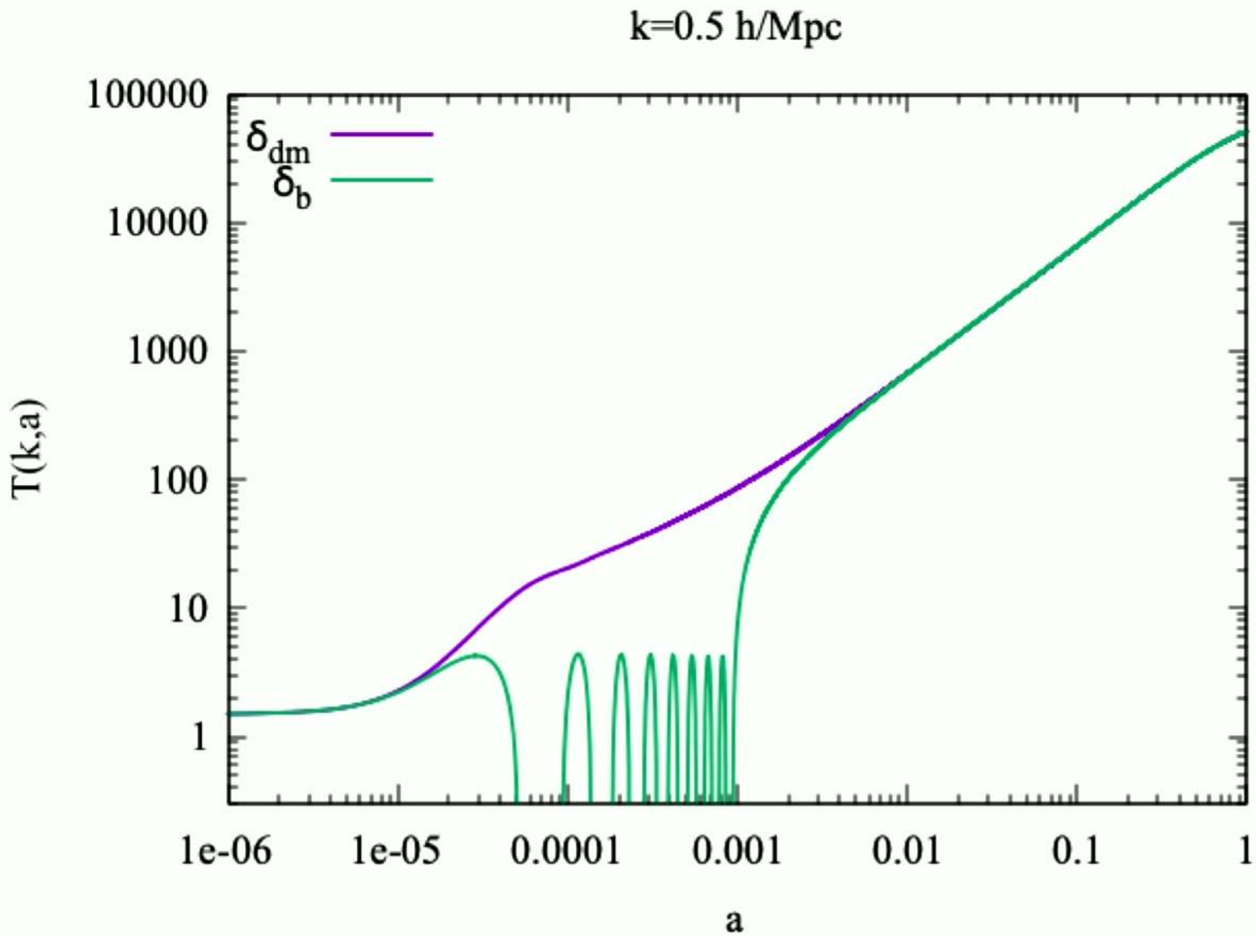

k=0.5 h/Mpc
1.5
1
0.5
0
-0.5
-1
-1.5
T(k,a)
Φ(k=0.5, a)
θ0(k=0.5, a)
0.0001 0.0002 0.0003 0.0004 0.0005 0.0006 0.0007 0.0008 0.0009
a

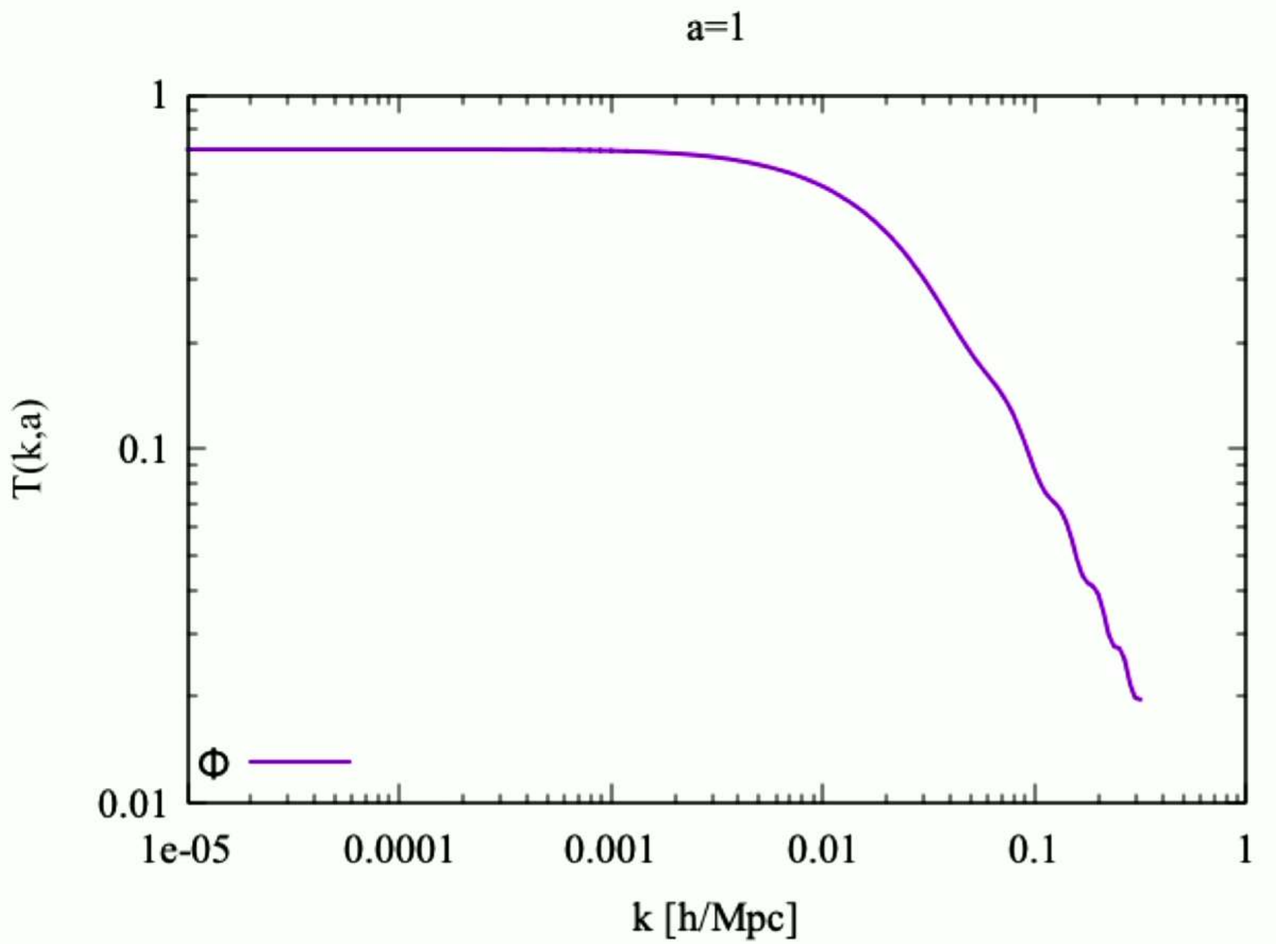


k=0.5 h/Mpc
Φ(k=0.5, a)
T(k,a)
1
0.1
0.01
0.001
1e-06
1e-05
0.0001
0.001
0.01
0.1
1
a

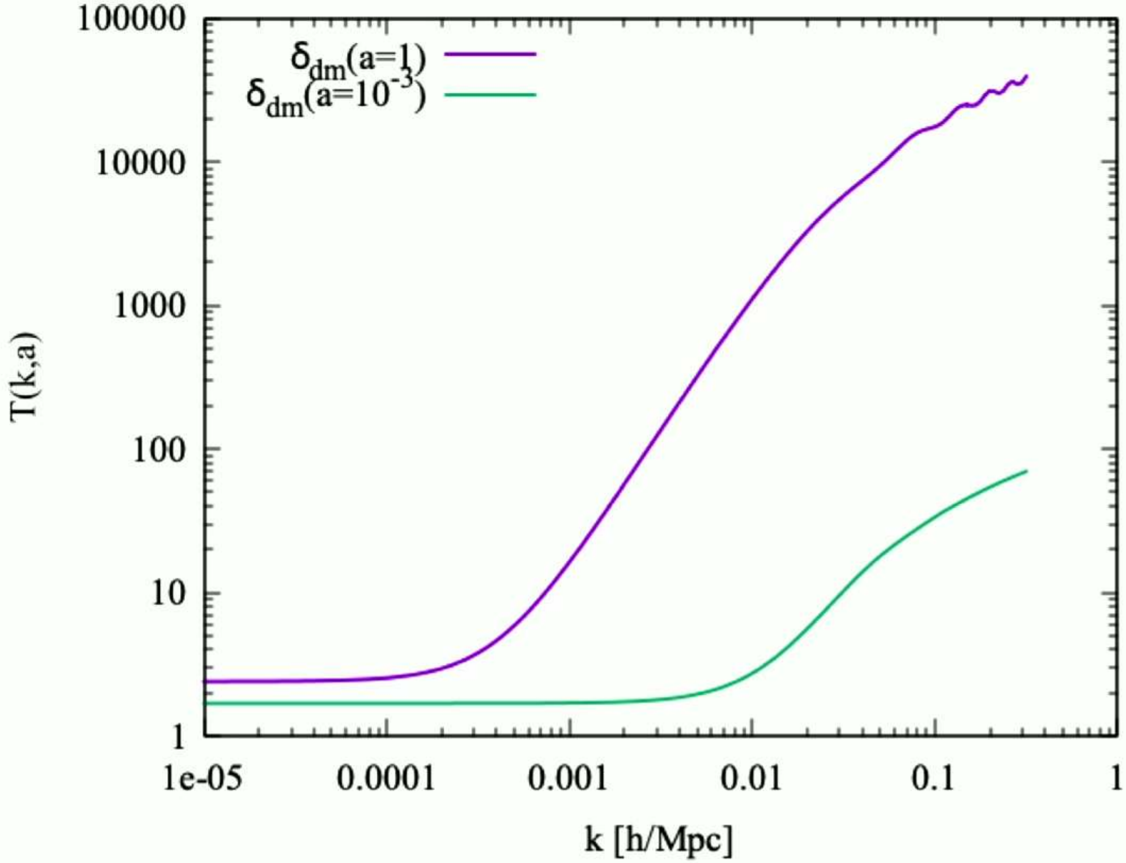

k=0.5 h/Mpc
δ_{dm}
δ_b
80
70
60
50
40
30
20
10
0
-10
T(k,a)
1e-06
1e-05
0.0001
a


k=10 h/Mpc
T(k,a)
a
δdm
200
180
160
140
120
100
80
60
40
20
0
1e-07
1e-06
1e-05
0.0001
0.001

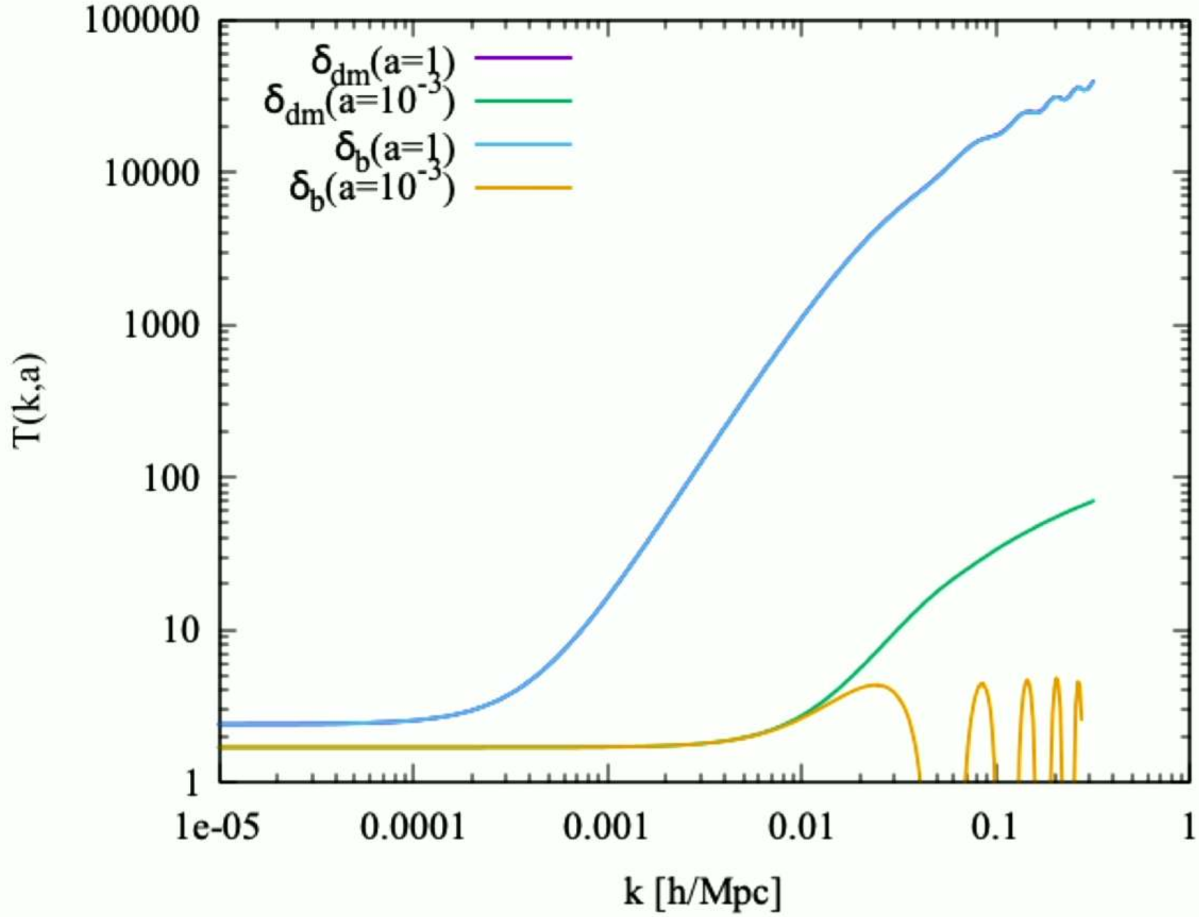

k=0.5 h/Mpc
100000
10000
1000
100
10
1
T(k,a)
δdm
δb
1e-06
1e-05
0.0001
0.001
0.01
0.1
1
a



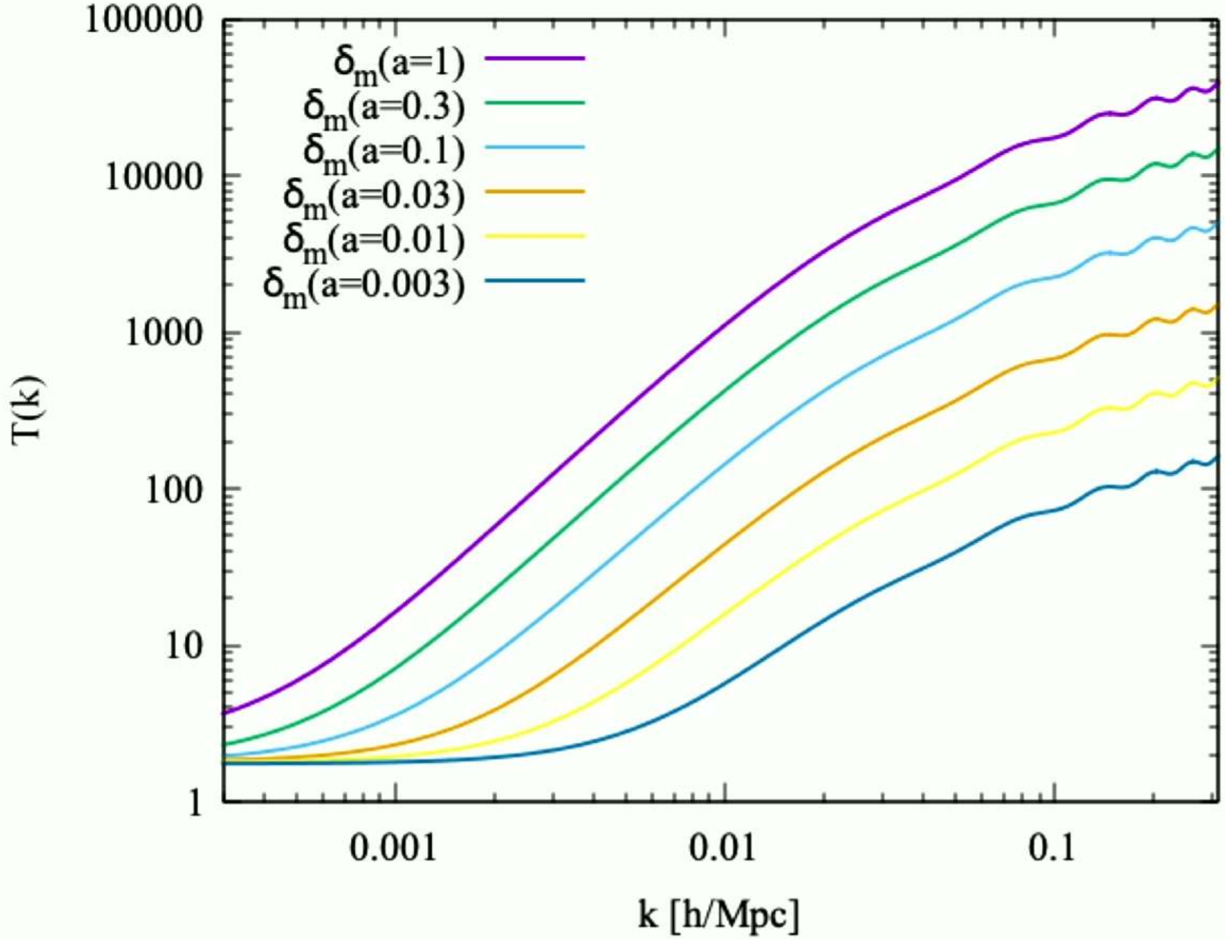
a=1
1
0.1
0.01
T(k,a)
Φ
1e-05
0.0001
0.001
0.01
0.1
1
k [h/Mpc]



100000
10000
1000
100
10
1
T(k,a)
$\delta_{dm}(a=1)$
$\delta_{dm}(a=10^{-3})$
1e-05
0.0001
0.001
0.01
0.1
1
k [h/Mpc]



100000
10000
1000
100
10
1
1e-05
0.0001
0.001
0.01
0.1
1
T(k,a)
k [h/Mpc]
$\delta_{dm}(a=1)$
$\delta_{dm}(a=10^{-3})$
$\delta_b(a=1)$
$\delta_b(a=10^{-3})$



100000
10000
1000
100
10
1
T(k)
δm(a=1)
δm(a=0.3)
δm(a=0.1)
δm(a=0.03)
δm(a=0.01)
δm(a=0.003)
0.001
0.01
0.1
k [h/Mpc]

ζ : gauge inv't.

PDF ~ Gaussian

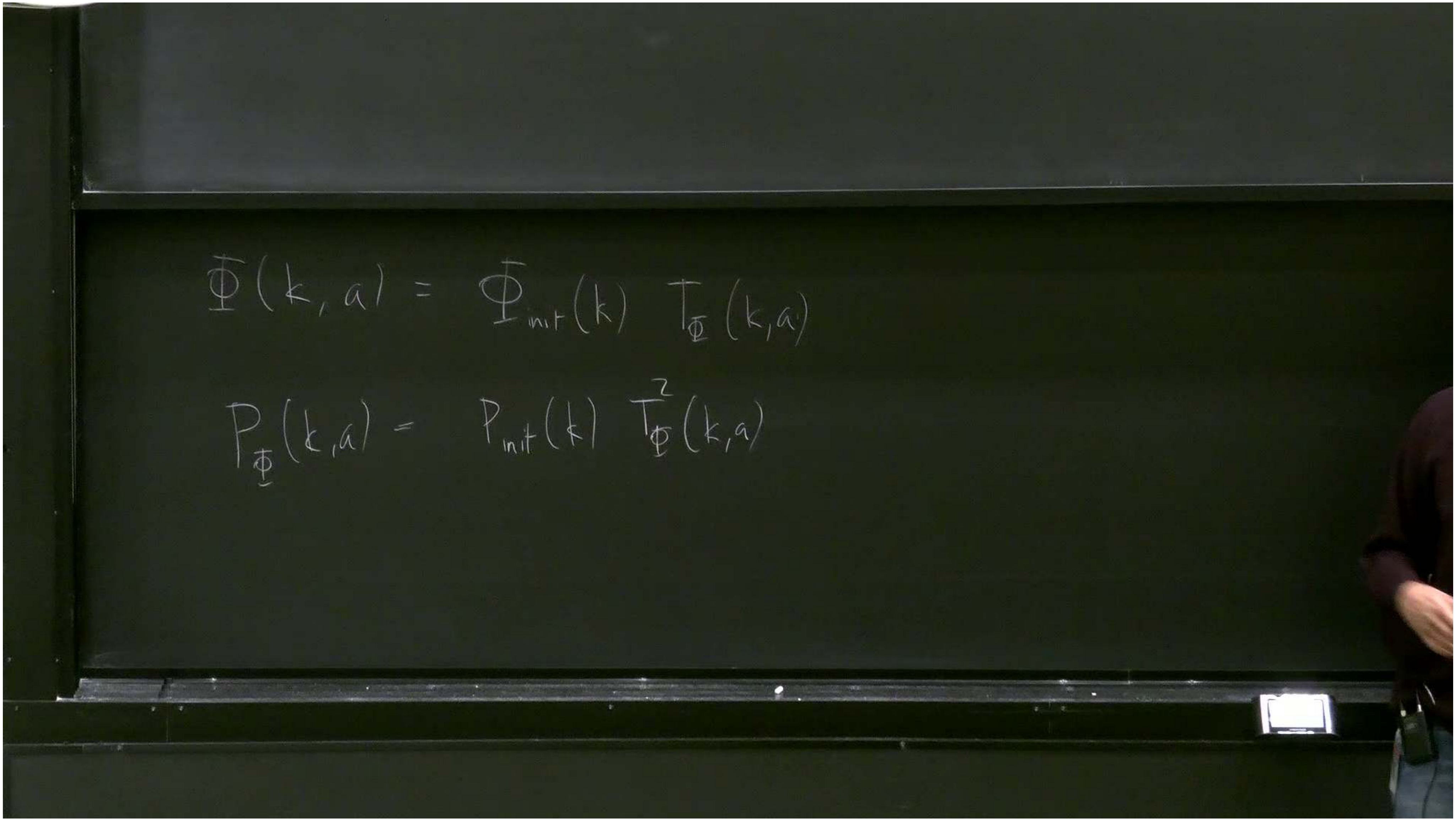
$$\Delta^2 \sim k^3 \langle \zeta^2 \rangle \sim k^{n_s - 1} \sim k^{-6\epsilon + 2\eta}$$

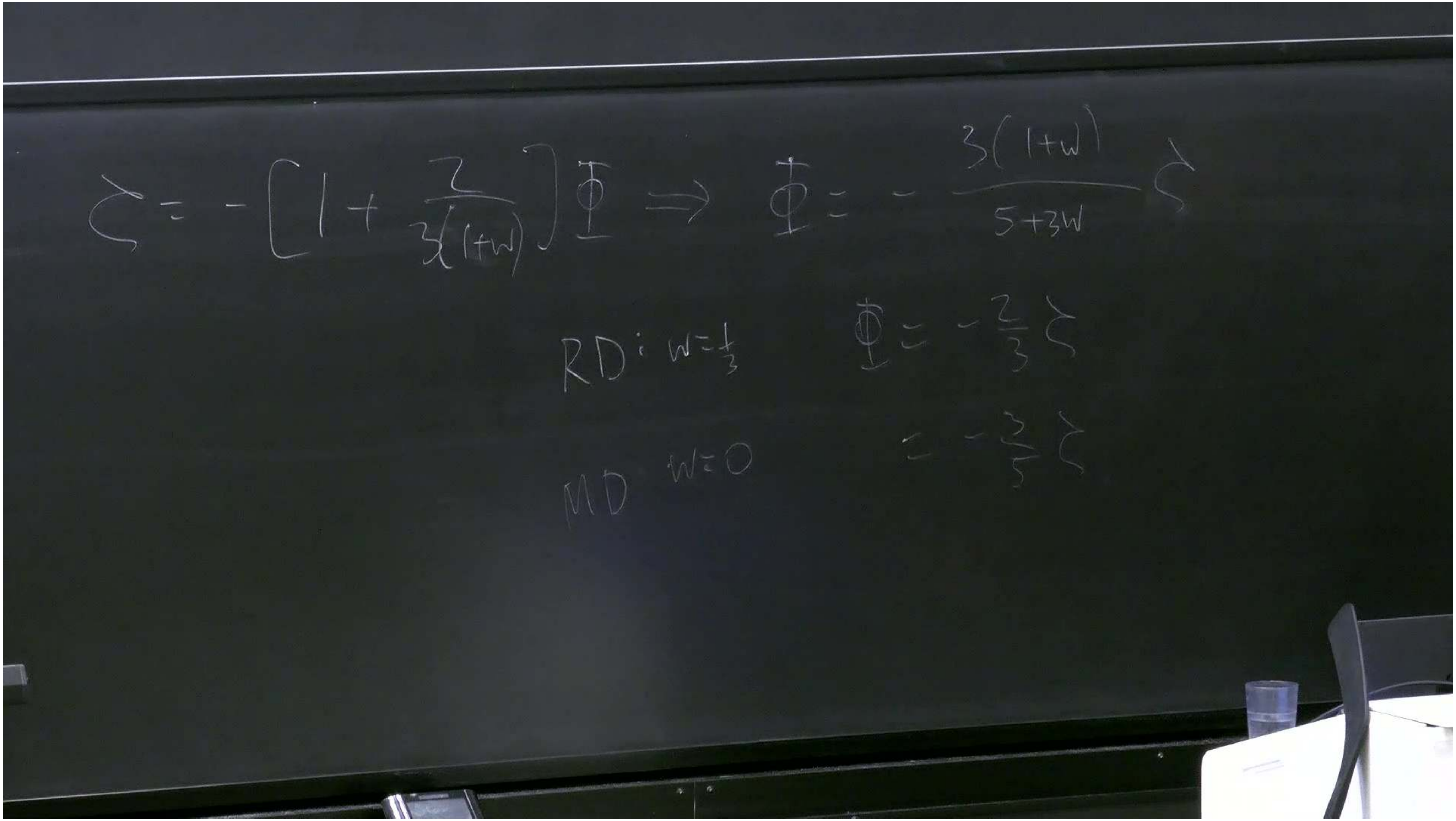
$\dot{\zeta} \to 0$ when $k \ll aH$

RD: $\Phi = -\frac{2}{3}\zeta$

$$\delta_{tot} = 2\Phi$$

$\Phi, \delta, \theta, \ldots$


$\Phi(k,a) = \Phi_{init}(k)\, T_\Phi(k,a)$
$P_\Phi(k,a) = P_{init}(k)\, T_\Phi^2(k,a)$


$\zeta = -\left[1 + \frac{2}{3(1+w)}\right]\Phi \Rightarrow \Phi = -\frac{3(1+w)}{5+3w}\zeta$
RD: $w = \frac{1}{3}$ $\Phi = -\frac{2}{3}\zeta$
MD $w = 0$ $= -\frac{3}{5}\zeta$

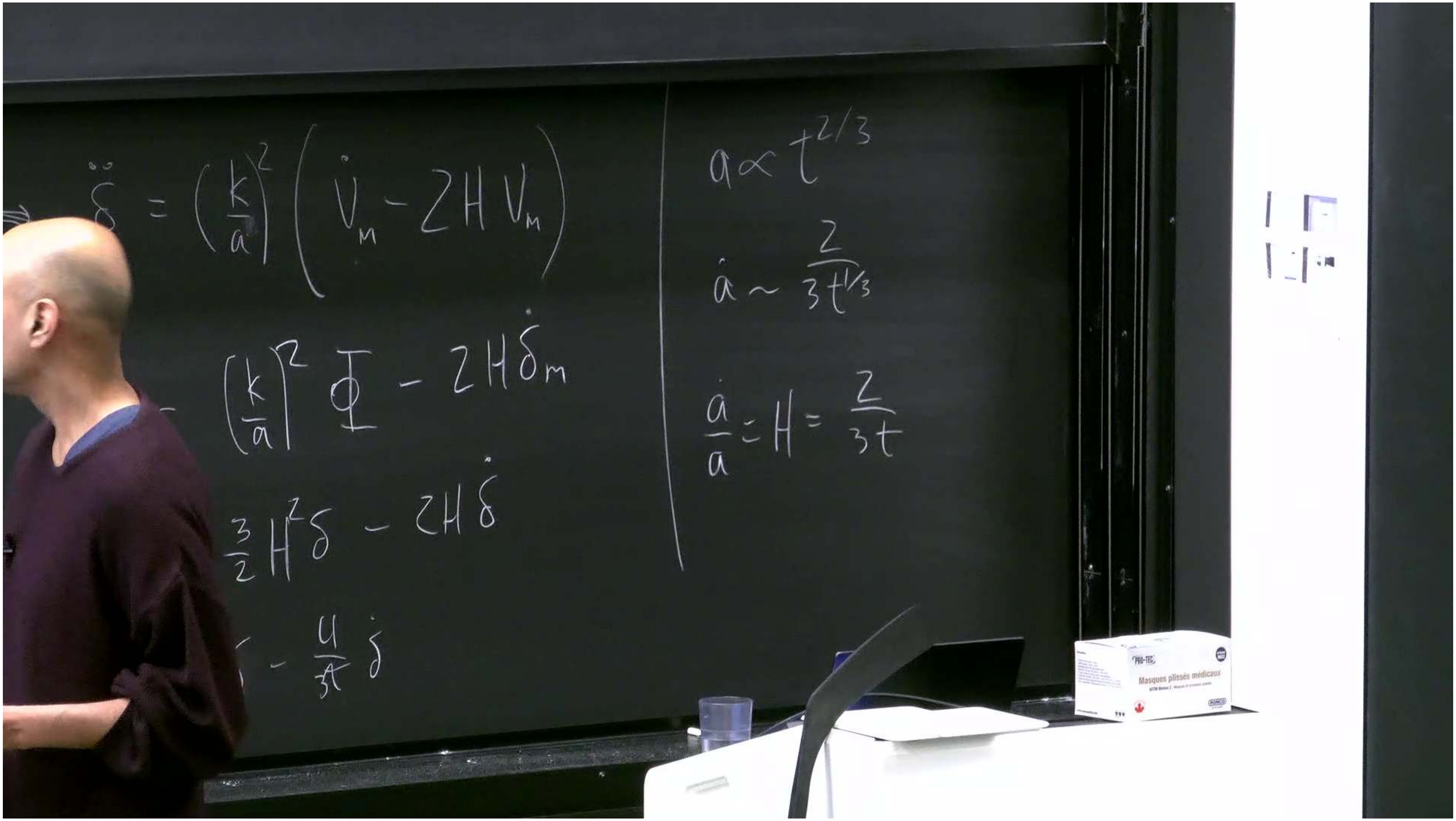
$$\dot{\delta}_m = \left(\frac{k}{a}\right)^2 V_m - \cancel{3\dot{\Phi}} \quad \Rightarrow \quad \ddot{\delta} = \left(\frac{k}{a}\right)^2 \left(\dot{V}_m - 2 H V_m \right)$$

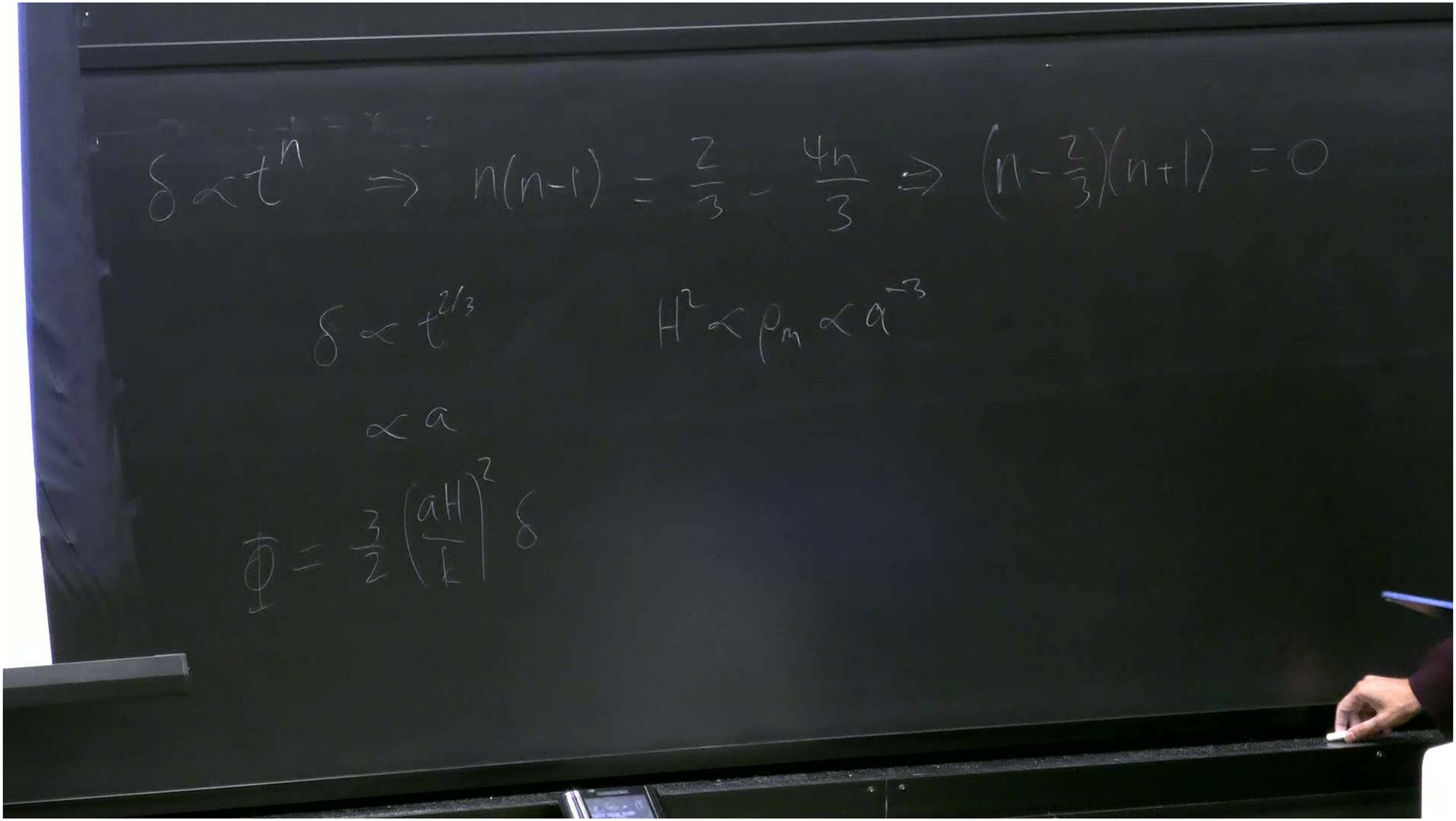
$$= \left(\frac{k}{a}\right)^2 \Phi - 2H\dot{\delta}_m$$

$$\dot{V}_m = \Phi$$

$$= \frac{3}{2} H^2 \delta - 2H\dot{\delta}$$

$$\left(\frac{k}{a}\right)^2 \Phi \approx 4\pi G \rho \delta = \frac{3}{2} H^2 \delta$$


$\ddot{\delta} = \left(\frac{k}{a}\right)^2 \left(\dot{V}_m - 2H V_m\right)$
$\left(\frac{k}{a}\right)^2 \Phi - 2H\dot{\delta}_m$
$\frac{3}{2}H^2\delta - 2H\dot{\delta}$
$- \frac{4}{3t}\dot{\delta}$
$a \propto t^{2/3}$
$\dot{a} \sim \frac{2}{3t^{1/3}}$
$\frac{\dot{a}}{a} = H = \frac{2}{3t}$
PRO-TEC
Masques plissés médicaux


δ ∝ t^n ⇒ n(n-1) = 2/3 - 4n/3 ⇒ (n - 2/3)(n+1) = 0
δ ∝ t^{2/3}
H^2 ∝ ρ_m ∝ a^{-3}
∝ a
Φ = 3/2 (aH/k)^2 δ

$$\dot{\theta}_0 = \frac{k^2}{a}\,\omega_1 - \dot{\Phi} \quad \Rightarrow$$

$$\frac{1}{a}\dot{\omega}_1 = \frac{H\omega_1}{1+R} + \frac{1}{3}\left(\Phi - \frac{\theta_0}{1+R}\right)$$

$$\frac{\partial}{\partial t} \to \frac{1}{a}\frac{\partial}{\partial \eta}$$

$$\ddot{\Theta}_0 = \frac{k^2}{a}\dot{\omega}_1 - \frac{\dot{a}}{a}\frac{k^2}{a}\omega_1 - \ddot{\Phi}$$

$$= k^2\left[\frac{H\omega_1}{1+R} + \frac{1}{3}\left(\Phi - \frac{\Theta_0}{1+R}\right)\right] - \frac{\dot{a}k^2}{a^2}\omega_1 - \ddot{\Phi}$$

$$= -\frac{R}{1+R}\frac{\dot{a}}{a}\left(\dot{\Theta}_0 + \dot{\Phi}\right) + \frac{k^2}{3}\left(\Phi - \frac{\Theta_0}{1+R}\right) - \ddot{\Phi}$$

$$\ddot{\Theta}_0 + $$

$$- \frac{\dot{a} k^2}{a^2} w_1 - \ddot{\Phi}$$

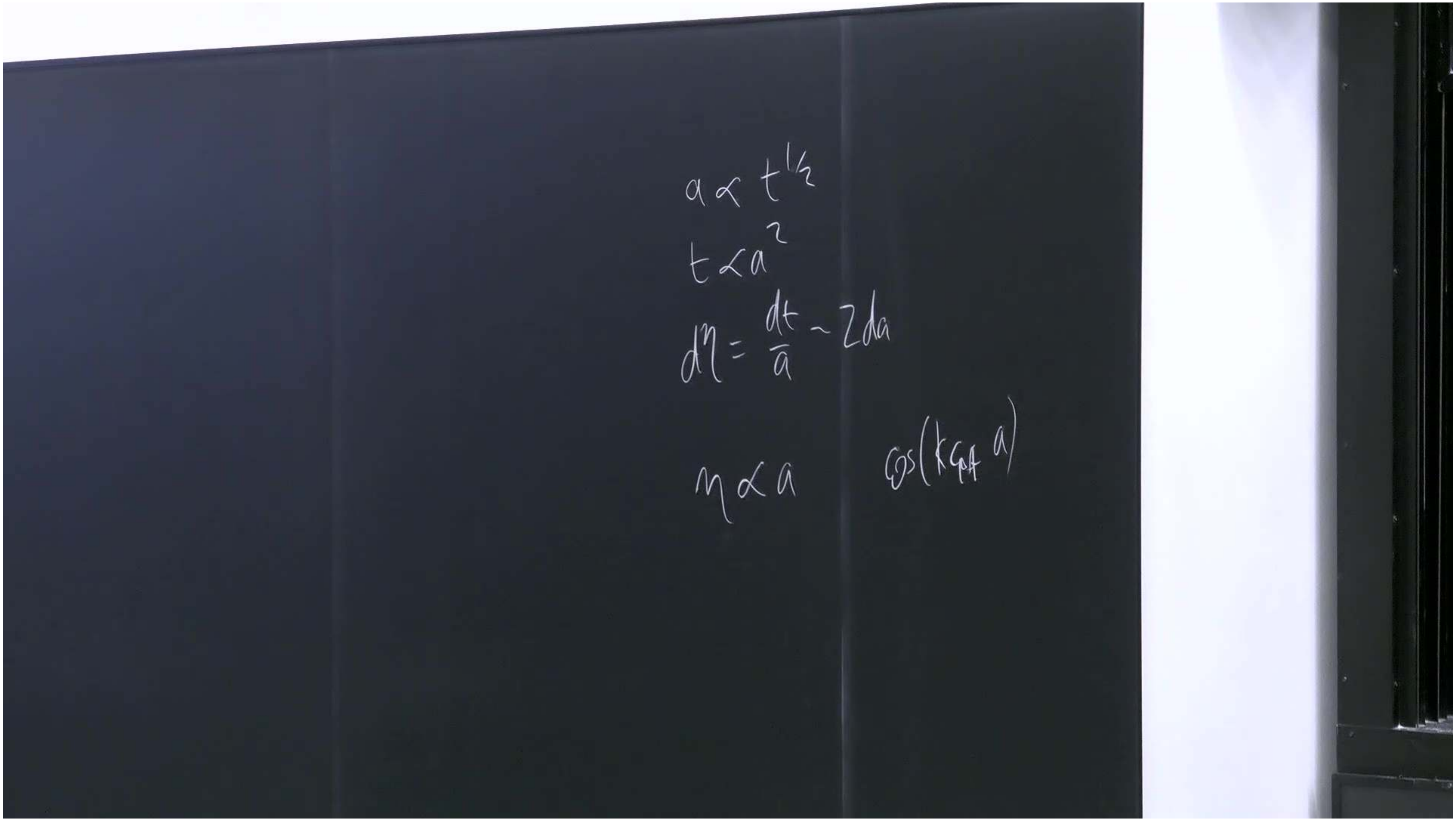
$$\Phi - \frac{\Theta_0}{1+R}\Big) - \ddot{\Phi}$$

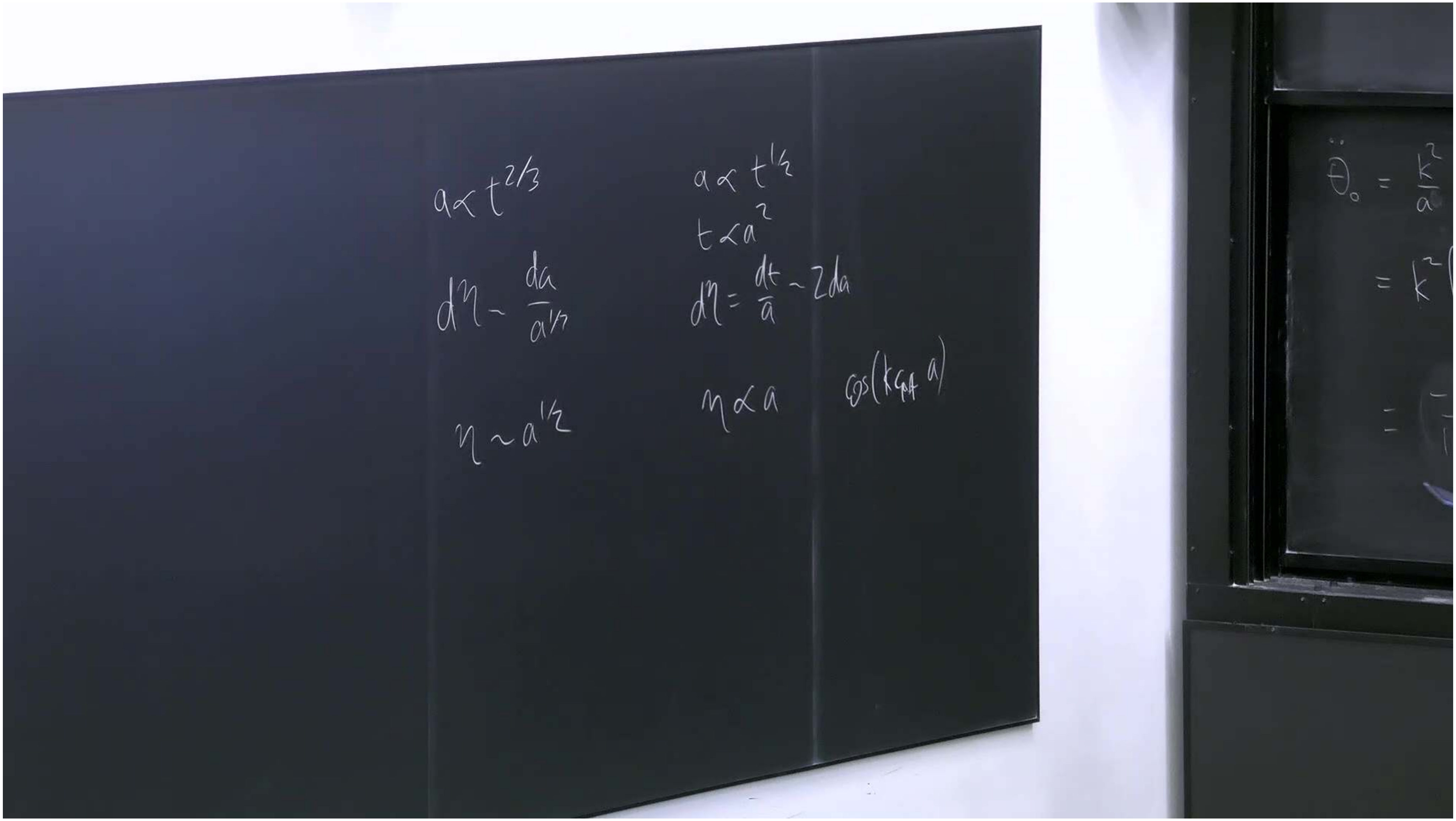
$$\ddot{\Theta}_0 + \frac{R}{1+R}\frac{\dot{a}}{a}\dot{\Theta} + k^2 c_{eff}^2 \Theta$$

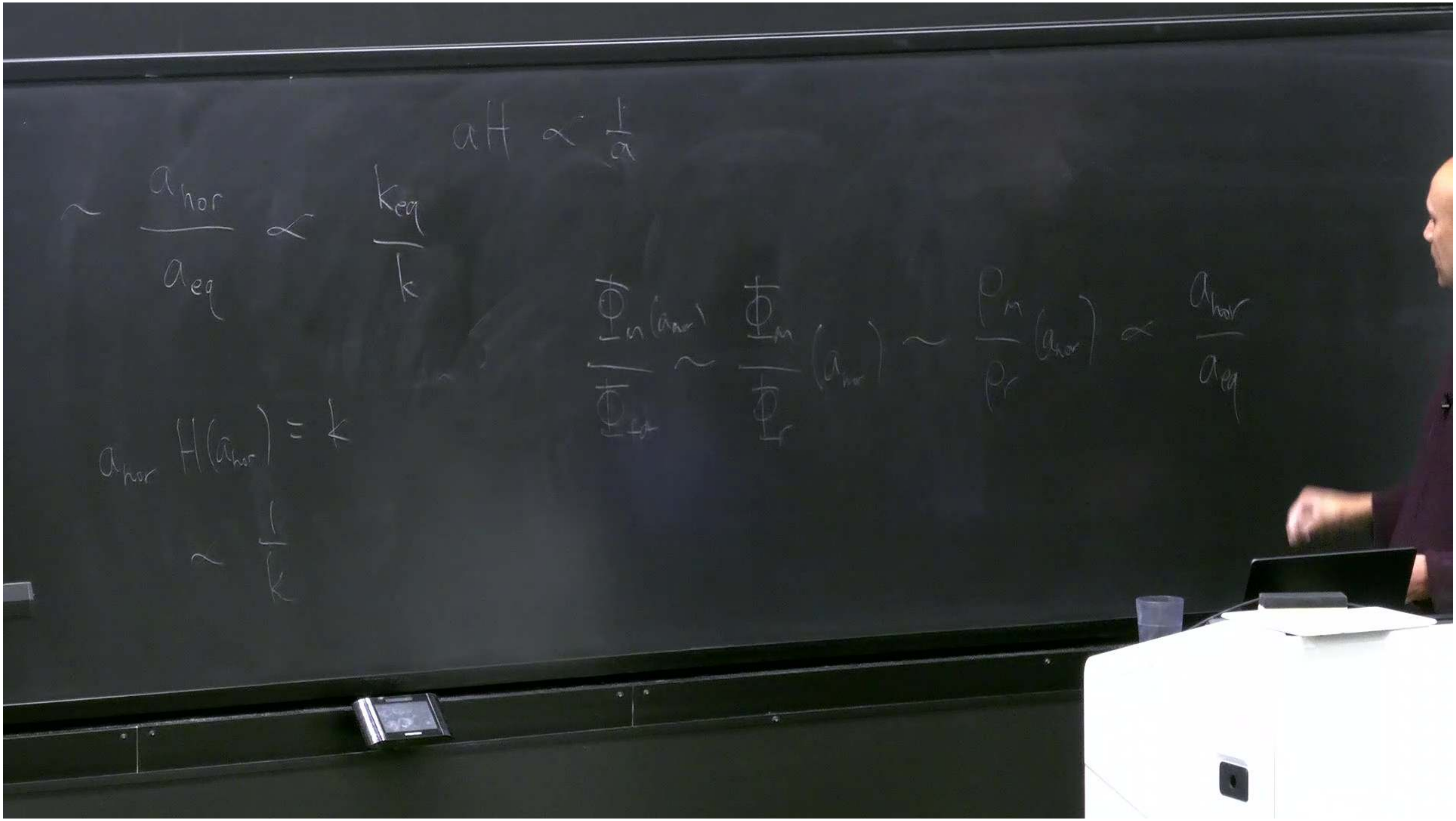
$$= -\ddot{\Phi} - \frac{R}{1+R}\frac{\dot{a}}{a}\dot{\Phi} + \frac{k^2}{3}\Phi$$

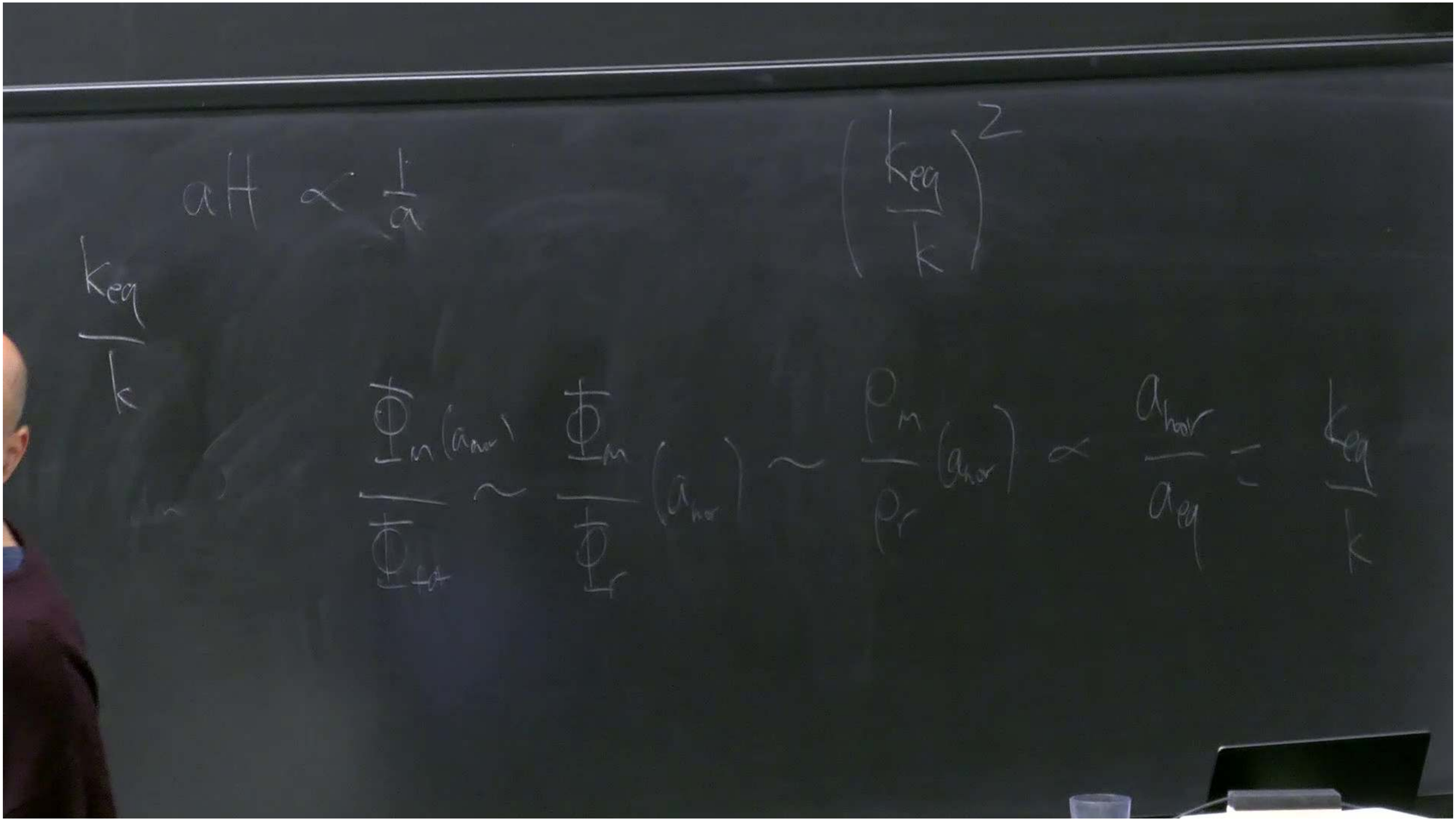
$$c_{eff}^2 = \frac{1}{3(1+R)}$$

$$\Theta_0 \sim \cos(k c_s \eta)$$


a ∝ t^{1/2}
t ∝ a^2
dη = dt/a ~ 2da
η ∝ a
cos(k a)


a ∝ t^2/3
dη ~ da/a^1/2
η ~ a^1/2
a ∝ t^1/2
t ∝ a^2
dη = dt/a ~ 2da
η ∝ a


$aH \propto \frac{1}{a}$
$\sim \frac{a_{hor}}{a_{eq}} \propto \frac{k_{eq}}{k}$
$\frac{\Phi_m(a_{hor})}{\Phi_{tot}} \sim \frac{\Phi_m}{\Phi_r}(a_{hor}) \sim \frac{\rho_m}{\rho_r}(a_{hor}) \propto \frac{a_{hor}}{a_{eq}}$
$a_{hor} H(a_{hor}) = k$
$\sim \frac{1}{k}$


$aH \propto \frac{1}{a}$
$\left(\frac{k_{eq}}{k}\right)^2$
$\frac{k_{eq}}{k}$
$\frac{\Phi_m(a_{hor})}{\Phi_{tot}} \sim \frac{\Phi_m}{\Phi_r}(a_{hor}) \sim \frac{\rho_m}{\rho_r}(a_{hor}) \propto \frac{a_{hor}}{a_{eq}} = \frac{k_{eq}}{k}$