Title: Lecture - Cosmology, PHYS 621

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Collection/Series: Cosmology (Elective), PHYS 621, March 31 - May 2, 2025

Subject: Cosmology

Date: April 24, 2025 - 2:00 PM

URL: https://pirsa.org/25040022

$\delta\phi$ eqn. of motion solved by

$$v(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right) \qquad \tau = -\frac{1}{aH} \sim$$

Early : $|k\tau| \gg 1$ inside horizon $\Rightarrow$ matches harmon

Late : $|k\tau| \ll 1$ bigger than horizon olved by

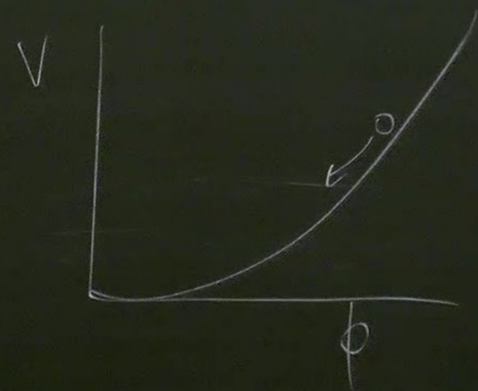
$$(1 - \frac{i}{k\tau})$$

$$\tau \simeq -\frac{1}{aH} \sim \text{apparent horizon}$$

inside horizon $\Rightarrow$ matches harmonic oscillator

bigger than horizon $\Rightarrow$ stop oscillating parent horizon

tor

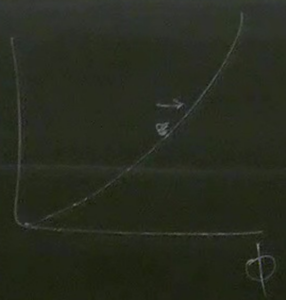


$$\langle \delta\phi^2 \rangle = \frac{\langle \chi^2 \rangle}{a^2} = \frac{|v|^2}{a^2} = \frac{1}{2a^2\tau^2k^3} = \frac{H^2}{2k^3}$$

$$\int \frac{d^3k}{(2\pi)^3} \langle |\delta\phi(k)|^2 \rangle = \int \frac{dk\, k^2\, 4\pi}{(2\pi)^3} \langle \delta\phi^2 \rangle$$

$$\int \frac{dk}{k} \Delta^2(k) \qquad \Delta^2(k) = \frac{k^3}{2\pi^2} \langle \delta\phi^2 \rangle = \left(\frac{H}{2\pi}\right)^2$$

scale invariant spectrum because $H \sim$ const



$$\epsilon = \frac{d}{dt}\left(\frac{1}{H}\right) = \frac{1}{2}\left(\frac{m_p V'}{V}\right)^2$$

$\delta\phi \to \Phi$, δ_m, etc.

$\delta\phi \to$ fluctuation of duration of inflation $\delta t \to \delta a$

$H = k$

$H \sim$ const

$$\delta t = \frac{\delta\phi}{\dot\phi} \qquad \frac{\delta a}{a} = H\,\delta t = H\frac{\delta\phi}{\dot\phi}$$

nearly

$$a^2 (1 + 2\Phi)\, \delta_{ij}\, dx^i dx^j$$

$$P_\zeta = \left(\frac{H}{\dot\phi} \right)^2$$

$$\zeta = -\Phi - HV \qquad \text{(Newtonian)}$$

$$= -\frac{H}{\dot\phi}\, \delta\phi \qquad \text{(spatially flat)}$$

$$P_\zeta = \left(\frac{H}{\dot\phi}\right)^2 \langle \delta\phi^2 \rangle = \left(\frac{H}{\dot\phi}\right)^2 \frac{H^2}{2k^3} = \frac{2\pi G H^2}{\varepsilon k^3}\Bigg|_{aH=k}$$

$$\dot\phi^2 = \frac{\varepsilon H^2}{4\pi G}$$

$$\frac{d\log P_\zeta}{d\log k} \equiv n_s - 4$$

ϕ $\quad a$

$$\boxed{n_s = 1 - 6\epsilon + 2\eta}$$

$$\langle \delta\phi^2 \rangle = \left(\frac{H}{\dot\phi}\right)^2 \frac{H^2}{2k^3} = \frac{2\pi G H^2}{\epsilon k^3}\Bigg|_{aH=k} \qquad \simeq 1$$

$$\frac{d\log P_\zeta}{d\log k} \equiv n_s - 4 \equiv -3 + 2\underbrace{\frac{d\log H}{d\log k}}_{-\epsilon} - \underbrace{\frac{d\log \epsilon}{d\log k}}_{4\epsilon - 2\eta}$$

observationally : $n_s \simeq 0.96$

rad : $\rho \propto a^{-4}$

mat $\quad a^{-3}$

Predictions of inflation

1. nearly scale inv't spectrum $n_s \lesssim 1$
2. adiabatic perturbations
3. Gaussian fluctuations

Evolution after inflation

$$\frac{\partial \Phi}{\partial t} - H\Psi = \frac{3}{2}(1+w)H^2 V$$

$$\zeta = -\Phi - \frac{\frac{\partial \Phi}{\partial t} - H\Psi}{\frac{3}{2}(1+w)H}$$

$$= -\left[1 + \frac{2}{3(1+w)}\right]\Phi$$

$$k \ll aH$$

$$\frac{\partial \zeta}{\partial \log a} \simeq + \frac{2w}{3(1+w)}\left(\frac{k}{aH}\right)^2 \Phi$$

$$\Rightarrow \zeta \sim \text{const}$$

Φ

infl. $w \simeq -1$ $\quad \Phi, \Psi \simeq 0$

RD $w \simeq \frac{1}{3}$ $\quad \Phi \simeq -\frac{2}{3}\zeta$

MD $w \simeq 0$ $\quad \Phi \simeq -\frac{3}{5}\zeta$

$$\Phi_{MD} = \frac{9}{10}\Phi_{RD}$$

$$= -\left[1 + \frac{\cdots}{3(1+w)}\right]\Phi$$

$$\frac{1}{a^2}\nabla^2\Phi + 3\left(H^2\Phi - H\frac{\partial\Phi}{\partial t}\right) = -4\pi G\,\delta\rho$$

$$3H^2\Phi = -4\pi G\,\delta\rho = -\frac{3}{2}H^2\delta_{tot} \Rightarrow \delta_{tot} = 2\Phi$$

$$\delta_{tot} = \frac{\rho_r\delta_r + \rho_m\delta_m + \dots}{\rho_r + \rho_m + \dots} \simeq \delta_r \quad \text{during RD}$$

$$\delta_m = \frac{3}{4}\delta_r$$

$$\frac{\partial \delta_{DM}}{\partial t} + \frac{1}{a} \vec{\nabla} \cdot \vec{v}_{dm} + 3 \frac{\partial \Phi}{\partial t} = 0 \longrightarrow \frac{\partial}{\partial t} \delta \simeq -3 \frac{\partial \Phi}{\partial t}$$

$$\frac{\partial \Theta_0}{\partial t} \simeq - \frac{\partial \Phi}{\partial t} \qquad \delta_\gamma = 4\,\Theta_0$$

$$\frac{\partial \Theta_0}{\partial t} + \frac{1}{a} \vec{\nabla} \cdot \vec{\Theta}_1 + \frac{\partial \Phi}{\partial t} = 0$$

$$\frac{\partial \delta_{dm}}{\partial t} = \frac{3}{4} \frac{\partial \delta_\gamma}{\partial t}$$