Title: Lecture - Cosmology, PHYS 621

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Subject: Cosmology

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$\delta\phi$ eqn. of motion solved by

$$v(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)$$

$$\tau \simeq -\frac{1}{aH}$$

Early: $|k\tau| \gg 1$ inside horizon $\Rightarrow$ matches harmon

Late : $|k\tau| \ll 1$ bigger than horizon

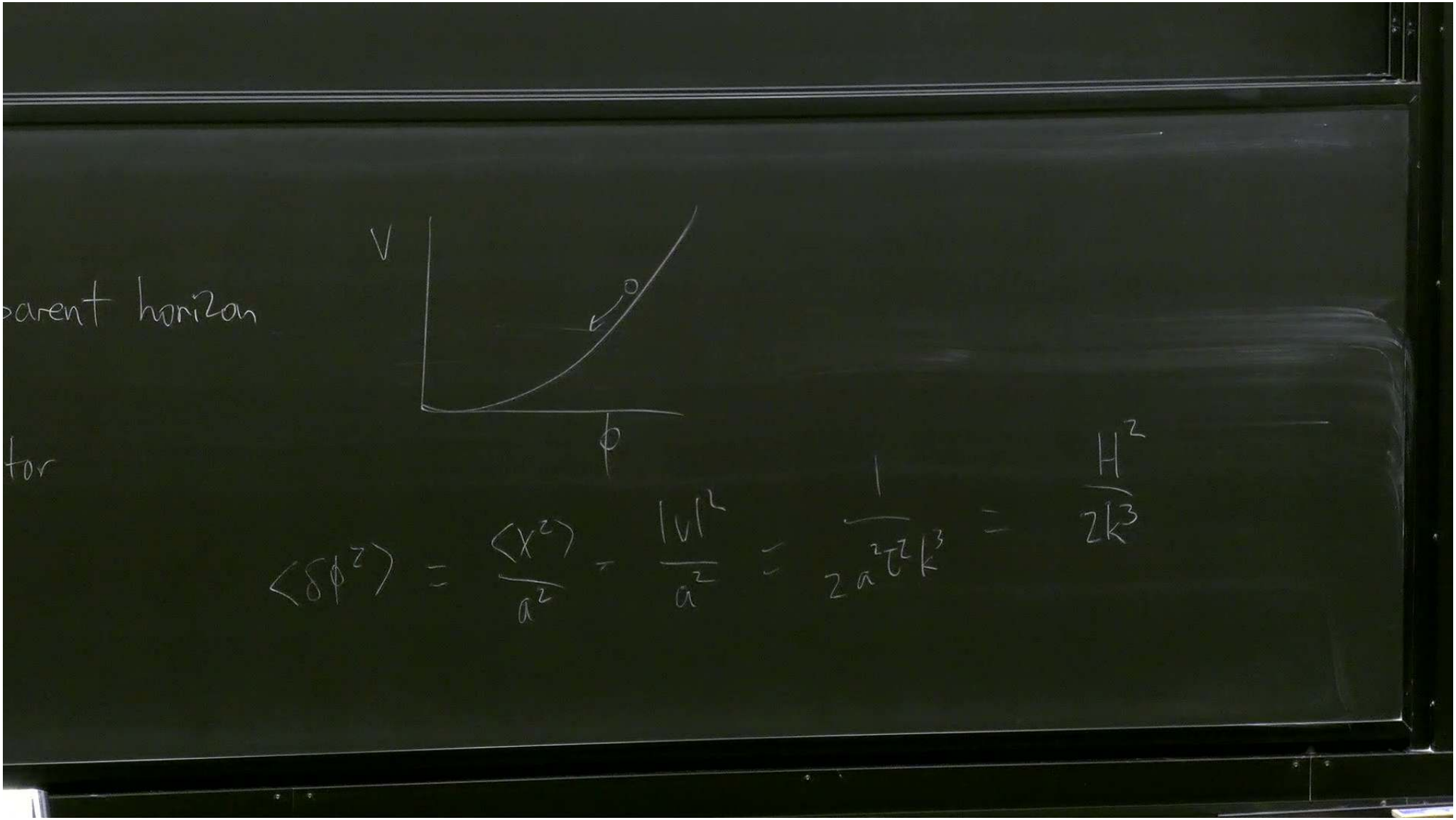
olved by

$$-\left(1-\frac{i}{k\tau}\right)$$

$\tau = -\frac{1}{aH}$ ~ apparent horizon

inside horizon $\Rightarrow$ matches harmonic oscillator

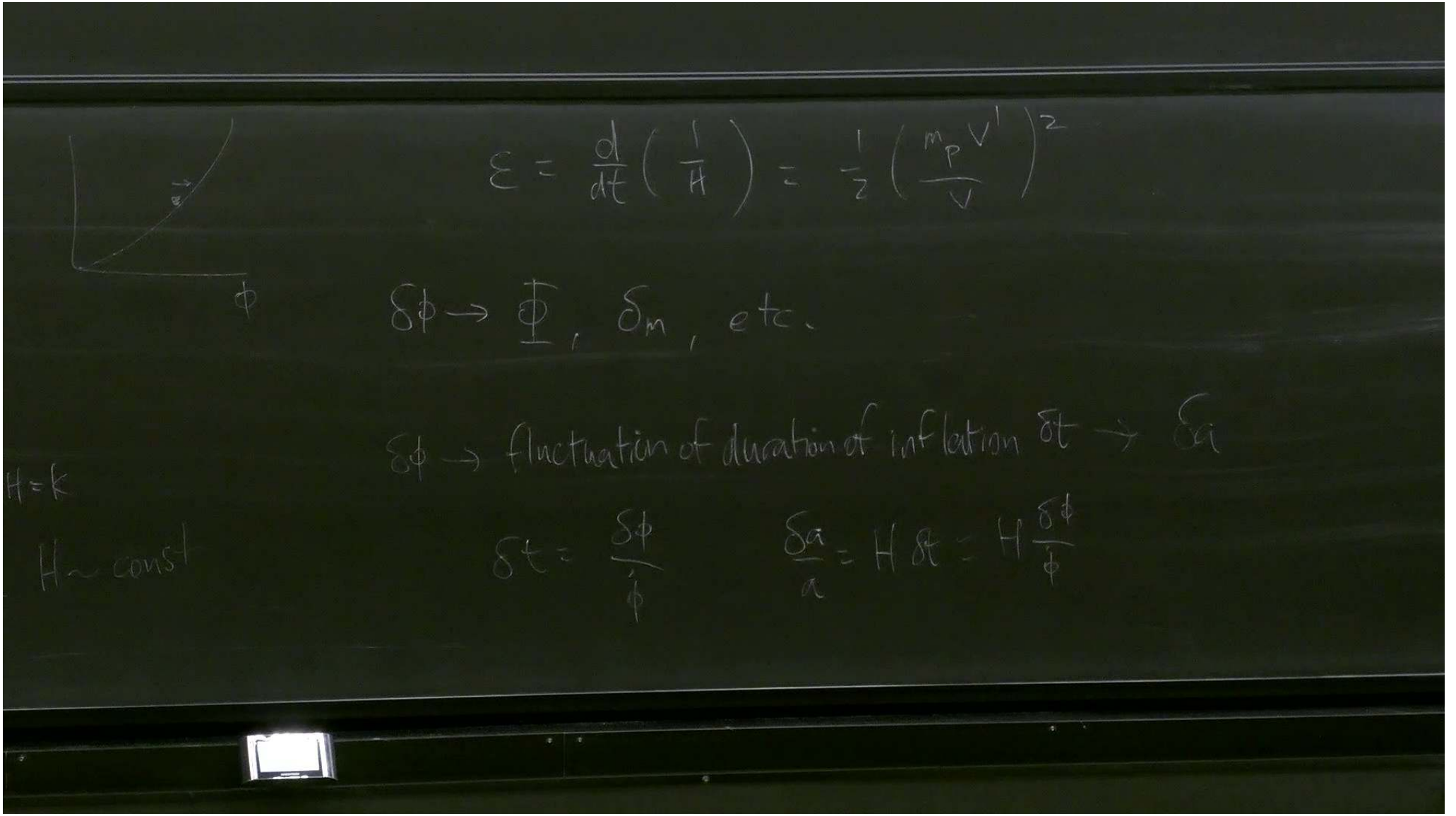
bigger than horizon $\Rightarrow$ stop oscillating


arent horizon
V
ϕ
tor
$\langle \delta\phi^2 \rangle = \frac{\langle x^2 \rangle}{a^2} = \frac{|v|^2}{a^2} = \frac{1}{2a^2\tau^2 k^3} = \frac{H^2}{2k^3}$

$$\int \frac{d^3k}{(2\pi)^3} \langle |\delta\phi(k)|^2 \rangle = \int \frac{dk\ k^2\ 4\pi}{(2\pi)^3} \langle \delta\phi^2 \rangle$$

$$\int \frac{dk}{k}\ \Delta^2(k) \qquad \Delta^2(k) = \frac{k^3}{2\pi^2} \langle \delta\phi^2 \rangle = \left(\frac{H}{2\pi}\right)^2$$

scale invariant spectrum because $H \sim$ const


$\varepsilon = \frac{d}{dt}\left(\frac{1}{H}\right) = \frac{1}{2}\left(\frac{m_p V'}{V}\right)^2$
ϕ
$\delta\phi \to \Phi$, δ_m, etc.
$\delta\phi \to$ fluctuation of duration of inflation $\delta t \to \zeta_q$
$H = k$
$H \sim$ const
$\delta t = \frac{\delta\phi}{\dot{\phi}}$
$\frac{\delta a}{a} = H\,\delta t = H\frac{\delta\phi}{\dot{\phi}}$

nearly scale-invariant

$$a^2(1+2\Phi)\,\delta_{ij}\,dx^i dx^j$$

$$\zeta = -\Phi - HV \qquad \text{(Newtonian)}$$

$$= -\frac{H}{\dot\phi}\,\delta\phi \qquad \text{(spatially flat)}$$

$$P_\zeta = \left(\frac{H}{\dot\phi}\right)^2$$

$$P_\zeta = \left(\frac{H}{\dot\phi}\right)^2 \langle \delta\phi^2 \rangle = \left(\frac{H}{\dot\phi}\right)^2 \frac{H^2}{2k^3} = \frac{2\pi G H^2}{\varepsilon k^3}\Bigg|_{aH=k}$$

$$\dot\phi^2 = \frac{\varepsilon H^2}{4\pi G}$$

$$\frac{d\log P_\zeta}{d\log k} \equiv n_s - 4$$

$$\langle \delta\phi^2 \rangle = \left(\frac{H}{\dot\phi}\right)^2 \frac{H^2}{2k^3} = \frac{2\pi G H^2}{\varepsilon k^3}\Bigg|_{aH=k} \simeq 1$$

$$n_s = 1 - 6\varepsilon + 2\eta$$

$$\frac{d\log P_\zeta}{d\log k} \equiv n_s - 4 \equiv -3 + 2\frac{d\log H}{d\log k} - \frac{d\log \varepsilon}{d\log k}$$

$$-\varepsilon \qquad 4\varepsilon - 2\eta$$

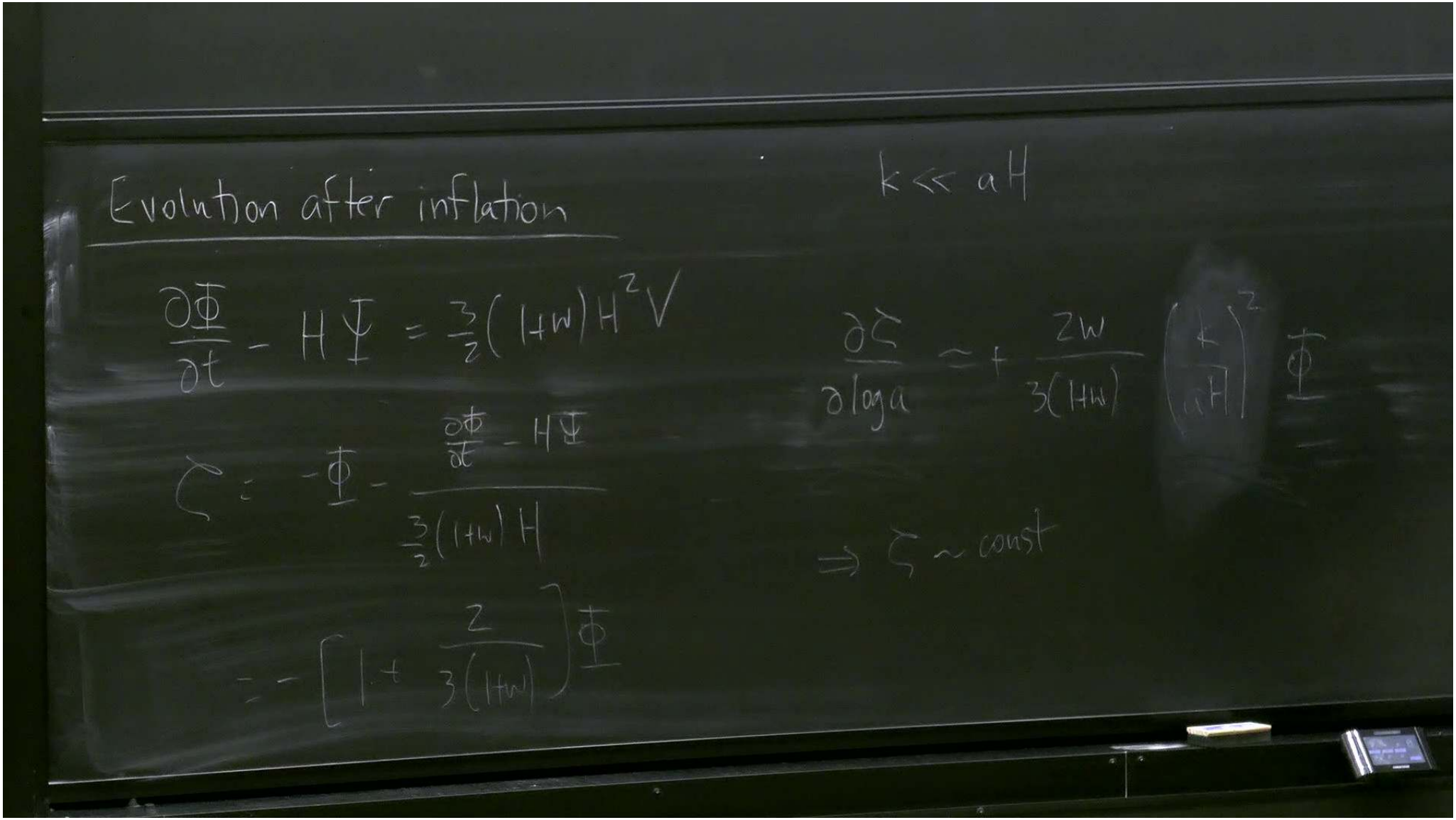
observationally : $n_s \simeq 0.96$

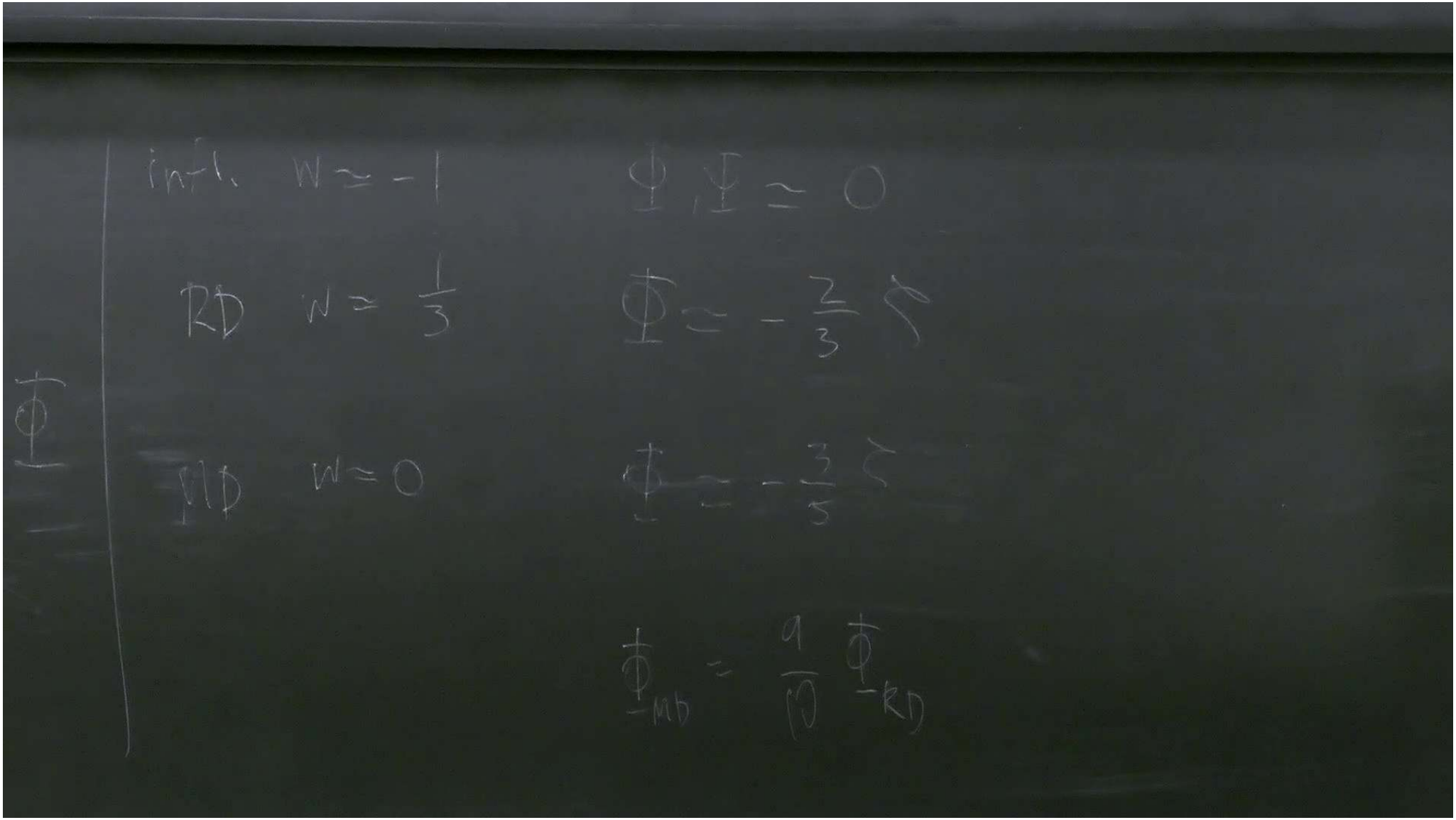
rad : $\rho \propto a^{-4}$

mat a^{-3}

Predictions of inflation

1. nearly scale invt. spectrum $|n_s - 1| \ll 1$
2. adiabatic perturbations
3. Gaussian fluctuations


Evolution after inflation
$k \ll aH$
$\frac{\partial \Phi}{\partial t} - H\Psi = \frac{3}{2}(1+w)H^2 V$
$\zeta = -\Phi - \frac{\frac{\partial \Phi}{\partial t} - H\Psi}{\frac{3}{2}(1+w)H}$
$= -\left[1 + \frac{2}{3(1+w)}\right]\Phi$
$\frac{\partial \zeta}{\partial \log a} = +\frac{2w}{3(1+w)}\left(\frac{k}{aH}\right)^2 \Phi$
$\Rightarrow \zeta \sim$ const


infl. $w \simeq -1$
$\Phi, \Psi \simeq 0$
RD $w \simeq \frac{1}{3}$
$\Phi \simeq -\frac{2}{3}\zeta$
MD $w = 0$
$\Phi \simeq -\frac{3}{5}\zeta$
$\Phi_{MD} \simeq \frac{9}{10}\Phi_{RD}$

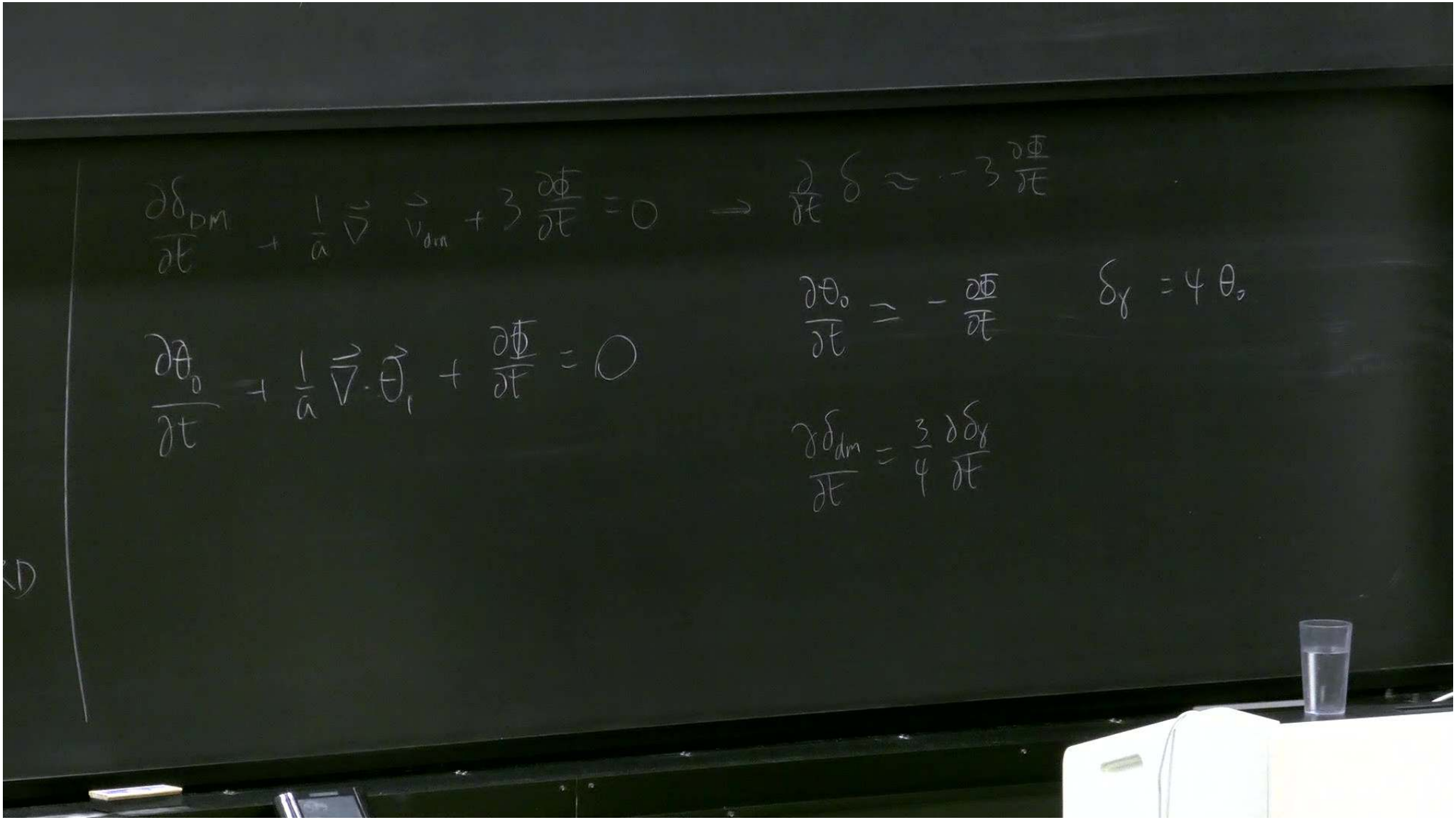
$$= -\left[1 + \frac{3(1+w)}{\quad}\right]\Phi$$

$$\frac{1}{a^2}\nabla^2\Phi + 3\left(H^2\Psi - H\frac{\partial\Phi}{\partial t}\right) = -4\pi G\,\delta\rho$$

$$3H^2\Psi = -4\pi G\,\delta\rho = -\frac{3}{2}H^2\delta_{tot} \Rightarrow \delta_{tot} = -2\Phi$$

$$\delta_{tot} = \frac{\rho_r\delta_r + \rho_m\delta_m + \cdots}{\rho_r + \rho_m + \cdots} \approx \delta_r \text{ during RD}$$

$$\delta_m = \frac{3}{4}\delta_r$$


$\frac{\partial \delta_{DM}}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\vec{v}_{dm} + 3\frac{\partial \Phi}{\partial t} = 0 \rightarrow \frac{\partial}{\partial t}\delta \approx -3\frac{\partial \Phi}{\partial t}$
$\frac{\partial \theta_0}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\vec{\theta}_1 + \frac{\partial \Phi}{\partial t} = 0$
$\frac{\partial \theta_0}{\partial t} \simeq -\frac{\partial \Phi}{\partial t}$
$\delta_\gamma = 4\theta_0$
$\frac{\partial \delta_{dm}}{\partial t} = \frac{3}{4}\frac{\partial \delta_\gamma}{\partial t}$