Title: Lecture - Cosmology, PHYS 621

Speakers: Neal Dalal

Collection/Series: Cosmology (Elective), PHYS 621, March 31 - May 2, 2025

Subject: Cosmology

Date: April 22, 2025 - 2:15 PM

URL: https://pirsa.org/25040021

Inflation

1. horizon
2. flatness
3. monopole

$$w \simeq -1 \Rightarrow a \propto e^{Ht}$$

$$\Omega_k \propto \frac{1}{a^2}$$

$$n \propto \frac{1}{a^3} \qquad T_{infl} < T_{GUT}$$

$$r_{hor} = \int d\eta = \int \frac{dt}{a} = \int \frac{da}{\dot{a}\, a} = \int \frac{da}{a^2 H} = \frac{1}{H} \int \frac{da}{a^2} = \frac{1}{H}\left(\frac{1}{a_{start}} - \frac{1}{a_{end}}\right)$$

$$\simeq \frac{1}{H a_{start}}$$

RD: $\rho_r \propto \frac{1}{a^4}$ $\quad H \propto \rho^{1/2} \propto \frac{1}{a^2} \Rightarrow r_{Hor} \propto a$

if H const.

How much inflation is needed?

today: $r_{LSS} \sim 10\,\mathrm{Gpc} \sim 3\cdot 10^{28}\,\mathrm{cm}$

assume $T_{infl} \lesssim 10^{15}\,\mathrm{GeV}$

$r_{hor} \sim \frac{1}{aH}$

$$\rho \sim g_* T^4$$

$$H^2 = \frac{\rho}{3 m_p^2} \Rightarrow N > 60 \text{ e-folds of inflation}$$

$$a_{infl} = \left(\frac{g_{*,0}}{g_{*,infl}}\right)^{1/3} \frac{T_0}{T_{infl}}$$

Slow roll scalar field inflation

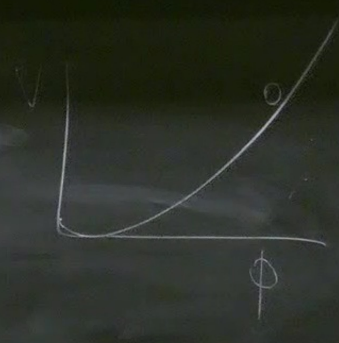
$$\phi : \quad \mathcal{L} = \frac{1}{2}|\partial\phi|^2 + V(\phi)$$

$$\Rightarrow T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi - \mathcal{L}\,g_{\mu\nu}$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + \cancel{\frac{1}{2a^2}|\nabla\phi|^2} + V(\phi)$$

$$a_{infl} = \left(\frac{g_{*,p}}{g_{*,infl}}\right)^{1/3} \frac{T_0}{T_{infl}}$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + \cancel{\frac{1}{2a^2}|\nabla\phi|^2} + V(\phi) \qquad P = \frac{1}{2}\dot{\phi}^2 - \cancel{\frac{1}{6a^2}|\nabla\phi|^2} - V(\phi)$$



need $P \simeq -\rho \Rightarrow \frac{\dot{\phi}^2}{2} \ll V(\phi)$ "slow roll"

$$\dot{\rho} + 3H(\rho + P) = 0 \Rightarrow \boxed{\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0}$$

small $\dot{\phi}$ require small V'

$$H^2 = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}\left(\frac{\dot\phi^2}{2} + V\right)$$

$$2H\dot H = \frac{8\pi G}{3}\left(\ddot\phi + V'\right)\dot\phi = \frac{8\pi G}{3}\left(-3H\dot\phi\right)\dot\phi = -8\pi G H\dot\phi^2$$

$$\dot H = -4\pi G\dot\phi^2 \Rightarrow |\dot H| \ll H^2 \rightarrow \frac{d}{dt}\left(\frac{1}{H}\right) \ll 1$$

$$\varepsilon = \frac{|\dot H|}{H^2} = \frac{d}{dt}\left(\frac{1}{H}\right) \ll 1 \qquad \varepsilon = \frac{3}{2}(1+w) \ll 1$$

$$\frac{\dot\phi^2}{2} \ll$$

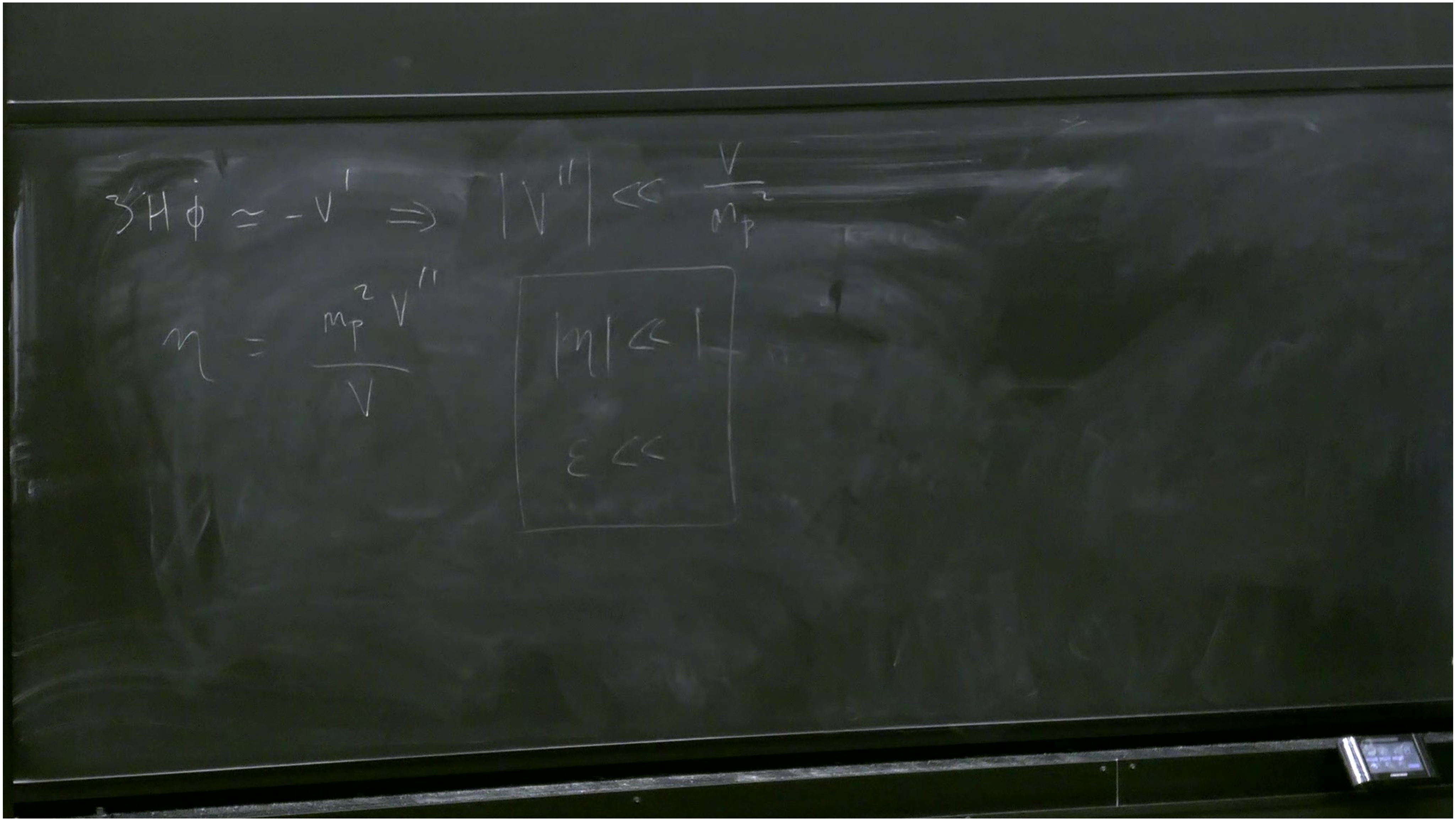
$$\frac{\dot{\phi}^2}{2} \ll V(\phi)$$

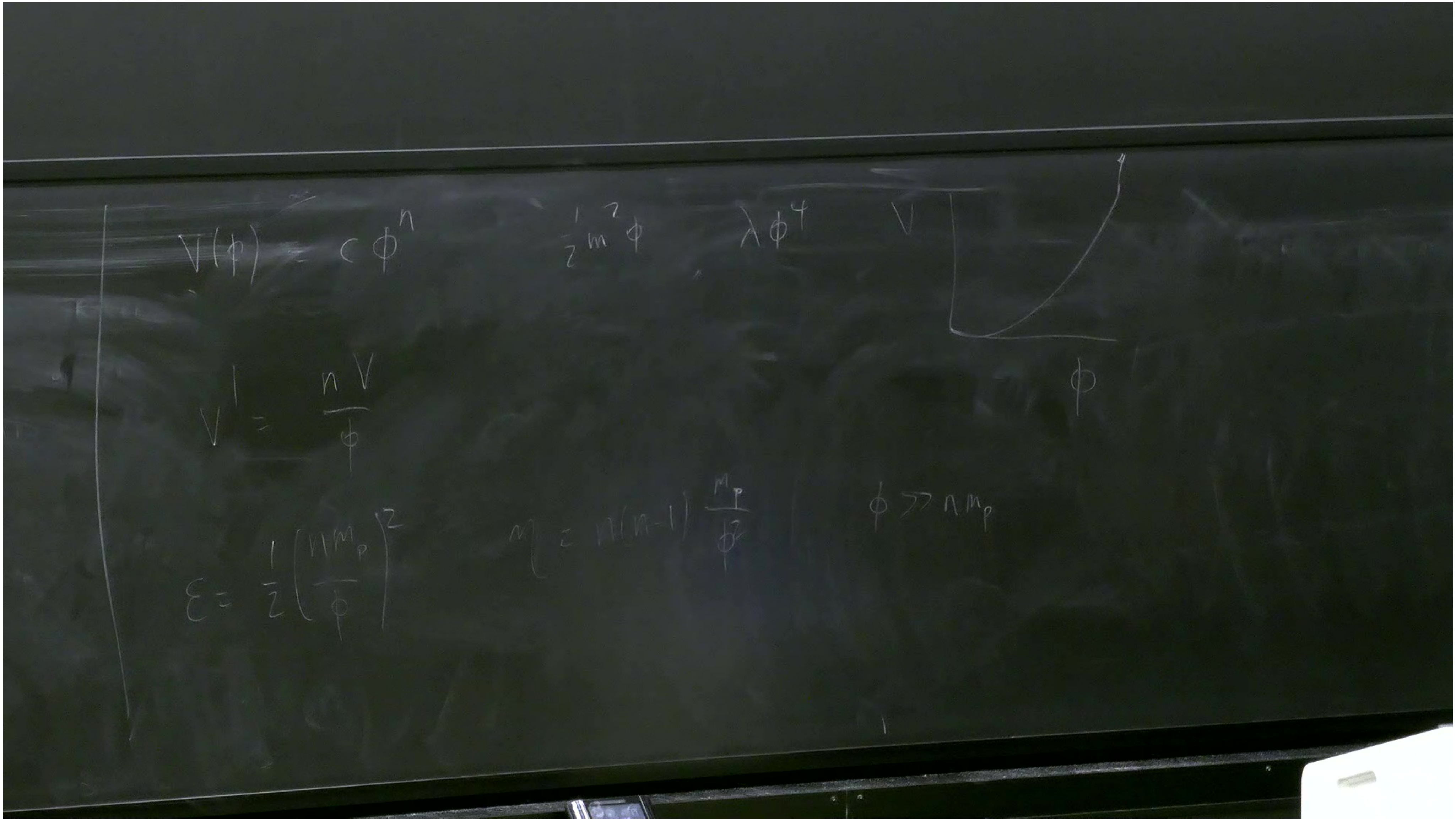
$$\ddot{\phi}\,\dot{\phi} \ll V'\dot{\phi}$$

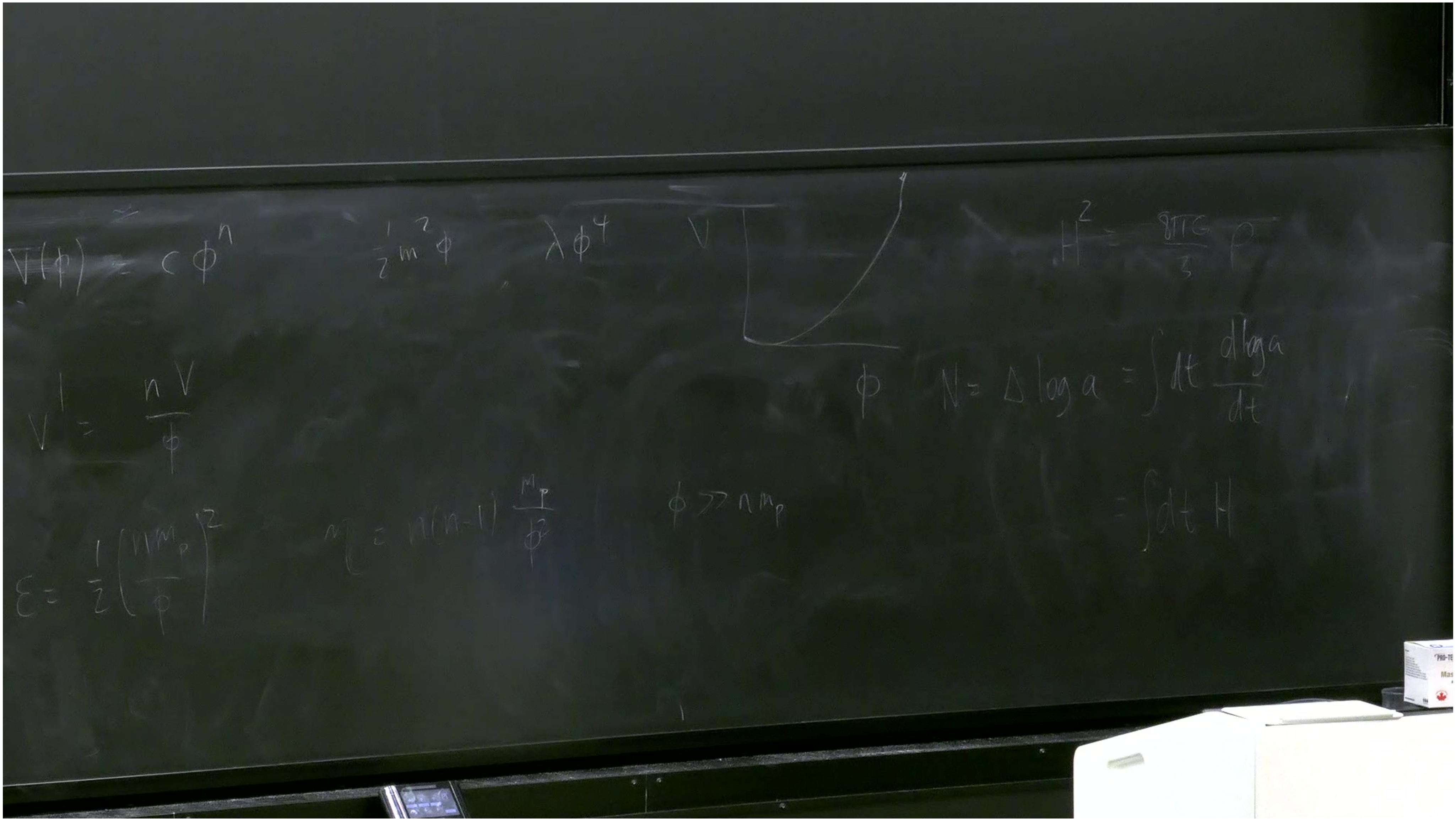
$$|\ddot{\phi}| \ll |V'|,\ |3H\dot{\phi}|$$

$$3H\dot{\phi} + V' \simeq 0 \Rightarrow \dot{\phi}^2 = \left(\frac{V'}{3H}\right)^2 = -\frac{\dot{H}}{4\pi G} = \frac{\epsilon H^2}{4\pi G}$$

$$\epsilon = -\frac{\dot{H}}{H^2} = 4\pi G\left(\frac{V'}{3H^2}\right)^2 = \frac{1}{16\pi G}\left(\frac{V'}{V}\right)^2 = \frac{1}{2}\left(\frac{m_p V'}{V}\right)^2 \ll 1$$


$3H\dot{\phi} \simeq -V' \Rightarrow |V''| \ll \frac{V}{m_p^2}$
$\eta = \frac{m_p^2 V''}{V}$
$|\eta| \ll 1$
$\varepsilon \ll$


$V(\phi) = c\phi^n$
$\frac{1}{2} m^2 \phi$
$\lambda \phi^4$
V
ϕ
$V' = \frac{nV}{\phi}$
$\varepsilon = \frac{1}{2}\left(\frac{n m_p}{\phi}\right)^2$
$\eta = n(n-1)\frac{m_p}{\phi^2}$
$\phi >> n m_p$

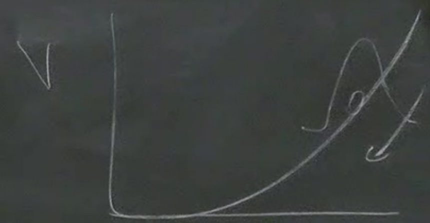

$V(\phi) = c\,\phi^n$
$\frac{1}{2} m^2 \phi^2$
$\lambda \phi^4$
V
ϕ
$H^2 = \frac{8\pi G}{3} \rho$
$N = \Delta \log a = \int dt \frac{d \log a}{dt}$
$= \int dt\, H$
$V' = \frac{n V}{\phi}$
$\phi \gg n\, m_p$
$\eta = n(n-1) \frac{m_p^2}{\phi^2}$
$\varepsilon = \frac{1}{2}\left(\frac{n\, m_p}{\phi}\right)^2$

$\epsilon = 2(\phi$

$$3H\dot{\phi} \simeq -V'$$

$$N = \int d\phi \, \frac{H}{\dot{\phi}} = \int d\phi \, \frac{3H^2}{V'} = \frac{1}{m_p^2} \int d\phi \, \frac{V}{V'} \simeq \frac{1}{n} \int_{\phi_{end}}^{\phi_{start}} d\phi \, \frac{\phi}{m_p^2} \simeq \frac{1}{2n} \left(\frac{\phi}{m_p} \right)^2 \geq 60$$

Perturbations



H.O. $\ddot{x} + \omega^2 x = 0$

$x = v(t)\, a + v^*(t)\, a^\dagger$

$v(t) \propto e^{\pm i\omega t}$

$[x, p] = i \Rightarrow |v|^2 = \frac{1}{2\omega}$

$v(t) = \frac{e^{-i\omega t}}{\sqrt{2\omega}}$

$\langle x^2$

$a_{infl} = \left(g_{*,infl} \right) T_{infl}$

$$\boxed{\langle x^2 \rangle = |v|^2 = \frac{1}{2\omega}}$$

$\phi = \phi_0 + \delta\phi$

$v^*(t)\, a^\dagger$

$\Rightarrow |v|^2 = \frac{1}{2\omega}$

$$\delta\ddot{\phi} + 3H\,\delta\dot{\phi} - \nabla^2\delta\phi + V''\delta\phi = 0$$

spatially flat: $ds^2 = -(1+2A)\,dt^2 - 2a\,B_i\,dx^i dt + a^2 \delta_{ij}\,dx^i dx^j$

$$\eta = \int \frac{dt}{a} = \int_{a_0}^{a} \frac{da}{a^2 H} = \frac{1}{a_0 H} - \frac{1}{aH}$$

$$\tau \equiv \eta - \eta_{end} = \frac{1}{a_{end} H} - \frac{1}{aH} \simeq - \frac{1}{aH}$$

$$|\tau| = \frac{1}{aH}$$

$$a(t) = \frac{e^{\cdots}}{\sqrt{2\omega}}$$

$$\int_{a_0}^{a} \frac{da}{a^2 H} = \frac{1}{a_0 H} - \frac{1}{aH}$$

$$= \frac{1}{a_{end} H} - \frac{1}{aH} \simeq -\frac{1}{aH}$$

$$\frac{1}{aH}$$

$$\frac{d^2 \delta\phi}{d\tau^2} - \frac{2}{\tau}\frac{d\delta\phi}{d\tau} - \nabla^2 \delta\phi = 0$$

$$\delta\phi = \frac{x}{a} \Rightarrow \frac{d^2 x}{d\tau^2} + \left(k^2 - \frac{2}{\tau^2}\right) x = 0$$

spatially flat: $ds^2 = -(1+2A)\,dt^2 - 2a B_i\, dx^i dt + a\, \delta_{ij}\, dx\, dx^j$

$$\nabla^2 \delta\phi = 0$$

$$\left(k^2 - \frac{2}{\tau^2}\right)\chi = 0$$

$$v(\tau) = \frac{e^{-i\omega\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)$$