Title: Lecture - Cosmology, PHYS 621

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Collection/Series: Cosmology (Elective), PHYS 621, March 31 - May 2, 2025

Subject: Cosmology

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Continuity

$$\frac{\partial \delta_d}{\partial t} = \left(\frac{k}{a}\right)^2 V_d - 3\frac{\partial \Phi}{\partial t}$$

$$\frac{\partial \delta_b}{\partial t} = \left(\frac{k}{a}\right)^2 V_b - 3\frac{\partial \Phi}{\partial t}$$

$$\frac{\partial \Theta_0}{\partial t} = \left(\frac{k}{a}\right)^2 w_1 - \frac{\partial \Phi}{\partial t}$$

Euler

$$\frac{\partial V_d}{\partial t} = \Phi$$

$$\frac{\partial V_b}{\partial t} = \Phi + \frac{4\rho_\gamma}{3\rho_b} n_e \sigma_T \left(3w_1 - \right.$$

$$\frac{\partial w_1}{\partial t} = H w_1 + \frac{\Phi - \Theta_0}{3} + n_e \sigma_T \left(\right.$$

Poisson

$$\frac{1}{a^2}\nabla^2\Phi + 3\left(H^2\Psi - H\frac{\partial\Phi}{\partial t}\right) = -4\pi G\,\delta\rho$$

$$- V_b)$$

$$\Psi = -\Phi$$

$$\left(\frac{V_b}{3} - w_1\right)$$

$- V_b)$

$+ \left(\frac{V_b}{3} - w_r\right)$

Poisson

$$\frac{1}{a^2}\nabla^2\Phi + 3\left(H^2\Psi - H\frac{\partial\Phi}{\partial t}\right) = -4\pi G\,\delta\rho$$

$$\Psi = -\Phi$$

$$\delta\rho = \bar{\rho}_d\delta_d + \bar{\rho}_b\delta_b + 4\bar{\rho}_r\Theta_0 + \dots$$

$$\frac{\partial\Phi}{\partial t} = H\left[\frac{1}{2}\left(\Omega_d\delta_d + \Omega_b\delta_b + 4\Omega_r\Theta_0 + \dots\right) - \left\{1 + \frac{1}{3}\left(\frac{k}{aH}\right)^2\right\}\right]\Phi$$

Solve numerically

$V_b \simeq 3\,\omega_1$

$$\Delta t < \frac{1}{n_e \sigma_T} \ll \frac{1}{H}$$

Tight coupling

$$V_b = 3\omega_1$$

$$V_b = 3\omega_1 - \quad \cdot \frac{R}{n_e \sigma_T}\left(3\frac{\partial \omega_1}{\partial t} - \Phi\right)$$

$$R = \frac{3\rho_b}{4\rho_\gamma}$$

$$V_b = 3\omega_1$$

$$\ll \frac{1}{H}$$

$$V_b = 3\omega_1 - \frac{R}{n_e \sigma_T}\left(3\frac{\partial \omega_1}{\partial t} - \Phi\right)$$

$$V_b = 3\omega_1 - \frac{R}{1+R}\frac{1}{n_e \sigma_T}\left(3\omega_1 - \theta_0\right)$$

$$\frac{\partial \omega_1}{\partial t} =$$

$$\rightarrow \frac{\partial \omega_1}{\partial t}$$

$$\cdot \frac{R}{n_e \sigma_T}\left(3\frac{\partial w_1}{\partial t} - \Phi\right)$$

$$\frac{\partial w_1}{\partial t} = H w_1 + \frac{\Phi - \Theta_0}{3} - R\left(\frac{\partial w_1}{\partial t} - \frac{\Phi}{3}\right)$$

$$\rightarrow \frac{\partial w_1}{\partial t} = \frac{H w_1}{1+R} + \frac{1}{3}\left[\Phi - \frac{\Theta_0}{1+R}\right]$$

$$t \to \log a \qquad \frac{\partial}{\partial t} \to H \frac{\partial}{\partial \log a}$$

$$V \to HV$$

$$\frac{k}{aH}$$

$\to \log a$ $\qquad \frac{\partial}{\partial t} \to H \frac{\partial}{\partial \log a}$

$V \to HV$

$\frac{k}{aH}$

CMB anisotropies

$$\frac{\Delta T}{T} = \Theta(\vec{x}, \hat{p}, t)$$

$$\Theta(x=0, \hat{p}, a=1)$$

$n_e \sigma_T ($

$$\frac{\partial \Theta}{\partial t} + \frac{\hat{p}}{a} \cdot \vec{\nabla}(\Theta + \Psi) + \frac{\partial \Phi}{\partial t} = n_e \sigma_T \left[\Theta_0 - \Theta(\vec{p}) + \vec{v}_b \cdot \hat{p} \right]$$

$$\frac{\partial \Theta}{\partial t} + \frac{\hat{p}}{a} \cdot i\vec{k}\Theta + n_e \sigma_T \Theta = -\frac{\hat{p}}{a} \cdot i\vec{k} \Psi - \frac{\partial \Phi}{\partial t} + n_e \sigma_T \left[\Theta_0 + \vec{v}_b \cdot \hat{p} \right]$$

$= 1)$

$n_e \sigma_T ($

$$\frac{\partial \Theta}{\partial t} + \frac{\hat{p}}{a} \cdot \vec{\nabla}(\Theta + \Psi) + \frac{\partial \Phi}{\partial t} = n_e \sigma_T \left[\Theta_0 - \Theta(\hat{p}) + \vec{v}_b \cdot \hat{p} \right]$$

$$\dot{\Theta} + \hat{p} \cdot i\vec{k}\,\Theta - \dot{\tau}\,\Theta = -\hat{p} \cdot i\vec{k}\,\Psi - \dot{\Phi} - \dot{\tau} \left[\Theta_0 + \vec{v}_b \cdot \hat{p} \right]$$

$= 1)$

$\frac{\partial}{\partial \eta}$

$\dot{} \equiv \frac{\partial}{\partial \eta}$

$n_e \sigma_T ($

$$\frac{\partial \Theta}{\partial t} + \frac{\hat{p}}{a} \cdot \vec{\nabla}(\Theta + \Psi) + \frac{\partial \Phi}{\partial t} = n_e \sigma_T \left[\Theta_0 - \Theta(\vec{p}) + \vec{v}_b \cdot \hat{p} \right]$$

$$\Theta e^{i\vec{k}\cdot\hat{p}\eta_0} = \int d\eta \left(\left[- \hat{p} \cdot i\vec{k} \Psi - \dot{\Phi} - \dot{\tau} \left[\Theta_0 + \vec{v}_b \cdot \hat{p} \right] \right] \right) \times e^{i\vec{k}\cdot\hat{p}\eta - \tau}$$

$$\int_0^{\eta_0} d\eta \; e^{i\vec{k}\cdot\hat{p}(\eta-\eta_0)-\tau} \, (-i\vec{k}\cdot\hat{p}\,\Psi) = -\Psi + \int_0^{\eta_0} d\eta \; e^{i\vec{k}\cdot\hat{p}(\eta-\eta_0)}$$

$$\int u\,dv = uv - \int v\,du$$

$$du = (\dot{\Psi} - \Psi\dot{\tau})e^{-\tau}d\eta$$

$$u = \Psi e^{-\tau}$$

$$v = -e^{i\vec{k}\cdot\hat{p}(\eta-\eta_0)}$$

$$dv = -i\vec{k}\cdot\hat{p}\; e^{i\vec{k}\cdot\hat{p}(\eta-\eta_0)}\,d\eta$$

$$\int_0^{\eta_0} d\eta\; e^{i\vec{k}\cdot\hat{p}(\eta-\eta_0)-\tau}\,\left(-i\vec{k}\cdot\hat{p}\,\Psi\right) = -\Psi + \int_0^{\eta_0} d\eta\; e^{i\vec{k}\cdot\hat{p}(\eta-\eta_0)}$$

$$= -\Psi + \int_0^{\eta_0} d\eta\; e^{i\vec{k}\cdot\hat{p}(\eta-\eta_0)}\Big[\Big($$

$$g(\eta) = \frac{d}{d\eta}e^{-\tau} = -\dot{\tau}e^{-\tau}$$

$$\int_0^{\eta_0} d\eta \, e^{i\vec{k}\cdot\hat{p}(\eta-\eta_0)-\tau} \cdot \left(-i\vec{k}\cdot\hat{p}\,\Psi\right) = -\Psi + \int_0^{\eta_0} d\eta \, e^{i\vec{k}\cdot}$$

$$= -\Psi + \int_0^{\eta_0} d\eta \, e^{i\vec{k}\cdot\hat{p}(\eta}$$

$$g(\eta) = \frac{d}{d\eta} e^{-\tau} = -\dot{\tau} e^{-\tau}$$

$$-\dot{\tau} = \frac{n_e \sigma_T}{a} \qquad e^{-\tau}$$

before recomb: g small

after recomb: g small

$$\int_0^{\eta_0} d\eta\, e^{i\vec{k}\cdot\hat{p}(\eta-\eta_0)-\tau}\,(-i\vec{k}\cdot\hat{p}\,\Psi) = -\Psi + \int_0^{\eta_0} d\eta\, e^{i\vec{k}\cdot}$$

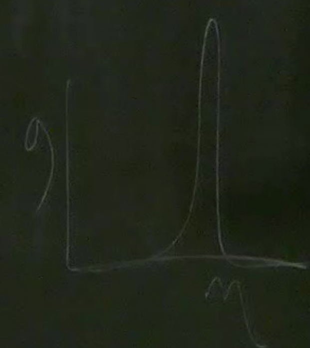
$$= -\Psi + \int_0^{\eta_0} d\eta\, e^{i\vec{k}\cdot\hat{p}(\eta}$$

$$g(\eta) = \frac{d}{d\eta} e^{-\tau} = -\dot{\tau} e^{-\tau}$$

$$-\dot{\tau} = \frac{n_e \sigma_T}{a} \qquad e^{-\tau}$$

before recomb'n g small

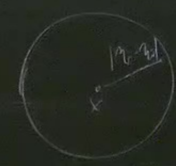
after recomb: g small



$$\Theta = -\Psi + e^{i\vec{k}\cdot\hat{p}(\eta_r-\eta_0)}\left[\Theta_0(\eta_r)+\Psi(\eta_r)+3\vec{\Theta}_1(\eta_r)\cdot\hat{p}\right] + \int_0^{\eta_0} d\eta\; e^{i\vec{k}\cdot\hat{p}(\eta-\eta_0)}\left[\dot{\Psi}-\dot{\Phi}\right]e^{-\tau}$$

MD: Ψ, Φ

$$\int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \qquad e^{i\vec{k}\cdot(\vec{x}+\hat{p}(\eta_r-\eta_0))}$$



$$\vec{x}_\eta = \vec{x} + \hat{p}(\eta-\eta_0)$$

$$\boxed{\Theta(\vec{x},\hat{p}) = -\Psi(\vec{x}) + \left[\Theta_0(\vec{x}_*)+\Psi(\vec{x}_*)+\underbrace{3\vec{\Theta}_1(\vec{x}_*)\cdot\hat{p}}_{\text{"Doppler"}}\right] + \underbrace{\int_0^{\eta_0} d\eta\left[\dot{\Psi}(\vec{x}_\eta)-\dot{\Phi}(\vec{x}_\eta)\right]e^{-\tau(x_\eta)}}_{\text{ISW}}}$$

$$\int_0^{\eta_0} d\eta \; e^{i\vec{k}\cdot\hat{p}(\eta-\eta_0)}\left[(\dot{\Psi}-\dot{\Phi})\,e^{-\tau}\right]$$

$$\vec{v}_b \simeq 3\vec{\Theta}_1$$

$$\vec{x}_\ell = \vec{x} + \hat{p}(\eta-\eta_0)$$

$$\left[\dot{\Psi}(\vec{x}_\ell) - \dot{\Phi}(\vec{x}_\ell)\right] e^{-\tau(x_\ell)}$$

MD: $\Psi, \Phi \sim$ const

1. at recomb. $z \sim 10^3$ $\Rightarrow$ early ISW
2. dark energy dom. $z < 1$ late ISW

Initial conditions

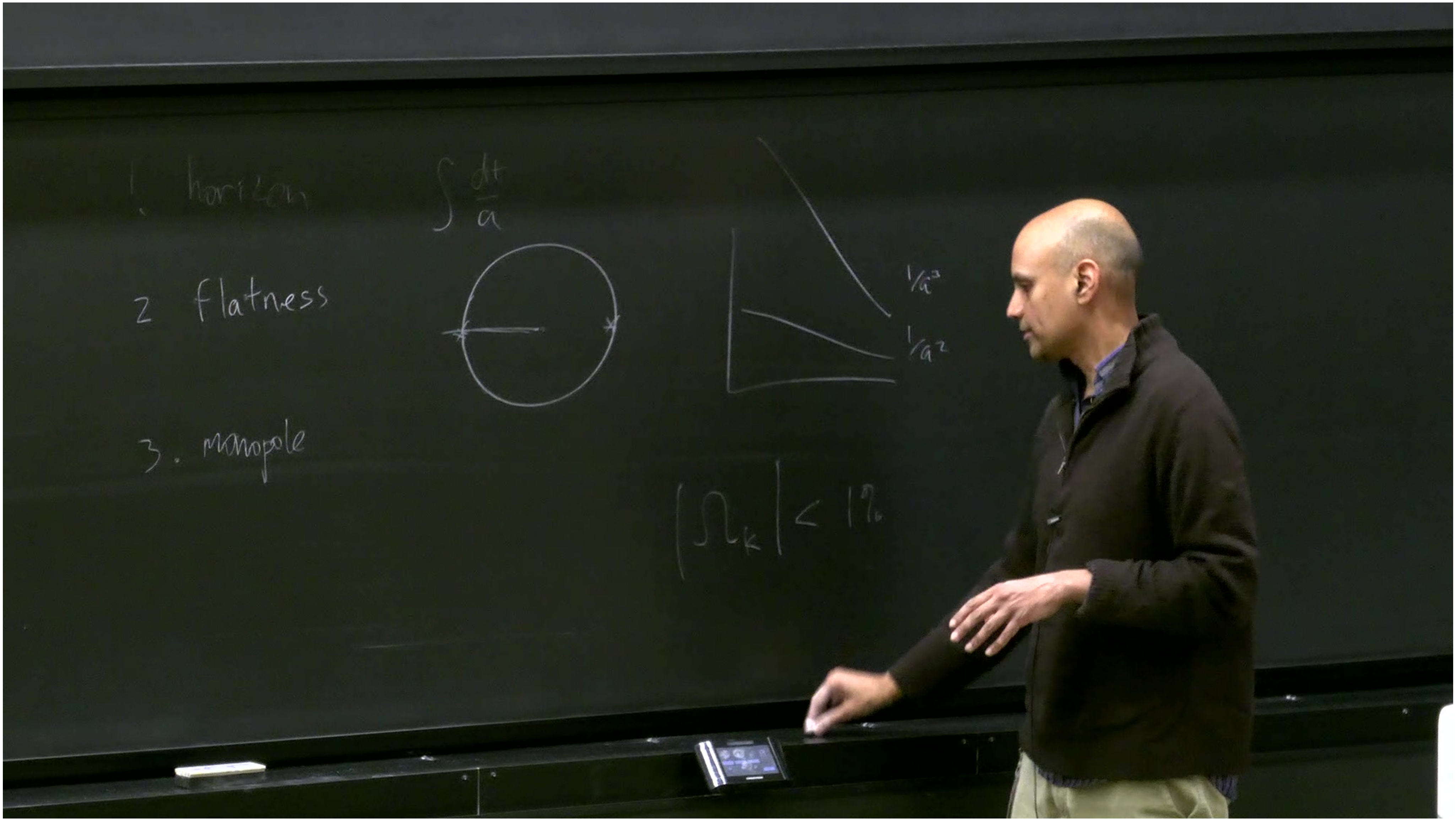
1. horizon

2. flatness

3. monopole

Inflation

$$w = -1 \Rightarrow a \propto e^{Ht}$$


1. horizon
$\int \frac{dt}{a}$
2 flatness
3. monopole
$1/a^3$
$1/a^2$
$|\Omega_k| < 1\%$