Title: Lecture - Cosmology, PHYS 621

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Subject: Cosmology

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baryons

$$\frac{df}{dt} = C[f]$$

$$\int \frac{d^3p}{(2\pi)^3} E \longrightarrow \quad \frac{\partial \rho_b}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\vec{q}_b + 3\left(H + \frac{\partial \Phi}{\partial t}\right)(\rho_b + P_b) = 0$$

baryons

$$\frac{df}{dt} = C[f]$$

$$\int \frac{d^3p}{(2\pi)^3}\, p\hat{p} \;\Rightarrow\; \frac{\partial \vec{q}_b}{\partial t} + 4H\vec{q}_b + \frac{1}{a}\vec{\nabla}\cdot\mathbb{T} + \frac{1}{a}\vec{\nabla}\Psi(\rho_b + P_b) = \frac{4}{3}\rho_\gamma n_e \sigma_T (3\vec{\Theta}_1 - \vec{v}_b)$$

$$\dot{\vec{v}} + \frac{1}{a}\vec{\nabla}\Psi(\rho_b + P_b) = \frac{4}{3}\rho_\gamma n_e \sigma_T (3\vec{\Theta}_1 - \vec{v}_b) \Rightarrow \frac{\partial \vec{v}_b}{\partial t} + H\vec{v}_b + \frac{1}{a}\vec{\nabla}\Psi = \frac{4\rho_\gamma}{3\rho_b} n_e \sigma_T \left(3\vec{\Theta}_1 - \vec{v}_b\right)$$

$\rho_b \vec{v}_b$

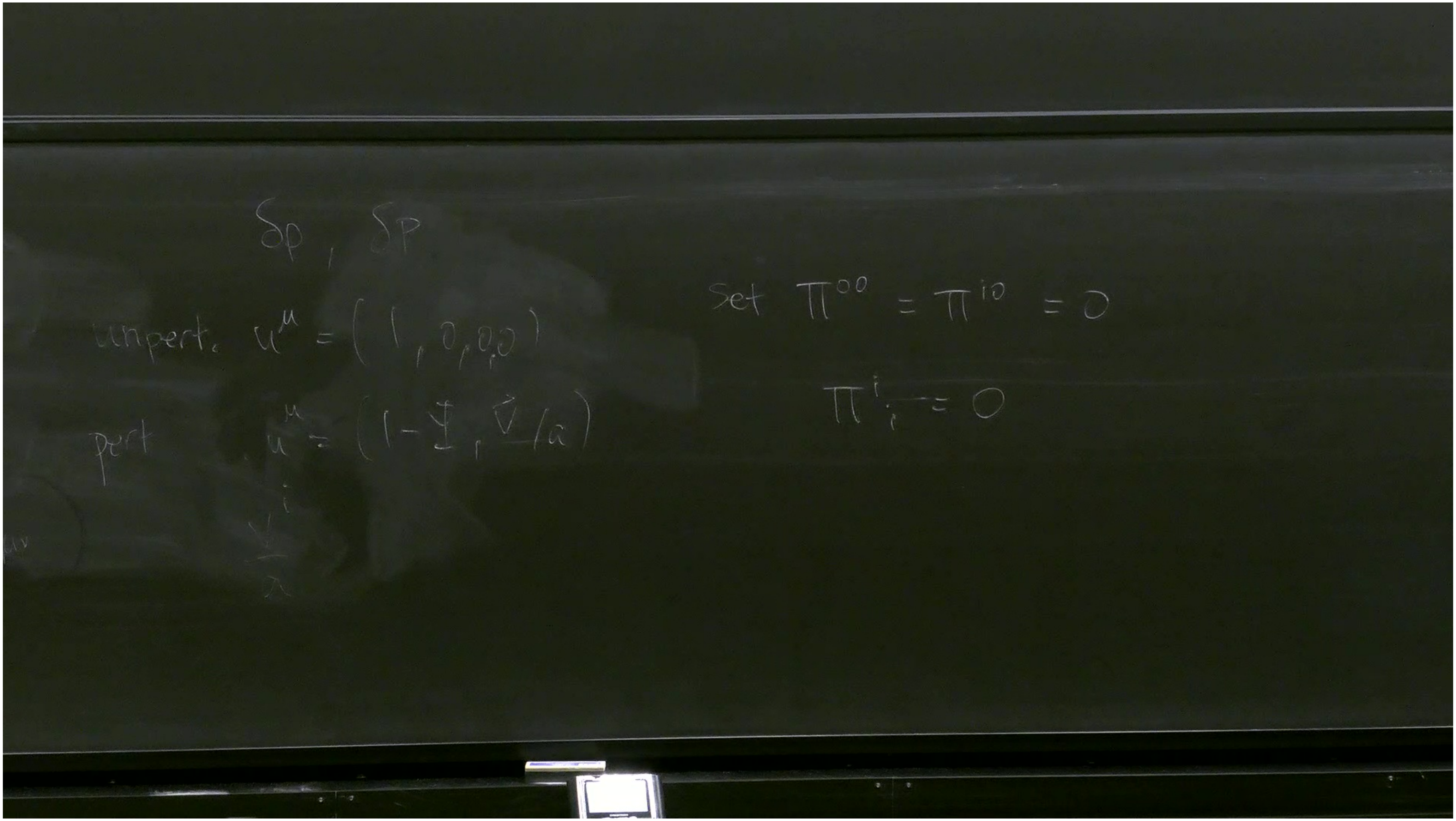
$$\frac{\partial \rho_b}{\partial t} + 3H\rho_b = 0$$

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

$$\delta\rho\,,\ \delta P$$

$$T^{\mu\nu} = g \int \frac{d^3p}{(2\pi)^3} f(\vec{p})\, \frac{p^\mu p^\nu}{E}$$

$$T^{\mu\nu} = (\rho + P)\, u^\mu u^\nu + P g^{\mu\nu} + \Pi^{\mu\nu}$$


δρ , δp
unpert. u^μ = (1, 0, 0, 0)
pert u^μ = (1 − Ψ, v⃗/a)
Set Π^00 = Π^i0 = 0
Π^i_i = 0

$\delta\rho\ ,\ \delta p$

unpert. $u^\mu = (1, 0, 0, 0)$

pert $u^\mu = (1-\Psi, \vec{v}/a)$

$\frac{v^i}{a}$

scalar: $\vec{v} = \frac{1}{a} \nabla$

$\vec{\nabla}$

Set $\Pi^{00} = \Pi^{i0} = 0$

$\Pi^i{}_i = 0$

$\frac{1}{a^2}\left(\partial_i \partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right) Q$

$a)$

$= \frac{1}{a} V$

Set $\Pi^{00} = \Pi^{i0} = 0$

$\Pi^i_i = 0$

$\frac{1}{a^2}\left(\partial_i \partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right) Q$

$\delta T^0_0 = -\delta\rho$

$\delta T^0_i = (\rho_0 + P_0)\, V_i$

$\delta T^i_j = \delta P\, \delta^i_j + \Pi^i_j$

$$\delta G^0{}_0 = \frac{2}{a^2}\nabla^2\Phi + 6\left(H^2\Psi - H\frac{\partial\Phi}{\partial\tau}\right)$$

$$= 8\pi G\, T^0{}_0 \implies \frac{1}{a^2}\nabla^2\Phi + 3\left(H^2\Psi - H\frac{\partial\Phi}{\partial\tau}\right) = -4\pi G\,\delta\rho$$

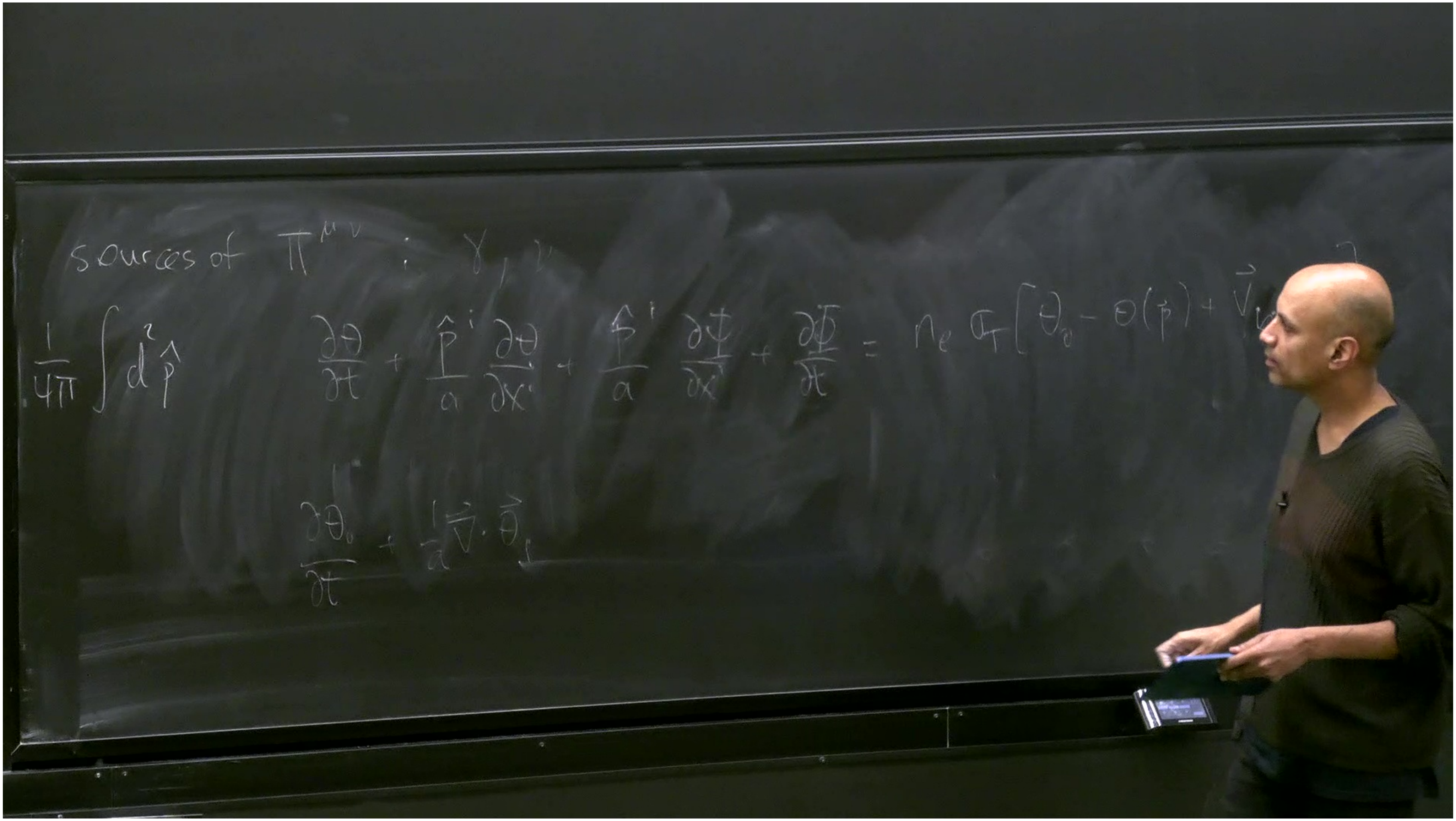
$$\frac{1}{a^2}\nabla^2\Phi = -4\pi G\left[\delta\rho - 3H(\rho_0 + P_0)V\right]$$

$$\delta G^0{}_i = \frac{2}{a}\frac{\partial}{\partial x^i}\left[\frac{\partial \Phi}{\partial t} - H\Psi\right] = 8\pi G T^0{}_i = 8\pi G(\rho_0 + P_0)\,\frac{1}{a}\frac{\partial V}{\partial x^i}$$

$$\frac{\partial \Phi}{\partial t} - H\Psi = 4\pi G(\rho_0 + P_0)V$$

$$\delta G^i{}_j = 8\pi G\,\delta T^i{}_j \Longrightarrow \frac{1}{a^2}\nabla^2(\Phi + \Psi) = 8\pi G\,\frac{1}{a}\nabla^2 Q$$

$$\Rightarrow \Phi + \Psi = 8\pi G Q \Rightarrow \Phi = -\Psi$$ if no anisotropic stress


sources of $\pi^{\mu\nu}$: γ, ν
$\frac{1}{4\pi}\int d^2\hat{p}$
$\frac{\partial\Theta}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial\Theta}{\partial x^i} + \frac{\hat{p}^i}{a}\frac{\partial\Psi}{\partial x^i} + \frac{\partial\Phi}{\partial t} = n_e\sigma_T\left[\Theta_0 - \Theta(\vec{p}) + \vec{v}\right.$
$\frac{\partial\Theta_0}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\vec{\Theta}_1$

sources of $\Pi^{\mu\nu}$: γ, ν

$$\frac{1}{4\pi}\int d^2\hat{p}\;\hat{p}\,\hat{p}\;\; \frac{\partial\Theta}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial\Theta}{\partial x^i} + \frac{\hat{p}^i}{a}\frac{\partial\Psi}{\partial x^i} + \frac{\partial\Phi}{\partial t} = n_e\sigma_T\left[\Theta_0 - \Theta(\hat{p}) + \vec{v}_b\cdot\hat{p}\right]$$

$$\frac{\partial\vec{\Theta}_1}{\partial t} - \frac{1}{3a}\vec{\nabla}\left(\Theta_0 + \Psi\right) + \frac{1}{a}\vec{\nabla}\cdot\vec{\vec{\Theta}}_2 = n_e\sigma_T\left(\frac{\vec{v}_b}{3} - \vec{\Theta}_1\right) \qquad \sim \frac{\Theta_1}{\lambda}$$

$$\frac{\partial\Theta_2}{\partial t} \qquad \nabla\Theta_1 \qquad \nabla\cdot\Theta_3 \qquad = -n_e\sigma_T\Theta_2$$

$l \gg l_{mfp}$

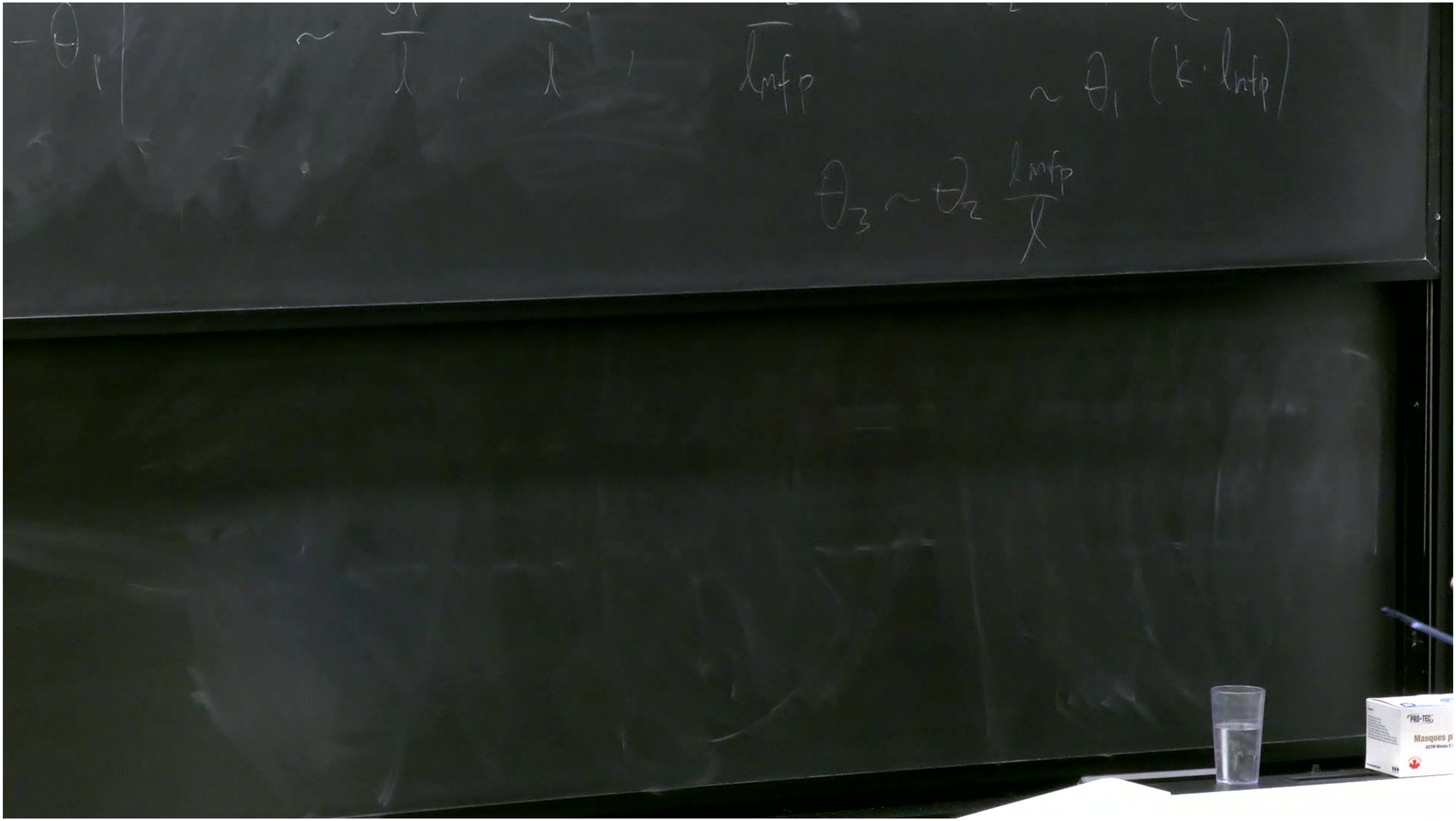
$l \ll l_{mfp}$ streaming $\Rightarrow$ large Θ_2

$\Theta_0 - \Theta(\vec{p}) + \vec{V}_b \cdot \hat{p} \,]$

$\Theta_2 \sim \Theta_1 \cdot \frac{l_{mfp}}{l}$

$\sim \Theta_1 \left(k \cdot l_{mfp}\right)$

$- \vec{\Theta}_1 \Big) \sim \frac{\Theta_1}{\lambda}, \quad \frac{\Theta_3}{\lambda}, \quad \frac{\Theta_2}{l_{mfp}}$


$\sim \theta_1 (k \cdot l_{mfp})$
$\theta_3 \sim \theta_2 \frac{l_{mfp}}{\ell}$

$$\Pi_{ij} = \frac{1}{a^2}\left(\nabla_i \nabla_j - \frac{1}{3}\delta_{ij}\nabla^2\right) Q$$

$$= -\frac{k^2}{a^2}\left(\hat{k}_i \hat{k}_j - \frac{1}{3}\delta_{ij}\right) Q$$

photons: $\Pi^i{}_j \simeq 0$ until recombination $(z \sim 10^3)$

neutrinos

$-\frac{1}{3}\delta_{ij}\nabla^2\Big) Q$

$-\frac{1}{3}\delta_{ij}\Big) Q$

recombination ($z\sim 10^3$)

neutrinos : $\Pi^i_j \simeq 0$ for $T \gtrsim 1$ MeV

$\Pi^i_j \neq 0$ $T <$ MeV

$\Rightarrow \Phi \neq -\Psi$

$\simeq -\Psi\left(1+\frac{2}{5}\Omega_\nu\right)$

$z<100$ $\Phi \simeq -\Psi$

coupled, linear PDE's

$\delta_d \,,\, \delta_b \,,\, \theta_0 \,,\, \ldots$

$\vec{v}_d \,,\, \vec{v}_b \,,\, \vec{\theta}_1 \,,\, \ldots$

$\vec{v}_d = \frac{1}{a} \vec{\nabla} V$

$\vec{\theta}_1 = \frac{1}{a} \vec{\nabla} w_1$

$\frac{\partial \delta_d}{\partial t} + \frac{1}{a} \vec{\nabla}^2 V_d + 3$

PDE's

$$\vec{V}_d = \frac{1}{a}\vec{\nabla} V$$

$$\vec{\Theta}_1 = \frac{1}{a}\vec{\nabla} w_1$$

$$\frac{\partial \delta_d}{\partial t} + \frac{1}{a^2}\nabla^2 V_d + 3\frac{\partial \Phi}{\partial t} = 0$$

$$\frac{\partial \delta_d(\vec{k})}{\partial t} - \frac{k^2}{a^2} V_d(\vec{k}) + 3\frac{\partial \Phi}{\partial t} = 0$$

Different k's independent!

$= \frac{1}{a} \vec{\nabla} V$

$$\frac{\partial \delta_d}{\partial t} + \frac{1}{a^2} \nabla^2 V_d + 3 \frac{\partial \Phi}{\partial t} = 0$$

$= \frac{1}{a} \vec{\nabla} w_1$

$$\frac{\partial \delta_d(\vec{k})}{\partial t} - \frac{k^2}{a^2} V_d(\vec{k}) + 3 \frac{\partial \Phi}{\partial t} = 0$$

Different $\vec{k}$'s independent! $\Rightarrow$ solve each $\vec{k}$ separately