Title: Lecture - Cosmology, PHYS 621

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Subject: Cosmology

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$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i} \cdot \frac{p\hat{p}^i}{aE} - \frac{\partial f}{\partial E}\left[\frac{p\hat{p}^i}{a}\frac{\partial \Psi}{\partial x^i} + \frac{p^2}{E}\left(H + \frac{\partial \Phi}{\partial t}\right)\right]$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i}\frac{p\hat{p}^i}{aE} - \frac{\partial f}{\partial E}\left[\frac{p\hat{p}^i}{a}\frac{\partial \Psi}{\partial x^i} + \frac{p^2}{E}\left(H + \frac{\partial \Phi}{\partial t}\right)\right]$$

$$g\int \frac{d^3p}{(2\pi)^3} \Rightarrow \frac{\partial n}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot(n_0\vec{v})$$

$$\frac{1}{a}\frac{\partial}{\partial x^i}\underbrace{\int \frac{d^3p}{(2\pi)^3} f \frac{p\hat{p}^i}{E}}_{\equiv n v^i}$$

$$\frac{1}{a}\frac{\partial \Psi}{\partial x}\int \frac{\partial f_0}{\partial E} p\hat{p}^i \frac{d^3p}{(2\pi)^3}$$

$$= 0$$

$$-\left(H + \frac{\partial \Phi}{\partial t}\right)$$

$$H + \frac{\partial\Phi}{\partial t}\Big)\Big] = C[f]$$

$= 0$

$$-\left(H + \frac{\partial\Phi}{\partial t}\right) g \int \frac{d^3p}{(2\pi)^3} \frac{\partial f}{\partial p}\, p$$

$$- \int 3p^2\, dp\, d\hat{p}^2 f$$

$$3\left(H + \frac{\partial\Phi}{\partial t}\right) n$$

$$\frac{\partial f}{\partial E} = \frac{\partial f}{\partial p}\frac{dp}{dE} = \frac{\partial f}{\partial p}\frac{E}{p}$$

$$\boxed{\frac{\partial n}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot(n_0\vec{v}) + 3\left(H + \frac{\partial\Phi}{\partial t}\right)n = 0}$$

$$\frac{\partial n_0}{\partial t} + 3Hn_0 = 0 \Rightarrow n_0 \propto a^{-3}$$

$$a^2(1+2\Phi)\,\delta_{ij}\,dx^i dx^j$$

$$n = n_0(1+\delta)$$

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\vec{v} + 3\frac{\partial \Phi}{\partial t} = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\vec{q} + 3\left(H + \frac{\partial \Phi}{\partial t}\right)(\rho + P) = 0$$

$$\vec{q} = (\rho_0 + P_0)\vec{v}$$

$$\frac{\partial \delta}{\partial t} + ($$

$$\rho = \rho_0$$

$$\frac{\partial \delta}{\partial t} + (1+w)\left[\frac{1}{a}\vec{\nabla}\cdot\vec{v} + 3\frac{\partial \Phi}{\partial t}\right] + 3H\left(c_s^2 - w\right)\delta = 0$$

$$w \equiv \frac{P_0}{\rho_0} \qquad c_s^2 \equiv \frac{\partial P}{\partial \rho}$$

$$\rho = \rho_0(1+\delta)$$

$$\rho = mn \qquad \delta_\rho = \delta_n$$

$$\Big] + 3H(c_s^2 - w)\delta = 0$$

$$\frac{p}{\rho}$$

$$\frac{\partial q^j}{\partial t} + 4Hq^j + \frac{1}{a}\frac{\partial}{\partial x^i}T^{ij} + \frac{1}{a}\frac{\partial \Phi}{\partial x}\Big(T^{ij} + p\,\delta^{ij}\Big] = 0$$

BBGKY hierarchy

$$\frac{\partial}{\partial t}T^{ij} = \ldots + \frac{1}{a}\frac{\partial}{\partial x^k}Q^{ijk} + \ldots$$

$T^{ij} = f(\rho, T)$

$p = f(\rho, T)$

Cold DM : $T^{ij} = 0$

$P = 0$

$$\frac{\partial \vec{v}}{\partial t} + H\vec{v} + \frac{1}{a}\vec{\nabla}\Phi = 0$$

photons & baryons

photons : $\frac{\Delta T}{T}$ instead of $\frac{\delta \rho}{\rho}$

$$f = f_0 + \delta f \qquad f(\vec{x}, \vec{p}, t) = e^{\frac{E}{T(1+\Theta)}}$$

$$f_0 = \frac{1}{e^{E/T} - 1}$$

photons & baryons

photons: $\frac{\Delta T}{T}$ instead of $\frac{\delta \rho}{\rho}$

$$f = f_0 + \delta f \qquad f(\vec{x}, \vec{p}, t) = \frac{1}{e^{\frac{E}{T(1+\theta)}} - 1}$$

$$f_0 = \frac{1}{e^{E/T} - 1} \qquad \theta = \theta(\vec{x}, \vec{p}, t)$$

$$\delta f \simeq \frac{\partial f_0}{\partial \Theta}\,\Theta = -E\,\frac{\partial f_0}{\partial E}\,\Theta$$

$$p = E$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i}\,\frac{\hat{p}^i}{a} - E\,\frac{\partial f}{\partial E}\left[\frac{\hat{p}^i}{a}\,\frac{\partial \Psi}{\partial x^i} + \left(H + \frac{\partial \Phi}{\partial t}\right)\right]$$

$$f \longrightarrow f_0 - E\,\frac{\partial f_0}{\partial E}\,\Theta$$

$$-E\,\frac{\partial f_0}{\partial E}\left[\frac{\partial \Theta}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial \Theta}{\partial x^i} - HE\,\frac{\partial \Theta}{\partial E} + \left(\frac{\hat{p}^i}{a}\frac{\partial \Psi}{\partial x^i} + \frac{\partial \Phi}{\partial t}\right)\right]$$

$C[f]$ $\qquad e^- + \gamma \to e^- + \gamma$

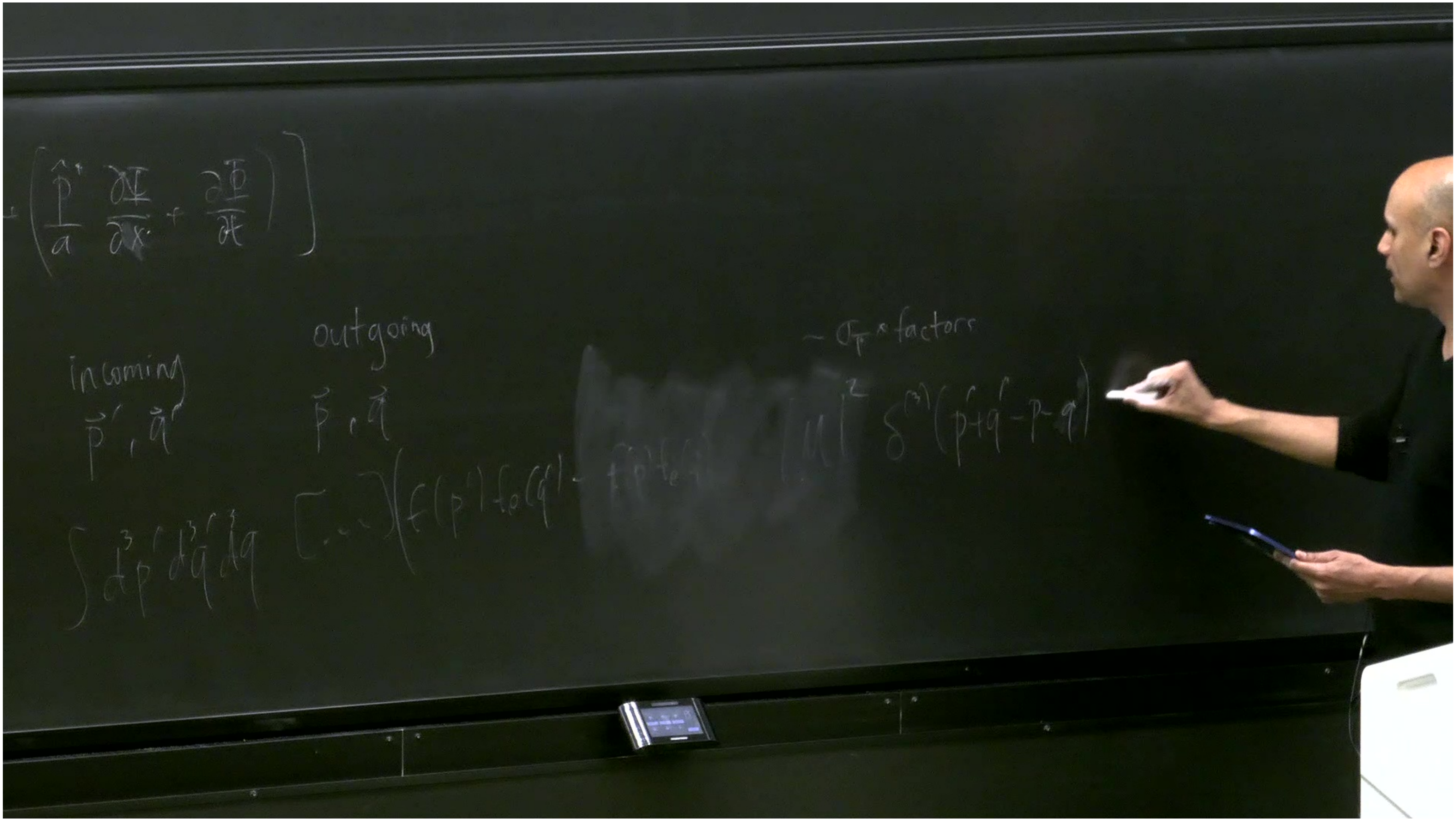
incoming $\qquad$ outgoing

$\vec{p}\,', \vec{q}\,' \qquad \vec{p}, \vec{q}$

$$\int d^3p'\, d^3q'\, d^3q \, [\,\ldots\,]\, f(p')\, f_0(q')$$

scattered into $\vec{p}$

away $\vec{p}$


incoming
outgoing
factors

$E_q \approx E_{q'}$ $\qquad$ $E_{q'} - E_q \approx \dfrac{q \cdot (\vec{p} - p')}{m_e} = v_e (\vec{p} - p')$

$\sim \sigma_T \times \text{factors}$

$\delta^3(\vec{p} + \vec{q} - p - q)\,\delta(E_p + E_q - E_p - E_q)$

$$E_p \simeq E_{p'}$$

$$E_q \simeq E_{q'} \qquad E_{q'} - E_q \approx \frac{\vec{q}\cdot(\vec{p}-p')}{m_e} = v_e(\vec{p}-p')$$

ng

$$\sim \sigma_T \times \text{factors}$$

$$f(p')f_e(q') - f(p)f_e(q) \qquad |M|^2\, \delta^{(3)}(p'+q'-p-q)\,\delta(E_{p'}+E_{q'}-E_p-E_q)$$

$$q' = q + p - p'$$

$C[f]$

$$\delta(E_p + E_q - E_{p'} - E_{q'}) \simeq \delta(E_p - E_{p'}) - \frac{d\delta}{dE} \times (E_{q'} - E_q)$$

$$\simeq \delta(E_p - E_{p'}) - \frac{d\delta}{dE} \times \vec{v}_e \cdot (\vec{p} - \vec{p}\,')$$

$$n_e \sigma_T \int d^3p' \, [\ldots] \cdot [f(p) - f(p')] \cdot \left[\delta(E_p - E_{p'}) - \frac{\partial \delta}{\partial E} \cdot \vec{v}_e \cdot (p - p')\right]$$

$$f \simeq f_0 - E \frac{\partial f_0}{\partial E} \theta$$

$$\boxed{C[f] = -E\frac{\partial f_0}{\partial E}\, n_e \sigma_T \left(\Theta_0 - \Theta(\hat{p}) + \vec{v}_e \cdot \hat{p} \right]}$$

$$\Theta_0 = \frac{1}{4\pi}\int d^2\hat{p}\ \Theta(E, \hat{p})$$

$$+ \vec{v}_e \cdot (p - p') \Big]$$

$$f \simeq f_0 - E\frac{\partial f_0}{\partial E}\Theta$$

$$g\int\frac{d^3p}{(2\pi)^3}\frac{df}{dt} = \frac{dn}{dt}$$

$$= g\int\frac{d^3p}{(2\pi)^3}\,C = g\int\frac{d^3p}{(2\pi)^3}\left(-E\frac{\partial f_0}{\partial E}n_e\sigma_T\left[\Theta_0 - \Theta(p) + \vec{v}_e\cdot\hat{p}\right]\right. = 0$$

$$4\pi\Theta_0 \qquad -4\pi\Theta_0$$

$$f = f_0 - E\frac{\partial f_0}{\partial E}\Theta$$

$$p = E$$

$$g\int\frac{d^3p}{(2\pi)^3}\frac{df}{dt}E\hat{p}^i =$$

$$= g\int\frac{d^3p}{(2\pi)^3}\, C E\hat{p}^i = g\int\frac{d^3p}{(2\pi)^3}\Big(-E^2\frac{\partial f_0}{\partial E}n_e\sigma_T\left[\Theta_0 - \Theta(p) + \vec{v}_e\cdot\hat{p}\right]\hat{p}^i = \frac{4}{3}\rho_\gamma$$

$p = E$

$\frac{\delta\rho}{\rho}$, v

$$g \int \frac{d^3p}{(2\pi)^3} \left(- E^2 \frac{\partial f_0}{\partial E} n_e \sigma_T \left[\Theta_0 - \Theta(p) + \vec{v}_e \cdot \hat{p} \right] \hat{p}^i = \frac{4}{3} \bar{\rho}_\gamma n_e \sigma_T \left(v_e^i - 3\Theta_1^i \right) \right.$$

$$\Theta_1 = \frac{1}{4\pi} \int \Theta \, \hat{p} \, d^2\hat{p}$$