Title: Lecture - Cosmology, PHYS 621

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Subject: Cosmology

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FRW $\quad g_{\mu\nu}(x,t)$

$f(x,p,)$ $\quad \rho, \; P$

$\delta\rho \rightleftarrows \delta g_{\mu\nu}$

$$\rho(x,t) = \rho_0(t) + \delta\rho(x,t)$$
$$= \rho_0(t) \cdot \left[1 + \delta(x,t)\right]$$

$$|\delta| \ll 1$$

$$g_{\mu\nu}(x,t) = g^{(FRW)}_{\mu\nu} + \delta g_{\mu\nu}(x,t) \qquad g^{(0)}_{\mu\nu} \rightarrow g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$$

$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + \underset{\substack{\shortparallel \\ r^2}}{D^2}\, d\Omega^2\right]$$

$$= -dt^2 + a^2 d\vec{x}^2$$

$$= a^2\left[-d\eta^2 + d\vec{x}^2\right] \qquad d\eta = \frac{dt}{a}$$

$$ds^2 = a^2(\eta)\left[-(1+A)\,d\eta^2 - 2B_i\,dx^i\,d\eta + (\delta_{ij} + h_{ij})\,dx^i\,dx^j\right]$$

Decompose into S, V, T

$$h_{ij} = C\,\delta_{ij}$$

$$B_i = \nabla_i B + \tilde{B}_i$$

$$\nabla \cdot \tilde{B} = 0$$

$$2 B_i \, dx^i \, d\eta + (\delta_{ij} + h_{ij}) \, dx^i dx^j \Big]$$

$$h_{ij} = C \delta_{ij} + \left(\nabla_i \nabla_j - \frac{\delta_{ij}}{3} \nabla^2\right) E + \frac{1}{2}\left(\nabla_i \hat{E}_j + \nabla_j \hat{E}_i\right) + \tilde{E}_{ij}$$

$\tilde{B}_i$

4 scalar (A, B, C, E) not all 10 dof's are dynamical

4 vector $(\tilde{B}_i, \hat{E}_i)$

2 tensor $\tilde{E}_{ij}$

$$x^\mu \to x'^\mu$$

$$(x^0, x^i) \to (x'^0, x'^i) = (x$$

$$g_{\mu\nu} \to g'_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}$$

$$= g_{\mu\nu} + h_{\mu\nu}$$

4 vector $(\tilde{B}_i, \hat{E}_i)$

2 tensor $\hat{E}_{ij}$

$$(x^0, x^i) \to (x'^0, x'^i) = (x^0 + T, x^i + L^i)$$

4 scalar − 2 scalar − 2 scalar d.o.f.

4 vector − 2 vector = 2 vector

2 tensor

$$\frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} g_{\alpha\beta}$$

$$g_{\mu\nu} + h_{\mu\nu}$$

$\tilde{E}_i)$

$(x^0 + T, x^i + L^i)$

2 scalar — 2 scalar d.o.f. couple to density pert.'s

2 vector = 2 vector (not produced, decay if present)

2 tensor (neglect for now)

Synchronous gauge : set $A = 0$

Conformal Newtonian: set $B = E = 0$

$$ds^2 = a^2(\eta)\left[-(1+A)\,d\eta^2 + (1+C)\,\delta_{ij}\,dx^i dx^j\right]$$

$$= a^2(\eta)\left[-(1+2\Psi)\,d\eta^2 + (1+2\Phi)\,\delta_{ij}\,dx^i dx^j\right]$$

example : particle's energy

$$p^2 = g_{ij} p^i p^j = a^2(1+2\Phi)\delta_{ij} C^2 \hat{p}_i \hat{p}_j = C^2 a^2(1+2$$

$$p^\mu \to E,\ p,\ \hat{p}$$

$$C = \frac{p}{a}(1-\Phi)$$

$$p^2 = g_{ij} p^i p^j$$

$$\delta_{ij} \hat{p}^i \hat{p}^j = 1$$

$$p^i = C \hat{p}^i$$

mple : particle's energy

$^{\mu} \to E, p, \hat{p}$

$^2 = g_{ij} p^i p^j$

$\delta_{ij} \hat{p}^i \hat{p}^j = 1$

$^i = C \hat{p}^i$

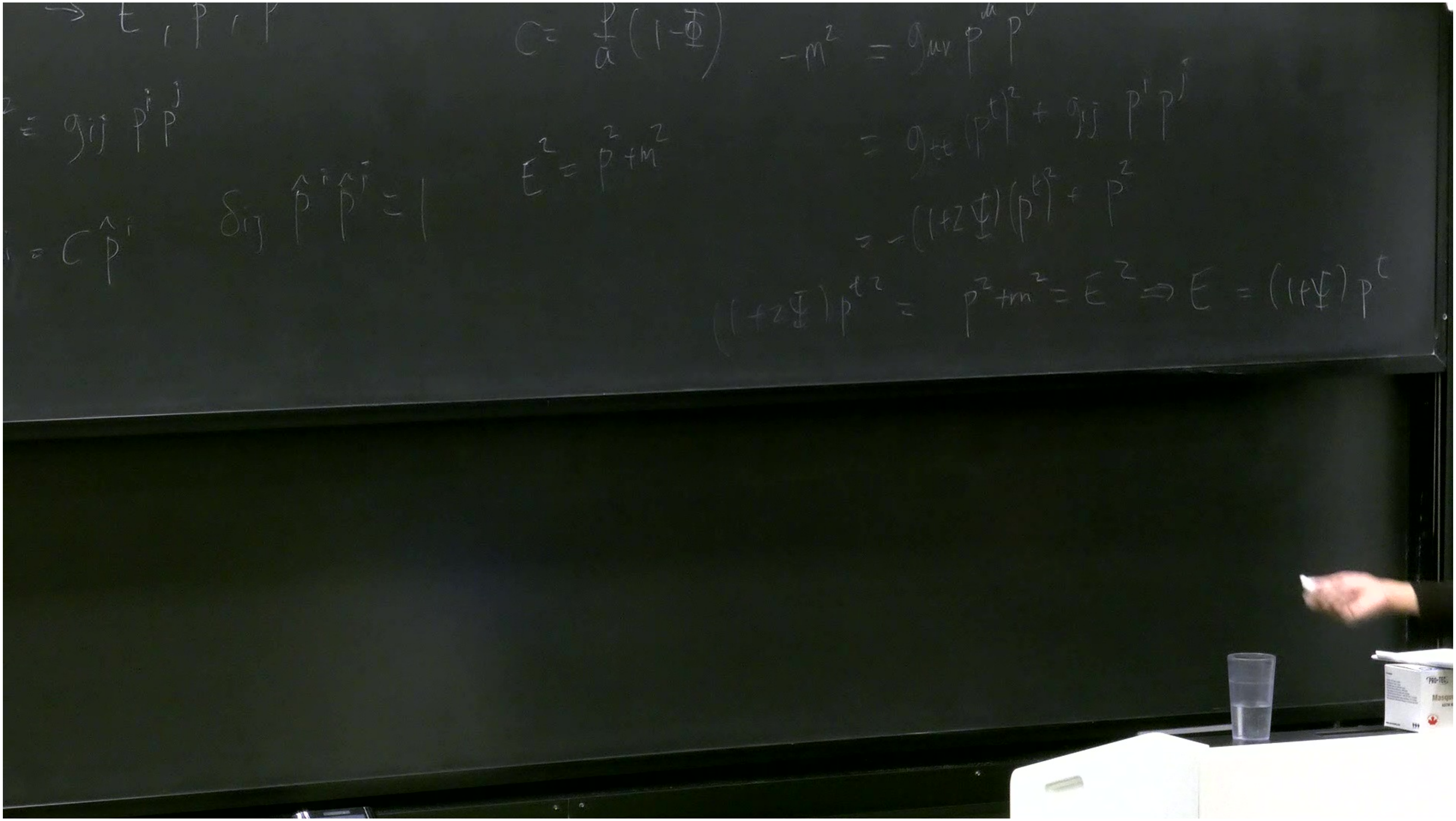
$$p^2 = g_{ij} p^i p^j = a^2 (1 + 2\Phi) \delta_{ij} C^2 \hat{p}_i \hat{p}_j = C^2 a^2 (1 + 2\Phi)$$

$$C = \frac{p}{a} (1 - \Phi)$$

$$E^2 = p^2 + m^2$$

$$-m^2 = g_{\mu\nu} p^\mu p^\nu$$

$$= g_{tt} (p^t)^2 + g_{ij} p^i p^j$$


$\rightarrow E, p, p$
$C = \frac{p}{a}(1-\Phi)$
$-m^2 = g_{\mu\nu} p^\mu p^\nu$
$= g_{ij} p^i p^j$
$E^2 = p^2 + m^2$
$= g_{tt} (p^t)^2 + g_{ij} p^i p^j$
$\delta_{ij} \hat{p}^i \hat{p}^j = 1$
$= -(1+2\Psi)(p^t)^2 + p^2$
$= C \hat{p}^i$
$(1+2\Psi) p^{t2} = p^2 + m^2 = E^2 \Rightarrow E = (1+\Psi) p^t$

$$\frac{dE}{dt} = \frac{d}{dt}\left[(1+\Psi)\, p^t \right] = \frac{d\Psi}{dt} p^t - (1+\Psi)\, \Gamma^t_{\mu\nu} \frac{p^\mu p^\nu}{p^t} = \left[\frac{\partial \Psi}{\partial t} + \frac{\partial \Psi}{\partial x} \right.$$

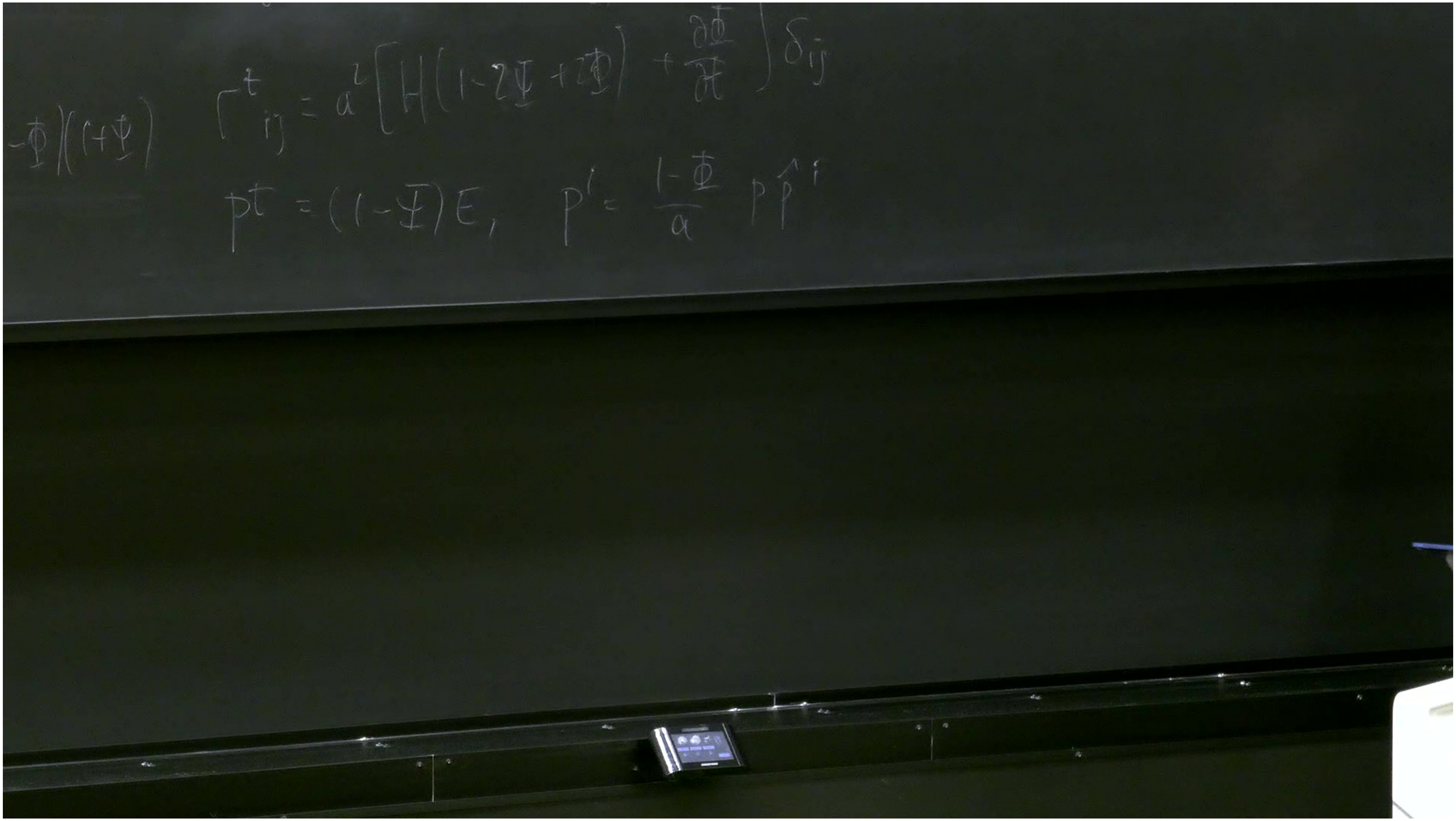
$$\frac{d\Psi}{dt} = \frac{\partial \Psi}{\partial t} + \frac{\partial \Psi}{\partial x^i} \frac{p\,\hat{p}^i}{aE}$$

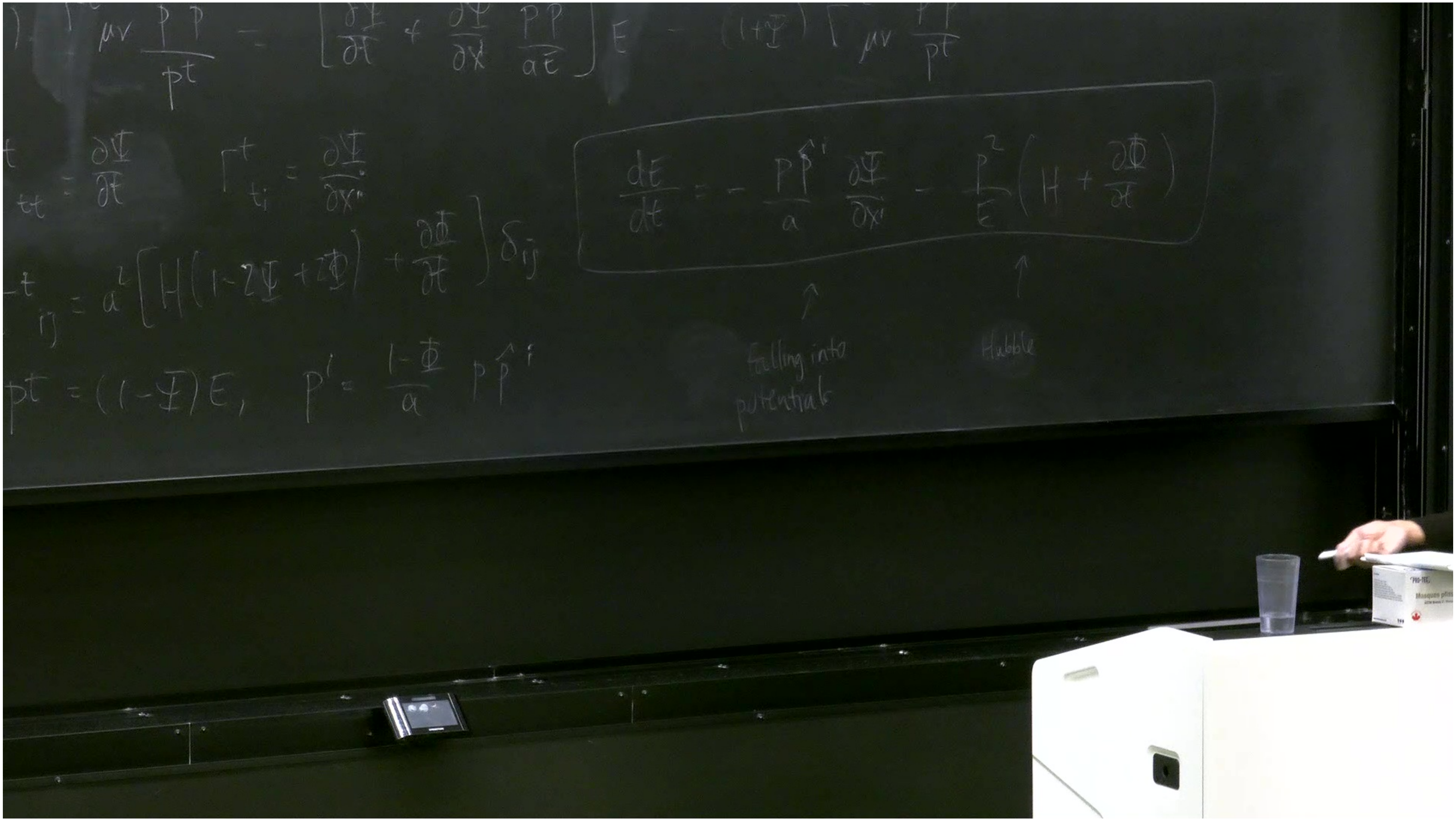
$$\frac{dx^i}{dt} = \frac{u^i}{u^t} = \frac{p^i}{p^t} = \frac{p\,\hat{p}^i}{aE}(1-\Phi)(1+\Psi)$$

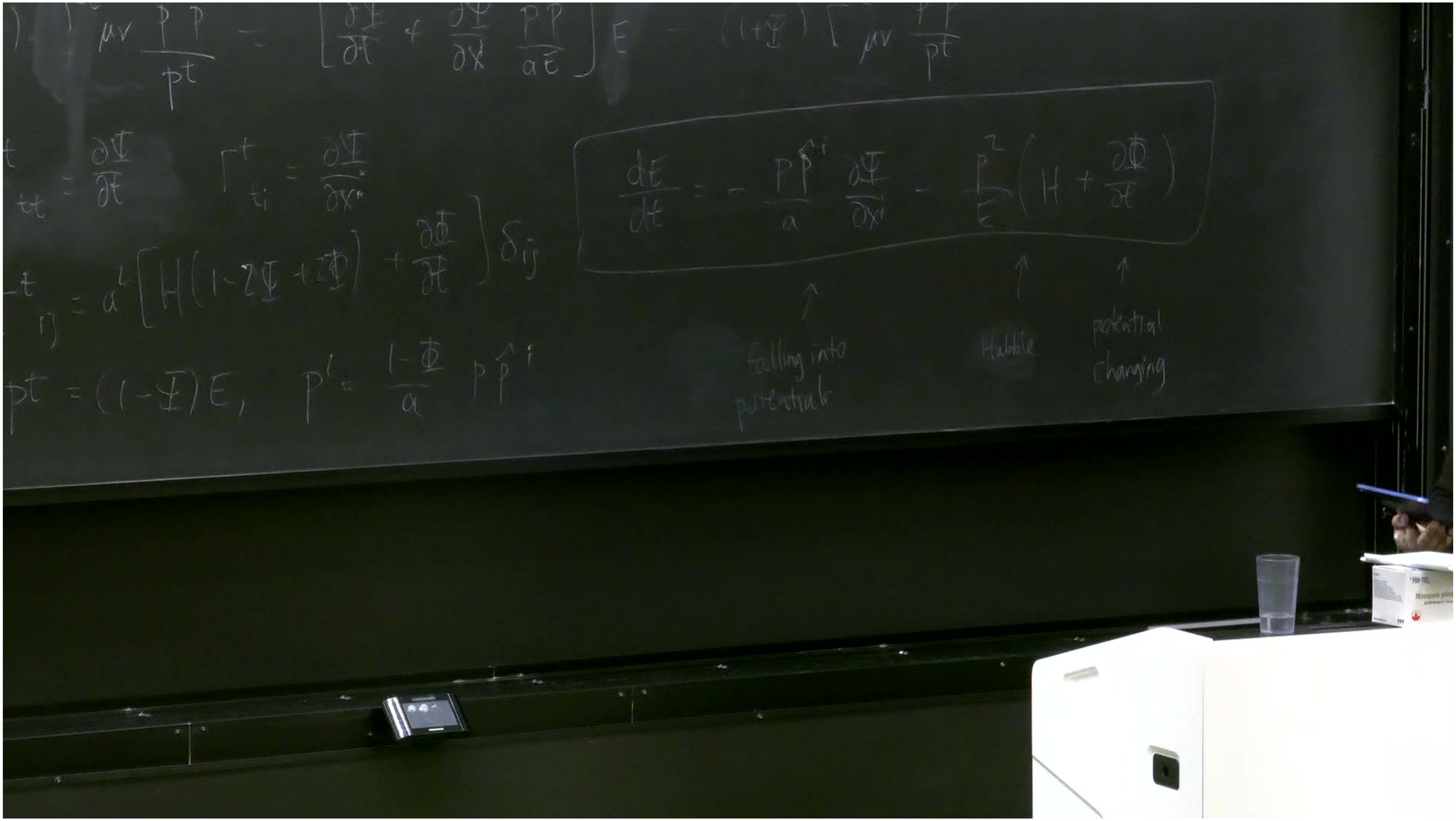
$$p^t - (1+\Psi)\,\Gamma^t_{\mu\nu}\frac{p^\mu p^\nu}{p^t} = \left[\frac{\partial\Psi}{\partial t} + \frac{\partial\Psi}{\partial x^i}\frac{p\hat{p}^i}{aE}\right]E - (1+\Psi)\,\Gamma^t_{\mu\nu}\frac{p^\mu p^\nu}{p^t}$$

$$\Gamma^t_{tt} = \frac{\partial\Psi}{\partial t} \qquad \Gamma^t_{ti} = \frac{\partial\Psi}{\partial x^i}$$

$$(1-\Phi)(1+\Psi) \qquad \Gamma^t_{ij} = a^2\left[H(1-2\Psi+2\Phi) + \frac{\partial\Phi}{\partial t}\right]\delta_{ij}$$


$-\Phi)(1+\Psi)$
$\Gamma^t_{ij} = a^2\left[H(1-2\Psi+2\Phi) + \frac{\partial\Phi}{\partial t}\right]\delta_{ij}$
$p^t = (1-\Psi)E, \quad p^i = \frac{1-\Phi}{a} p \hat{p}^i$


$\frac{dE}{dt} = -\frac{p\hat{p}^i}{a}\frac{\partial\Psi}{\partial x^i} - \frac{p^2}{E}\left(H + \frac{\partial\Phi}{\partial t}\right)$
falling into potential
Hubble
$\Gamma^t_{ti} = \frac{\partial\Psi}{\partial x^i}$
$p^t = (1-\Psi)E, \quad p^i = \frac{1-\Phi}{a} p\hat{p}^i$


$\frac{dE}{dt} = -\frac{p\hat{p}^i}{a}\frac{\partial\Psi}{\partial x^i} - \frac{p^2}{E}\left(H + \frac{\partial\Phi}{\partial t}\right)$
falling into potentials
Hubble
potential changing
$p^t = (1-\Psi)E, \quad p^i = \frac{1-\Phi}{a} p\hat{p}^i$

$$\frac{dx}{dt} = \frac{u}{u^t} = \frac{1}{p^t} = \frac{1}{aE}(1-\Phi)(1+\Psi)$$

$$p^t = (1-\Psi)E, \quad p^i = \frac{1-\Phi}{a} p \hat{p}^i$$

Perturbed Boltzmann eqn.

$$\frac{df}{dt} = C[f]$$

$$f(\vec{x}, \vec{p}) = f(E) = \frac{1}{e^{E/T} \pm 1}$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i}\frac{dx^i}{dt} + \frac{\partial f}{\partial E}\frac{dE}{dt} + \frac{\partial f}{\partial \hat{p}^i}\frac{d\hat{p}^i}{dt}$$

$$f(x, p) = f_0(\vec{p}) + \delta f(\vec{x}, \vec{p})$$

$$\frac{dx}{dt} = \frac{p\hat{p}^i}{aE}(1-\Phi+\Psi)$$

$$\frac{dx^i}{dt} = \frac{u^i}{u^t} = \frac{p^i}{p^t} = \frac{p\hat{p}^i}{aE}(1-\Phi)(1+\Psi) \qquad \Gamma^t_{ij} = a^2\left[H(1-2\Psi+2\Phi) + \frac{\partial\Phi}{\partial t}\right]\delta_{ij}$$

$$p^t = (1-\Psi)E, \qquad p^i = \frac{1-\Phi}{a}\, p\hat{p}^i$$

falling into potential

potential changing

Perturbed Boltzmann eqn.

$$\frac{df}{dt} = C[f]$$

$$f(\vec{x},\vec{p}) = f(E) = \frac{1}{e^{E/T} \pm 1}$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i}\frac{dx^i}{dt} + \frac{\partial f}{\partial E}\frac{dE}{dt} = \boxed{\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i}\frac{p\hat{p}^i}{aE} + \frac{\partial f}{\partial E}\left[\frac{p\hat{p}^i}{a}\frac{\partial\Psi}{\partial x^i} + \frac{p^2}{E}\left(H + \frac{\partial\Phi}{\partial t}\right)\right]}$$

$$f(\vec{x},\vec{p}) = f_0(\vec{p}) + \delta f(\vec{x},\vec{p})$$

$$\frac{dx}{dt} = \frac{p\hat{p}^i}{aE}$$