Title: Lecture - Cosmology, PHYS 621

Speakers: Neal Dalal

Collection/Series: Cosmology (Elective), PHYS 621, March 31 - May 2, 2025

Subject: Cosmology

Date: April 07, 2025 - 2:00 PM

URL: https://pirsa.org/25040015

$T \gg m$

$$n \simeq \frac{gT^3}{2\pi^2} \underbrace{\int_0^\infty \frac{x^2\,dx}{e^x \pm 1}}_{\zeta(3) \times \left\{\begin{array}{ll} 2 & \text{bosons} \\ \frac{3}{2} & \text{fermions} \end{array}\right.}$$

$\zeta(3) = 1.2$

$$\rho = g \int \frac{d^3p}{(2\pi)^3} F(E$$

sons

rmions

$$\rho = g \int \frac{d^3p}{(2\pi)^3} f(E)\, E$$

$$\simeq g \frac{\pi^2}{30} T^4 \begin{cases} 1 & \text{bosons} \\ \frac{7}{8} & \text{fermions} \end{cases}$$

$$\frac{\rho}{n} = \begin{cases} 2.7\,T & B \\ 3.15\,T & F \end{cases}$$

$$P = g \int \frac{d^3p}{(2\pi)^3} f(E) \frac{p^2}{3E} \qquad p = E$$

$$P = \rho/3$$

cold $T << m$

$$E^2 = p^2 + m^2$$

$$n = \frac{g}{2\pi^2} \int_0^\infty dp \, \frac{p^2}{e^{E/T} \pm 1} = \frac{g}{2\pi^2} \int_m^\infty dE \, \frac{\sqrt{E^2 - m^2} \, E \, e^{-E/T}}{1 \pm e^{-E/T}}$$

$$E = m + \varepsilon$$

$$\frac{g}{2\pi^2} \int_0^\infty d\varepsilon \, \frac{\sqrt{2m\varepsilon + \varepsilon^2} \, (m+\varepsilon) \, e^{-(m+\varepsilon)/T}}{1 \pm e^{-m/T} e^{-\varepsilon/T}} =$$

$\varepsilon \ll m$ $\qquad$ $\varepsilon = Tx$

Boltzmann suppression

T

$$= g\,\frac{e^{-m/T}}{2\pi^2}\int_0^\infty d\varepsilon\,\sqrt{2m\varepsilon}$$

$$= g\,\frac{e^{-m/T}}{2\pi^2}\sqrt{2m^3T^3}\int_0^\infty dx\,x^{1/2}e^{-x}$$

$$\Gamma\left(\tfrac{3}{2}\right) = \frac{\sqrt{\pi}}{2}$$

$$\boxed{= g\,e^{-m/T}\left(\frac{mT}{2\pi}\right)^{3/2}}$$

$$\rho \simeq m n$$

$$\frac{p^2}{3E} = \frac{E^2 - m^2}{3E} \simeq \frac{2m\varepsilon}{3m} = \frac{2}{3}\varepsilon = \frac{2}{3} T x$$

$P =$

$$P = g \frac{e^{-m/T}}{2\pi^2} \sqrt{2m^3 T^5} \quad \frac{2}{3} \int_0^\infty x^{3/2} e^{-x} \, dx$$

$$\Gamma\left(\frac{5}{2}\right)$$

$\frac{2}{3} T x$

$$= g \frac{e}{2\pi^2} \sqrt{2m^3 T} \int_0 dx\, x$$

$$\Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2}$$

$$\Delta n = n(\text{part}) - n(\text{anti-p})$$

$$= n(\mu=0) \cdot 2\sinh\left(\frac{\mu}{T}\right)$$

$$P = g \frac{e^{-m/T}}{2\pi^2} \sqrt{2m^3 T^5} \; \frac{2}{3} \int_0^\infty x^{3/2} e^{-x} dx$$

$$\Gamma\left(\frac{5}{2}\right) = \frac{3}{2}\Gamma\left(\frac{3}{2}\right)$$

$$= n T$$

$$e^{E/T} \longrightarrow e^{(E-\mu)/T}$$

$\mu \neq 0$

$T \ll m \Rightarrow P \ll \rho$

$\mu \ll m \quad e^{\mu/T} n$

$$= \frac{\pi^2}{30}\left[\sum_{rel,B} g_i T_i^4 + \frac{7}{8}\sum_{rel,f} g_i T_i^4\right]$$

$$\frac{7}{8}\sum_{rel,f} g_i \left(\frac{T_i}{T}\right)^4$$

evolution of T : T evolves adiabatically

$s = \text{const} \quad ds = 0$

$$\frac{\partial \rho_{tot}}{\partial t} + 3H\left(\rho_{tot} + P_{tot}\right) = 0$$

$$\frac{dT}{dt} = \frac{\partial \rho_{tot}/\partial t}{\partial \rho_{tot}/\partial T}$$

$$dE + PdV - TdS = 0$$

$$dE = d(\rho V)$$

$$d(PV) = VdP + PdV$$

$$d((\rho + P)V) - VdP = TdS$$

$$dS = \frac{1}{T}\left\{ d(\rho+P)V - VdP \right\}$$

$$\frac{dP}{dT} = \frac{\rho + P}{T}$$

$$dS = \frac{1}{T}\left\{d(\rho+P)V - VdP\right\} = \frac{1}{T}d\left[(\rho+P)V\right] - V(\rho+P)\frac{dT}{T^2} = d\left[\frac{(\rho+P)V}{T}\right]$$

$$\frac{dP}{dT} = \frac{\rho+P}{T}$$

$$\boxed{S = \frac{\rho+P}{T}V.}$$

$$s = \frac{S}{V} = \frac{\rho+P}{T} = \frac{4}{3}\frac{\rho}{T} = g\frac{2\pi^2}{45}$$

$$\rho = g\frac{\pi^2}{30}T\begin{cases}1\\ 7/8\end{cases}$$

$$\frac{dS}{dt} = 0$$

$$P = \rho/3$$

$d((\rho+P)V) - VdP = TdS$

$\frac{dS}{dt} = 0$

$P = \rho/3$

$$S_{tot} = \sum_{rel} s_i = \frac{2\pi^2}{45}\left[\sum_{rel,B} g_i T_i^3 + \frac{7}{8}\sum_{rel,f} g_i T_i^3\right] \equiv g_{*s}\frac{2\pi^2}{45}T^3$$

$$S = sV \propto \boxed{g_{*s} T^3 a^3 = \text{const}}$$

if $T < m_i$: then g_{*s} decreases

if g_{*s} const $\Rightarrow T \propto \frac{1}{a}$

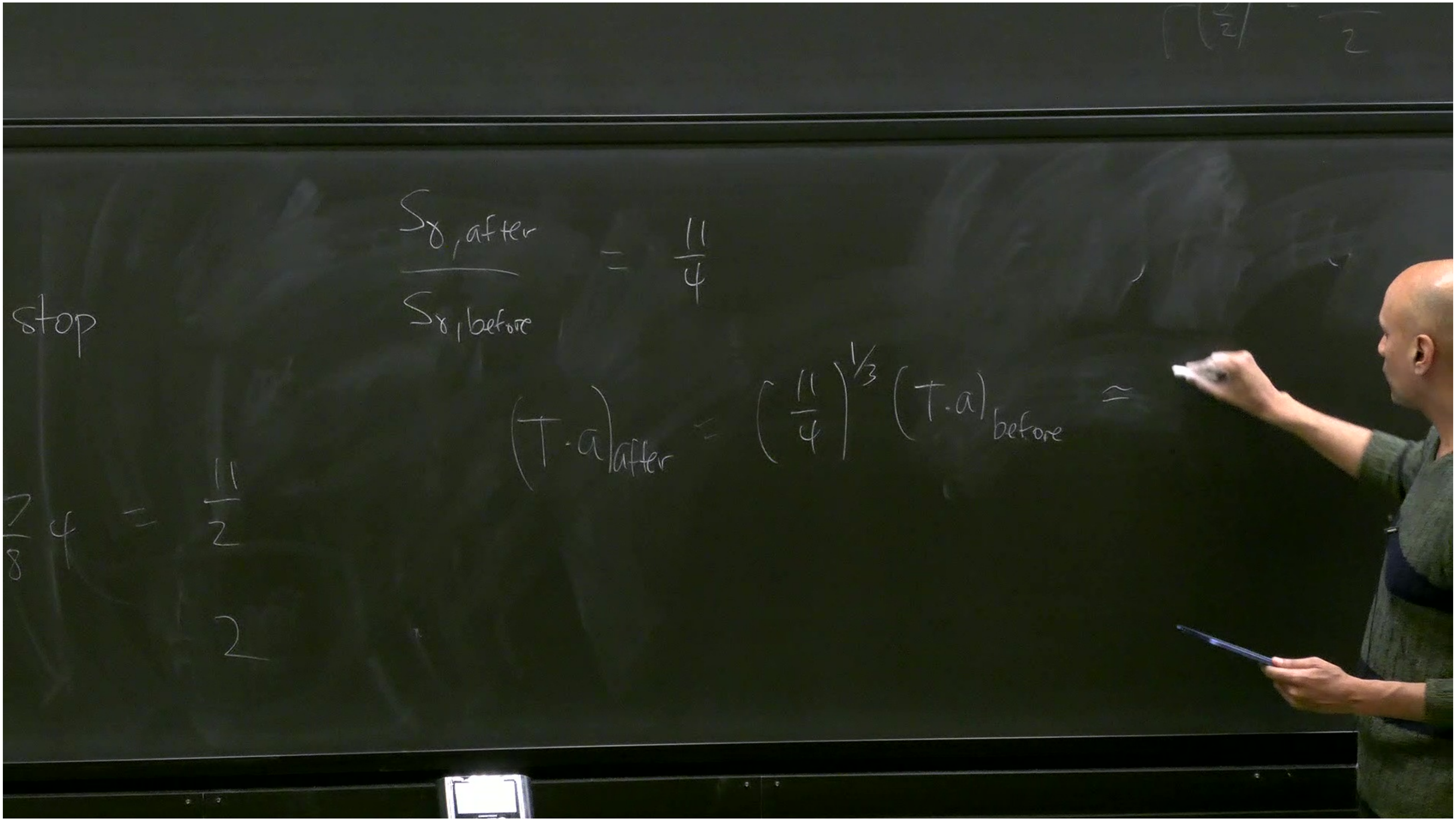
$m_e = 0.511\ \mathrm{MeV}$

$T_\gamma \lesssim 0.1\ \mathrm{MeV}$ $\quad 2\gamma \to e^+e^-$ stop

$e^+e^- \to 2\gamma$

before annihil. $\quad g_{\gamma s} + g_{es} = 2 + \frac{7}{8}4 = \frac{11}{2}$

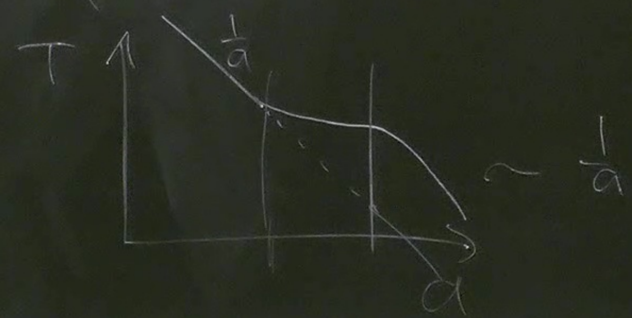
after $\quad g_{\gamma s} = 2$


stop
$\frac{S_{\gamma, after}}{S_{\gamma, before}} = \frac{11}{4}$
$(T \cdot a)_{after} = \left(\frac{11}{4}\right)^{1/3} (T \cdot a)_{before} \simeq$
$\frac{7}{8} 4 = \frac{11}{2}$
2

$$(T\cdot a)_{\text{after}} = \left(\frac{11}{4}\right)^{1/3} (T\cdot a)_{\text{before}} \simeq 1.4\,(T a)_{\text{before}}$$

$$\frac{7}{8}\,4 = \frac{11}{2}$$

$$2$$



$$T \ll m \Rightarrow p \ll \rho$$

$\mu \neq 0$ $\quad e^{E/T} \longrightarrow e$

$\mu \ll m$ $\quad e^{\mu/T}\, n$

Out of equil

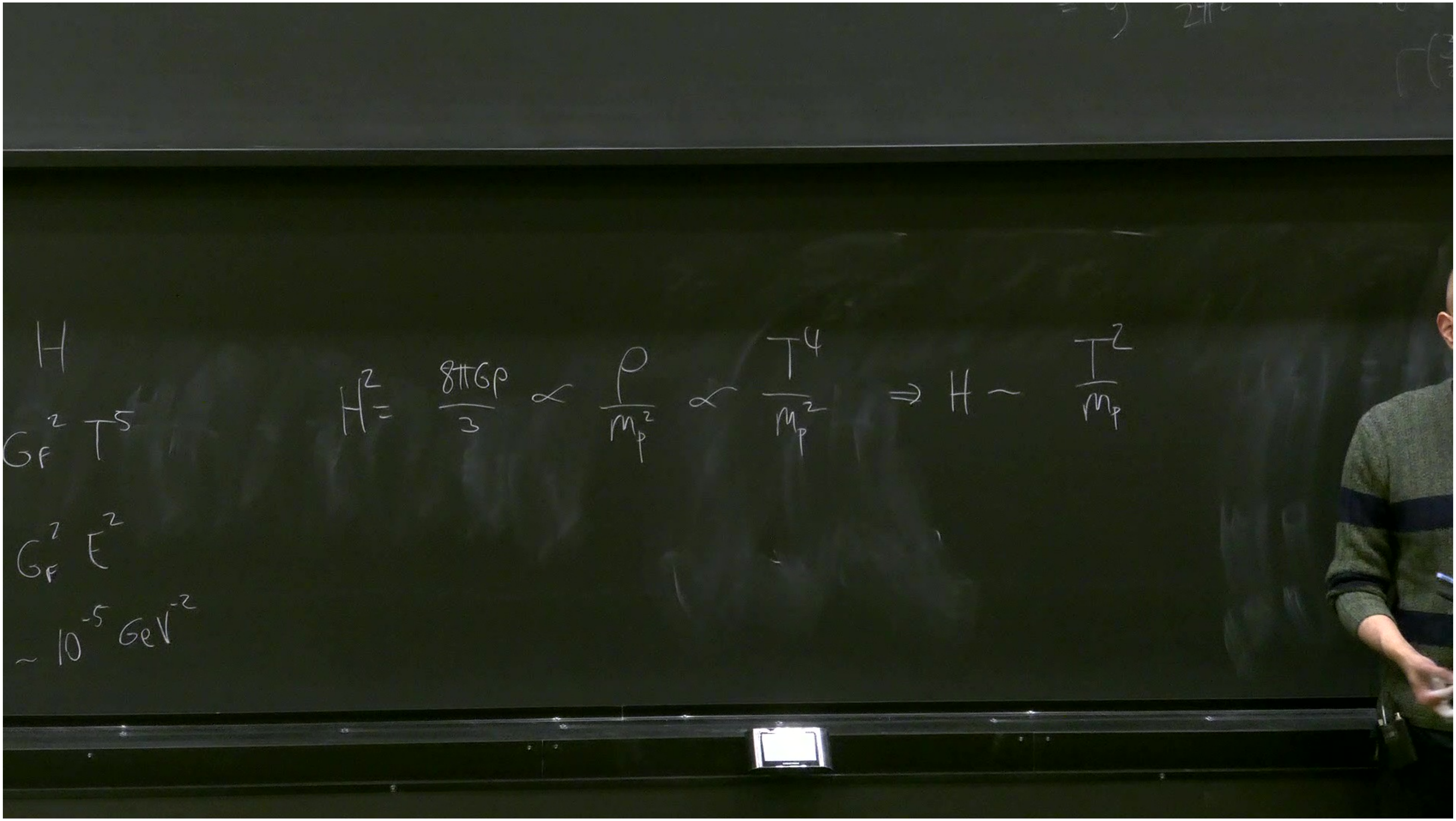
assuming $\Gamma \gg H$

$\Gamma \sim n \sigma c \propto G_F^2 T^5$

$H^2 = \frac{8\pi G \rho}{3} \propto \frac{\rho}{m_p^2} \propto$

$n \propto T^3 \qquad \sigma \propto G_F^2 E^2$

$G_F \sim 10^{-5}\,\mathrm{GeV}^{-2}$


H
$H^2 = \frac{8\pi G\rho}{3} \propto \frac{\rho}{m_p^2} \propto \frac{T^4}{m_p^2} \Rightarrow H \sim \frac{T^2}{m_p}$
$G_F^2 T^5$
$G_F^2 E^2$
$\sim 10^{-5}\ GeV^{-2}$

"weak freeze out"

$T < T_f \Rightarrow$

H

$G_F^2 T^5$

$G_F^2 E^2$

$\sim 10^{-5}\,\mathrm{GeV}^{-2}$

$$H^2 = \frac{8\pi G\rho}{3} \propto \frac{\rho}{m_p^2} \propto \frac{T^4}{m_p^2} \Rightarrow H \sim \frac{T^2}{m_p}$$

$$\frac{\Gamma}{H} \propto \frac{G_F^2 T^5}{T^2/m_p} \sim G_F^2 m_p T^3 \simeq 1 \Rightarrow T_f \sim \left(G_F^2 m_p\right)^{-1/3} \simeq 1\ \mathrm{MeV}$$

$$\Gamma\left(\tfrac{3}{2}\right) = \frac{\sqrt{\pi}}{2}$$

out'

$T < T_f \Rightarrow \nu$'s decouple

$$\frac{\rho}{m_p^2} \propto \frac{T^4}{m_p^2} \Rightarrow H \sim \frac{T^2}{m_p}$$

$$\sim G_F^2 m_p T^3 \simeq 1 \Rightarrow T_f \sim \left(G_F^2 m_p\right)^{-1/3} \simeq 1 \text{ MeV}$$

out'

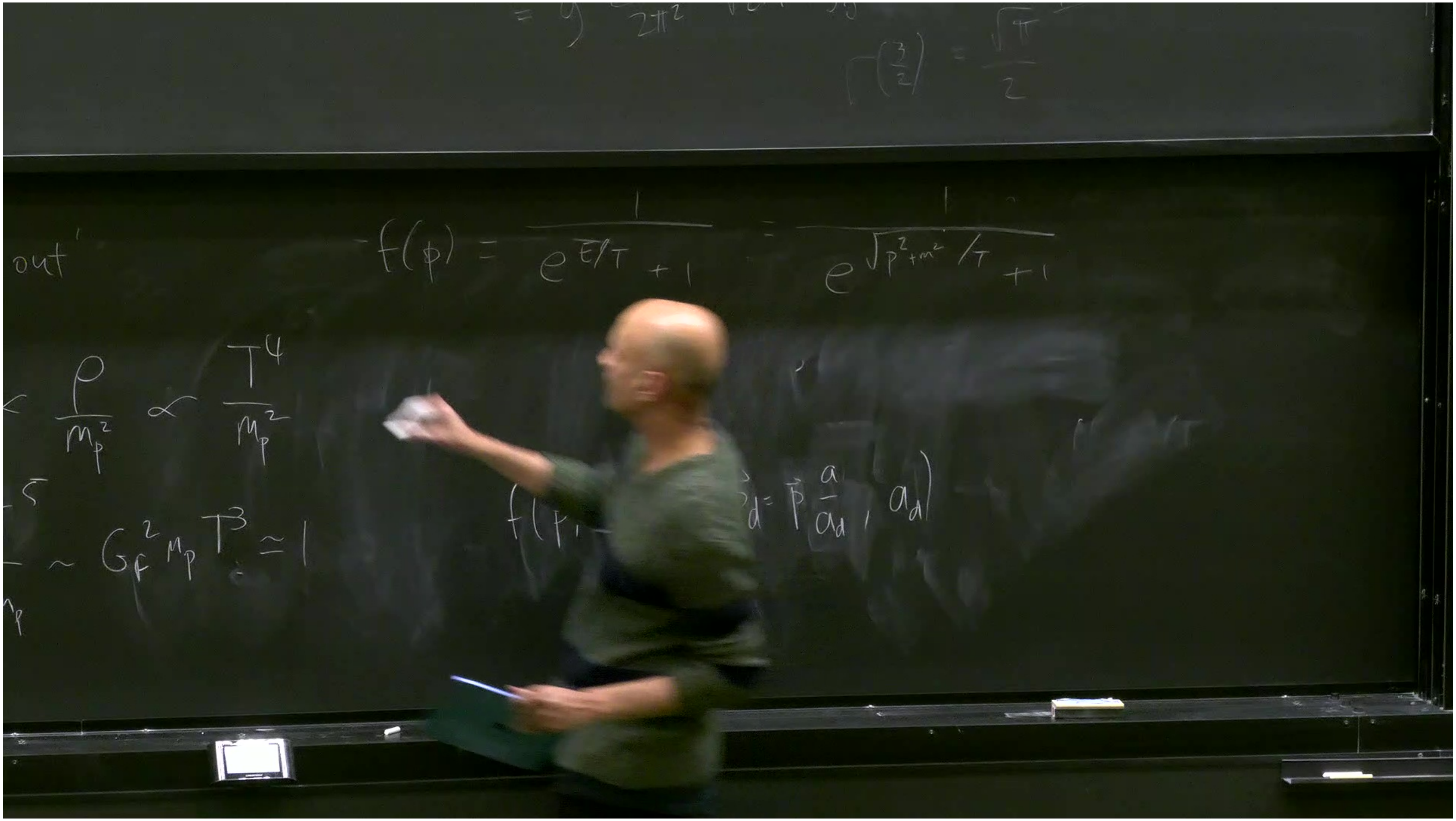
$T < T_f \Rightarrow \nu$'s decouple

$$\vec{p}(a) = \vec{p}_d \times \frac{a_d}{a}$$

$$< \frac{\rho}{m_p^2} \propto \frac{T^4}{m_p^2}$$

$$f(\vec{p}, a) = f\left(\vec{p}_d = \vec{p}\,\frac{a}{a_d},\ a_d\right)$$

$$\sim G_F^2 m_p T^3 \simeq 1$$


out
f(p) = 1/(e^{E/T} + 1) = 1/(e^{√(p²+m²)/T} + 1)
ρ/m_p² ∝ T⁴/m_p²
~ G_F² m_p T³ ≃ 1

$$\Gamma\left(\tfrac{3}{2}\right) = \frac{\sqrt{\pi}}{2}$$

out before $f(p) = \dfrac{1}{e^{E/T}+1} = \dfrac{1}{e^{\sqrt{p^2+m^2}/T}+1}$

$$\frac{\rho}{m_p^2} \propto \frac{T^4}{m_p^2} \longrightarrow \frac{1}{e^{\sqrt{p^2\left(\frac{a}{a_d}\right)^2+m^2}/T}+1} \neq f(E)$$

$$\sim G_F^2 m_p T^3 \simeq 1$$

$$f(\vec{p}, a) = f\left(\vec{p}_d = \vec{p}\,\frac{a}{a_d},\ a_d\right)$$