Title: Lecture - Cosmology, PHYS 621

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Collection/Series: Cosmology (Elective), PHYS 621, March 31 - May 2, 2025

Subject: Cosmology

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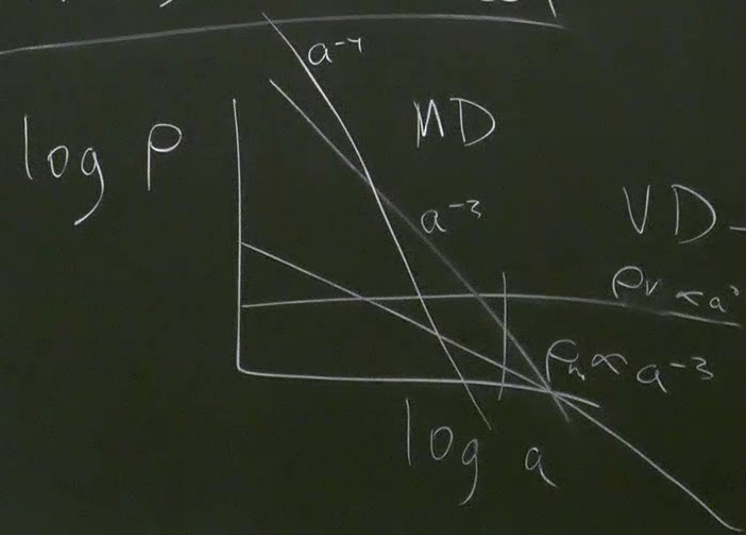
URL: https://pirsa.org/25040014

ΛCDM model



$\Omega_{v0} \simeq 0.7$

$\Omega_{m0} \simeq 0.3$

$\Omega_{r0} \simeq 10^{-4}$

$a \propto e^{H_0 t}$ (VD)

$a \propto t^{2/3}$ (MD)

$\frac{\rho_m}{\rho_r} \sim 3000$

$\propto a$

$z_{eq} \sim 3000$

$$\frac{n_\gamma}{n_B} \sim 2\times 10^9$$

$$T_{CMB} \simeq 2.73K$$

$z \gtrsim 1000$ $\qquad H + \gamma \longrightarrow e^- + p^+$

almost no atoms, instead ions + e^-'s

$T_{CMB} \gg 1\ \mathrm{MeV}$ no nuclei

$> 100\ \mathrm{MeV}$ no nucleons

almost no atoms, instead ...

Thermal history

distribution $f(\vec{x},\vec{p})\,\frac{d^3x\,d^3p}{(2\pi\hbar)^3} \sim$ # particles at $\vec{x},\vec{p}$

$c=\hbar=1$

$$\rho(\vec{x}) = g\int\frac{d^3p}{(2\pi)^3}E$$

$$n(\vec{x}) = g\int\frac{d^3p}{(2\pi)^3}f(\vec{x},\vec{p})$$

$g=2s+1$

$$E^2 = p^2+m^2$$

ermal history

istribution $f(\vec{x}, \vec{p}) \, \frac{d^3x \, d^3p}{(2\pi\hbar)^3} \sim$ # particles at $\vec{x}, \vec{p}$

$c = \hbar = 1$

$\rho(\vec{x}) = g \int \frac{d^3p}{(2\pi)^3} E \, f(x, p)$

$n(\vec{x}) = g \int \frac{d^3p}{(2\pi)^3} f(\vec{x}, \vec{p})$

$g = 2s+1$

$E^2 = p^2 + m^2$

$$T^{\mu}{}_{\nu}(\vec{x}) = g \int \frac{d^3p}{(2\pi)^3} \frac{p^{\mu} p_{\nu}}{E} f(x, p)$$

f_e,

$$P = \frac{1}{3} \mathrm{Tr}(T^i{}_j) = g \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E} f(x, p)$$

$f_e, f_\gamma, = f(\vec{p}, t)$ $\qquad \frac{d\vec{p}}{dt} + H\vec{p} = 0$

evolution of f governed by Boltzmann eqn

if $C = 0$

$$\frac{df_i}{dt} = C_i[\vec{f}]$$

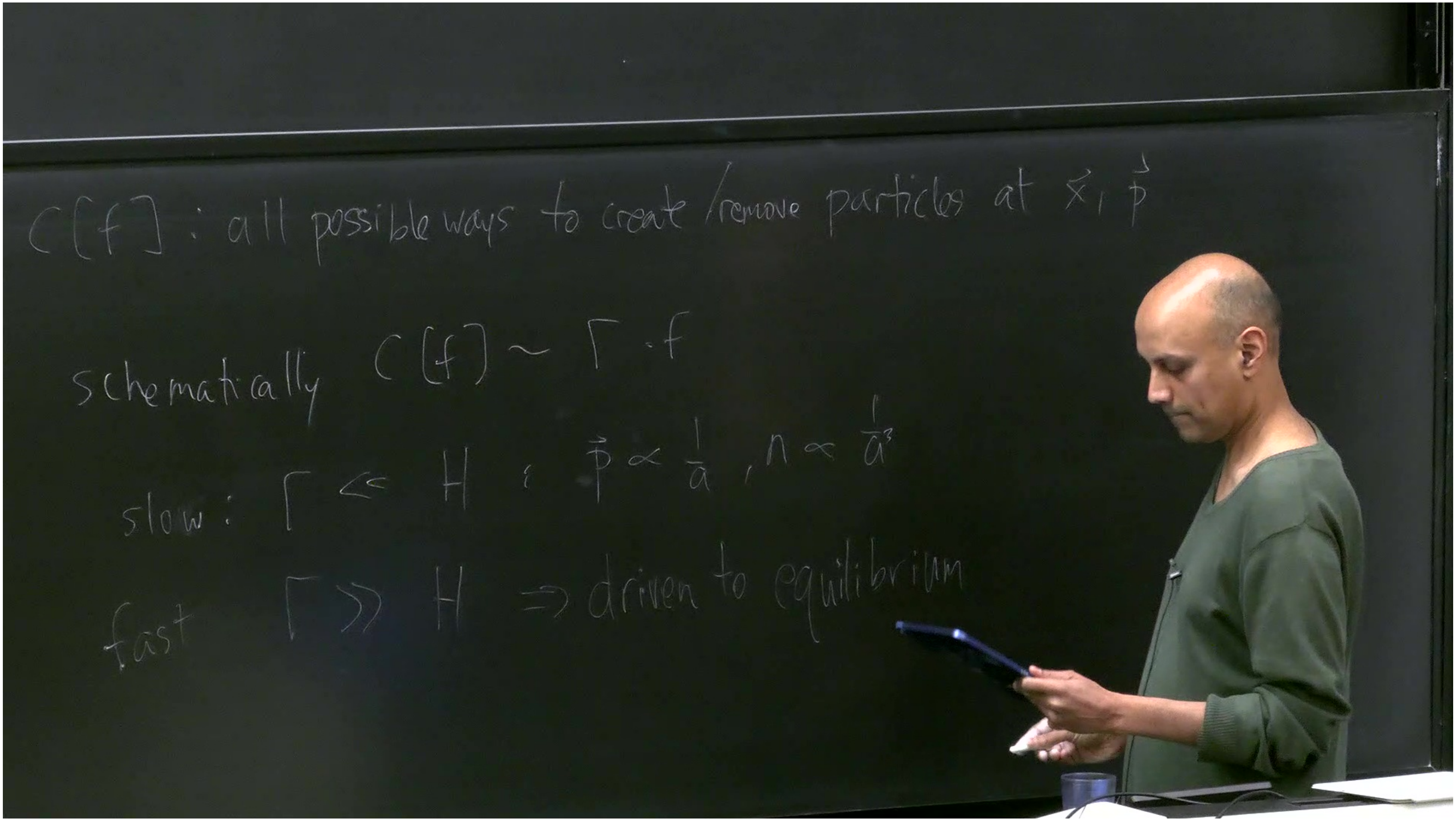
$$= \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \vec{x}} \cdot \frac{d\vec{x}}{dt} + \frac{\partial f}{\partial \vec{p}} \cdot \frac{d\vec{p}}{dt} = \frac{\partial f}{\partial t} - H\vec{p} \cdot \frac{\partial f}{\partial \vec{p}} = 0$$

Suppose $C[f] = 0$

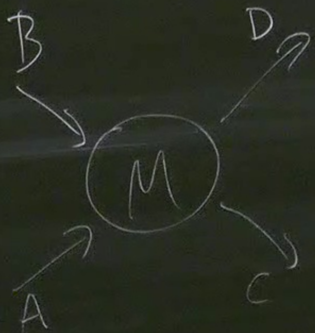
$$g \int \frac{d^3p}{(2\pi)^3} \left[\frac{\partial f}{\partial t} - H p^i \frac{\partial f}{\partial p^i} \right] E = 0$$

$$\frac{\partial \rho}{\partial t} + H \, g \int \frac{d^3p}{(2\pi)^3} f \frac{\partial}{\partial p_i}(p^i E) = \frac{\partial \rho}{\partial t} + 3H(\rho + P) = 0$$

$$3H \quad g \int \frac{d^3p}{(2\pi)^3} f \left(E + \frac{p^2}{3E} \right)$$


$C[f]$: all possible ways to create/remove particles at $\vec{x}, \vec{p}$
schematically $C[f] \sim \Gamma \cdot f$
slow: $\Gamma \ll H$; $\vec{p} \propto \frac{1}{a}$, $n \propto \frac{1}{a^3}$
fast $\Gamma \gg H \Rightarrow$ driven to equilibrium

$$f(\vec{p}) = \frac{1}{e^{(E-\mu)/T} \mp 1} \qquad \begin{matrix} b \\ F \end{matrix}$$

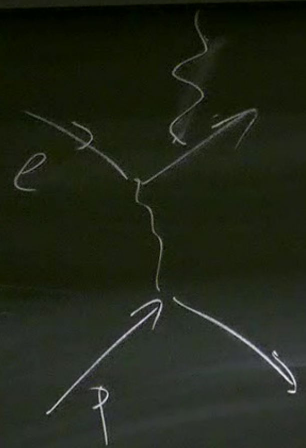


$$\int dp_A\, dp_B\, dp_C\, dp_D\, |M|^2 \delta_D(p_A + p_B - p_C - p_D)\left[-f_A f_B(\right.$$

$$\int dp_A\, dp_B\, dp_C\, dp_D\, |M|^2 \delta_D(p_A+p_B-p_C-p_D)\left[-f_A f_B (1\pm f_C)(1\pm f_D) + (1\pm f_A)(1\pm f_B) f_C f_D\right]$$

$T_A = T_B = T_C = T_D$ kinetic equil.

$\mu_A + \mu_B = \mu_C + \mu_D$ chemical equil



$$\mu_p + \mu_e = \mu_p + \mu_e + \mu_\gamma$$

$$\mu_\gamma = 0$$

$$e^+ + e^- \to 2\gamma$$

relativistic : $T \gg m$

cold $T \ll m$

$\gg H$

$\ll H$

$$E^2 = p^2 + m^2$$

$$E\,dE = p\,dp$$

$$= g \int \frac{dp\, p^2}{2\pi^2} f = \frac{g}{2\pi^2} \int \frac{dE\, E \sqrt{E^2 - m^2}}{e^{E/T} \mp 1} \simeq \frac{gT^3}{2\pi^2} \underbrace{\int \frac{dx\, x^2}{e^x \mp 1}}$$