Title: Lecture - Cosmology, PHYS 621

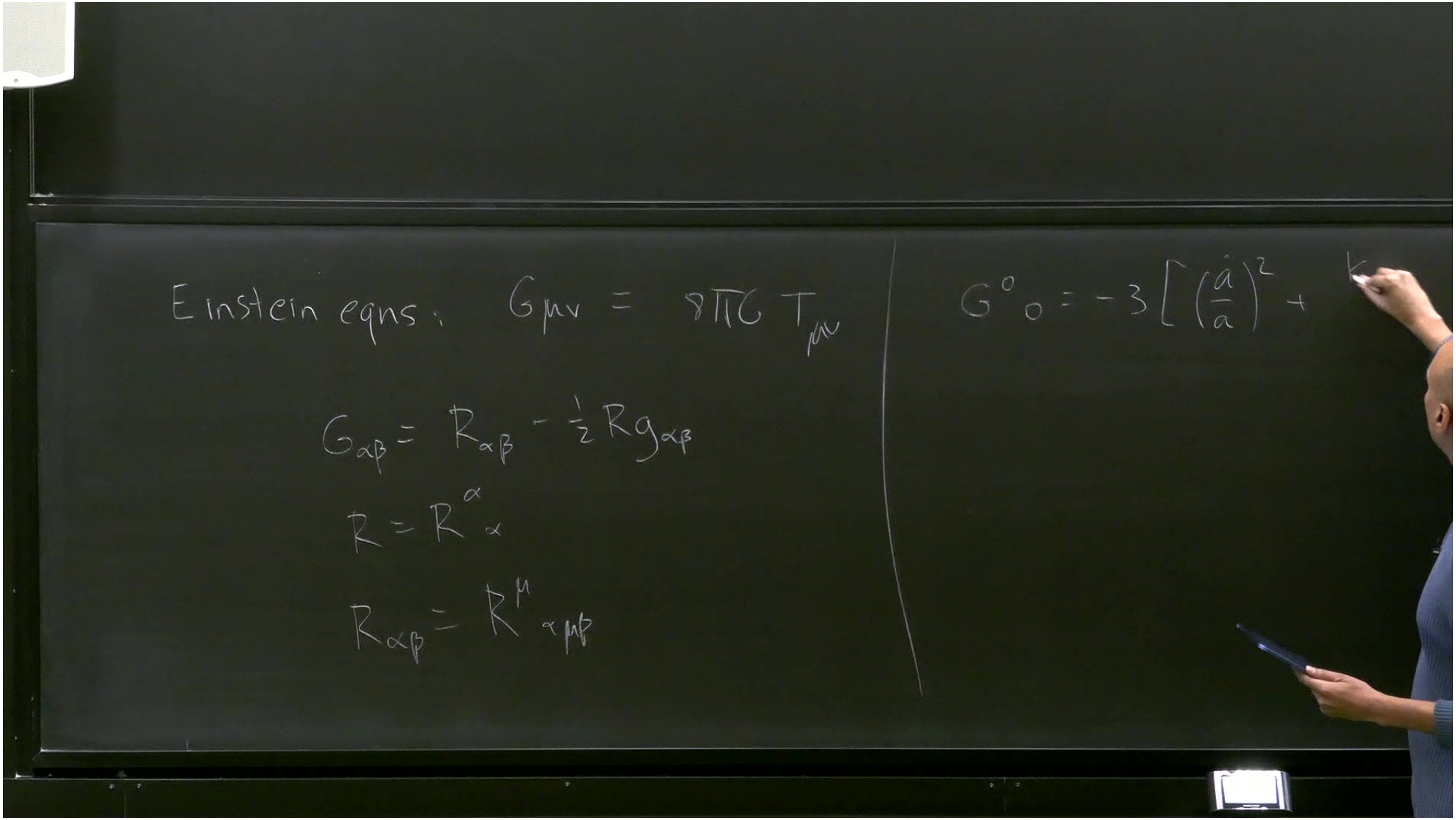
Speakers: Neal Dalal

Collection/Series: Cosmology (Elective), PHYS 621, March 31 - May 2, 2025

Subject: Cosmology

Date: April 01, 2025 - 2:00 PM

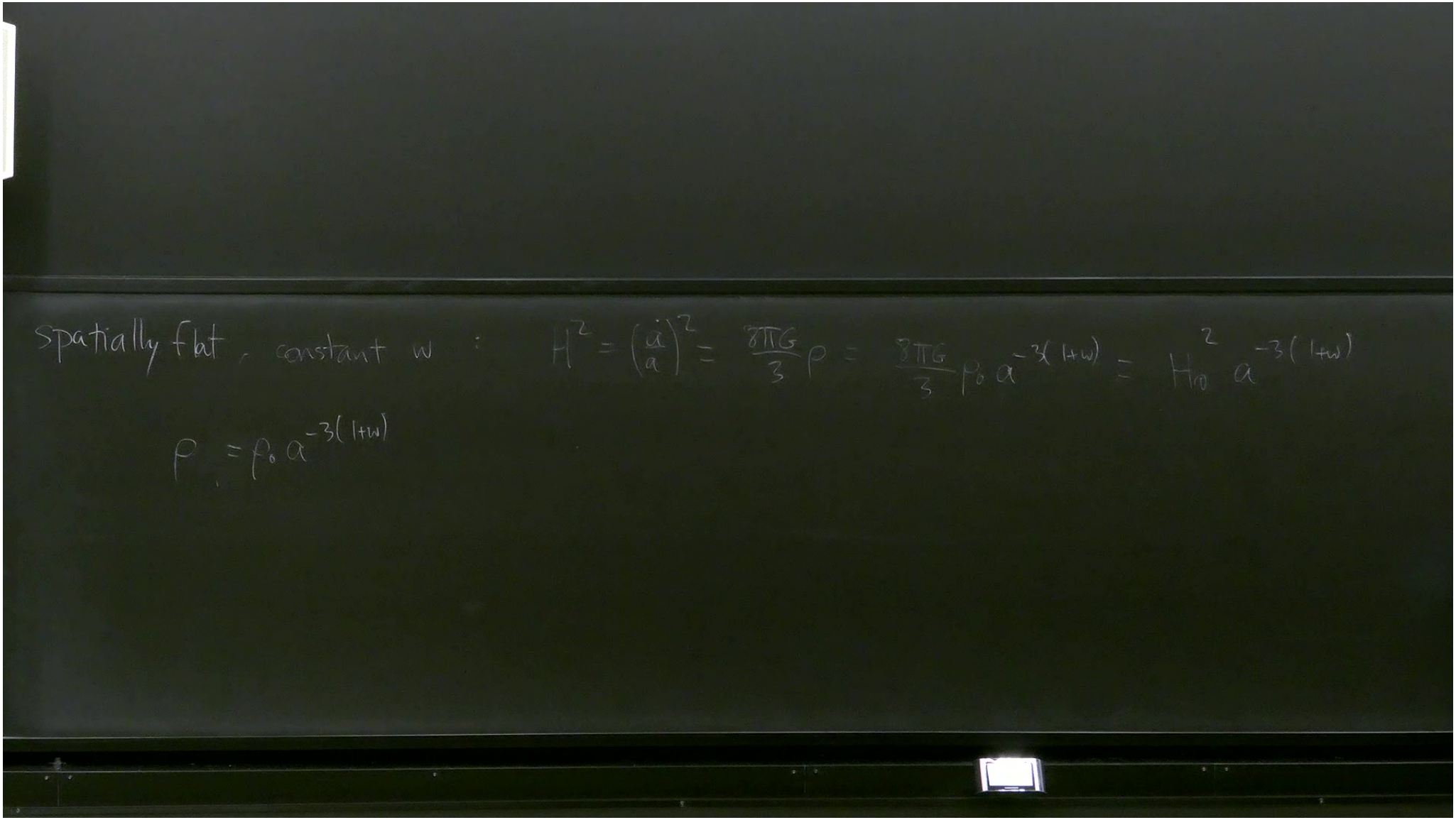
URL: https://pirsa.org/25040013


Einstein eqns: $G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$
$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta}$
$R = R^{\alpha}{}_{\alpha}$
$R_{\alpha\beta} = R^{\mu}{}_{\alpha\mu\beta}$
$G^{0}{}_{0} = -3\left[\left(\frac{\dot{a}}{a}\right)^2 + k\right.$

$$G^0{}_0 = -3\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2R^2}\right] = 8\pi G(-\rho) \Rightarrow \boxed{H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2R^2}}$$

$$G^i{}_j = -\left[2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2R^2}\right]\delta^i{}_j = 8\pi G P \delta^i{}_j$$

$$\boxed{\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\rho + 3P\right]}$$


spatially flat, constant w : $H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}\rho_0 a^{-3(1+w)} = H_0^2 a^{-3(1+w)}$
$\rho = \rho_0 a^{-3(1+w)}$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}\rho_0 a^{-3(1+w)} = \left| H_0 a^{-\frac{3}{2}(1+w)} = \left(\frac{\dot{a}}{a}\right) \right.$$

NR: $w=0$ $a \propto t^{2/3}$

R: $w=\frac{1}{3}$ $a \propto t^{1/2}$

$$\int H_0 \, dt = \int da \, a^{\frac{3}{2}(1+w)-1}$$

$$H_0 t = \frac{2}{3(1+w)} a^{\frac{3}{2}(1+w)} \qquad w \neq -1$$

$$a(t) = \left[\frac{3}{2}(1+w) H_0 t\right]^{\frac{2}{3(1+w)}}$$

$$\rho_0 a^{-3(1+w)} = \left| H_0 \; a^{-\frac{3}{2}(1+w)} = \left(\frac{\dot{a}}{a}\right) \right.$$

$$\int H_0 \, dt = \int da \; a^{\frac{3}{2}(1+w)-1}$$

$$H_0 t = \frac{2}{3(1+w)} a^{\frac{3}{2}(1+w)} \qquad w \neq -1$$

$$a(t) = \left[\frac{3}{2}(1+w) H_0 t\right]^{\frac{2}{3(1+w)}}$$

$$\left. w = -1 \right.$$

$$H_0 t = \int \frac{da}{a} = \log a + C$$

$$\Rightarrow a \propto e^{H_0 t}$$

Spatially flat, constant w : $H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}\rho_0$

$\rho = \rho_0 a^{-3(1+w)}$

NR: $w=0$ $a \propto t^{2/3}$

R: $w=\frac{1}{3}$ $a \propto t^{1/2}$

$w=-1 \rightarrow P=-\rho$

$$T^\mu_\nu = \begin{pmatrix} -\rho & & & \\ & P & & \\ & & P & \\ & & & P \end{pmatrix}$$

$T^{\mu}_{\ \nu} = \begin{pmatrix} \rho & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$

Multi-component

Critical density $H^2 = \frac{8\pi G \rho}{3}$

$\rho_{crit} = \frac{3H^2}{8\pi G}$

$\Omega_m = \frac{\rho_m}{\rho_{crit}}$

$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{crit}} = \frac{\Lambda}{3H^2}$

$\Omega_k = -\frac{k}{H^2 R^2 a^2} \Rightarrow R =$

$$a(t) = \left[\frac{3}{2}(1+w) H_0 t\right]^{2/(1+w)}$$

$$\Omega_m + \Omega_r + \dots + \Omega_K \equiv 1$$

$$\rho_m = \rho_{m0}\, a^{-3}$$

$$\rho_r = \rho_{r0}\, a^{-4}$$

$$\frac{H^2}{H_0^2} = \left[\Omega_{m0} a^{-3} + \Omega_{r0} a^{-4} + \dots\right] + \frac{\Omega_{K0}}{a^2}$$

$$\frac{1}{|\Omega_{K0}|^{1/2} H_0}$$

$\Omega_m > 1 \Rightarrow \Omega_k < 0$

$$H^2 = H_0^2 \left[\Omega_{m0} a^{-3} + \Omega_k a^{-2} \right] = H_0^2 \left[(1+\delta_0) a^{-3} - \delta_0 a^{-2} \right]$$

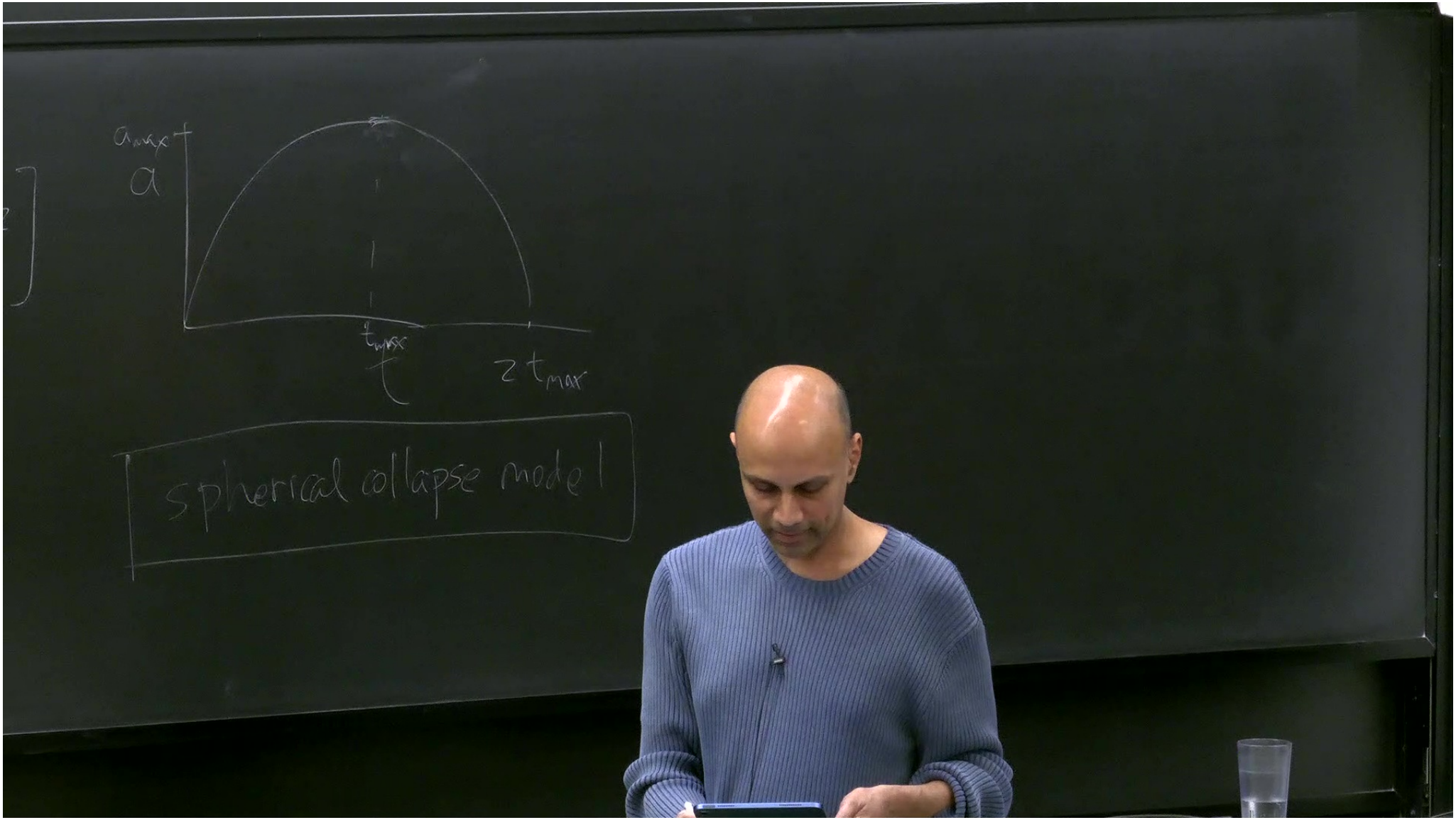
$\Omega_{m0} = 1 + \delta_0$ at $a = 1$

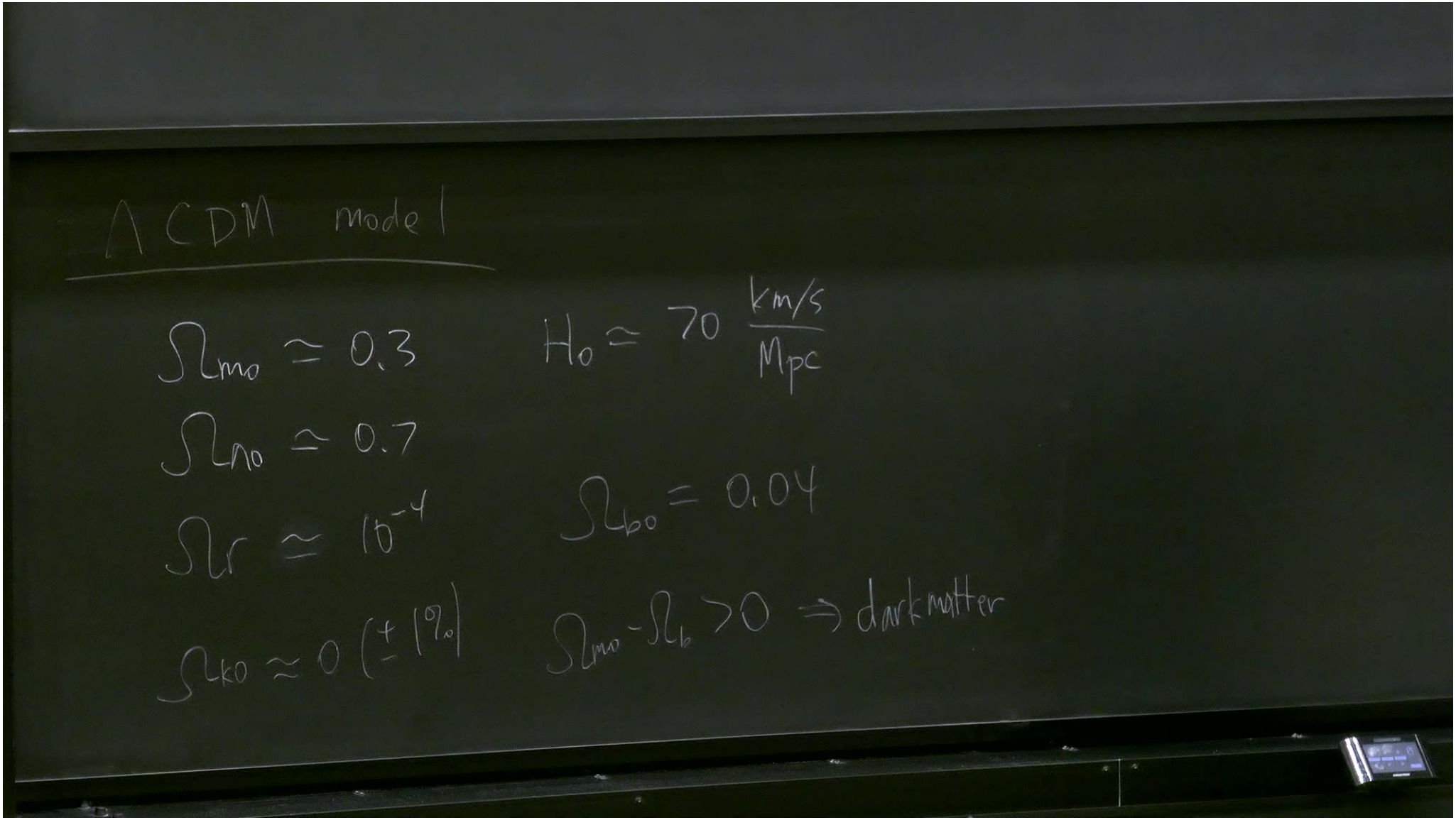
$\Omega_{k0} = -\delta_0$

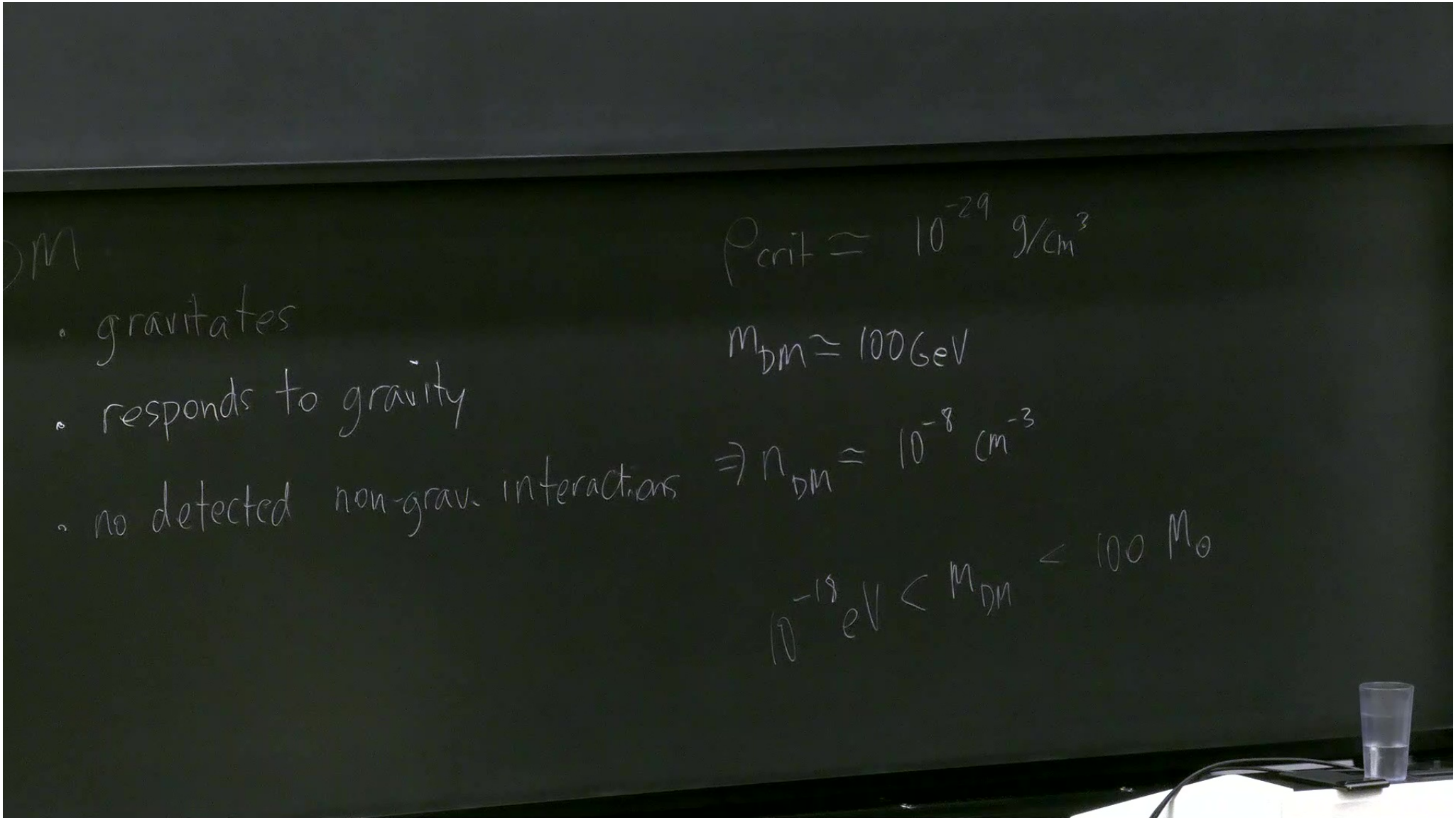
$$a_{max} = \frac{1+\delta_0}{\delta_0}$$

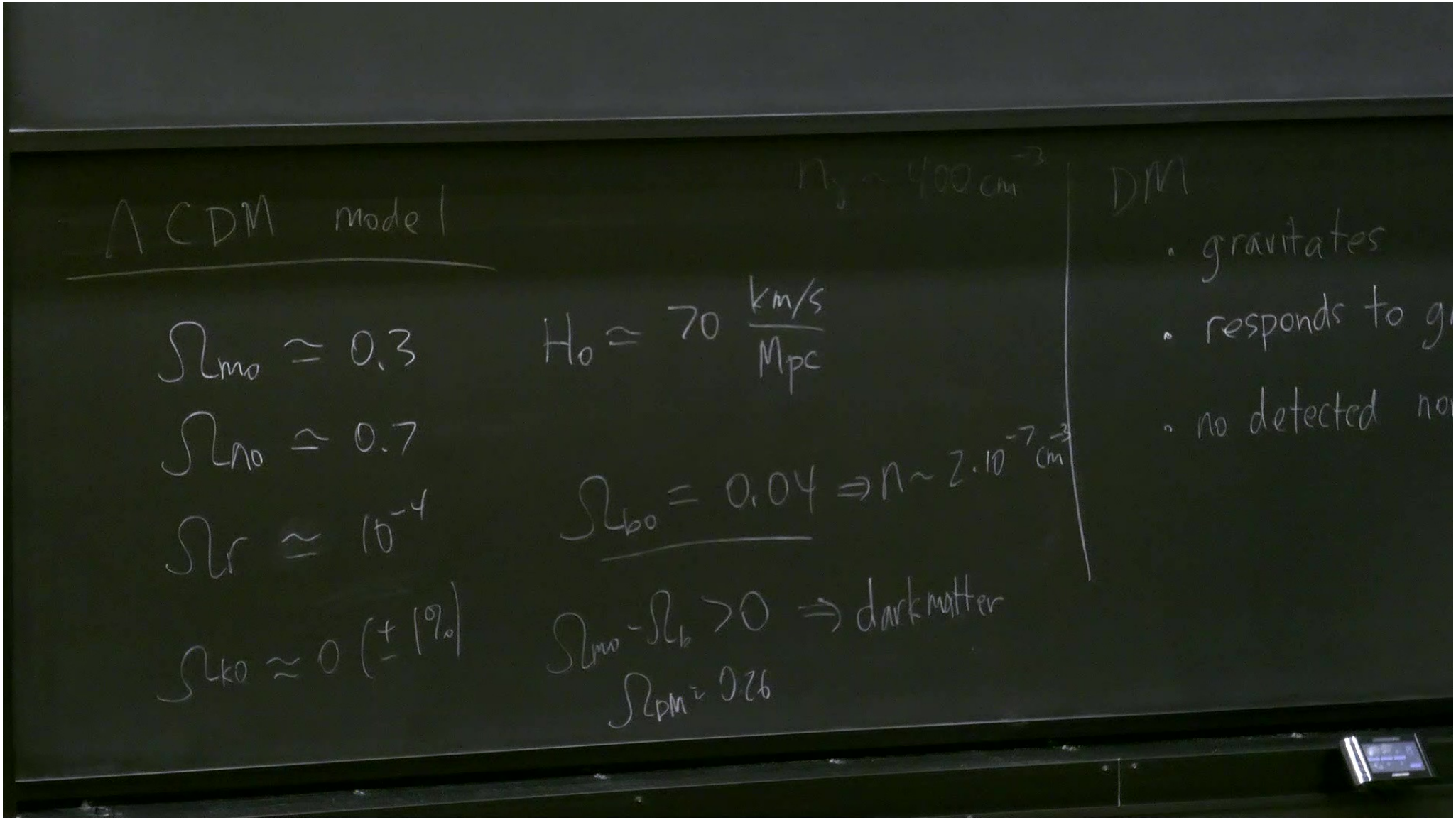
$$\dot{a}^2 = H_0 \left[\frac{1+\delta_0}{a} - \delta_0 \right]$$

$$H_0 \int_0^{t_{max}} dt = \int_0^{a_{max}} \frac{da}{\sqrt{(1+\delta_0)/a - \delta_0}} = \frac{\pi}{2} \frac{1+\delta_0}{\delta_0^{3/2}}$$


a_max
a
t_max
t
2 t_max
spherical collapse model


ΛCDM model
$\Omega_{m0} \simeq 0.3$
$H_0 \simeq 70 \frac{km/s}{Mpc}$
$\Omega_{\Lambda 0} \simeq 0.7$
$\Omega_r \simeq 10^{-4}$
$\Omega_{b0} = 0.04$
$\Omega_{k0} \approx 0 \ (\pm 1\%)$
$\Omega_{m0} - \Omega_b > 0 \Rightarrow$ dark matter


DM
• gravitates
• responds to gravity
• no detected non-grav. interactions
$\rho_{crit} \simeq 10^{-29}\ g/cm^3$
$m_{DM} \simeq 100\ GeV$
$\Rightarrow n_{DM} \simeq 10^{-8}\ cm^{-3}$
$10^{-19}\ eV < m_{DM} < 100\ M_\odot$


ΛCDM model
$n_\gamma \sim 400\,cm^{-3}$
$\Omega_{m0} \simeq 0.3$
$H_0 \simeq 70\ \frac{km/s}{Mpc}$
$\Omega_{\Lambda 0} \simeq 0.7$
$\Omega_r \simeq 10^{-4}$
$\Omega_{b0} = 0.04 \Rightarrow n \sim 2 \cdot 10^{-7}\,cm^{-3}$
$\Omega_{k0} \approx 0\ (\pm 1\%)$
$\Omega_{m0} - \Omega_b > 0 \Rightarrow$ dark matter
$\Omega_{DM} \simeq 0.26$
DM
• gravitates
• responds to g
• no detected