Title: Lecture - Quantum Matter, PHYS 777

Speakers: Chong Wang

Collection/Series: Quantum Matter (Elective), PHYS 777, March 31 - May 2, 2025

Subject: Condensed Matter

Date: April 29, 2025 - 9:00 AM

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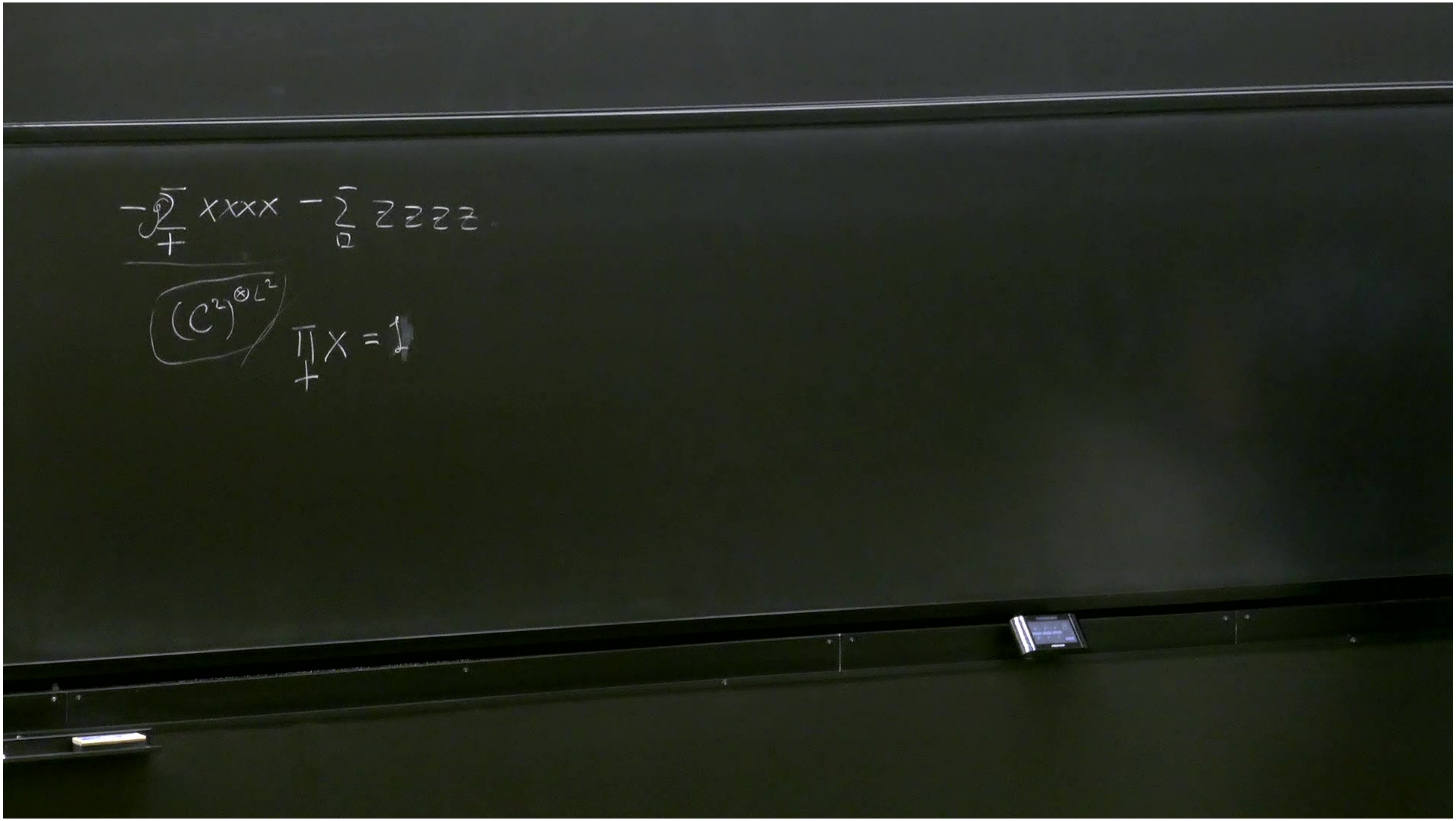


$-\sum_{+} XXXX - \sum_{\square} ZZZZ$
$(\mathbb{C}^2)^{\otimes L^2}$
$\prod_{+} X = 1$

Quantum Hall Effect (QHE). Ref. e.g. David Tong's

Today: Integer QHE. Response of Insulator

(2+1)d : $S_{Maxwell}[A_\mu] = \frac{1}{4e^2}\int F_{\mu\nu}^2$

Chern-Simons term: $S_{CS}[A_\mu] = i\frac{k}{4\pi}\int d^3x \, \underline{\epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda}$ (=

Breaks time-reversal & reflection. $x_1 \to -x_1$, A_1

Physically: Hall conductivity: $j_\mu = \frac{k}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda$

on QHE.

Gauge invariant: $A \to A + \partial\varphi$.

$$CS[A] \to CS[A] + i\frac{k}{4\pi}\int d^3x\, \underbrace{\partial_\mu(\epsilon_{\mu\nu\lambda}\,\phi\,\partial_\nu A_\lambda)}_{\text{total derivative.}}$$

Closed manifold ✓

$$\ldots A = A dA)$$

$$\begin{pmatrix} \ldots_x \\ \ldots_y \end{pmatrix} = \begin{pmatrix} 0 & -\frac{k}{2\pi} \\ \frac{k}{2\pi} & 0 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \end{pmatrix} \qquad \sigma_{xy} = \frac{k}{2\pi}.$$

$$j_i = \sigma_{ij} E_j$$

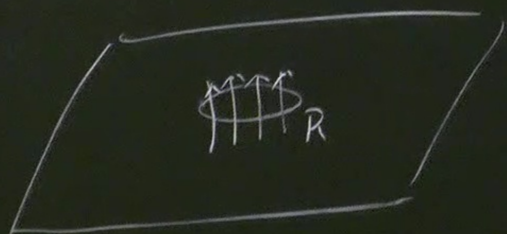
$$\rho = \frac{k}{2\pi} B$$

Outside R.

...n $k \in \mathbb{Z}$.

$|\psi_0\rangle \to |\psi_1\rangle$

"Flux-threading argument"



slowly turn on B within R, $t: 0 \to t_1$

$$\Phi = \int_R d^2x\, B = \begin{cases} 0 & t=0 \\ \underset{\underset{\frac{hc}{e}}{\parallel}}{2\pi} & t=t_1 \end{cases}$$

Thm: if insulator is "non-fractionalized", then $k \in \mathbb{Z}$.

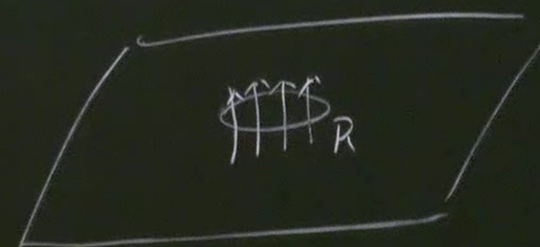
"Invertible"

"Unique gapped g.s."

"All local excitations are locally created".

"Flux-threading argument".

sl

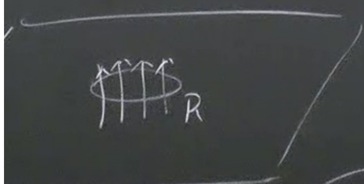


EB.

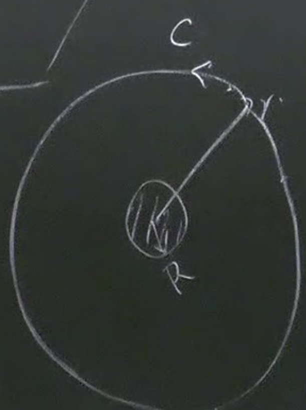
x-threading argument."

$|\psi_0\rangle \to |\psi_1\rangle$

slowly turn on B within R, $t: 0 \to t_1$



$$\Phi = \int_R d^2x\, B = \begin{cases} 0 & t=0 \\ \underset{\displaystyle \frac{hc}{e}}{\overset{\shortparallel}{2\pi}} & t=t_1 \end{cases}$$



$$\oint_C A\cdot dl = \underline{2\pi}$$

$$A_\theta = \frac{2\pi}{2\pi r} = \partial_\theta \frac{\theta}{r}$$

$$\vec{A} = \nabla \cdot \theta \quad \text{pure gauge}$$

Outside R. System is gauge equivalen

During: $\vec{A} \sim \frac{1}{|R|} << \Delta$

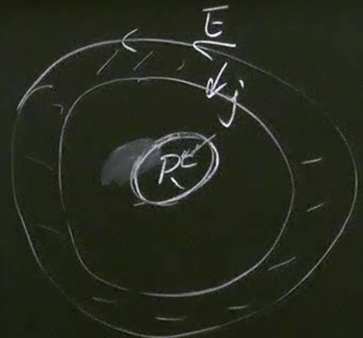
If $R >> \frac{1}{\Delta}$

$\Rightarrow$ Adiabatic, perturbative

$\Rightarrow$ After t_1, system goes back to g.s. outside R.

$$\Rightarrow \boxed{|\psi_1\rangle = \mathcal{O}_R |\psi_0\rangle}$$

Properties of $\mathcal{O}_k$? How much charge?



$$\vec{A} = \frac{\phi(t)}{2\pi r}\cdot\hat{\theta} \Rightarrow \vec{E} = \frac{\partial_t \phi}{2\pi r}\hat{\theta}.$$

$$\Rightarrow \vec{j} = \frac{k}{2\pi}\hat{z}\times\vec{E} = -\frac{k}{2\pi}\frac{\partial_t\phi}{2\pi r}\hat{r}.$$

$$\oint_C \vec{j}\cdot\hat{n} = \frac{k}{2\pi}\partial_t\phi.$$ Total charged flow through: $\Delta Q = k$.

$\Rightarrow \mathcal{O}_k$ carries charge k. $\Rightarrow k\in\mathbb{Z} \Rightarrow$ Hall conductivity quantized. σ_x

Measured in semiconductor/graphene structures.

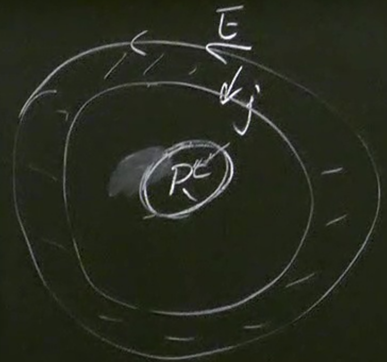
accurate enough to define h.

-ow through: $\Delta Q = k$.

-ductivity quantized. $\sigma_{xy} = k \cdot \frac{1}{2\pi} = k\left(\frac{e^2}{h}\right)$

Properties of $\mathcal{O}_k$? How much charge?

Measured



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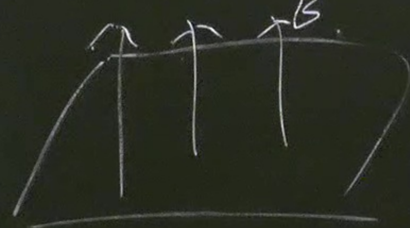
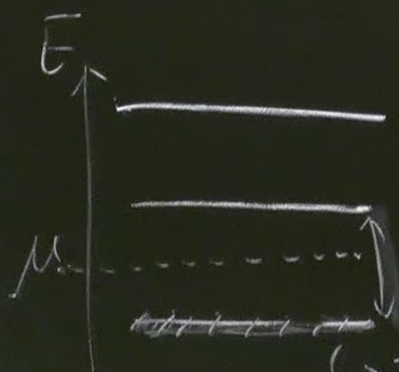
Canonical example: filled Landau level.



2DEG

$$H = \frac{1}{2m}(p - A)^2$$

$$\nabla \times A = B$$



$\omega_c = B/m$

↳ Degeneracy $= \Phi / \Phi_0 = \frac{\int B \, d^2x}{2\pi}$

Gapped unique g.s. at $\nu = \frac{N_e}{\Phi/\Phi_0} = \frac{2\pi\rho_e}{B} = 0, 1, 2 \ldots$

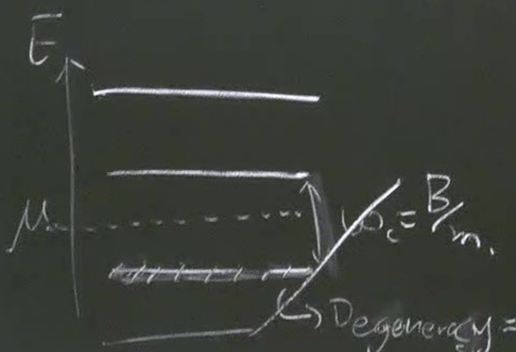
Adding $\Phi_0 = 2\pi \Rightarrow$ one extra electron in g.s. $\Rightarrow O_{P_1} \sim C^{\dagger} \Rightarrow k = 1$.

Comment: B field, translation & Galilean symmetry makes calculations simple.

But not essential

IQHE, Once formed, is stable against perturbations.

$\nabla \times A = B$



$\omega_c = B/m$

$\hookrightarrow$ Degeneracy $= \Phi/\Phi_0 = \frac{\int B\, d^2x}{2\pi}$

$V(x) \sim x \sim k_y$

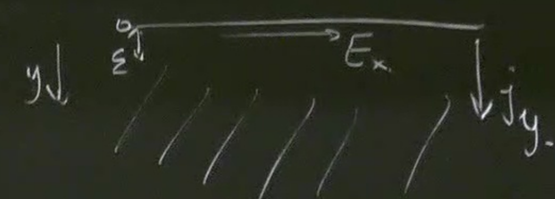
Comment: B field. translation & Galilean

But not essential

IQHE, Once formed, is stable a

Edge state:

(Anomaly in QFT)



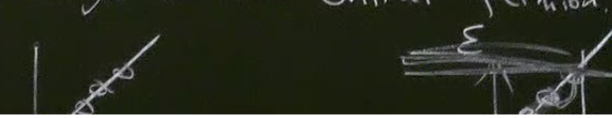
On edge: $\underline{\partial_t \rho + \partial_x j_x} = -\partial_y j_y = \frac{k}{2\pi}$

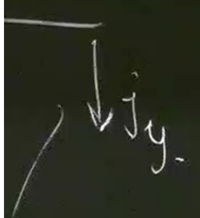
① Edge should be gapless. (otherw

② $U(1)$ symm. on edge is broke

('t Hooft anomaly)

Simplest edge state: chiral fermion.





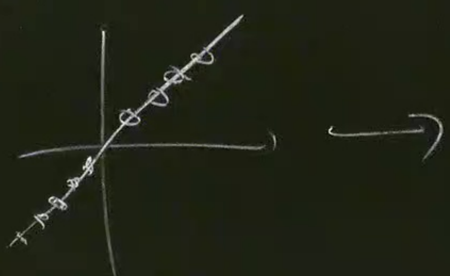
On edge. $\partial_t \rho + \partial_x j_x = -\partial_y j_y = \frac{k}{2\pi} E_x$ ✓

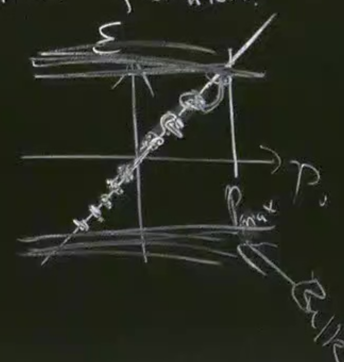
① Edge should be gapless. (otherwise no response for small E_x)

② U(1) symm. on edge is broken under background gauge field.

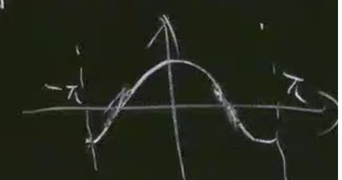
('t Hooft anomaly)

Simplest edge state: chiral fermion. $H = \int \psi^\dagger i\partial_x \psi = \sum_p p\, \psi^\dagger_p \psi_p$





"Half" of what's allowed



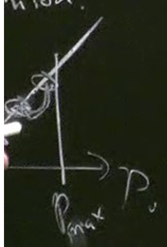
$= \underbrace{\frac{k}{2\pi} E_x}$ ✓

(otherwise no response for small E_x)

is broken under background gauge field

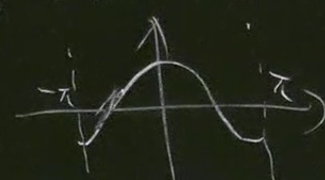
ly)

nion.



$$H = \int \psi^\dagger i\partial_x \psi = \sum_p p\, \psi_p^\dagger \psi_p$$

"Half" of what's allowed on a strictly 1d lattice . . . . . c_2



"Newton's law"

$$\frac{dp}{dt} = E_x$$

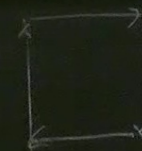
$$\text{Total charge} = \sum_{p>0} \langle c_p^\dagger c_p \rangle = \frac{|p_{max}|}{2\pi/L}$$

$$|p_{max}| = E_x t$$

$$\Rightarrow \partial_t \rho = \frac{E_x}{2\pi} \quad ✓ \; (k=1)$$

(Chiral anomaly in (1+1)d)

σ_{xy} computable for free fermion insulators. (TKNN)



turn on.

$$A_x = \frac{\theta_x(t)}{L_x}, \quad A_y = \frac{\theta_y(t)}{L_y}.$$

turn on $(\theta_x, \theta_y): (0,0) \to (0, 2\pi) \to (2\pi,$

$$\int CS(A) = \frac{i}{4\pi}\int (\theta_y \partial_t \theta_x - \theta_x \partial_t \theta_y)\, dt.$$

$(0, 0) \longrightarrow (0, 2\pi) \longrightarrow (2\pi, 2\pi) \longrightarrow (2\pi, 0) \longrightarrow (0, 0)$

In momentum space

$$(p_x, p_y) \longrightarrow \left(p_x, p_y + \frac{2\pi}{L_y}\right) \longrightarrow \left(p_x + \frac{2\pi}{L_x}, p_y + \frac{2\pi}{L_y}\right) \longrightarrow \left(p_x + \frac{2\pi}{L_x}, p_y\right) \longrightarrow (p_x, p_y)$$

$\square \rightsquigarrow$ Berry phase picked up $\phi_p = \int_{\square} b \, d^2x$

Berry curvature

"momentum space mag. field".

Bloch state: $c_{p} = \sum_{r} e^{i\vec{p}\cdot\vec{r}} u_p^a c_a(r)$

(e.g. $a = \uparrow, \downarrow$)

$|u_p^a\rangle = \begin{pmatrix} u_p^\uparrow \\ u_p^\downarrow \end{pmatrix}$

$(2\pi, 2\pi) \to (2\pi, 0) \to (0, 0)$

mentum space

$(p_x, p_y + \frac{2\pi}{L_y}) \longrightarrow (p_x + \frac{2\pi}{L_x}, p_y + \frac{2\pi}{L_y}) \longrightarrow (p_x + \frac{2\pi}{L_x}, p_y) \longrightarrow (p_x, p_y)$

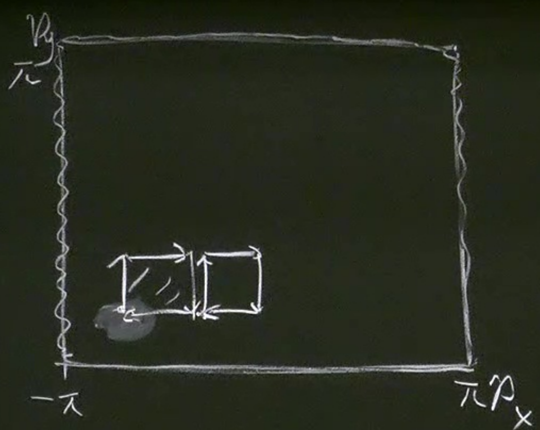
$\square \rightsquigarrow$ Berry phase picked up $\phi_p = \int_{\square} b \, d^2x$

Berry curvature

"momentum space mag. field".

$\hookrightarrow$ Degeneracy $= \Phi/\Phi_0 = \dfrac{\int B\, d^2x}{2\pi}$

$V(x) \sim x \sim k_y$



Momentum space: T^2

$$\Phi = \sum_{p \in T^2} \phi_p = \int_{T^2} b\, d^2p = 2\pi \cdot \underset{\in \mathbb{Z}}{C}$$ "1st Chern number"

$$\Rightarrow \sigma_{xy} = \frac{C}{2\pi} \quad (\text{TKNN})$$

Why is this natural?

$C \neq 0 \Rightarrow$ Band. $\underbrace{\{|u_p\rangle, p \in T^2\}}_{\text{"U(1) principal bundle"}}$ is "nontrivial". i.e.

$\underbrace{|u_p\rangle \text{ cannot be defined globally (single patch)}}_{\text{"No global section"}}$

If $|u_p\rangle$ globally defined, can do inverse Fourier transform.

$\hat{c}_r \sim \sum e^{-i\vec{p}\cdot\vec{r}} c_p$ would be a local fermion.

$|G.S.\rangle = \prod_r \hat{c}_r^\dagger |0\rangle$ Product state.

$$\Phi_p = \int_{T^2} b \, d^2p = 2\pi \cdot C \quad \in \mathbb{Z} \text{ "1st Chern number"}$$

$$\sigma_{xy} = \frac{C}{2\pi} \quad \text{(TKNN)}$$

is this natural?

$\Rightarrow$ Free fermion band theory:

Classifying topo. phases $\simeq$ classifying band structure as a fibre bundle.

$C \neq 0 \Rightarrow \underbrace{\text{Band. } \{|u_p\rangle, p}_{\text{"U(1) principal}}$

If $|u_p\rangle$ globally defined

$$\hat{C}_r \sim \sum e^{-i\vec{p}\cdot\vec{r}} c_p$$

$$|G.S.\rangle = \prod_r \hat{C}_r^+ |$$