Title: Lecture - Quantum Matter, PHYS 777

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Subject: Condensed Matter

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Ginzburg-Landau $\mathcal{L}[\Phi, A_\mu] \xrightarrow[\langle \Phi \rangle \neq 0]{}$ Goldstone-Higgs $\mathcal{L}[\theta, A_\mu] = \frac{K}{2}(\partial$

Comment: in a real superconductor $\Phi \sim cc$ two electrons a.k.a. Coop

on

Why this is prefered ← "Pairing mecha

Universality.

Good metals, understood. electron-p

$$|\partial\phi|^2 + r|\phi|^2 + \lambda|\phi|^4$$

"Unconventional S.C." "High-T_c Superc

$\rangle \neq 0$ $\longrightarrow$ Goldstone-Higgs $\mathcal{L}[\theta, A_\mu] = \frac{K}{2}(\partial\theta - A)^2$.

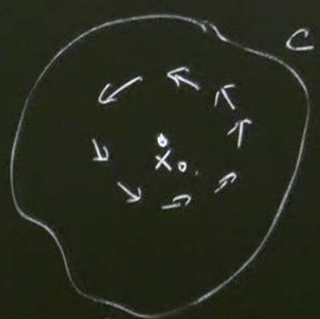
ctor. $\Phi \sim CC$ two electrons a.k.a. Cooper pair.

Why this is prefered $\leftarrow$ "Pairing mechanism"

Good metals, understood. electron-phonon coupling. $\leftarrow$ BCS. theory.

"Unconventional S.C." "High-T_c Superconductor"

Vortex: θ can have a topological defect.



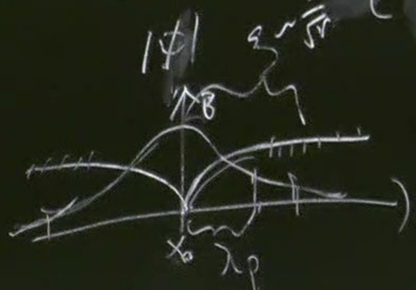
2d.

$$\oint_C \vec{\nabla}\theta \cdot d\ell = 2\pi n, \quad n \in \mathbb{Z}.$$

$$\vec{\nabla} \times \vec{\nabla}\theta = 2\pi n \cdot \delta^{(2)}(x - x_0)$$

θ is not defined at x_0

$(|\phi| e^{i\theta} \to 0$ as $x \to x_0)$



$\Rightarrow$ Vortex cannot be created/des

In a S.C. vortex costs ene

For a dynamical gauge field

For larg C, $\quad A \to A + \partial\theta$

$A = \partial\theta$

$$\int B\, d^2x = \int A \cdot d\ell = \int \partial\theta \cdot d\ell = 2\pi n.$$

$$\frac{K}{2}(\partial\theta - A)^2 + (\partial \times A)^2.$$

...pological defect.

...n., $n \in \mathbb{Z}$. $\Rightarrow$ Vortex cannot be created/destroyed locally.

In a S.C., vortex costs energy, but is stable once formed.

...n. $\delta^{(2)}(x - x_0)$

For a dynamical gauge field A

...ed at x_0

$A \to A + \partial\theta$ illegal with vortex

...0 as $x \to x_0$]

For larg C; $B = \nabla \times A = \nabla \times \partial\theta = 2\pi n \, \delta(x - x_0)$

$A = \partial\theta$

$\frac{K}{2}(\partial\theta - A)^2 + (\partial \times A)^2$. minimize if $A = \partial\theta$,

Put back Maxwell

$\int B \, d^2x = \int A \cdot dl = \int \partial\theta \, dl = 2\pi n$.

Charge-Vortex duality. (Refs: David Tong's notes on Gauge Theory. Chp 8).

Claim: U(1) Goldstone theory (superfluid) is dual to a $\underbrace{\text{gauge theory}}_{\text{U(1) Maxwell}}$

(2+1)d

$$Z = \int D\theta \, e^{-\int d^3x \frac{K}{2}(\partial\theta)^2} = \int D\theta \, DA_\mu \, Df_{\mu\nu} \, e^{-\int \frac{K}{2} A_\mu^2} \, \underline{e^{i\int \frac{\epsilon_{\mu\nu\lambda}}{2\pi} f_{\mu\nu} (A_\lambda - \partial_\lambda \theta)}}$$

$$\underset{A_\mu}{\overset{\text{integrate out}}{=}} \int Df_{\mu\nu} \, \underline{D\theta} \, e^{-\int \frac{1}{2} \frac{1}{4\pi^2 K} f_{\mu\nu}^2} \, \underline{e^{i\int \frac{\epsilon_{\mu\nu\lambda}}{2\pi} \theta_\lambda \partial f_{\mu\nu}}}$$

$$\int D\theta \Rightarrow \epsilon_{\mu\nu\lambda} \partial_\mu f_{\nu\lambda} = 0$$

n Gauge Theory. Chp. 8) "T-duality"

a gauge theory $\quad \vartheta$ Compact $\Rightarrow \oint_{C_2} f_{\mu\nu} = 2\pi n$

U(1) Maxwell

$$\int \frac{\epsilon_{\mu\nu\lambda}}{2\pi} f_{\mu\nu} \left(A_\lambda - \partial_\lambda \vartheta\right)$$

$$e^{i\int \frac{\epsilon_{\mu\nu\lambda}}{2\pi} \vartheta_\lambda \partial f_{\mu\nu}} \overset{\int D\vartheta}{=} \int e^{-\int \frac{1}{4e^2}(\partial_\mu a_\nu - \partial_\nu a_\mu)^2} Da_\mu$$

$$\int D\vartheta \Rightarrow \epsilon_{\mu\nu\lambda}\partial_\mu f_{\nu\lambda} = 0$$

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$$

(2+1)d only has 1 polarization $\simeq$ 1 mass scalar.

(2+1)d photon $\equiv$ Goldstone mode.

Conserved U(1) current $j_\mu = K\partial_\mu \theta \longleftrightarrow$ Maxwell. $j_\mu = \frac{\epsilon_{\mu\nu\lambda}}{2\pi}\partial_\nu a_\lambda$, $\partial_\mu j_\mu = 0$.

$j_0 = \frac{b}{2\pi}$, $\vec{j} = \frac{\hat{z}\times\vec{e}}{2\pi}$

Quantized charge. $\longleftrightarrow$ Quantized. $\int d^2x \, \frac{b}{2\pi} = n \in \mathbb{Z}$. "Compact" U(1) gauge field.

$\simeq$ 1 mass scalar.

$\equiv$ Goldstone mode.

$\partial_\mu \theta \longleftrightarrow$ Maxwell. $j_\mu = \frac{\epsilon_{\mu\nu\lambda}}{2\pi} \partial_\nu a_\lambda$, $\partial_\mu j_\mu = 0$.

$j_0 = \frac{b}{2\pi}$, $\vec{j} = \frac{\hat{z} \times \vec{e}}{2\pi}$

$\longleftrightarrow$ Quantized. $\int d^2x \frac{b}{2\pi} = n \in \mathbb{Z}$. "Compact" U(1) gauge field. Dirac quantization.

$\simeq$ 1 mass scalar.

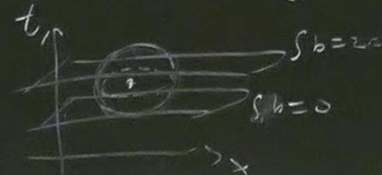
ε Goldstone mode.

$\partial_\mu \theta \longleftrightarrow$ Maxwell. $j_\mu = \frac{\varepsilon_{\mu\nu\lambda}}{2\pi} \partial_\nu a_\lambda$, $\partial_\mu j_\mu = 0$.

$j_0 = \frac{b}{2\pi}$, $\vec{j} = \frac{\hat{z} \times \vec{e}}{2\pi}$

$\longleftrightarrow$ Quantized. $\underline{\int d^2x \frac{b}{2\pi} = n \in \mathbb{Z}}$. "Compact" U(1) gauge field. Dirac quantization.

$\longleftrightarrow$ An operator changes $\int d^2x \frac{b}{2\pi}$: (Spacetime) monopole (instanton)



$\Rightarrow$ (2+1)d Maxwell theory is a condensate (superfluid) of monopoles.

...ton in (2+1)d only has 1 polarization $\simeq$ 1 mass scalar.

(2+1)d photon $\equiv$ Goldstone mode.

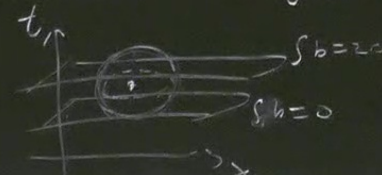
Conserved U(1) current $j_\mu = K\partial_\mu\theta$ $\longleftrightarrow$ Maxwell. $j_\mu = \frac{\epsilon_{\mu\nu\lambda}}{2\pi}\partial_\nu a_\lambda$, $\partial_\mu j_\mu = 0$

Quantized charge. $\longleftrightarrow$ Quantized. $j_0 = \frac{b}{2\pi}$, $\vec{j} = \frac{\hat{z}\times\vec{e}}{2\pi}$

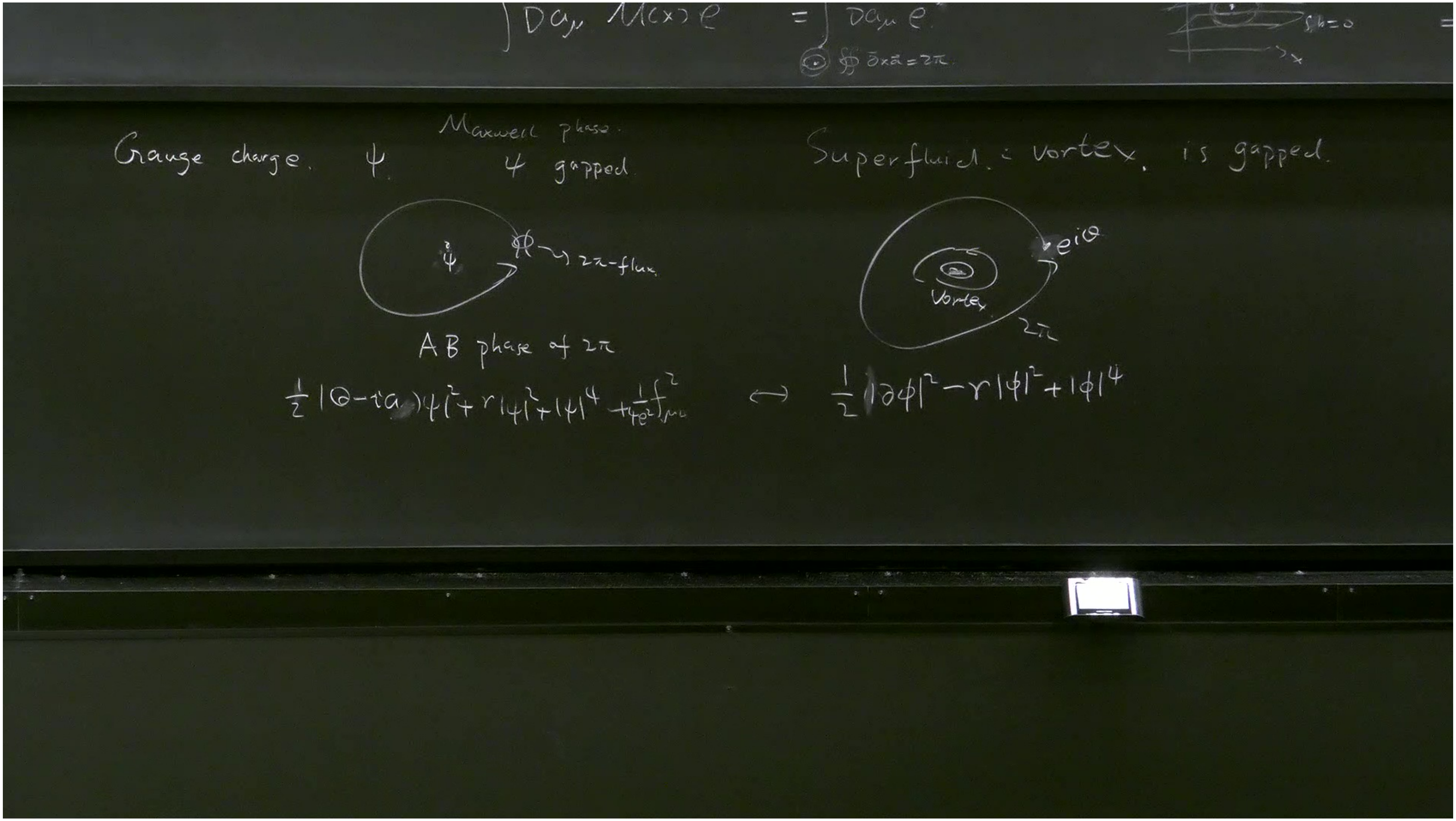
$\underline{\int d^2x \, \frac{b}{2\pi} = n \in \mathbb{Z}}$. "Compact" U(1) g...

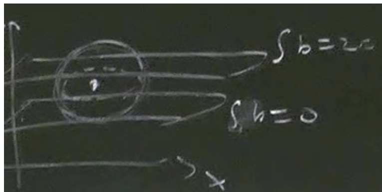
Charged operator. $e^{i\theta}$ $\longleftrightarrow$ An operator changes $\int d^2x \, \frac{b}{2\pi}$: (Spacetim...

$$\int Da_\mu \, M(x) \, e^{-S[a]} = \int_{\oint \vec{\partial}\times\vec{a} = 2\pi} Da_\mu \, e^{-S[a]}$$



$\Rightarrow$ (2+1)d Maxwe...


$\int Da_\mu M(x) e = \int Da_\mu e$
$\oint \nabla \times \vec{a} = 2\pi$
Gauge charge. ψ.
Maxwell phase.
ψ gapped
Superfluid: vortex. is gapped.
$\psi \to 2\pi$-flux
AB phase of 2π
$e^{i\theta}$
Vortex
2π
$\frac{1}{2}|(\partial - ia)\psi|^2 + r|\psi|^2 + |\psi|^4 + \frac{1}{4e^2} f_{\mu\nu}^2 \leftrightarrow \frac{1}{2}|\partial\phi|^2 - r|\phi|^2 + |\phi|^4$



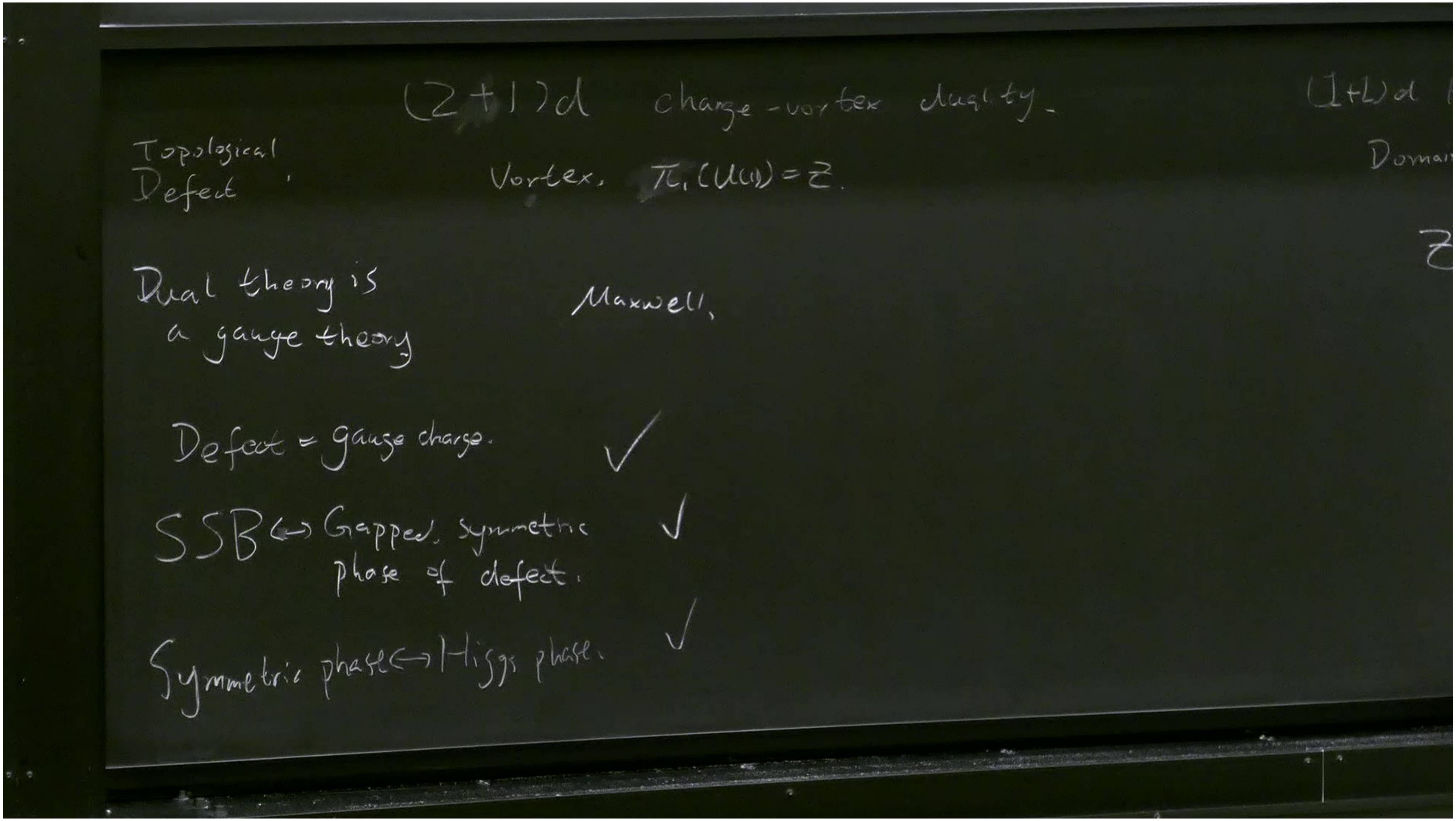
$\Rightarrow$ (2+1)d Maxwell theory is a condensate (superfluid) of monopoles.

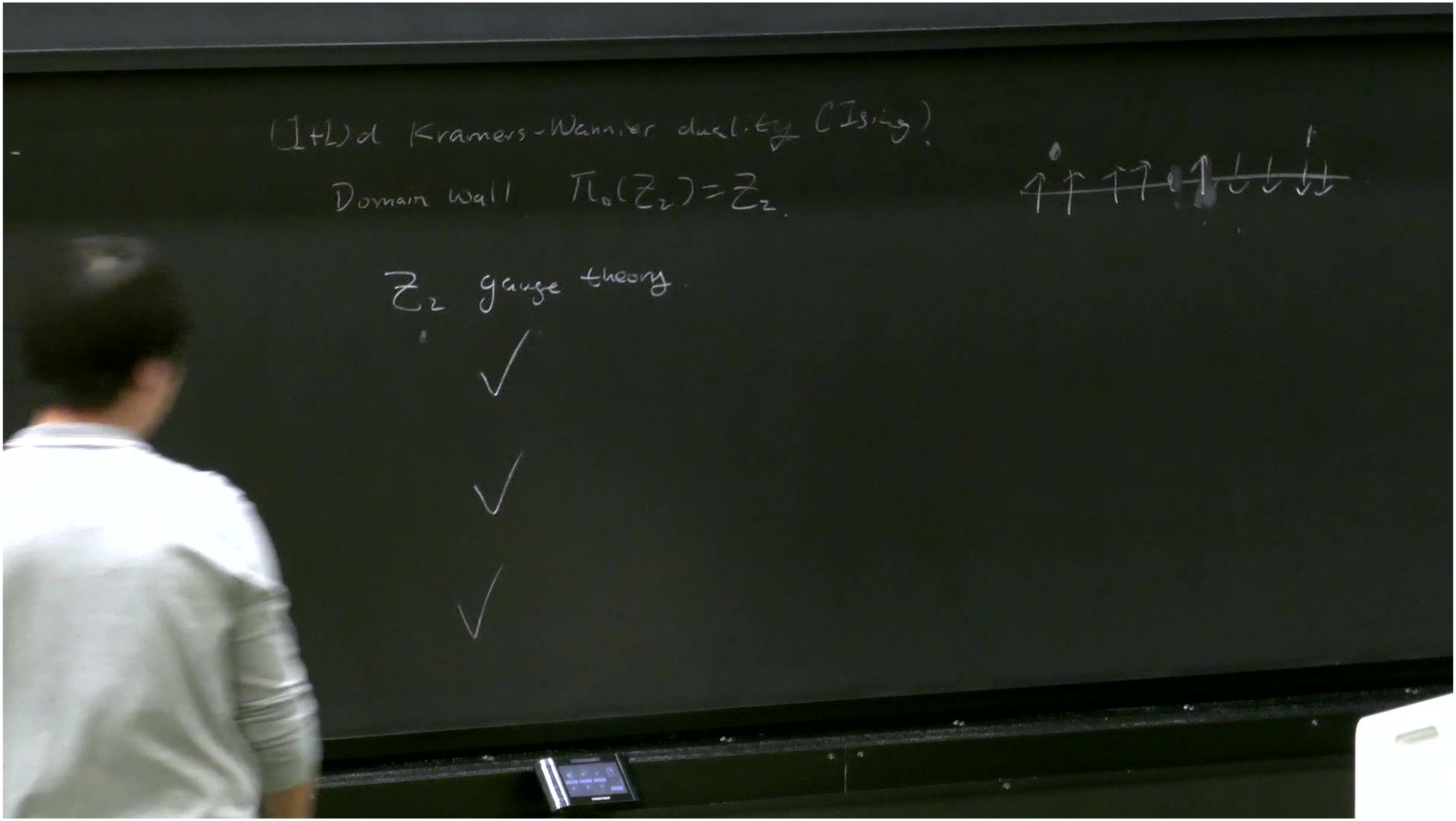
is gapped.

$r<0$. Insulator phase, trivially gapped.

Gauge theory. $\underline{\langle\phi\rangle \neq 0}$, gapped vacuum (Higgs).

$\Rightarrow$ Insulator is a condensate of vortices.


(2+1)d charge-vortex duality.
(1+1)d
Topological Defect :
Vortex, $\pi_1(U(1)) = \mathbb{Z}$.
Domai
Dual theory is a gauge theory
Maxwell,
Defect = gauge charge. ✓
SSB ↔ Gapped, symmetric phase of defect. ✓
Symmetric phase ↔ Higgs phase. ✓


(1+1)d Kramers-Wannier duality (Ising)
Domain wall $\pi_0(\mathbb{Z}_2)=\mathbb{Z}_2$.
$\mathbb{Z}_2$ gauge theory.

Symmetric phase ⟷ Higgs phase. ✓

Charged operator ⟷ Instantons.

Monopole

Boundary-condition changing op.

Bonus: Maxwell $+ \lambda[\mathcal{M}(x) + \mathcal{M}^*(x)]$ ⟷ $(\partial\theta)^2 + \lambda(e^{i\theta} + e^{-i\theta})$

$\sim \theta^2$

Photon becomes gapped due to instanton ⟷ Gapped

⇒ Gauge charge confined. ⟷ r

$E \sim O(|r|)$

Polyakov mechanism.