Title: Lecture - Quantum Matter, PHYS 777

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Collection/Series: Quantum Matter (Elective), PHYS 777, March 31 - May 2, 2025

Subject: Condensed Matter

Date: April 24, 2025 - 10:15 AM

URL: https://pirsa.org/25040010

$U(1)$: Insulator. Superfluid, Superconductor.

Ginzburg–Landau theory

Effective field theory. in term $\phi(x,t)$ Smoothly varying in spacetime. (e

$U(1)$ symmetry: $\phi(x,t) \to e^{i\alpha}\phi(x,t)$

$$Z[A] = \int D\phi\, D\phi^* \, e^{-\int d^dx\, d\tau\, \mathcal{L}[\phi, A_\mu]}$$

A_μ : background/probe gauge field.

$$= e^{-F[A]}$$

$$\mathcal{L}[\phi, A] = i\phi^*(\partial_t - iA_t)\phi + \frac{1}{2}|(\nabla - iA)\phi|^2 + \frac{r}{2}|\phi|^2 + \frac{\lambda}{4}|\lambda|$$

ace time. (energy/momentum cut off. $\sim \Lambda <<$ lattice scale)

ield.

higher order in ϕ, ∂.

$\frac{r}{2}|\phi|^2 + \frac{\lambda}{4}|\lambda|^4 + \dots$

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$$\mathcal{L}[\phi, A] = \underbrace{i\phi^*(\partial_t - iA_t)\phi}_{\text{Sometimes extra symm condition } |(\partial_t - iA_t)\phi|^2} + \frac{1}{2}|(\vec{\nabla} - i\vec{A})\phi|^2 + \frac{r}{2}|\phi|^2 + \frac{\lambda}{4}|\lambda$$

Consider $|r| \gg 0$: mean field.

$r \gg 0$. Vacuum (g.s.) fluctuates around $\langle\phi\rangle = 0$. $U(1)$ not spontaneously broken.

Correlation length $\xi \sim 1/\sqrt{r}$. $\langle \phi^*(x,t)\,\phi(0,t)\rangle \sim e^{-x/\xi}$

Energy gap $\Delta \sim r$ ($\sqrt{r}$ if relativistic)

$F(A)$ should be local theory at length $\gg \xi$, energy $\ll \Delta$.

Maxwell

$$F(A) = \int c\,(\partial_t \vec{A} - \vec{\partial} A_t)^2 + c\,(\partial_i A_j - \partial_j A_i)^2 + o(\partial, A)$$

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Dielectric response

...ntanously broken.

$\ldots \rangle \sim e^{-x/\xi}.$

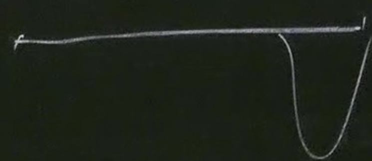
$\ldots < \Delta.$

Some response theory:

$B = 0.$

Current density $j_i = -\frac{\delta F(A)}{\delta A_i} = C \cdot \partial_t \vec{E} = \sigma \cdot \vec{E}$

$\Rightarrow \sigma(\omega) = i\, C \cdot \omega.$



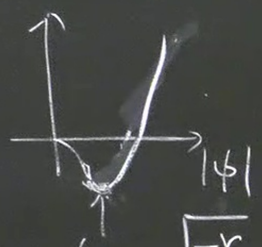
① Non-dissipative.

② Vanishes as $\omega \to 0$ (DC limit)

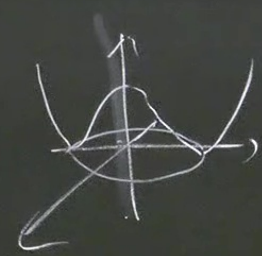
$\Rightarrow$ Insulator.

$\Rightarrow$ Gapped system are insulators

$r \ll 0$ minimize $V(\phi)$



$$\phi_0 = \sqrt{\frac{-r}{\lambda}}$$



SSB of $U(1)$

Expand around ϕ_0, $\phi = (\phi_0 + \underbrace{\delta\rho}_{\text{real}}) e^{i\theta}$

Amplitude mode $\delta\rho$ is gapped. → inte

$\Rightarrow \mathcal{L}[\theta] = \frac{K}{2}\left[(\partial_t \theta)^2 + (\nabla\theta)^2\right]$ θ :

$U(1)$: $\theta(x) \to \theta(x) + \alpha$. $\cancel{\cos\theta}$, $\cancel{\sin\theta}$

Compactness: $\theta \sim \theta + 2\pi$. $\cancel{\theta}$, $\cancel{\theta^2}$

Gol

Superfluidity:

real

$(\phi_0 + \delta\rho)e^{i\theta}$

apped. → integrate it out!

θ: gapless mode, $E \sim |k|$

"Goldstone mode".

Goldstone theorem: such gapless mode always exit for continuous SSB.

Superfluidity

Put back gauge field $\mathcal{L} = \frac{K}{2}(\partial_\mu \theta - A_\mu)^2$.

If I turn on $A_x = \text{const}$, $\mathcal{L}$ minimized by $\partial_x \theta = A_x$.

$\Rightarrow$ G.S. has current $j_x = K \partial_x \theta = K A_x$ per field $\mathcal{L} = \frac{K}{2}(\partial_\mu \theta - A_\mu)^2$.

$\int dt\ j \cdot E = 0$.

$A_x = \text{const}$. $\mathcal{L}$ minimized by $\partial_x \theta = A_x$.

$\Rightarrow$ G.S. has current $j_x = K \partial_x \theta = K A_x$ persistent (non-dissipative)

$A_x = \frac{E_x}{i\omega} \Rightarrow \sigma(\omega, \vec{q}=0) = \frac{K}{i\omega}$

① Non-dissipative

② Diverges as $\omega \to 0$

Superconductor

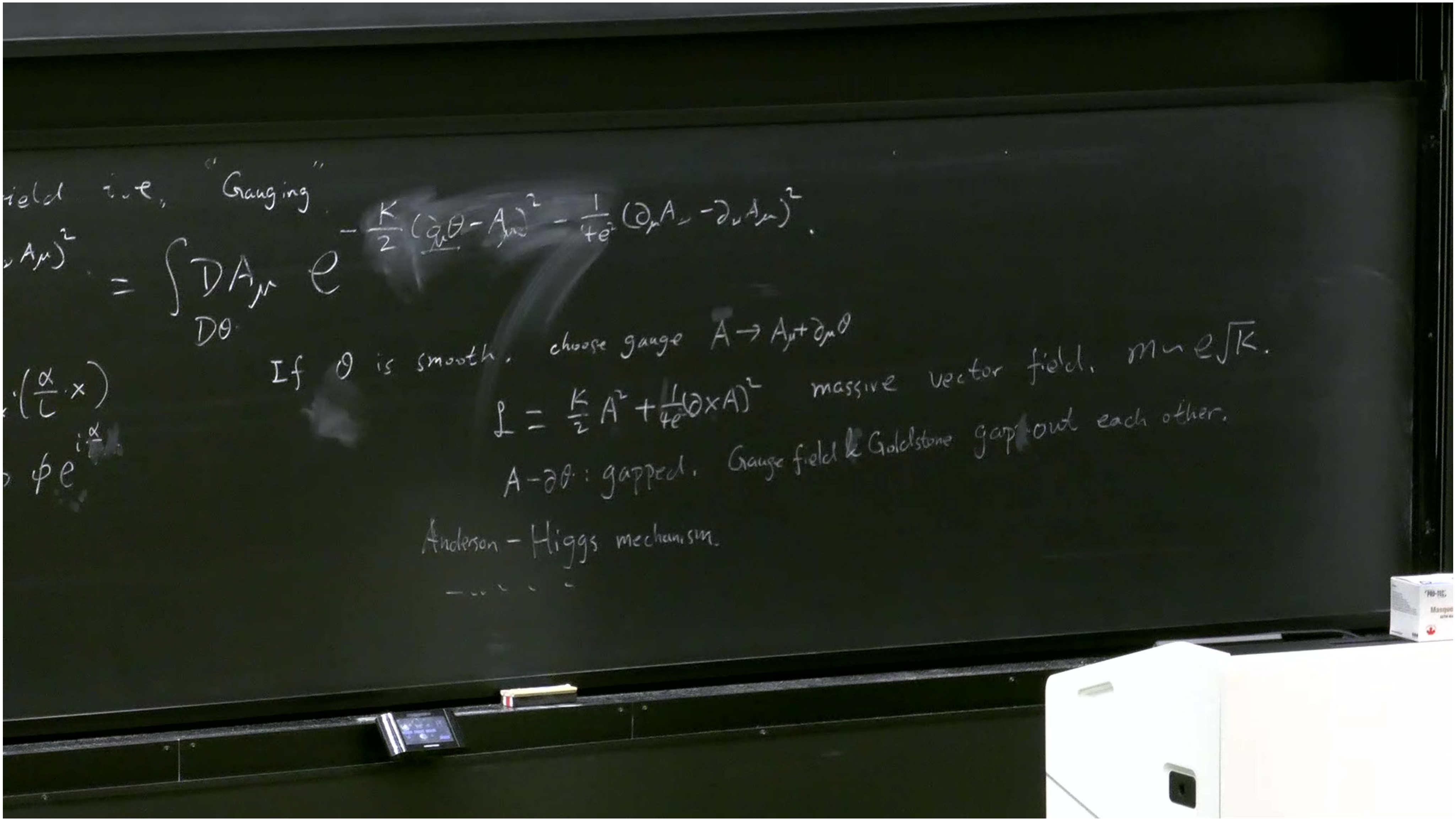
Now upgrade A_μ to a dynamical gauge field i.e. "Gauging"

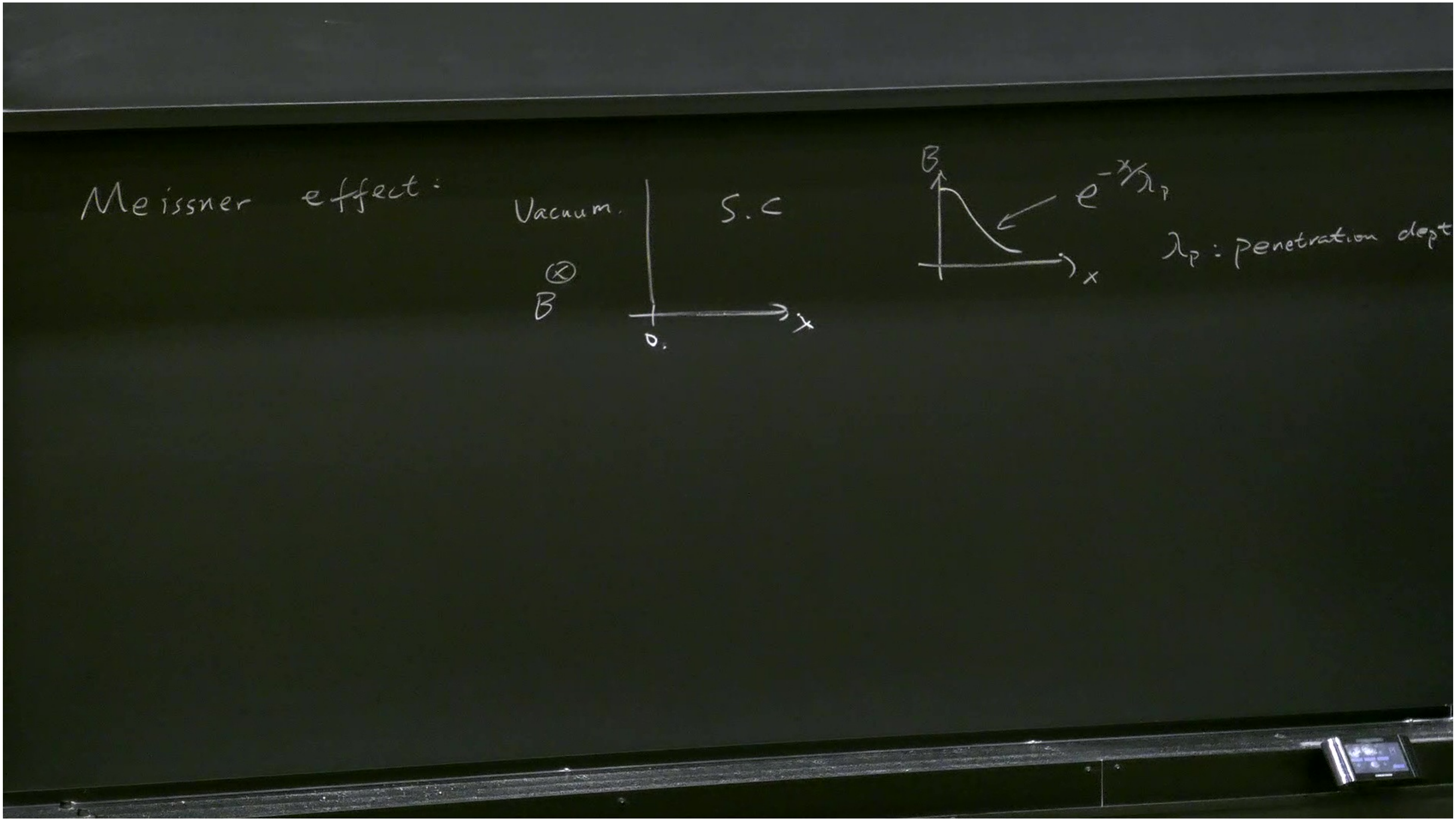
$$Z = \int D\phi\, DA_\mu\, e^{-S[\phi, A_\mu]}\, e^{-\frac{1}{4e^2}\int(\partial_\mu A_\nu - \partial_\nu A_\mu)^2} = \int DA_\mu\, e^{-F[A] - F_{Maxwell}[A]}$$

A

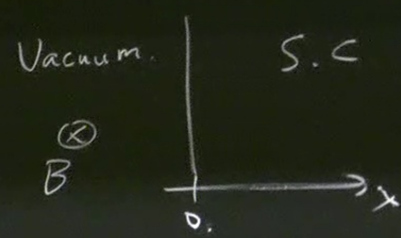
$$A_x = \frac{\alpha}{L} = \partial_x\left(\frac{\alpha}{L}\cdot x\right)$$

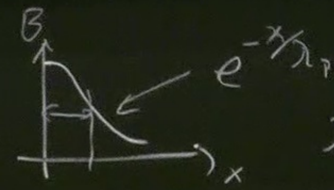
$$\phi \to \phi e^{i\alpha}$$


ield i.e. "Gauging"
$= \int_{D\theta} DA_\mu \, e^{-\frac{K}{2}(\partial_\mu\theta - A_\mu)^2 - \frac{1}{4e^2}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2}$.
If θ is smooth. choose gauge $A \to A_\mu + \partial_\mu\theta$
$\mathcal{L} = \frac{K}{2}A^2 + \frac{1}{4e^2}(\partial\times A)^2$ massive vector field. $m \sim e\sqrt{K}$.
$A-\partial\theta$: gapped. Gauge field & Goldstone gap out each other.
Anderson - Higgs mechanism.


Meissner effect:
Vacuum.
S.C
B
0.
x
B
x
e^{-x/λ_p}
λ_p : penetration dept

Meissner effect:





λ_p : penetration depth $\sim \dfrac{1}{e\sqrt{K}}$