Title: Lecture - Quantum Matter, PHYS 777

Speakers: Chong Wang

Collection/Series: Quantum Matter (Elective), PHYS 777, March 31 - May 2, 2025

Subject: Condensed Matter

Date: April 22, 2025 - 9:00 AM

URL: https://pirsa.org/25040009

Toric code. $H = -\sum_{+} XXXX - \sum_{\square} ZZZZ$.

$Z_2^{(0)} \times Z_2^{(0)}$ (1-form) symmetry. Z_c  X_c 

$\tilde{X} = UXU^\dagger$

$X|\uparrow\uparrow\uparrow\uparrow\ldots\rangle = |\uparrow\uparrow\uparrow\uparrow\ldots\rangle$?

$|\uparrow\uparrow\uparrow\ldots\rangle \pm |\downarrow\downarrow\downarrow\ldots\rangle$?

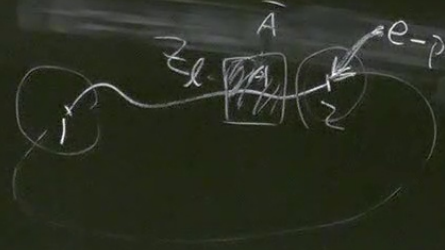
$H = -\sum X_i + g\sum Z$

$$H = -\sum_{+} XXXX - \sum_{\square} ZZZZ.$$

$Z_2^{(0)} \times Z_2^{(0)}$ (1-form) symmetry: Z_c $X_{c'}$

Local commutation: Z X $= (-1)$ Z X (a.k.a "braiding statistics", "anomaly", ...)

Consequences: (1) Open strings creates excitation at 1, 2. Excitation energy $\Delta E \sim O(1)$ (not $\sim O(1/|1|)$). "Deconfined"



"confined"

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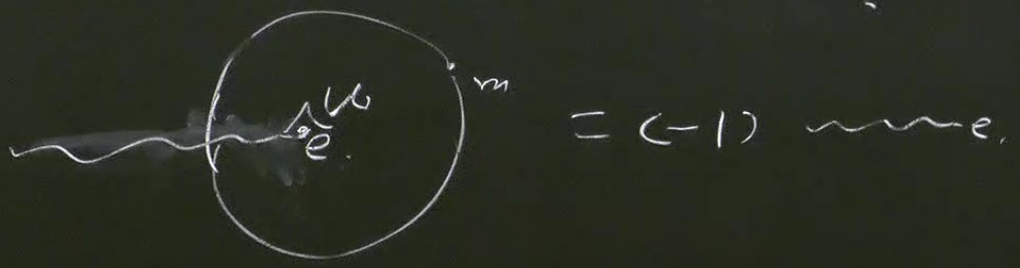
X_ℓ m-excitation $\bar{A}$ Z_ℓ e-particle 1 2

(2) e & m mutual statistics.



e m $:=$ ϵ ϵ : fermion

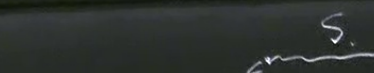
(3) e, m, ϵ cannot be created locally.

$Z_1|\psi\rangle \neq U_1 U_2 |\psi\rangle$ follows from mutual statistics.

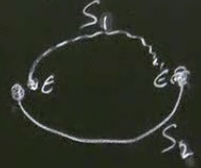
Local excitations, cannot be created locally. "Anyons".

What is statics?

It should a property of operators moving particles around, i.e. string operators



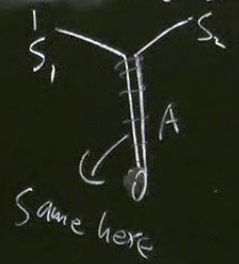
Naive attempt:



$S_1 \cdot S_2 |\psi\rangle = e^{i\theta} |\psi\rangle$?

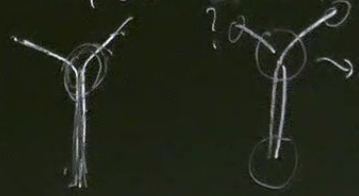
This depend on how I cut open S. (analogue of e^{iEt})

One ways that works:

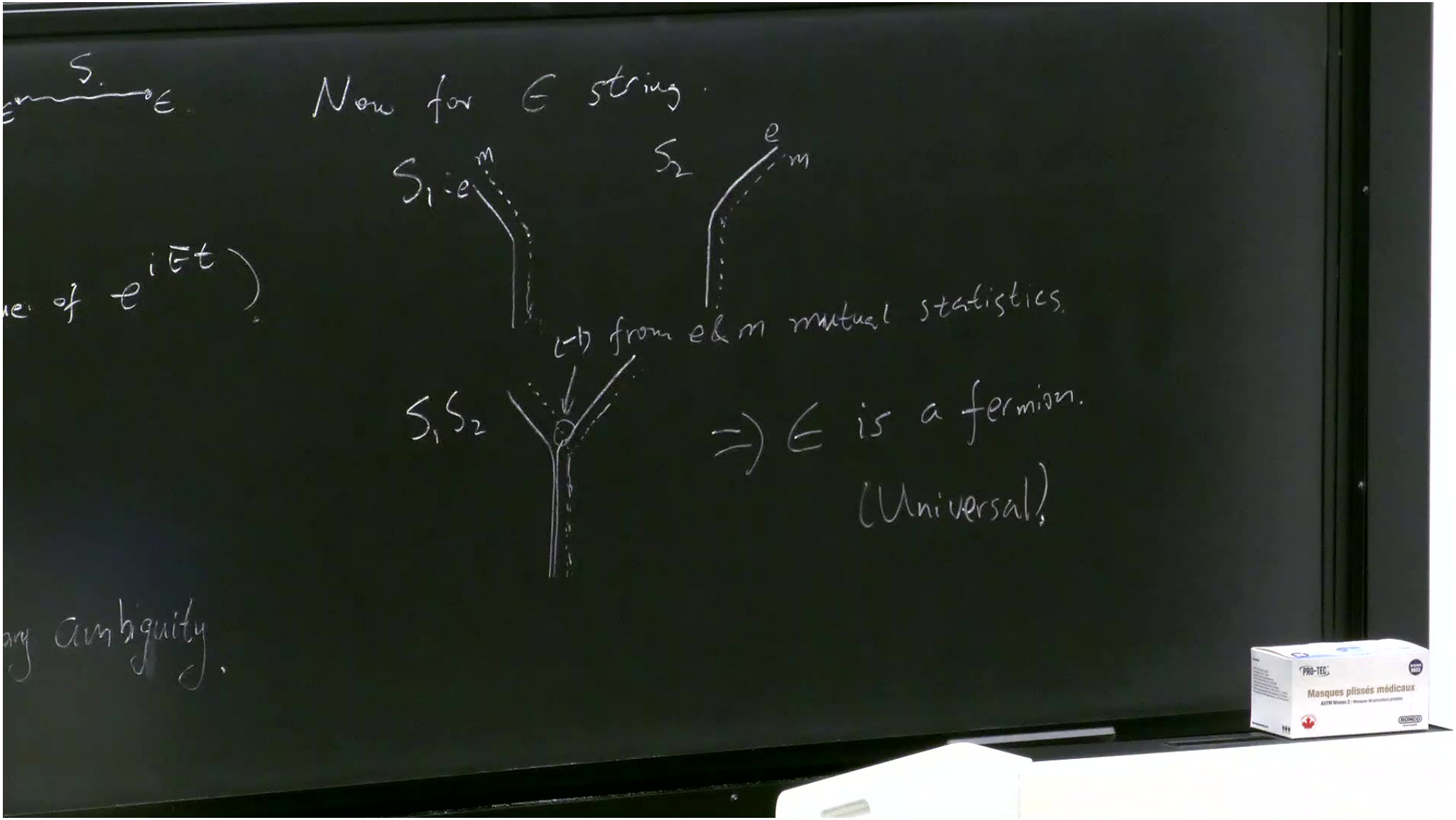


S_1, S_2 identical on segment A.

$S_1 S_2 = e^{i\theta} S_2 S_1$ if local fermion, $S_1 = \gamma_1 \gamma_0$, $S_2 = \gamma_2 \gamma_0$



differ by exchange $1 \leftrightarrow 2$. Not sensitive to boundary ambiguity.


Now for ∈ string.
S_1 : e m
S_2 e m
(-1) from e & m mutual statistics.
$S_1 S_2$
⇒ ∈ is a fermion.
(Universal!)
ue of $e^{i\bar{E}t}$)
ary ambiguity.
PRO-TEC
Masques plissés médicaux

Z_2 gauge theory interpretation:

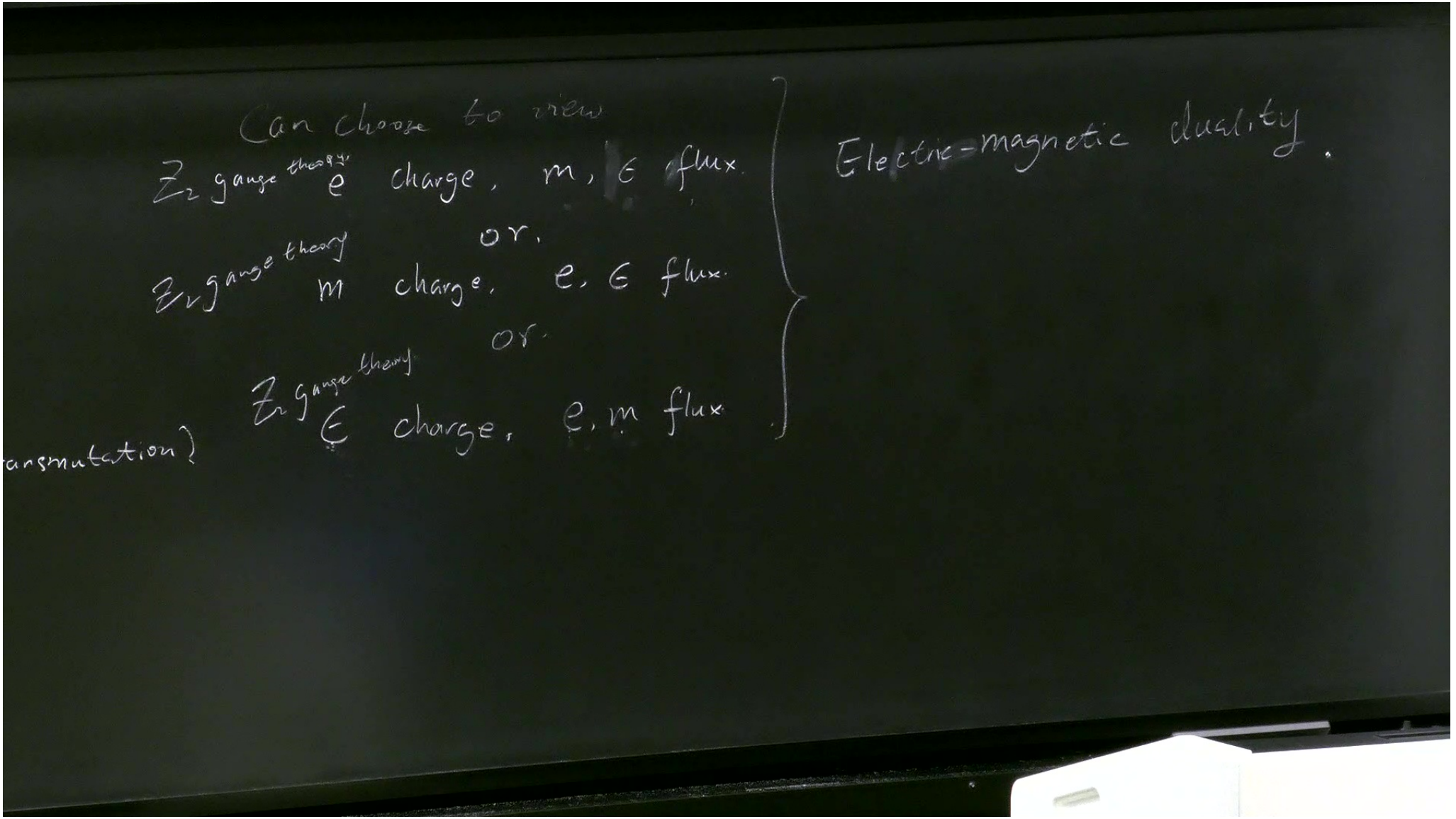
Z_e : e-particle. flip 

X_m : m-particle flip 

ZX : ϵ-particle: charge-flux bound state.

Fermion from flux-attachment. (statistics transmutation)

(recall Jordan-Wigner)


Can choose to view
Z_2 gauge theory: e charge, m, ϵ flux.
or.
Z_2 gauge theory: m charge, e, ϵ flux.
or.
Z_2 gauge theory: ϵ charge, e, m flux.
Electric-magnetic duality.
transmutation)

"confined"

local excitation

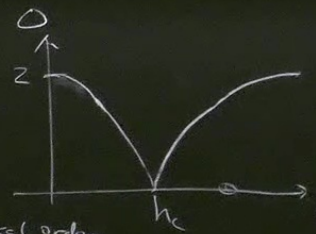
How to kill toric code.

$$H = -\sum_{+} XXXX - \sum_{\square} ZZZZ - h\sum_i Z_i$$

$+ \sum X$

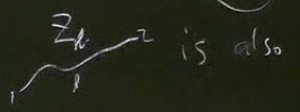
e — e

↑ Create e-e pair

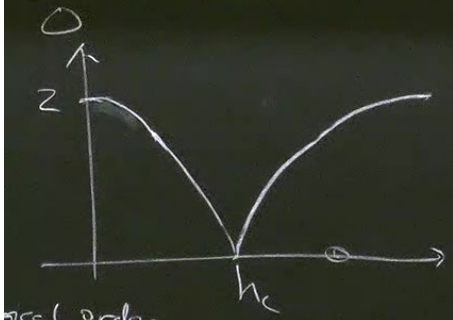


If $h \gg 1$, $|GS\rangle \approx |\uparrow\uparrow\uparrow\uparrow\ldots\uparrow\rangle$ SRE, no topological order.

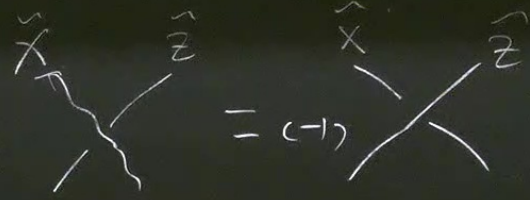
The Z_C  is still a good 1-form sym.



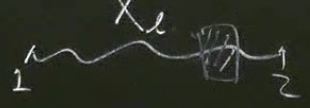
is also a symmetry $Z_\ell |GS\rangle = |GS\rangle$. (Under U_{FD}, $\tilde{Z}_\ell |GS\rangle = V_1 V_2 |GS\rangle$.)



gical order.

$\nexists$ X-like sym. 

e for such $\hat{X}$, ...

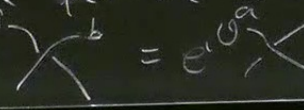


$\hat{X}|G.S\rangle$. excitation energy $\sim O(|l|)$

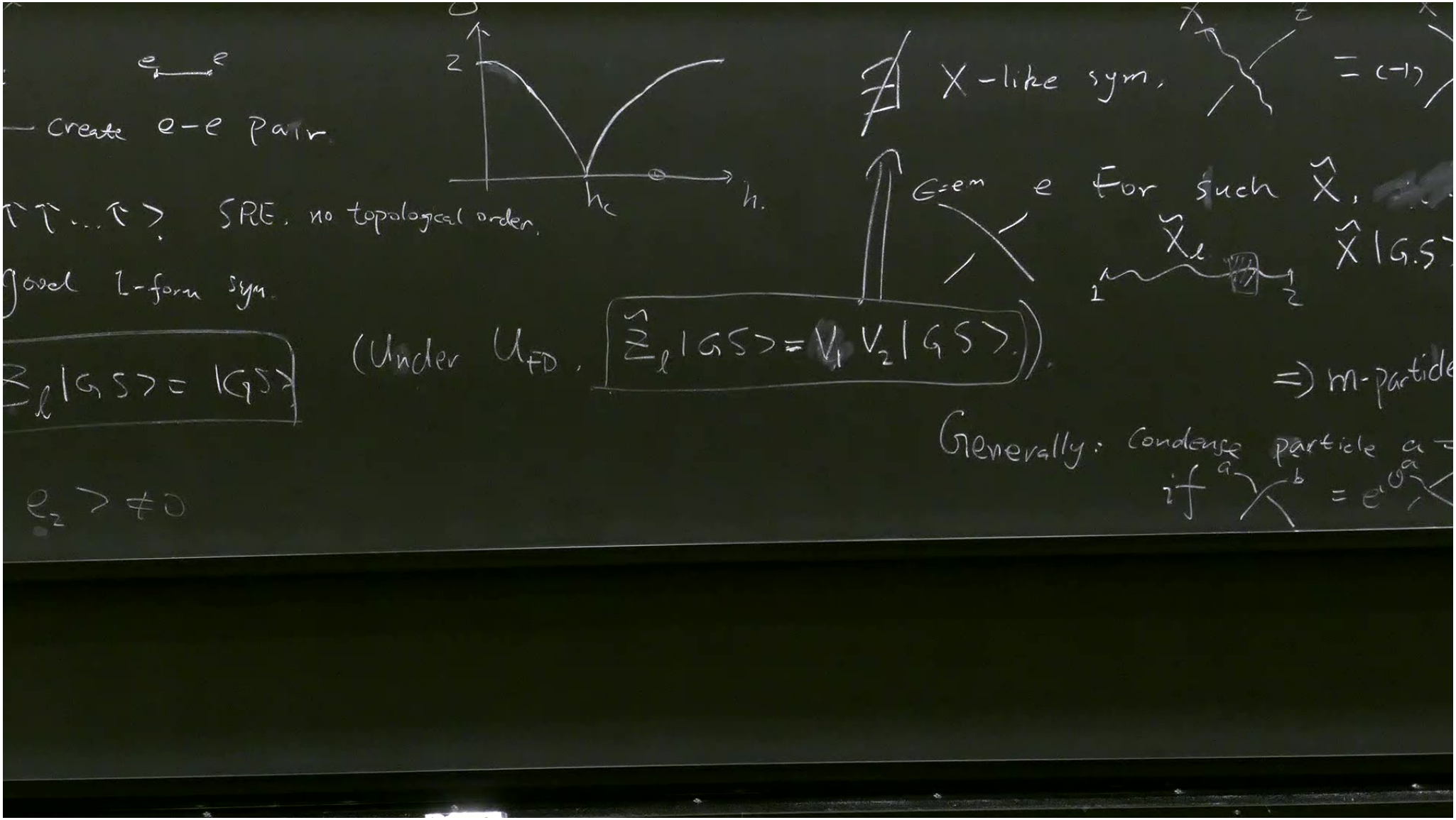
der U_{FD}. $\hat{Z}_l|GS\rangle = V_1 V_2|GS\rangle$.)

$\Rightarrow$ m-particle "confined".

Generally: Condense particle a $\Rightarrow$ Confines particle b.

if 

, $e^{i\theta} \neq 1$


e e
create e-e pair
|TT...T>: SRE. no topological order.
good 1-form sym.
$\hat{\Sigma}_\ell |GS\rangle = |GS\rangle$
(Under U_{FD}, $\hat{\Sigma}_\ell |GS\rangle = V_1 V_2 |GS\rangle$)
$|e_2\rangle \neq 0$
z
h
h_c
$\nexists$ X-like sym.
$= (-1)$
G=em e For such $\hat{X}$,
$\hat{X}_\ell$
$\hat{X}|GS\rangle$
1
2
$\Rightarrow$ m-particle
Generally: Condense particle a
if
a
b
$= e^{i\theta^a}$

$\langle e_1 e_2 \rangle \neq 0$

at h_c: Ising critical point (2+1 d) coupled to Z_2 gauge field.

$$\int_{b.c} \sum D\phi\, e^{-\int d^{2+1}x \left(\frac{1}{2}(\partial_\mu \phi)^2 + m\phi^2 + \mu\phi^4\right)}$$

Z_2 gauge field gapped.

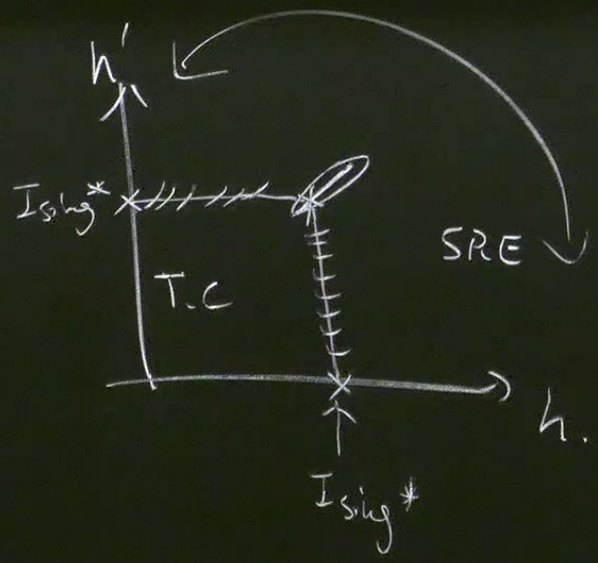
$\Rightarrow$ Critical exponents same as Ising.

$\dim(\phi^2)_{\text{Ising}}$ ϕ not a local operator.

"Ising/Z_2" "Ising*"

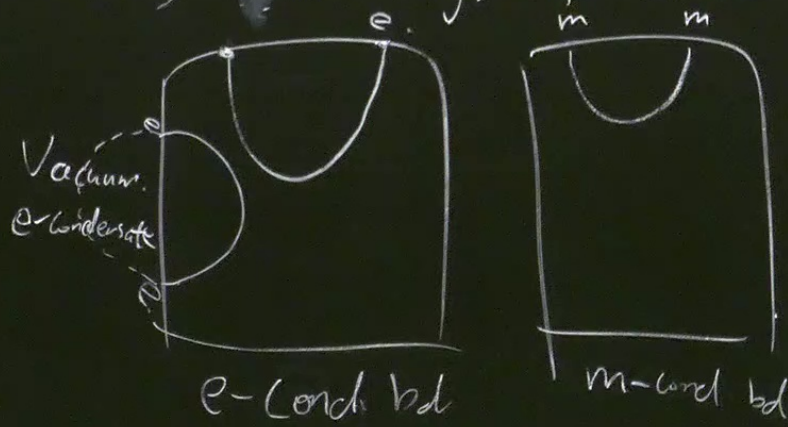
$H_{T.C} - h\sum Z - h'\sum X$

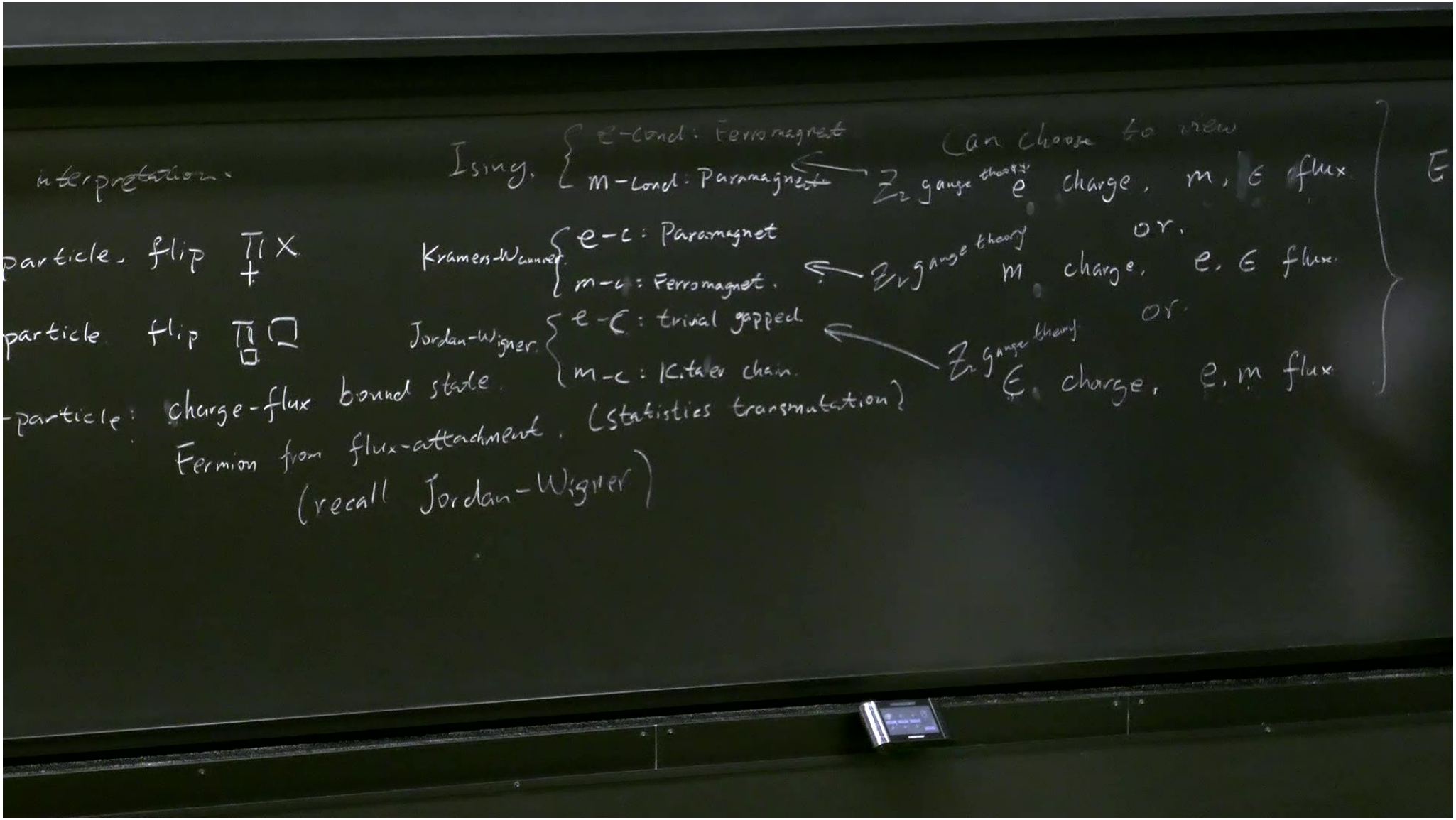
↳ condensing m. confine e.



Two ways of confining toric code.

⇒ two ways of have gapped boundary




interpretation:
Ising. { e-cond: Ferromagnet
m-cond: Paramagnet
Can choose to view
Z_2 gauge theory
e charge, m, ε flux
or.
m charge, e, ε flux.
or.
ε charge, e, m flux
particle. flip
Kramers-Wannier { e-c: Paramagnet
m-c: Ferromagnet.
Z_2 gauge theory
particle flip
Jordan-Wigner { e-ε: trival gapped
m-c: Kitaev chain.
Z_2 gauge theory
-particle: charge-flux bound state.
Fermion from flux-attachment. (Statistics transmutation)
(recall Jordan-Wigner)