Title: Lecture - Quantum Matter, PHYS 777

Speakers: Chong Wang

Collection/Series: Quantum Matter (Elective), PHYS 777, March 31 - May 2, 2025

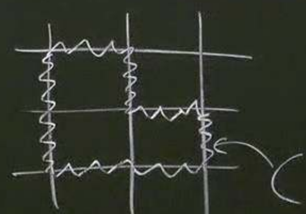
Subject: Condensed Matter

Date: April 17, 2025 - 9:00 AM

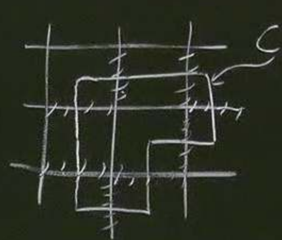
URL: https://pirsa.org/25040008

Toric Code. $H = -\sum_{+} XXXX - \sum_{\square} ZZZZ.$

1-form Symm: $(\mathbb{Z}_2^{(1)} \times \mathbb{Z}_2^{(1)})$



$Z_C := \prod_{e \in C} Z_e$



$X_{C'} := \prod_{e \in C'} X_{C'}$

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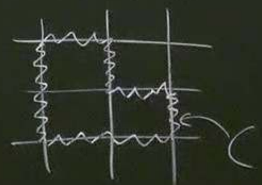

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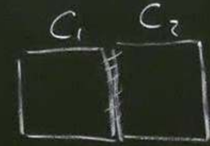
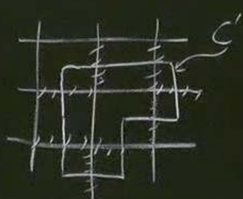


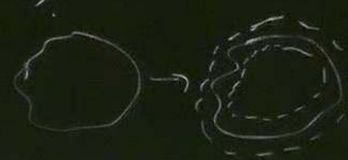
$Z_{C_1} Z_{C_2} = Z_{C_1 \circ C_2}$

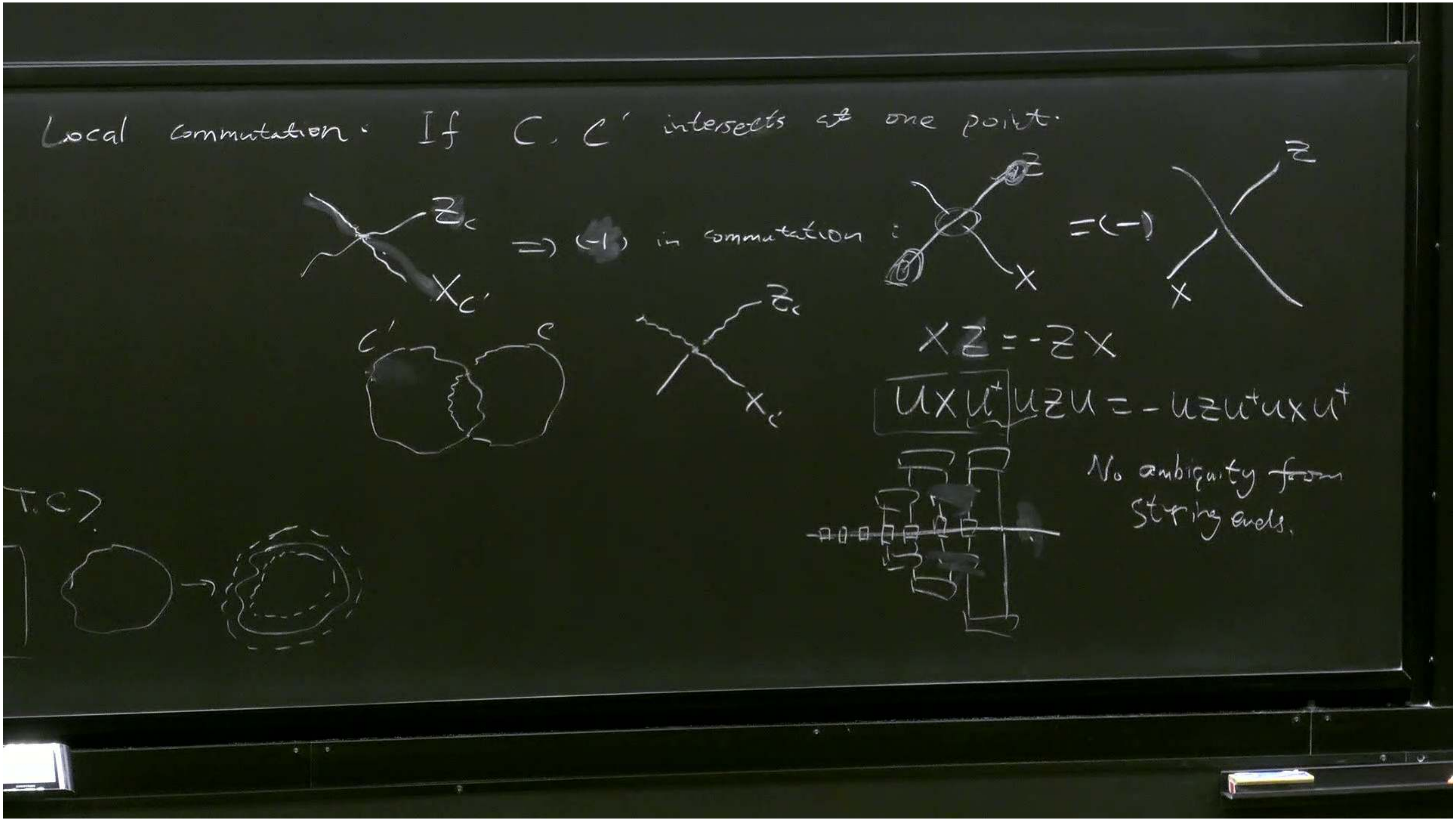


$X_{C'} := \prod_{e \in C'} X_{C'}$

$|T, C_1\rangle \longrightarrow U_{FD}|T, C\rangle$

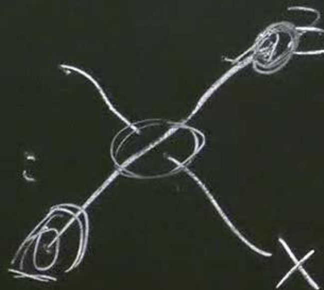
$g_C \to U_{FD}\, g_C\, U_{FD}^{\dagger}$




Local commutation: If C, C' intersects at one point.
Z_c
X_c'
=> (-1) in commutation :
Z
X
=(-)
Z
X
Z_c
X_c'
C'
C
XZ = -ZX
UXU† UZU = -UZU†UXU†
No ambiguity from string ends.
T.C.>

C' intersects at one point.

(-1) in commutation :

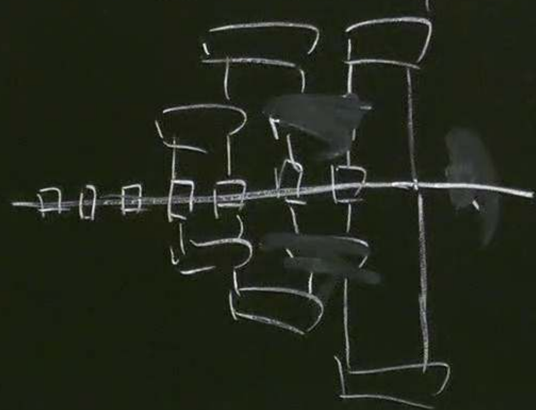


$= (-1)$

$$XZ = -ZX$$

$$\boxed{UXU^\dagger}\, UZU = -UZU^\dagger UXU^\dagger$$

No ambiguity from string ends.

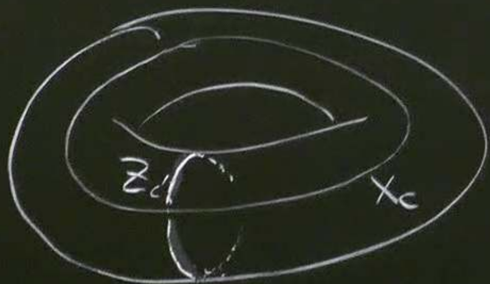


Consequences: (1) $X_c Z_{c'} = -Z_{c'} \underline{X_c}$

$(Z_c X_{c'} = -X_{c'} Z_c)$

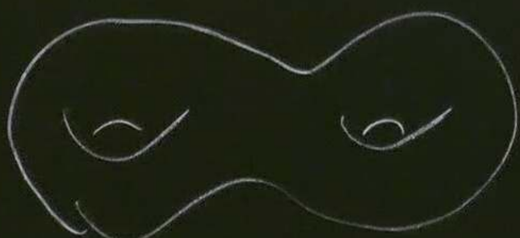
$Z_{c'}$ charged under X

acting by $Z_{c'}$ chang



$$|\psi\rangle = X_c|\psi\rangle$$

$$\langle\psi|(Z_{c'}|\psi\rangle) = 0$$



X_c.
$Z_{c'}$)

$Z_{c'}$ charged under X_c.

acting by $Z_{c'}$ changes X_c charge.

$$\left.\begin{array}{l} |\psi\rangle = X_c|\psi\rangle \\ \langle\psi|(Z_{c'}|\psi\rangle) = 0 \end{array}\right\}$$

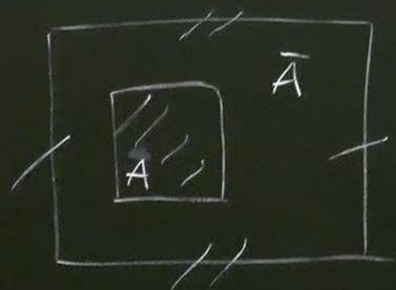
4-fold g.s.d. on torus.

genus-g Riemann surface. $gsd = 4^g$

"Topological order"

$\mathcal{H}_G$.

Thm. any two states $|\psi_1\rangle, |\psi_2\rangle \in \mathcal{H}_G$ are locally indistinguishable.



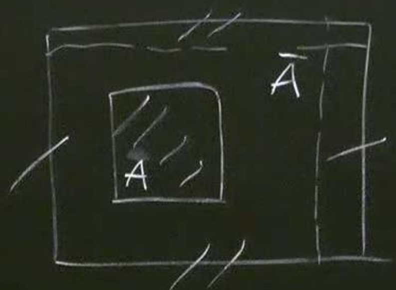
$$\rho_1 = tr_{\bar{A}} |\psi_1\rangle\langle\psi_1|$$

$$\rho_2 = tr_{\bar{A}} |\psi_2\rangle\langle\psi_2|.$$

Claim: $\rho_1 = \rho_2$.

Proof: HW.

Thm. any two states $|\psi_1\rangle, |\psi_2\rangle \in \mathcal{H}_G$ are locally indistinguishable.



$$\rho_1 = \mathrm{tr}_{\bar{A}} |\psi_1\rangle\langle\psi_1|$$

$$\rho_2 = \mathrm{tr}_{\bar{A}} |\psi_2\rangle\langle\psi_2|.$$

Claim: $\rho_1 = \rho_2$.

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$\mathcal{H}_G$.

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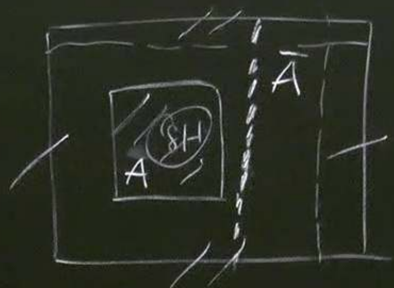


$$\rho_1 = \mathrm{tr}_{\bar{A}} |\psi_1\rangle\langle\psi_1|$$

Claim: $\rho_1 = \rho_2$.

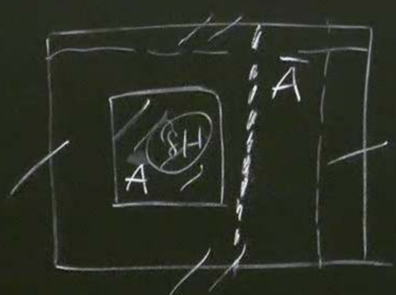
Proof: HW.

$$\rho_2 = \mathrm{tr}_{\bar{A}} |\psi_2\rangle\langle\psi_2|$$

$$\delta E \sim e^{-L/\xi}.$$

Consequence: local perturbation cannot lift g.s.d. $\delta H = \sum_i H_i$.

Thm. any two states $|\psi_1\rangle, |\psi_2\rangle \in \mathcal{H}_G$ are locally indistinguishable.



$\rho_1 = tr_{\bar{A}} |\psi_1\rangle\langle\psi_1|$ Claim: $\rho_1 = \rho_2$. Proof: HW.

$\rho_2 = tr_{\bar{A}} |\psi_2\rangle\langle\psi_2|$

$\delta E \sim e^{-L/\xi}$. (Ising

Consequence: local perturbation cannot lift g.s.d. $\delta H = \sum_i H_i$.

Recall: Cat/GHZ/Ferro $|\pm\rangle = |\uparrow\uparrow\uparrow\ldots\rangle \pm |\downarrow\downarrow\downarrow\ldots\rangle$

$|\uparrow\uparrow\ldots\uparrow\rangle$ vs. $|\downarrow\downarrow\downarrow\ldots\rangle$ Distinguishable!

$\rho_\pm = \left(\frac{1}{2} |\uparrow\uparrow\ldots\rangle\langle\uparrow\uparrow\ldots| + \frac{1}{2} |\downarrow\downarrow\ldots\rangle\langle\downarrow\downarrow\ldots| \right.$ $\Rightarrow$ G.S.D. lifted by δH

$\mathcal{H}_G$.

are locally indistinguishable. $\Rightarrow \nexists\, O_{local} : O_{local}|\psi_1\rangle = |\psi_2\rangle$.

$|\psi_1\rangle \pm |\psi_2\rangle$

Claim: $\rho_1 = \rho_2$. Proof: HW.

$|\uparrow\uparrow\uparrow\ldots\rangle$

$\pm_{12}|\downarrow\downarrow\downarrow\ldots\rangle$

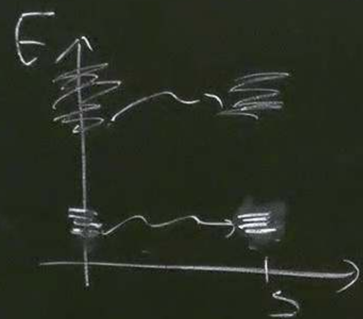
$\delta E \sim e^{-L/\xi}$. (Ising model $\delta \sim e^{-L^d/\xi^d}$)

g.s.d. $\delta H = \sum_i H_i$.

$\ldots\rangle$

$\rho_\pm = \left(\frac{1}{2}|\uparrow\uparrow\ldots\rangle\langle\uparrow\uparrow\ldots| + \frac{1}{2}|\downarrow\downarrow\ldots\rangle\langle\downarrow\downarrow\ldots|\right. \Rightarrow$ G.S.D. lifted by $\delta H \sim Z$

$$H = H_0 + \delta H(s)$$



$$U_{\text{Quasi-adiabatic}} = \mathcal{T} e^{-i\int_0^1 \tilde{H}(s)\,ds} = U_{FD}$$

$\tilde{H}$ quasi local $\Rightarrow$ do not mix different g.s.

Consequence: T.C. $|GS\rangle$ is LRE. $\neq U_{FD}|{+}{+}{+}\ldots\rangle$

If $\underline{U_{FD}^{\dagger}|GS\rangle} = \underline{|{+}{+}{+}\ldots\rangle}$

$\Rightarrow \exists\ |\psi\rangle : \langle {+}{+}{+}\ldots|\psi\rangle$

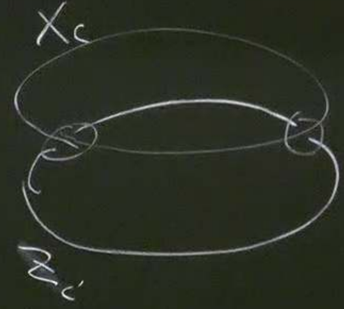
but on any contractible A: $\rho_A = |{+}{+}{+}\ldots\rangle\langle{+}{+}{+}\ldots|$

$\Rightarrow |\psi\rangle = |{+}{+}{+}\ldots\rangle$

$\mathbb{Z}_2^{(1)}$ symm is "spontaneously broken". Order parameter: $Z_{C'}$

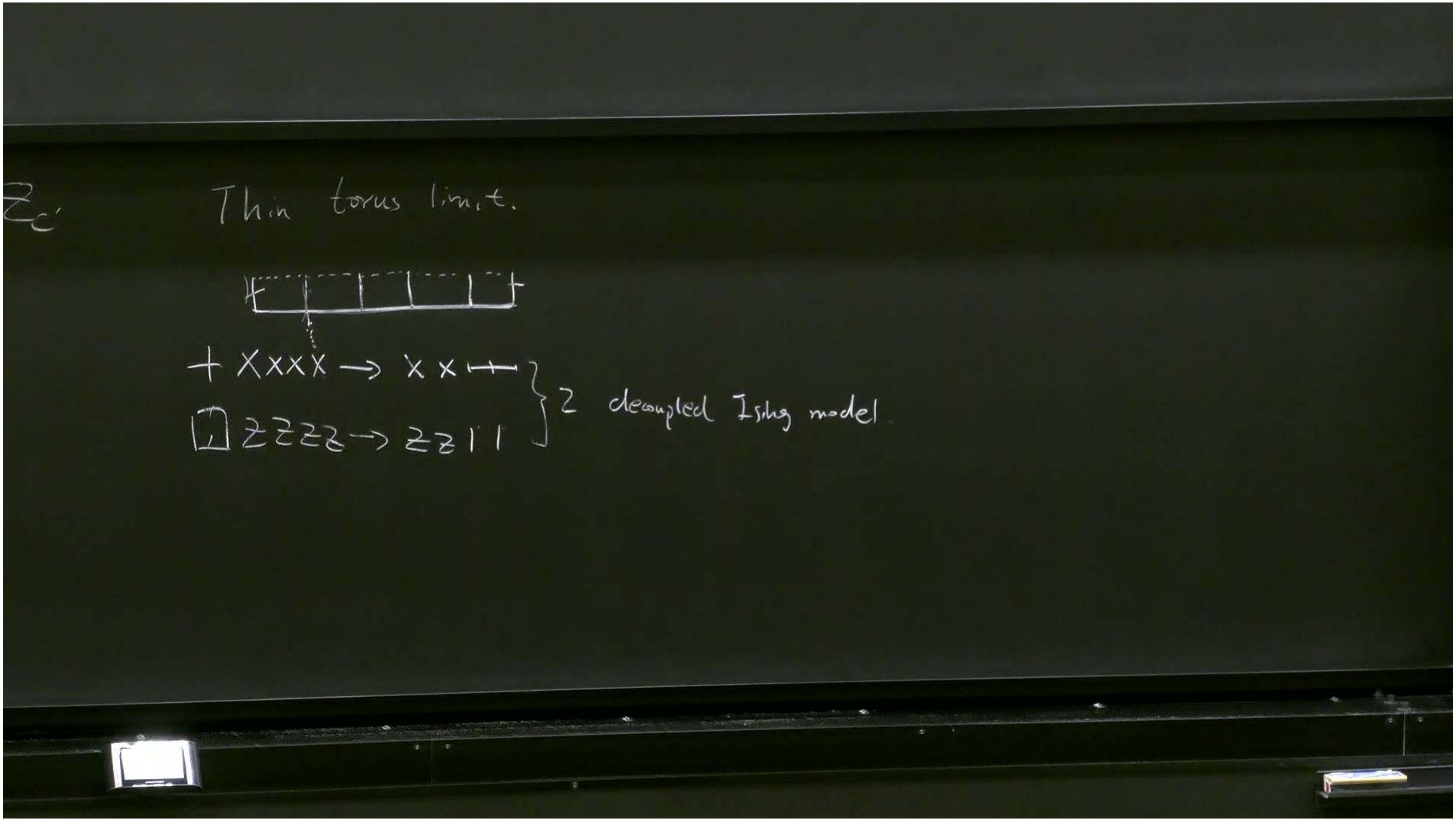
$\uparrow$

X_C



vs.

1d Z_{sing}


Thin torus limit.
+ XXXX → XX++
ZZZZ → ZZ11
2 decoupled Ising model

Open strings:

$= (-1)$

Open strings create ex

Z_c flipps $\prod_{+} X$,

(e - particle)

g_{ij}

1. Mutual statistics

m

Z

e

$= (-1)$

X

ϵ has $n + \frac{1}{2}$ angular momentum

$\Rightarrow$ ϵ is fermion.

m

e

$= \|$

ϵ

1

2

3

$g_{23}\, g_{12} \stackrel{?}{=} g_{13}$

Z $\quad$ Z

$= (-1)$

X $\quad$ $X.$

Open strings create excitations at ends.

Z_c flipps $\prod_{+} X$ (e-particle), $X_{c'}$ flipps $\prod_{\square} Z$ (m-particle).

ϵ has $n+\frac{1}{2}$ angular momentum

$\Rightarrow$ ϵ is fermion ✓

g_{ij} moves particle from site i to j.

1 $\quad$ 2 $\quad$ 3

$$g_{23}\, g_{12} \stackrel{?}{=} g_{13}$$

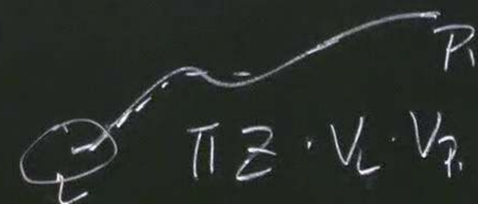
m

e

$\parallel$

ϵ

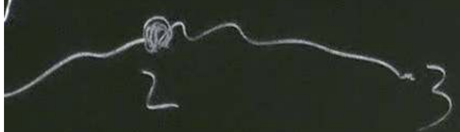
Open strings create excitations at ends.



$\prod Z \cdot V_L \cdot V_{P_1}$

Z_c flipps $\prod_{+} X$ (e-particle), $X_{c'}$ flipps $\prod_{\square} Z$ (m-particle).

g_{ij} moves particle from site i to j.



$g_{12} \stackrel{?}{=} g_{13}$