Title: Lecture - Quantum Matter, PHYS 777

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Collection/Series: Quantum Matter (Elective), PHYS 777, March 31 - May 2, 2025

Subject: Condensed Matter

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Today: (1) Wrap up 1d stuff.

- SPT edge state & symmetry fractionalization
- Kitaev chain: topological response & Ising model.

(2) Move on to 2d: Toric Code (Topological order)

1d SPT (e.g. Cluster chain) open b.c. L $\check{g}_L$ $\check{g}_R$ R.

Symmetry localization $g|\Psi\rangle = \boxed{\check{g}_L}\,\check{g}_R|\Psi\rangle$, $g \in G$ ← Symmetry gp.

e.g. Cluster chain $|\psi\rangle = Z_i X_{i+1} Z_{i+2} |\psi\rangle$

$$\Rightarrow X_{even}|\psi\rangle = \underbrace{\boxed{Z_1}}_{\check{X}_{e,L}}\ \underbrace{Z_{2L-1}X_{2L}}_{\check{X}_{e,R}}|\psi\rangle$$

$$X_{odd}|\psi\rangle = \underbrace{\boxed{X_1 Z_2}}_{\check{X}_{o,L}}\ \underbrace{Z_{2L}}_{\check{X}_{o,R}}|\psi\rangle$$

$$\check{X}_{e,L}\check{X}_{o,L} = (-1)\check{X}_{o,L}\check{X}_{e,L}$$

Projective representation $\Longrightarrow$ $\{\check{g}_L\}$ form representation of G only up to some phase factor.

Consequence of sym. frac: $\nexists$ 1d rep. $\Rightarrow$ g.s. cannot be unique.

Generally: $\check{g}\cdot\check{h} = \nu(g,h)\,\widecheck{gh}$, $\nu(g,h): G\times G \longrightarrow U(1)$.

Gauge transform: $\check{g} \to \check{g}\cdot\mu(g)$. $\mu(g): G \longrightarrow U(1)$

$$\nu(g,h) \to \boxed{\nu(g,h)\,\frac{\mu(gh)}{\mu(g)\cdot\mu(h)} \simeq \nu(g,h)}$$

Constrain from associativity: $g_1(g_2g_3) = (g_1g_2)g_3$

$$\Rightarrow \boxed{\nu(g_1, g_2g_3)\cdot\nu(g_2,g_3) = \nu(g_1,g_2)\cdot\nu(g_1g_2, g_3)}$$

ep. $\Rightarrow$ g.s. cannot be unique.

$,h): G \times G \longrightarrow U(1)$.

$): G \longrightarrow U(1)$

$\boxed{\simeq \nu(g,h)}$

$) = (g_1 g_2) g_3$

$\boxed{= \nu(g_1, g_2) \cdot \nu(g_1 g_2, g_3)}$

2nd gp cohomology.

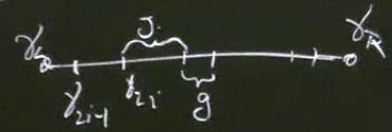
$\nu \in H^2(G, U(1))$: classifies all 1d SPT in bosonic systems.

e.g. $H^2(SO(3), U(1)) = \mathbb{Z}_2$.

Kitaev chain: $H = -J \sum i \gamma_{2i} \gamma_{2i+1}$

Trivial chain: $H = -g \sum i \gamma_{2i-1} \gamma_{2i} =$

Topological response

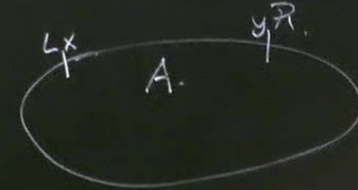


$$\hat{c} = \gamma_L + i\gamma_R \begin{cases} |0\rangle : \hat{c}|0\rangle = 0 \\ \hat{c}^{\dagger}|0\rangle \end{cases}$$

2-fold g.s.d. $\delta \sim e^{-L/\xi}$

$H = -J\sum i\gamma_{2i}\gamma_{2i+1}$

Trivial chain: $H = -g\sum i\gamma_{2i-1}\gamma_{2i} = -g\sum_i c_i^{\dagger}c_i$



Topological response

Fermion Z_2 sym: $F = \prod_i (-1)^{c_i^{\dagger}c_i} = \prod_i i\gamma_{2i-1}\gamma_{2i}$.

$$F_A = \prod_{i\in A} i\gamma_{2i-1}\gamma_{2i}$$

Trivial: $F_A|\Psi\rangle = |\Psi\rangle$

Kitaev chain, $i\gamma_{2i}\gamma_{2i+1}=1$ on G.S. $\Rightarrow F_A|\psi\rangle = -i\gamma_{2x-2}\gamma_{2y}|\psi\rangle$.

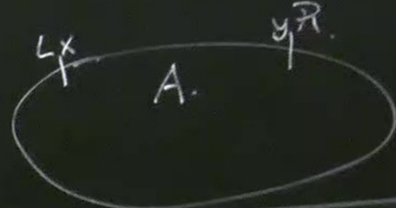
$|0\rangle = 0$

$\sim e^{-L/\xi}$

hain : $H = -g\sum i\gamma_{2i-1}\gamma_{2i} = -g\sum_i c_i^\dagger c_i$

Twist b.c. $\pm(-J)\, i\underbrace{\gamma_{2L}\gamma_1}$

G.S. changes total $\langle F \rangle$



$(-1)^{c_i^\dagger c_i} = \prod_i i\gamma_{2i-1}\gamma_{2i}$.

$\boxed{\text{Kitaev: } Z_2 \text{ flux induces fermion charge.}}$

Local fermion operation

$\uparrow$

G.S. $\Rightarrow F_A|\psi\rangle = -i\gamma_{2x-2}\gamma_{2y}|\psi\rangle$. $\Rightarrow |\hat{\psi}\rangle = U_{FD}|\psi\rangle$: $F_A|\psi\rangle = -i\,\underbrace{\tilde{\gamma}_L}\,\tilde{\gamma}_R|\psi\rangle$

Ising model: gauging F.

$$H = -J\sum_{i=1}^{L-1} i\gamma_{2i}\gamma_{2i+1} - g\sum_{i=1}^{L} i\gamma_{2i-1}\gamma_{2i} - J\, i\gamma_{2L}\gamma_1\tau^x_L$$

Charge constraint: $\underline{F = \prod_{i=1}^{L} i\gamma_{2i-1}\gamma_{2i} = \tau^x_L}$ (= total Ising charge)

$J > g$, we have ferromagnet. 2-fold g.s.d. $\delta \sim e^{-L/\xi}$.

(1) Open b.c. γ_L, γ_R, $\delta \sim e^{-L/\xi}$ ✓

(2) Closed chain: If τ^x change sign, (1) Charge constraint $\Rightarrow$ F also changes sign
(2) Topo response $\Rightarrow$ g.s. also changes F.

ing charge)

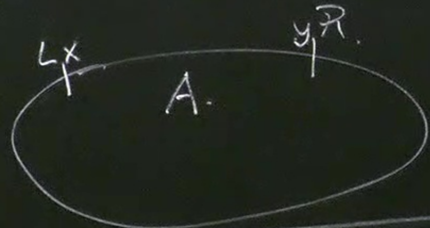
$= $ also changes sign } $\because$ $\Rightarrow$ You stay at g.s. $\Rightarrow$ $\tau_x = \pm 1$ degenerate.

S. also changes F.

$= -g\sum_i i\gamma_{2i-1}\gamma_{2i} = -g\sum_i c_i^{\dagger} c_i$

Twist b.c. $\pm(-J)\, i\underbrace{\gamma_{2L}\gamma_1}$

G.S. changes total $\langle F \rangle$



$i\gamma_{2i-1}\gamma_{2i}$.

Local fermion operation

$\uparrow$

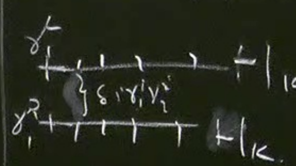
ev: $\mathbb{Z}_2$ flux induces fermion charge.

$|\psi\rangle = -i\gamma_{2x-2}\gamma_{2y}|\psi\rangle. \Rightarrow |\hat{\psi}\rangle = U_{FD}|\psi\rangle : F_A|\psi\rangle = -i\tilde{\gamma}_L\tilde{\gamma}_R|\psi\rangle$

Comment: Kitaev is not quite an SPT. Fermion $\mathbb{Z}_2$ symmetry cannot be broken $\delta H \sim (c_i + c_i^\dagger)$ nonsense.

Topological response $\Rightarrow$ $|\Psi\rangle \neq U_{FD}|\text{trivial}\rangle$ $\Rightarrow$ $|\text{Kitaev}\rangle$ is LRE

$\delta i\gamma_i^1\gamma_i^2$ However, $|\text{Kitaev}\rangle \otimes |\text{Kitaev}\rangle = U_{FD}|\text{Trivial}\rangle$



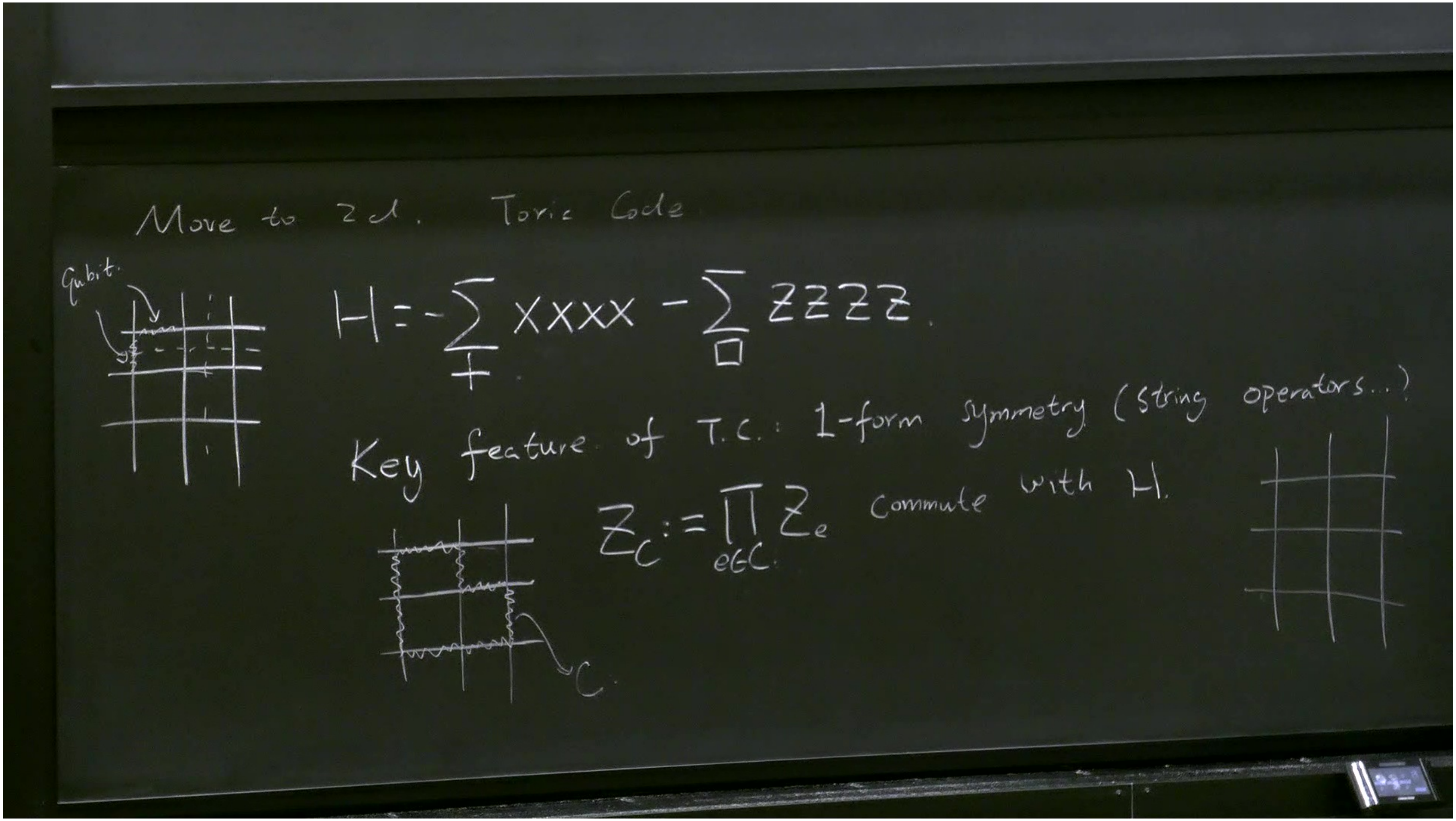
"Invertible topological order": $|\Psi\rangle \otimes |\tilde{\Psi}\rangle = U_{FD}|\text{Trivial}\rangle$

)

hanges sign

hanges F_i

$\Rightarrow$ You stay at g.S. $\Rightarrow$ $\tau_x = \pm 1$ degenerate.


Move to 2d. Toric Code
Qubit.
$H := -\sum_{+} XXXX - \sum_{\square} ZZZZ.$
Key feature of T.C.: 1-form symmetry (String operators...)
$Z_C := \prod_{e \in C} Z_e$ commute with H.
C.

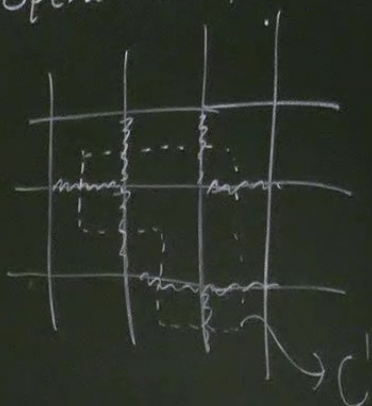
c', $\underbrace{Z_c, X_{c'}}$ are $\mathbb{Z}_2$ symmetries. But like ordinary sym. (e.g. Ising $X = \prod_i X_i$)

act on 1d submanifolds. called "1-form symmetry". (p-form sym. act on (d-p) manifolds)

Ordinary sym "=" 0-form sym.

string operators...)

H.



$$X_{c'} := \prod_{e \in c'} X_e .$$ Commute with H.

(2) Topo response $\Rightarrow$ g. S. also changes F

form Symmetry is robust under U_{FD}

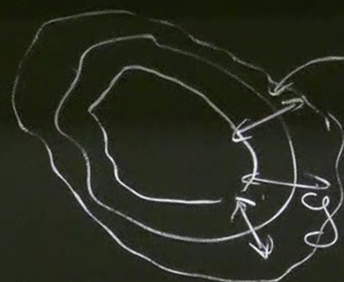
Pick a g.s. of T.C $|\psi_0\rangle$. consider $|\psi\rangle = U_{FD}|\psi_0\rangle$

Sym: $g_{\{\}} |\psi_0\rangle = |\psi_0\rangle$

$$\tilde{g}|\psi\rangle = \underbrace{U_{FD}\, g\, U_{FD}^{\dagger}}_{\tilde{g}} U_{FD}|\psi_0\rangle = |\psi\rangle \Rightarrow \tilde{g} = U_{FD}\, g\, U_{FD}^{\dagger} \text{ is a sym of } |\psi$$

$\tilde{g}\cdot\tilde{h} = \widetilde{gh}$ e.g. $g^2 = I$, $\tilde{g}^2 =$

also changes t



$l \sim$ light cone of U_{FD}

$\Rightarrow \mathbb{Z}_2^{(1)} \times \mathbb{Z}_2^{(1)}$ sym robust under U_{FD}

"Emergent symm":

$\tilde{g} H \neq H \tilde{g}$

is a sym of $|\psi\rangle$?

$g^2 = I$, $\tilde{g}^2 = U g U^{\dagger} U g U^{\dagger} = I$

Toric Code

$\forall\, C, C'$, $\underbrace{Z_C, X_{C'}}$ are $\mathbb{Z}_2$ symmetries. But I...

act on 1d submanifolds. called "1-form symme...

Ordinary sym "=" 0-for...

$$\ldots XXXX - \sum_{\square} ZZZZ.$$

$$Z_{C_1} \cdot Z_{C_2} = Z$$

...eature of T.C.: 1-form symmetry (string operators...)

$$Z_C := \prod_{e \in C} Z_e$$ commute with H.

$$X_{C'} := \prod_{e \in C'} X_e.$$ Commute...

C

g

C'