Title: Lecture - Quantum Matter, PHYS 777

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Collection/Series: Quantum Matter (Elective), PHYS 777, March 31 - May 2, 2025

Subject: Condensed Matter

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Ref on SPT: Lecture notes on "Quantum Phases of Matter" by John McGreevy

Cluster chain: $H = -\sum_i Z_i X_{i+1} Z_{i+2}$. $Z_2 \times Z_2$ sym: $X_{even/odd} = \prod_{i=even/odd} X_i$

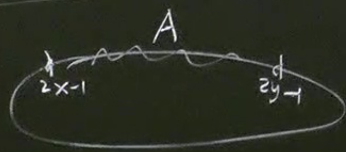
g.s. $|\text{Cluster}\rangle = Z_i X_{i+1} Z_{i+2} |\text{Cluster}\rangle$

$$= \sum_{\substack{\{s_i\} \\ i=1,\dots,L/2}} \bigotimes_i |Z_{2i-1} = s_i\rangle \, |X_{2i} = s_i s_{i+1}\rangle.$$

Domain wall condensation with charge decoration.

...y John McGreevy

...n/odd $= \prod_{i=\text{even/odd}} X_i$



Domain wall creation operator (for odd spin)

$$X_A^{odd} := \prod_{2x-1 \le 2i-1 \le 2y-1} X_{2i-1} \quad \left(\text{Recall in KW duality} \sim \tilde{Z}_{2x-1} \tilde{Z}_{2y-1} \right)$$

In Ising: Domain wall condensate

$$\sim \langle \tilde{Z}_{2x-1} \tilde{Z}_{2y-1} \rangle \to \text{const.}$$

Trivial Ising chain $|0\rangle = |++ + \dots +\rangle$.

$$X_A^{odd} |0\rangle = |0\rangle$$

Cluster chain $|Cl\rangle$

"Topological response"

$$X_A^{odd} |Cl\rangle = Z_{2x-2} Z_{2y} |Cl\rangle \quad (*)$$

Ref on SPT: Lecture notes on "Quantum Phases of Matter" by John

Cluster chain: $H = -\sum_i Z_i X_{i+1} Z_{i+2}$. $\quad Z_2 \times Z_2$ sym: $X_{\text{even/odd}} = \prod_{i=e}$

g.s. $\underline{|\text{Cluster}\rangle = Z_i X_{i+1} Z_{i+2} |\text{Cluster}\rangle}$

$$= \sum_{\substack{\{s_i\} \\ i=1,\dots,L/2}} \bigotimes_i |Z_{2i-1} = s_i\rangle \, |X_{2i} = s_i s_{i+1}\rangle.$$

Domain wall condensation with charge decoration.

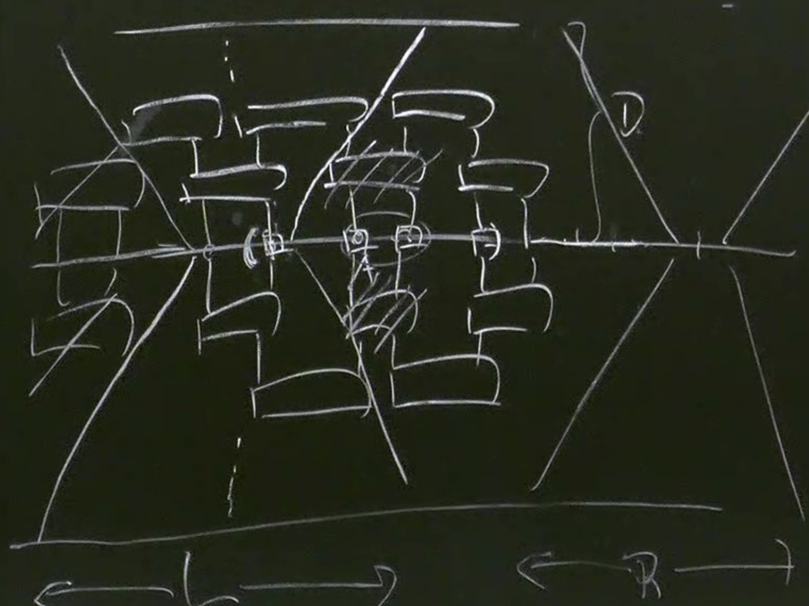
$Z_0 X_1 \cancel{Z_2} \cancel{Z_2} X_3 \cancel{Z_4} \cancel{Z_4} X_5 Z_6 |Cl\rangle = |Cl\rangle$

We can show that $|\mathrm{Cluster}\rangle \neq U_{FD}^{sym} |0\rangle$.

$$\forall\, U_{FD}^{sym} \equiv U \qquad U^{\dagger} X_A^{odd} U = V_L\, V_R\, X_A^{odd}$$

V_L: local unitary around left boundary of A

Symmetry: $X^{\dagger}_{odd/even} V_L X_{odd/even} = V_L$.



$U_{FD}^{Sym}\,|0\rangle.$

$U = V_L \, V_R X_A^{odd}.$

$\downarrow$

local unitary
around left boundary of A.

Symmetry: $X^{\dagger}_{odd/even} V_L X_{odd/even} = V_L.$

$|\psi\rangle = U_{FD}^{Sym}|++\dots\rangle$

$X_A^{odd}|\psi\rangle = U \underbrace{U^{\dagger} X_A^{odd} U}|++\dots\rangle$

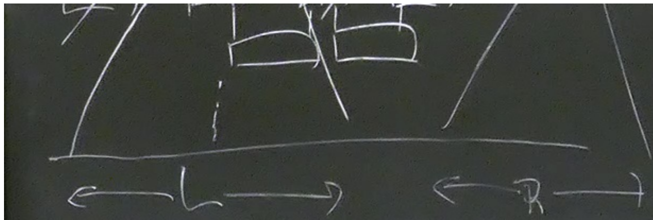
$= U\, V_L V_R \underbrace{\cancel{X^{odd}}}|++\dots\rangle$

$= \tilde{V}_L \tilde{V}_R \underbrace{U|+++\dots\rangle}.$ $\quad \tilde{V}_L = U V_L U^{\dagger}$: local

$= \tilde{V}_L \tilde{V}_R |\psi\rangle.$

Similarly. If $|\phi\rangle = U_{FD}^{Sym}|\text{Cluster}\rangle.$

$\Rightarrow \boxed{X_A^{odd}|\phi\rangle = \tilde{Z}_L \, \tilde{Z}_R|\phi\rangle,}$ $\tilde{Z}_L$ local, charged under X^{even}: $X^{even}\tilde{Z}_L X^{even\dagger} = -\tilde{Z}_L.$

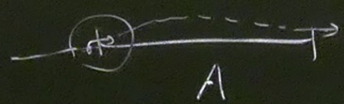


Similarly, if $|\phi\rangle = U_{FD}^{sym}|\mathrm{Cluster}\rangle$.

$\Rightarrow \boxed{X_A^{odd}|\phi\rangle = \check{Z}_L\,\check{Z}_R|\phi\rangle,}$ (*) $\quad \check{Z}_L$ local, charged under X^{even}: $X^{even}\check{Z}_L X$

Near L (or R), effect of X_A^{odd} $\simeq$ changing b.c. on Z^{odd}. p.b.c. $\Leftrightarrow$ anti-p.b.c.

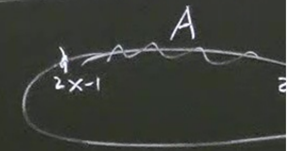
(*) changing b.c. (on Z^{odd}) flips X^{even} charge



Explicitly check for H_{cl}. anti-p.b.c. for Z^{odd}: $-Z_{2L-1}X_{2L}Z_1 \longrightarrow +Z_{2L-1}X_{2L}Z_1$.

Lecture notes on "Quantum Phases of Matter" by John McGreevy

$$H = -\sum_i Z_i X_{i+1} Z_{i+2}. \qquad \mathbb{Z}_2 \times \mathbb{Z}_2 \text{ sym: } X_{\text{even/odd}} = \prod_{i=\text{even/odd}} X_i$$



g.s. $\underline{|\text{Cluster}\rangle = Z_i X_{i+1} Z_{i+2} |\text{Cluster}\rangle}$ for $i \neq 2L-1$. for $i = 2L-1$

$|C\rangle = -Z_{2L-1} X_{2L} Z_1 |C\rangle$

$$= \sum_{\substack{\{s_i\} \\ i=1,\dots L/2}} \bigotimes_i |Z_{2i-1} = s_i\rangle \, |X_{2i} = s_i s_{i+1}\rangle.$$

Domain wall condensation with charge decoration.

"Topological response"

$Z_0 X_1 \cancel{Z_2} \cdot \cancel{Z_2} X_3 \cancel{Z_4} \cdot \cancel{Z_4} X_5 Z_6 |C\rangle = |C\rangle$

omain wall condensation with charge decoration.

$$\cancel{Z_1 X_2 Z_3} \cdot \cancel{Z_3 X_4 Z_5} \cdot \cancel{Z_5 X_6 Z_7} \, |C\rangle = |C\rangle$$

$$\sum_i Z_i \, Z_{i+1}$$

p.b.c.

$$Z_{L+1} := Z_1$$

anti-pbc.

$$Z_{L+1} := -Z_1$$

$$Z_L Z_{L+1} = -Z_L Z_1$$

L

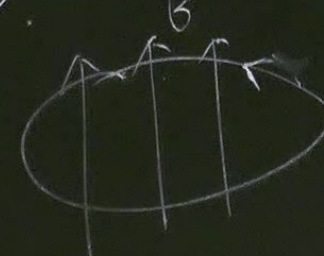
R

Near L (or R), effect of X_A^{odd} $\simeq$ changing b.c. on Z^{odd}. p.b.c $\Leftrightarrow$ anti-p.b.

(*) Changing b.c. (on Z^{odd}) flipps X^{even} charge

Explicitly check for H_{cl}. anti-p.b.c. for Z^{odd} : $-Z_2$

$$\prod X_{even} |cl'\rangle = - |cl'\rangle .$$



$e^{i\int \vec{A}\cdot d\vec{l}}$

$\Rightarrow$ Gauge field $\longrightarrow$ Charge response

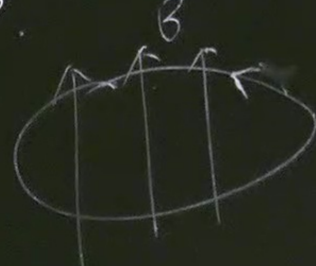
(*)

...ct of $X_A^{odd.}$ $\simeq$ changing b.c. on $Z^{odd.}$ p.b.c $\Leftrightarrow$ anti-p.b.c.

(*) changing b.c. (on Z^{odd}) flipps X^{even} charge

Explicitly check for H_{cl}. anti-p.b.c. for Z^{odd} : $-Z_{2L-1}X_{2L}Z_1 \longrightarrow +Z_{2L-1}X_{2L}Z_1$.

$$\prod X_{even}\,|cl'\rangle = -|cl'\rangle.$$



$$e^{i\int \vec{A}\cdot d\vec{l}}$$

Gauge field $\longrightarrow$ Charge response

$Z_1 X_2 Z_3 \cdot Z_3 X_4 Z_5 \cdot Z_i X_i Z_i \, |Cl\rangle = |Cl\rangle$

Edge mode: Intuition: Sensitive to b.c $\Rightarrow$ edge degeneracy

$$H_{cluster} = -\sum_{i=1}^{2L-2} Z_i X_{i+1} Z_{i+2} \quad \text{(open b.c.)}$$

Each bd h

Boundary operators commuting with H:

Left: $Z_1, X_1 Z_2$ Right: $Z_{2L}, Z_{2L-1} X_{2L}$

Charged under X_{odd}, X_{even}

$\longleftrightarrow$

anti-commute.

$|\rangle = |00\rangle$

$\Rightarrow$ edge degeneracy $\quad g\, Z_{2L-1} X_{2L} Z_1$

g_{0i}

pen b.c.) Each bd hosts a qubit protected by symmetry

g.s. degeracy 2×2

$, Z_{2L-1} X_{2L}$

From topological response:

More generally, symmetr $g = \otimes_i g_i$

On SPT: $g_A |\Psi\rangle = \boxed{V_{\partial A}} |\Psi\rangle$ $\rightarrow$ Can be used to characterize SPT

$X_e |\psi\rangle =$

$X_o |\psi\rangle =$

$\Rightarrow \hat{Z}_{o,L} \hat{Z}_{e,L}$

Cluster: $\hat{Z}_{o,L} = Z_1, \hat{Z}$

Bulk-bd correspondence

$g Z_{2L-1} X_{2L} Z_1$

$g_{0,i}$

hosts a qubit protected by symmetry

$\Rightarrow$ g.s. degeneracy 2×2

l response:

mmetr $g = \otimes_i g_i$

T: $g_A|\psi\rangle = \boxed{V_{\partial A}}|\psi\rangle$ $\longrightarrow$ Can be used to characterize SPT (Ref. Else, Nayak, 1409.5436)

$X_e|\psi\rangle = \underline{\check{Z}_{o,L}}\,\hat{Z}_{o,R}|\psi\rangle$

$X_o|\psi\rangle = \underline{\check{Z}_{e,L}}\,\hat{Z}_{e,R}|\psi\rangle$

$\Rightarrow \hat{Z}_{o,L}\hat{Z}_{e,L} = -\hat{Z}_{e,L}\hat{Z}_{o,L}$

Cluster: $\check{Z}_{o,L} = Z_1$, $\check{Z}_{e,L} = X_1 Z_2$.

Bulk-bd correspondence

Real life: Similar SPT $\mathbb{Z}_2\times\mathbb{Z}_2 \to SO(3)$ realized in Spin-1 and

$H = \sum J \vec{S}_i$

Higher dim. lots realization in free fermions, a.k.a.

In 2 weeks, topological response : Quantum Hall effect.

$\times \mathbb{Z}_2 \rightarrow SO(3)$) realized in Spin-1 antiferromagnetic Heisenberg model.

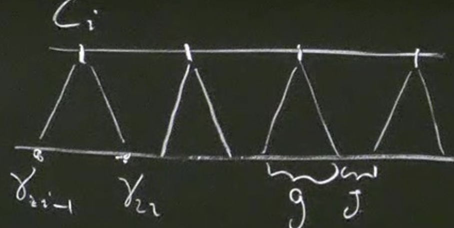
$$H = \sum J \, \vec{S}_i \cdot \vec{S}_{i+1} \quad \leftarrow \text{spin-1}$$

realization in free fermions, a.k.a. "Topological insulators".

logical response : Quantum Hall effect.

Another example of topo. resp.: Kitaev chain.

Fermion chain



$$H = -J \underline{\sum_{i=1}^{2L} i\gamma_{2i}\gamma_{2i+1}} - g\sum_{i=1}^{2L} i\gamma$$

For $J < g$, $|G.S.\rangle = \prod_i c_i^{+}$

What about $J > g$. Take $g \to 0$

taev chain.

$$H = -J \sum_{i=1}^{2L} i\gamma_{2i}\gamma_{2i+1} - g \sum_{i=1}^{2L} i\underbrace{\gamma_{2i-1}\gamma_{2i}}_{}$$

$-c_i^\dagger c_i$

$J < g$, $|G.S.\rangle = \prod_i c_i^\dagger |0\rangle$ "Trivial (SRE)"

about $J > g$. Take $g \to 0$ $\quad H = -\sum_{i=1}^{L} i\gamma_{2i}\gamma_{2i+1}$, $\quad |\Psi\rangle = i\gamma_{2i}\gamma_{2i+1}|\Psi\rangle$

Open b.c. $\quad H = -\sum_{i=1}^{L-1} i\gamma_{2i}\gamma_{2i+1}$, $\quad \gamma_1, \gamma_{2L}$ commute with H

$\tilde{c} = \frac{\gamma_1 + i\gamma_{2L}}{2}$, $\mathcal{H} = \begin{cases} |0\rangle, \ \tilde{c}|0\rangle = 0 \\ |1\rangle := \tilde{c}^\dagger |0\rangle \end{cases}$ $\dim(\mathcal{H}) = 2$.

$\Rightarrow$ If $\mathcal{H} \cancel{\simeq} \mathcal{H}_L \otimes \mathcal{H}_R$, $\dim = \dim_L^2$, $\dim_L = \sqrt{2}$

$\mathcal{H}$ not defined locally, 2-fold degeneracy cannot be lifted locally.

$\delta H \sim \tilde{c}^\dagger \tilde{c} \sim i\gamma_1\gamma_{2L}$

$\delta E \sim e^{-L/\xi}$.

...ivial (SPE)"

$\sum i\gamma_{2i}\gamma_{2i+1}$, $|\Psi\rangle = i\gamma_{2i}\gamma_{2i+1}|\Psi\rangle$

$\gamma_{2i}\gamma_{2i+1}$, γ_1, γ_{2L} commute with H