Title: Lecture - Quantum Matter, PHYS 777

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Collection/Series: Quantum Matter (Elective), PHYS 777, March 31 - May 2, 2025

Subject: Condensed Matter

Date: April 10, 2025 - 10:15 AM

URL: https://pirsa.org/25040005

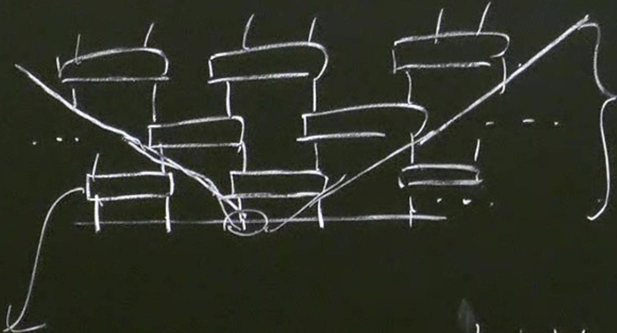
Recap: ① Lieb-Robinson bound: $\|[A_i(t), B_j]\| \leq \text{Const} \cdot e^{-c|i-j|}$ for $|i-j| \geq v_{LR}\, t$

② Two gapped $|\psi_1\rangle \simeq |\psi_2\rangle \iff |\psi_2\rangle = U_{\psi_2}|\psi_1\rangle = \mathcal{T} e^{-i\int_0^1 \tilde{H}(s)\,ds}|\psi_1\rangle$

$\underline{H(s),\ s\in[0,1]}$

③ U_{ψ_1} can be efficiently simulated using Q.C. with low depth

Quantum Circuit:



Depth = D

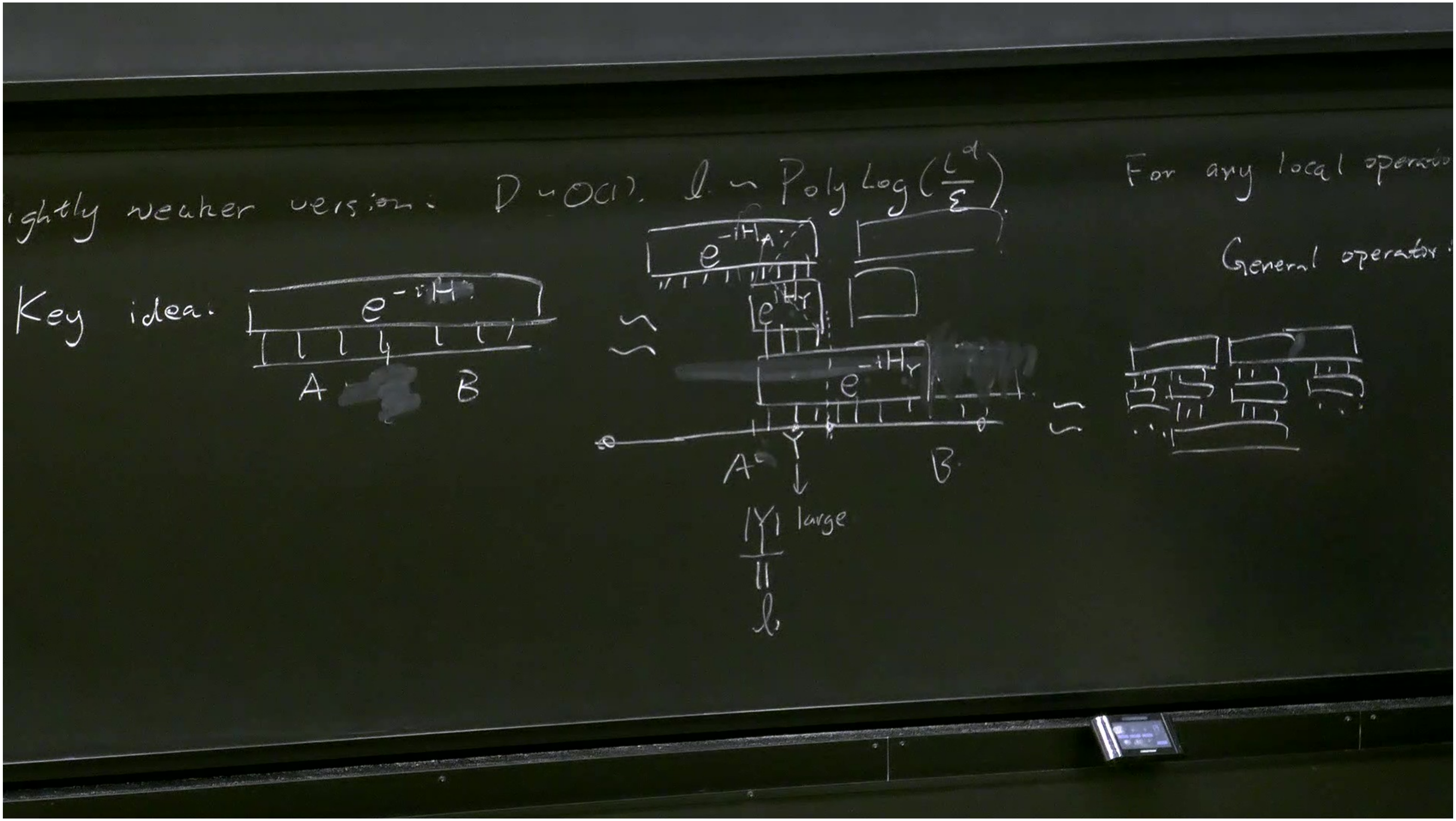
Unitary, width ~ l

Light cone size ~ $D \cdot l$

Setup:

Setup: local $H(s)=\sum_z H_z(s)$, $\|H_z\|=0$ if $|z|\geq$ Const., $U=\mathcal{T}e^{-i\int \tilde{H}(s)\,ds}$.

Thm: U can be simulated in Depth $O(\mathrm{polylog}\,\frac{L^d}{\varepsilon})$, $\|V-U\|\leq\varepsilon$.


ightly weaker version: D ~ O(1), l ~ PolyLog(L^d/ε)
For any local operato
General operator:
Key idea:
e^{-iH}
A
B
e^{-iH_A}
e^{-iH_Y}
e^{-iH_Y}
A
Y
B
|Y| large
l

For any local operator O_i: effect: e^{-iH} to $e^{-l\cdot c}$.

General operator: $\sum \lambda_a O_1^{(a)} O_2^{(a)} O_3^{(a)} \cdots O_L$ error $\underline{L e^{-l\cdot c}}$.

$$U O_1 O_2 U^\dagger = U O_1 U^\dagger U O_2 U^\dagger$$

$D=3$, Gate size l.

Error $L e^{-l\cdot c} \le \varepsilon \Rightarrow l \sim \mathrm{Polylog}\,\frac{L}{\varepsilon}$

$$\boxed{|\psi_1\rangle \simeq |\psi_2\rangle \quad \text{iff} \quad \exists\, U_{FD} : |\psi_1\rangle = U_{FD}|\psi_2\rangle .}$$

- All product states belong to same phase ("trivial phase")

$$|\text{prod}\rangle = |a_1\rangle \otimes |a_2\rangle \otimes \cdots |a_L\rangle .$$

$$|0\rangle = |\rightarrow\rangle \otimes |\rightarrow\rangle \otimes \cdots |\rightarrow\rangle$$

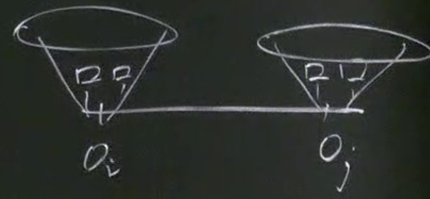
- If $|\psi\rangle = U_{FD}|0\rangle$ for some U_{FD}, $|\psi\rangle$ is "Short-range entangled" (SRE)

Immediate consequence:

$$\langle O_i O_j \rangle_c = \langle (O_i - \langle O_i \rangle)(O_j - \langle O_j \rangle)\rangle \sim e^{-|i-j|/\xi}$$ (0 for strictly local gates)

for $|i-j| > l \cdot D$

Proof: $$\underline{\langle \psi | O_i O_j | \psi \rangle} = \langle 0 | \underbrace{U_{FD}^\dagger O_i U_{FD}}_{\tilde{O}_i} \underbrace{U_{FD}^\dagger O_j U_{FD}}_{\tilde{O}_j} |0\rangle$$



if $|i-j| \gg l \cdot D$

$$= \langle 0 | \tilde{O}_i | 0 \rangle \langle 0 | \tilde{O}_j | 0 \rangle = \underline{\langle O_i \rangle_\psi \langle O_j \rangle_\psi}$$

If $\nexists\, U_{FD}$ s.t. $|\psi\rangle = U_{FD}|0\rangle$: $|\psi\rangle$ is <u>Long-range entangled</u> (LRE)

Example. Ferromagnet / Cat / GHZ state

$$|\uparrow\uparrow\uparrow \dots \uparrow\rangle + |\downarrow\downarrow\downarrow \dots \downarrow\rangle$$

$$\langle Z_i Z_j \rangle = 1, \quad \langle Z_i \rangle = \langle Z_j \rangle = 0. \quad \langle Z_i Z_j \rangle_c \to \text{const} \neq 0$$

Critical point. $\langle Z_i Z_j \rangle_c \to \dfrac{1}{|i-j|^{\#}}$

-range entangled (LRE)

Next weak: Examples of LRE
not from $\langle O_i O_j \rangle_c$.
"Topological order".

$\rangle_c \to \text{const} \neq 0$

Back to SRE, can be interesting if $\exists$ symmetry.

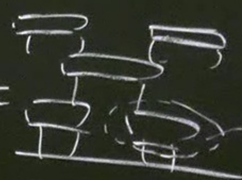
If $|\psi\rangle = U_{FD}|0\rangle$, symmetry G. It may be the case that $\Rightarrow$

individual gates in U_{FD} has to break G.

$G|\psi\rangle = |\psi\rangle$

$G|0\rangle = |0\rangle$

$G \underline{U_{FD}} G^\dagger = U_{FD}$

$U_{FD} =$  $G u_i G^\dagger \neq u_i$

Back to SRE, can be interesting if $\exists$ symmetry.

If $|\psi\rangle = U_{FD}|0\rangle$, symmetry G. It may ~~be~~ the case that individual gates in U_{FD} has to

$\downarrow$

$|\psi\rangle = \rho e^{-i}$

$G|\psi\rangle = |\psi\rangle$

$G|0\rangle = |0\rangle$

$G \underline{U_{FD}} G^\dagger = U_{FD}$

$U_{FD} =$  $\rightarrow G u_i G^\dagger \neq u_i$

If $|\psi_1\rangle = U_{FD}|\psi_2\rangle$, gates in U_{FD} are symmetric

$\Rightarrow$ $|\psi_1\rangle, |\psi_2\rangle$ belong to same SPT phase.

ase that $\Rightarrow$ $|\psi\rangle$ is a (nontrivial) Symmetry-protected topological (SPT) state.

has to break G.

$\downarrow$

$|\psi\rangle = \mathcal{P} e^{-i\int_0^1 H(s)\,ds}|0\rangle$, but $H(s) = \sum_z \underbrace{H_z(s)}_{\text{has to break } G \text{ for some } s.}$

Example: Cluster chain:

1d qubit chain, $L = \text{even}$

$\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry: $X_{even} = \prod_{i=even} X_i$ $\quad X_{odd} = \prod_{i=odd} X_i$

$$H_{cl} = -\sum_i Z_i X_{i+1} Z_{i+2}.$$

vs. trivial chain $H_0 = -\sum X_i$

From Tutorial $H_{cl} = U_{FD}^{+} H_0 U_{FD}$, $\quad U_{FD} = \overbrace{\prod_i \underbrace{CZ_{i,i+1}}_{\text{break } \mathbb{Z}_2 \times \mathbb{Z}_2}}^{\mathbb{Z}_2 \times \mathbb{Z}_2 \text{ symmetric.}}$

$\Rightarrow$ G.S. of H_{cl} is unique & SRE

• If 14

Imm

Proof

break $Z_2 \times Z_2$.

$\Rightarrow$ G.S. of H_{cl} is unique & SRE

$|\rightarrow\rangle = |+\rangle$; $|\leftarrow\rangle = |-\rangle$

$$|\text{Cluster}\rangle = |\uparrow + \uparrow - \downarrow + \downarrow \ldots\rangle + |\uparrow - \downarrow + \downarrow - \uparrow \ldots\rangle + \ldots$$

$$= \sum_{\substack{\{S_j\} \\ j=1\ldots L/2}} \bigotimes_{2j-1} |Z_{2j-1} = S_j\rangle \bigotimes_{2j} |X_{2j} = S_{2j-1} S_{2j+1}\rangle.$$

$\rightarrow$ In comparison: $|++++\ldots\rangle = |\uparrow + \uparrow + \downarrow + \downarrow \ldots\rangle + |\uparrow + \downarrow + \downarrow + \uparrow \ldots\rangle$

Symmetric state is a condensate of domain walls. (KW duality: $\langle \tilde{Z}_i \tilde{Z}_j \rangle \neq 0$)

What's nontrivial about $|\text{Cluster}\rangle$: domain walls of Z_{odd} are "decorated" with charge of X_{even}.