Title: Lecture - Quantum Matter, PHYS 777

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Collection/Series: Quantum Matter (Elective), PHYS 777, March 31 - May 2, 2025

Subject: Condensed Matter

Date: April 08, 2025 - 9:00 AM

URL: https://pirsa.org/25040004

Locality in Quantum Systems, Gapped Phas

Refs. → Chen, Gu, Wen 1004.3835 (Appendix. co

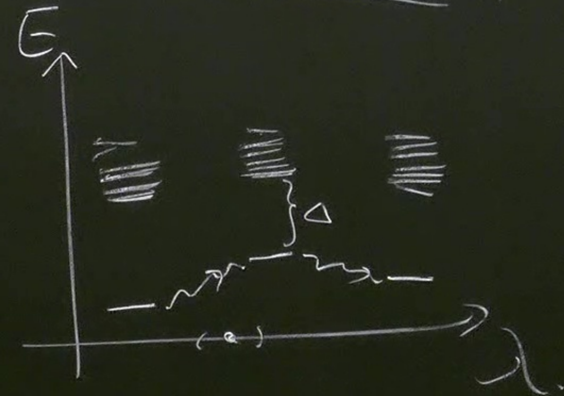
→ Hastings, 1008.5137

→ Haah, Hastings, Kothari, Low 1801.03922.

...apped Phases, Circuit Complexity.

(Appendix. concise)

...01.03922.



$\Delta(\lambda) \geq \Delta_0 > 0$.

results : (1) Lieb-Robinson bound. Locality $\to$ Causality.

(2) Quasi-abiabaticity. $|\psi_1\rangle \simeq |\psi_2\rangle \iff |\psi_2\rangle = U_{q.l.} |\psi_1$

(3). U_{ql} can be simulated using a quantum circuit with error ε, in depth $D \sim O(\mathrm{PolyLog}\,\frac{L^d}{\varepsilon})$

Typically, $\varepsilon \sim \mathrm{Poly}(\frac{1}{L}) \Rightarrow D \sim \mathrm{PolyLog}(L)$ "$\sim$" $O(1)$. "F

$\rightarrow$ Causality w/ relativity

$|\psi_2\rangle = U_{q.l.}|\psi_1\rangle$ $\quad U_{q.l.} = \mathcal{T} e^{-i\int_0^1 \tilde{H}(s)\cdot ds}$, $\tilde{H}(s)$ quasilocal

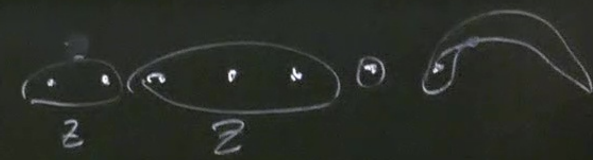
circuit

$\log \frac{L^d}{\varepsilon})$

) "$\sim$" $O(1)$. "Finite-depth circuit".

Lieb-Robinson bound

Setup: lattice spins (e.g. qubit).



$H = \sum_Z H_Z$. Z: local regions in space.

$\|H_Z\|$ shrinks exponentially with size of Z, $|Z|$

$\|\cdot\|$: operator norm: largest $|$eigenvalue$|$.

Consider a local operator A_i under Heisenberg evolution

$$A_i(t) = e^{-iHt} A_i e^{iHt}.$$

Q: How fast does $A_i(t)$ spread?

Consider another B_j, j far from i.

$\| [A_i(t), B_j] \|$. if small for $\forall B_j$.

$A_i(t)$ has not spreaded to j.

Useful identity: $A_i(t) = A_i + \underline{it[A_i, H]} + \frac{(it)^2}{2!}[[A_i, H], H]$

Small t: need H_Z s.t. $i, j \in Z \Rightarrow \|[A_i(t), B_j]\|$

Things are different if $\|\underbrace{[[[A_i, H], H], H] \ldots]}_{n}$ is

$\frac{t^n}{n!} \sim \left(\frac{et}{n}\right)^n$ need $\boxed{t}$

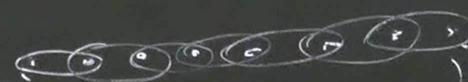
$n! \sim \left(\frac{n}{e}\right)^n \cancel{\sqrt{n}}$

Rigorously

$$\frac{(it)^2}{2!}[[A_i,H],H] + \dots \frac{(it)^n}{n!}\underbrace{[\dots[A_i,H],H],\dots H]}_{n} + \dots$$

$$\Rightarrow \|[A_i(t),B_j]\| \sim e^{-|i-j|\cdot c}$$

$\underbrace{\dots,H],H]\dots]}_{n}$ is not small: $n \sim \underbrace{d}_{\text{Const}} \cdot |i-j|$

i  j

$|i-j| > v_{LR}\cdot t$.

$$\left(\frac{et}{n}\right)^n \quad \text{need} \quad \boxed{t \sim \frac{n}{\lambda}} \sim \frac{|i-j|}{\underbrace{v_{LR}}_{\text{Lieb-Robinson velocity}}}$$

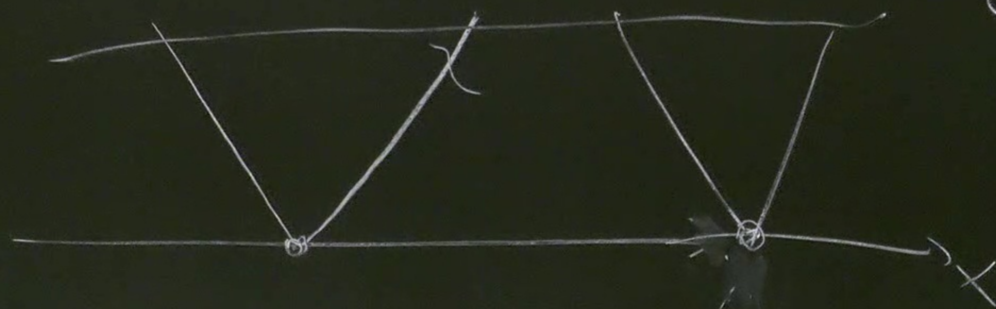
$$\left(\frac{n}{e}\right)^n \cancel{\sqrt{n}}$$

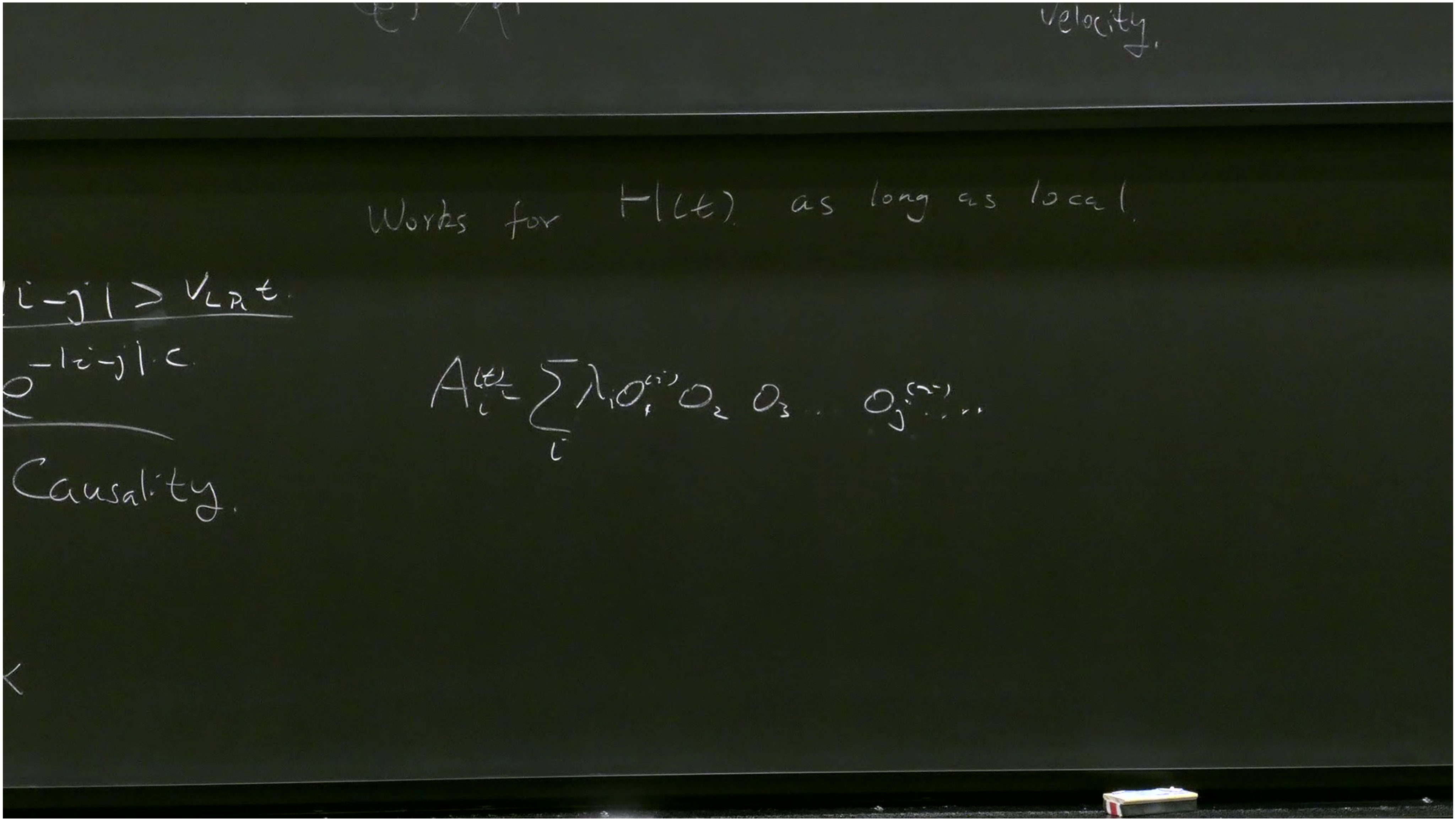
Rigorously proven statement.

$\exists$ const $V_{LR} > 0$, s.t. for $|i-j| > V_{LR}\, t$

$$\|[A_i(t), B_j]\| \leq \text{const} \cdot e^{-|i-j| \cdot c}$$

Causality.



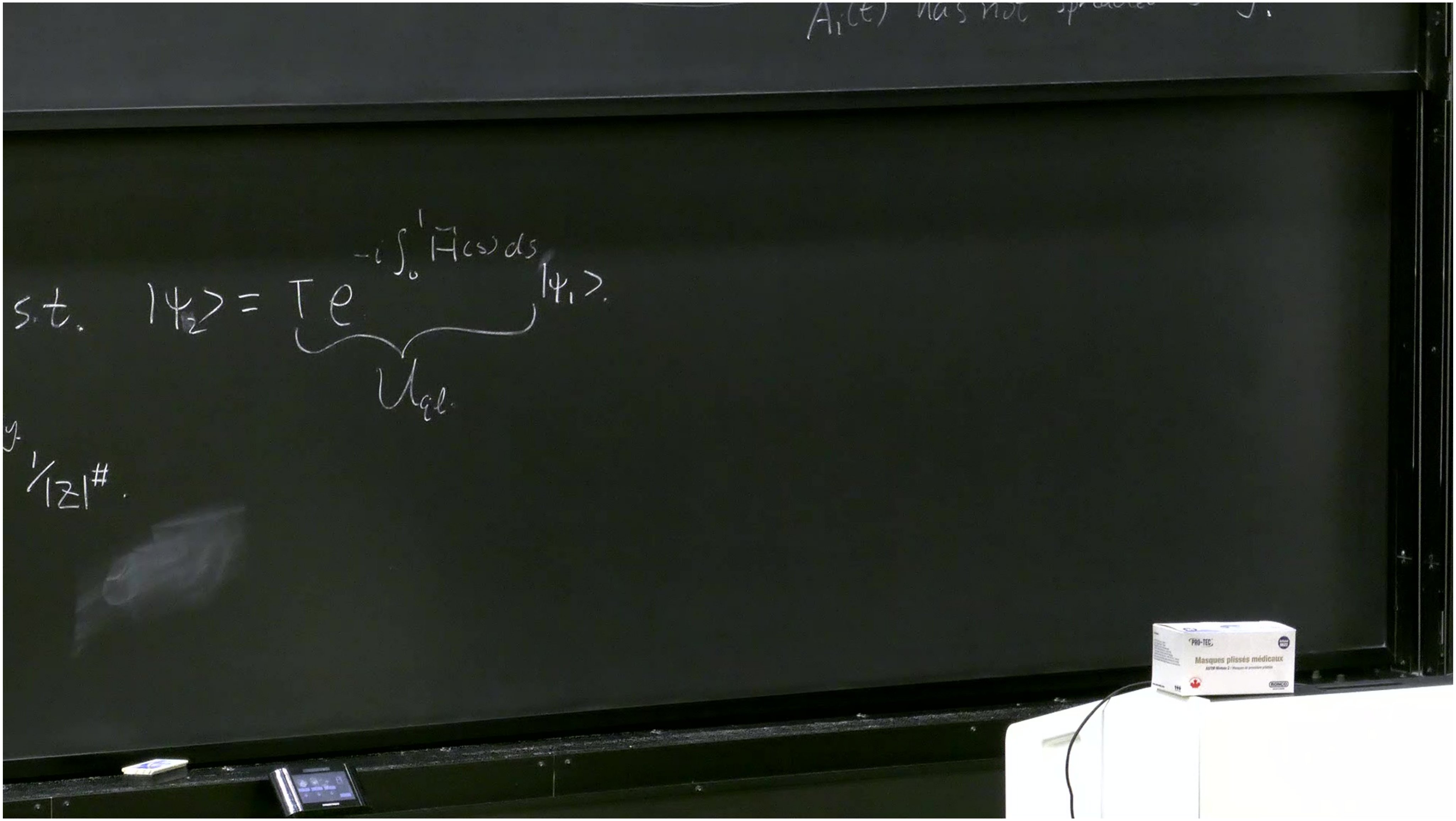

velocity.
Works for H(t) as long as local.
|i-j| > v_LR t.
e^{-|i-j|·c}
Causality.
A_i(t) = Σ_i λ_i O_1^(i) O_2 O_3 ... O_j^(n) ...

Quasi-adiabatic continuity

Thm: $|\psi_1\rangle \simeq |\psi_2\rangle$ iff $\exists$ $\underbrace{\text{quasilocal}}$ $\tilde{H}(s)$

Two gapped

$\|H_Z\|$ decays superpolynomially.
i.e. faster than any $1/|Z|^{\#}$.


$A_1(t)$ has not
s.t. $|\psi_2\rangle = \underbrace{T e^{-i\int_0^1 \tilde{H}(s)\,ds}}_{U_{ql}} |\psi_1\rangle$.
$1/|Z|^{\#}$.
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$(\frac{n}{e})^n$ $\cancel{\sqrt{\pi n}}$

Lieb-Robinson velocity.

$\Leftarrow$ (easy). Suppose $|\psi_1\rangle$ gapped g.s. of H_1

& $\exists$ q.l. $\tilde{H}(s)$. $|\psi_1\rangle \to |\psi_2\rangle$.

Define $\mathcal{U}(s) = T e^{-i\int_0^s \tilde{H}(s')\,ds'}$

$H(s) = \underline{\mathcal{U}(s)}\, H_1\, \underline{\mathcal{U}^\dagger(s)}$ gapped for all s.

$|\psi_2\rangle = \mathcal{U}(s=1)|\psi_1\rangle$ is gapped g.s. of $H(s=1)$

$H_Z(s) \sim \mathcal{U}(s) H_Z\, \mathcal{U}^\dagger(s)$: (quasi) local

$\Rightarrow$ (Nontrivial) Need to construct $\tilde{H}(s)$ from $H(s)$

Naive attempt $\tilde{H}(s) = \tau H(s)$ for some $\tau \gg \frac{1}{\Delta}$

Need to modify $\tilde{H}$: quasi-adiabatic evolution. Failure rate from each excited state c_i suppressed.

$\sum_i \to$ diverges with L.

Inspiration from 1st order perturbation

$$i \partial_s |\psi_0(s)\rangle = \sum_{i \neq 0} \underbrace{\frac{i}{E_0(s) - E_i(s)}}_{= \int iF(t)\, e^{i(E_i(s)-E_0(s))t}\, dt} \langle \psi_i(s)|\partial_s H(s)|\psi_0(s)\rangle |\psi_i(s)\rangle$$

$F(t)$ satisfies (a) $\tilde{F}(\omega) = \frac{-1}{\omega}$ for all $\omega \geq \Delta$

(b) $\hat{F}(\omega=0)=0$

$$= i\underbrace{\int dt\, F(t)\, e^{iH(s)t}\,\partial_s H(s)\, e^{-iH(s)t}}_{\tilde{H}(s)}\,|\psi_0(s)\rangle.$$

demand (c) $F(t)$ decay superpolynomially for $t \gg \frac{1}{\Delta}$

$\xrightarrow{\text{LR bound}}$ $\tilde{H}(s)$ quasi-local

Comment: $\Delta > 0$ crucial.

Suppose $\Delta = 0$ $\quad \hat{F}(\omega) = \frac{1}{\omega}$.

Fact: such $F(t)$ exists $\square$

t $\partial_s H(s) e^{-iH(s)t} |\psi_0(s)\rangle$.

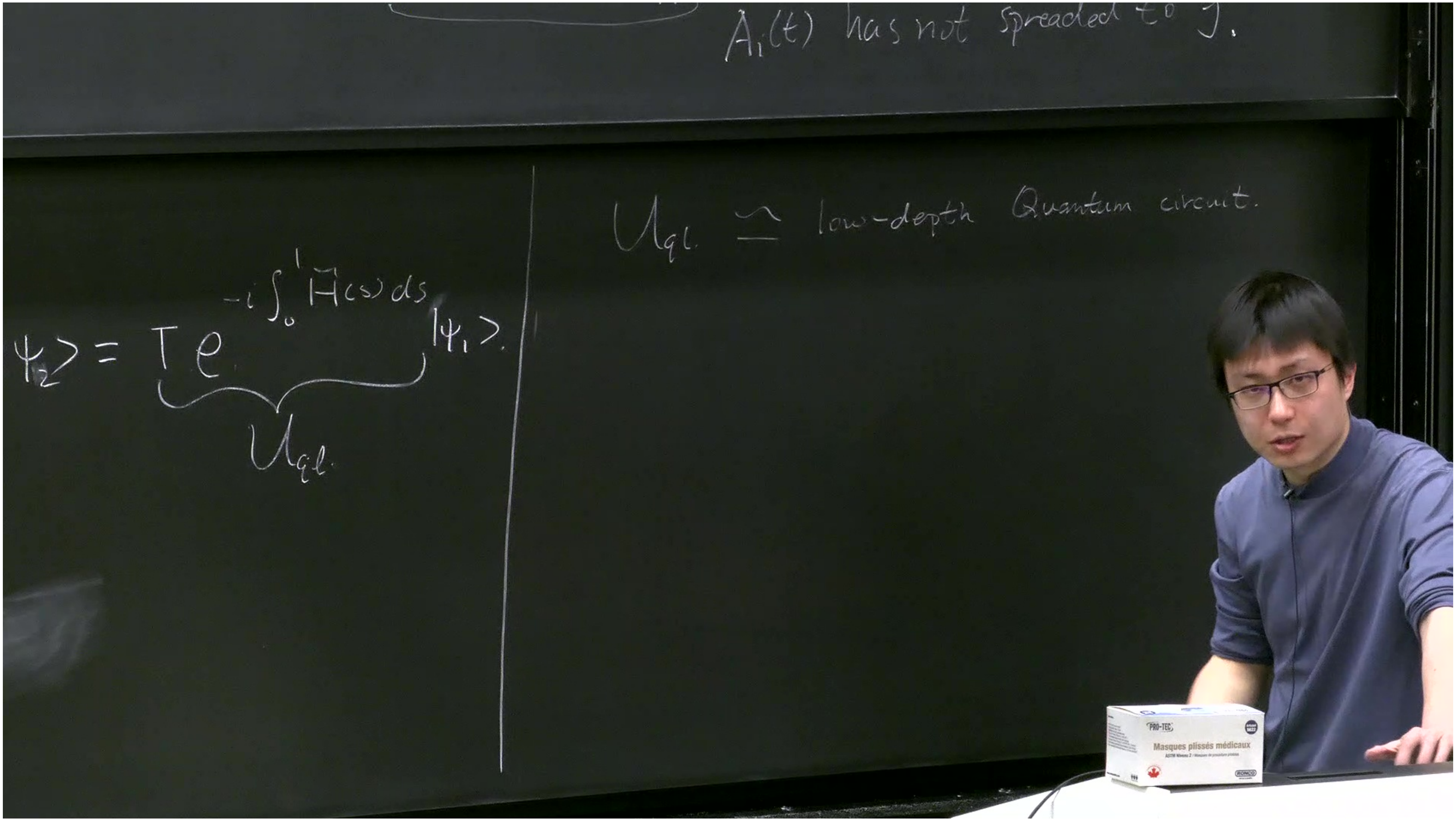
$\tilde{H}(s)$

omially for $t \gg \frac{1}{\Delta}$

cal.

Comment: $\Delta > 0$ crucial.

Suppose $\Delta = 0$ $\tilde{F}(\omega) = \frac{-1}{\omega}$, $F(t) = \mathrm{sgn}(t) \Rightarrow \tilde{H}$ non-local.


$A_1(t)$ has not spreaded to j.
$|\psi_2\rangle = \underbrace{T e^{-i\int_0^1 \tilde{H}(s)\, ds}}_{U_{ql}} |\psi_1\rangle$.
$U_{ql} \simeq$ low-depth Quantum circuit.
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