Title: The Standard Model from the Shadow of a 5 Dimensional Cube

Speakers: Cole Coughlin

Collection/Series: Training Programs (TEOSP)

Subject: Other

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Abstract:

In this episode, we continue our exploration of Lie Groups and weight diagrams of their representations as a means of understanding the very specific field content and Gauge Groups of the Standard Model. Previously we presented a novel perspective of the Electroweak sector of the Standard Model as a shadow of 3 dimensional polytopes, which correspond to representations of a rank 3 group, and who's shadow corresponds to a specific embedding of the Electroweak Gauge Group. While interesting, we highlighted the theories inability to include colour as a simple extension, which motivated our search for higher rank unifying groups. Today, we present an algorithm for determining the Gauge quantum numbers of all SM chiral fermions from nothing but a projection of a 5 dimensional cube. We will discuss how previous unifications such as Georgi-Glashow, Pati-Salam, and others can be seen as different aspects of this rank 5 group and what conditions are used to fix the embedding corresponding to unbroken symmetries of the Standard Model. While our approach does violate common necessary conditions for Grand Unification, we use these difficulties to motivate our search for connections between the violation of spacetime symmetries such as Parity, and symmetry breaking in the Gauge sector.

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Polytopes and the Standard Model

Cole Coughlin

(March 2025)

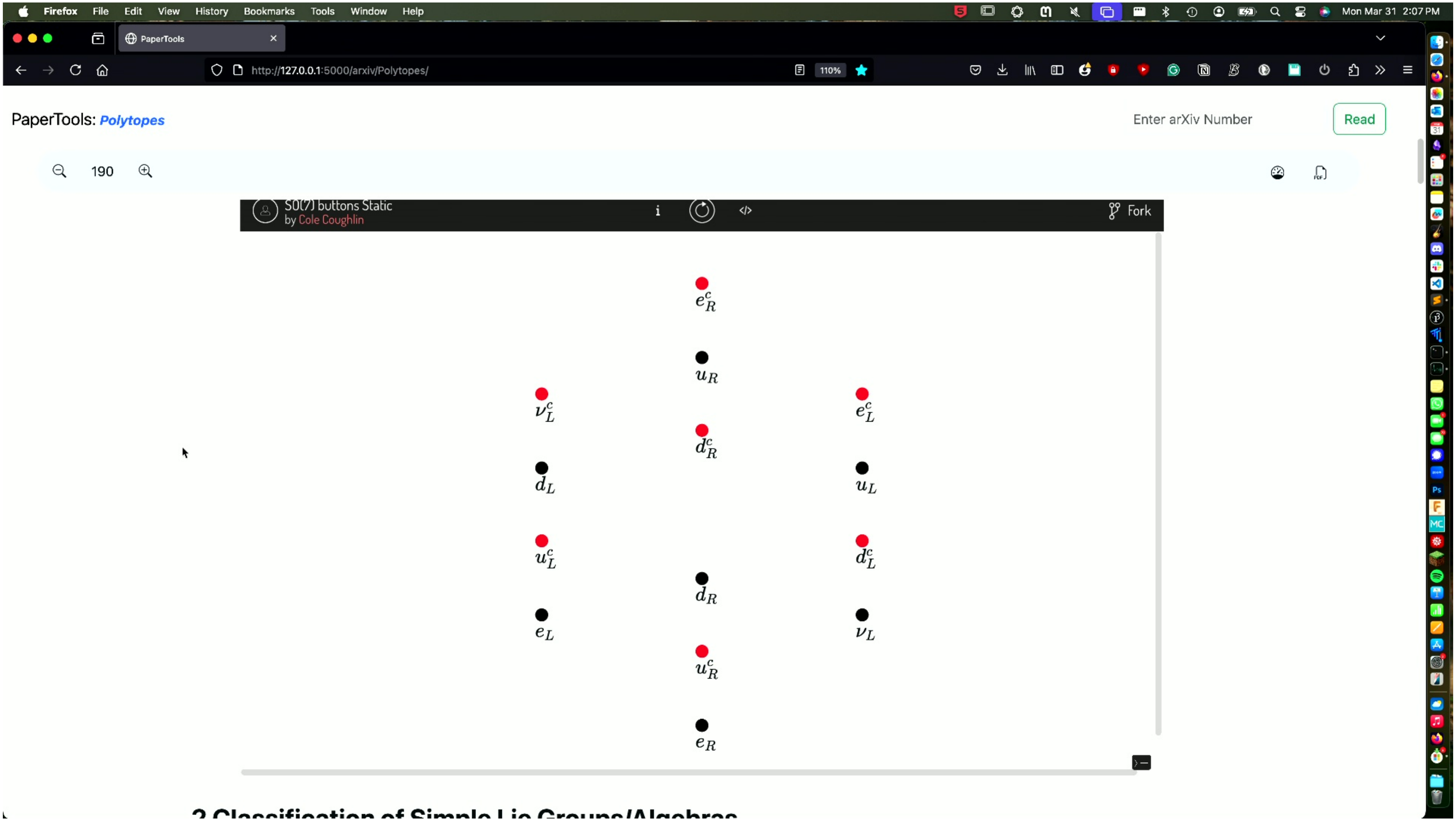
1 Why *This* Standard Model

"The Standard Model (SM) may be defined as the renormalizable field theory with gauge group $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$, with 3 generations of fermions in the representation

$$(\mathbf{3},\mathbf{2})_{1/3} + (\bar{\mathbf{3}},\mathbf{1})_{-4/3} + (\bar{\mathbf{3}},\mathbf{1})_{2/3} + (\mathbf{1},\mathbf{2})_{-1} + (\mathbf{1},\mathbf{1})_{2},$$

and a scalar Higgs doublet H transforming as $(\mathbf{1},\mathbf{2})_1$." - PDG [2]

The Standard Model contains 19 free parameters. The 3 couplings of the gauge groups g_1, g_2, g_3 corresponding to $SU(3)_C \times SU(2)_L \times U(1)_Y$ respectively. 9 masses for the charged fermions $e, \mu, \tau, u, d, c, s, t, b$ as well as 3 angles and one CP violating phase for the CKM mixing matrix. And finally the Higgs mass and quartic coupling as well as a CP violating phase in the QCD sector which is currently consistant with 0. The Standard Model can be made to include neutrino masses by adding 3 more masses as well as the appropriate mixing angles and phases.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2 Classification of Simple Lie Groups/Algebras

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Lie Algebra is the tangent space of the Lie group at the origin.

Complete Reducibility - Every finite dimensional representation of a compact semisimple Lie group decomposes into a direct sum of irreducible representations.

Cartan Subalgebra - Every simple lie algebra g contains a Cartan subalgebra $h \in g$ such that adH is diagonalizable for all $H \in h$. The dimension of h coincides with the rank of g. As described in [3] we may choose a basis of g

$$\left\{h_1, \cdots, h_1, g_1, g_{-1}, g_2, g_{-2}, \cdots, g_{\frac{z-1}{2}}, g_{-\frac{z-1}{2}}\right\}$$

satisfying

1. $\left[h_i, g_j\right] = \lambda_j^j g_j$ (no sum), $\lambda_i^j \in \mathbf{R}$
2. $\left[h_i, h_j\right] = 0$
3. $\left[g_j, g_{-j}\right] \in h$

This defines the Root System of the algebra

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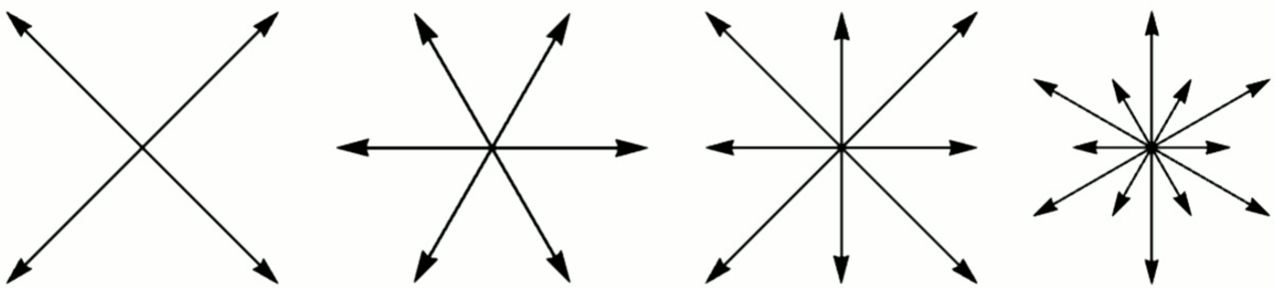

All simple Lie groups have been classified

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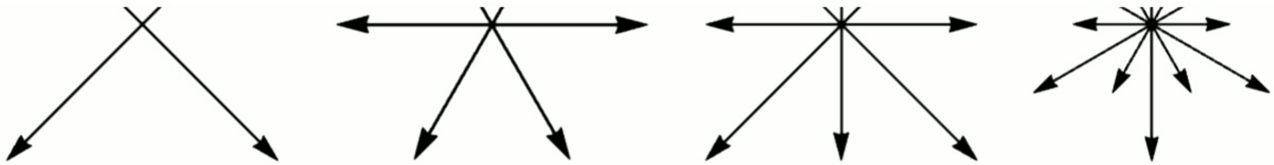

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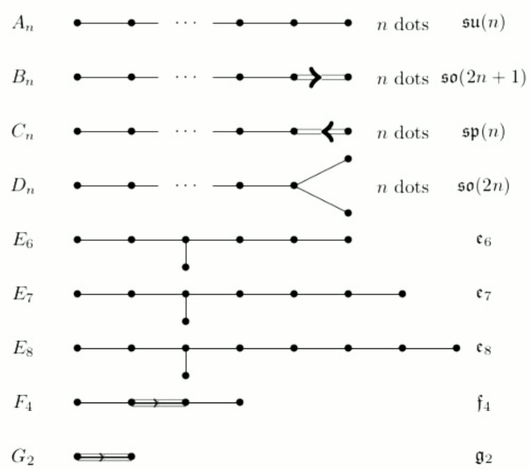

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3 Broken History of the Standard Model

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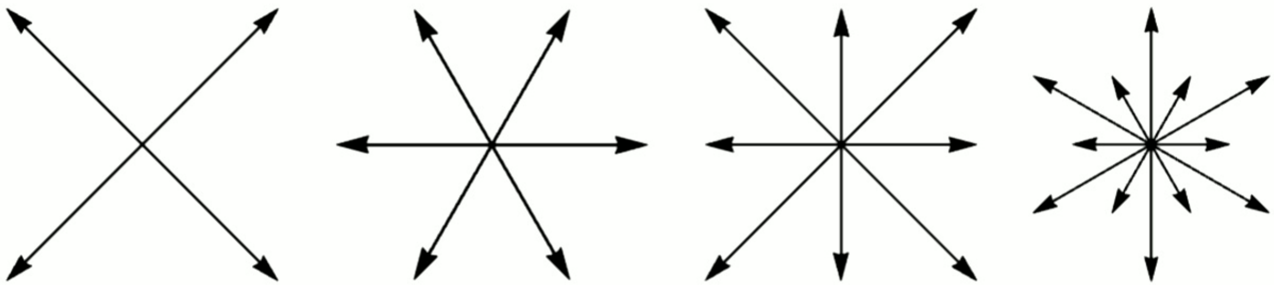

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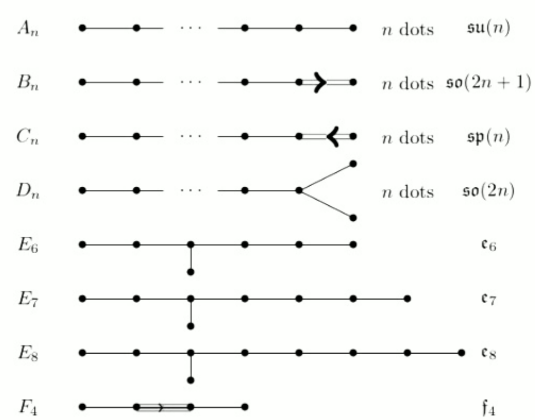

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G_2  $\mathfrak{g}_2$

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3 Broken History of the Standard Model

3.1 Gell-Mann and the Eightfold way

By the 1960s, over 100 particles were discovered which seemed just as fundamental as the proton and neutron. It was difficult to imagine that this was a fundamental description of nature with hundreds of free parameters, thus this era was deemed the Particle Zoo.

The conserved quantity Isospin described by an $SU(2)_{\text{Iso}}$ was previously discovered but other particles were decaying strangely so a new conserved charge was proposed $U(1)_S$.

Better Together
by Cole Coughlin

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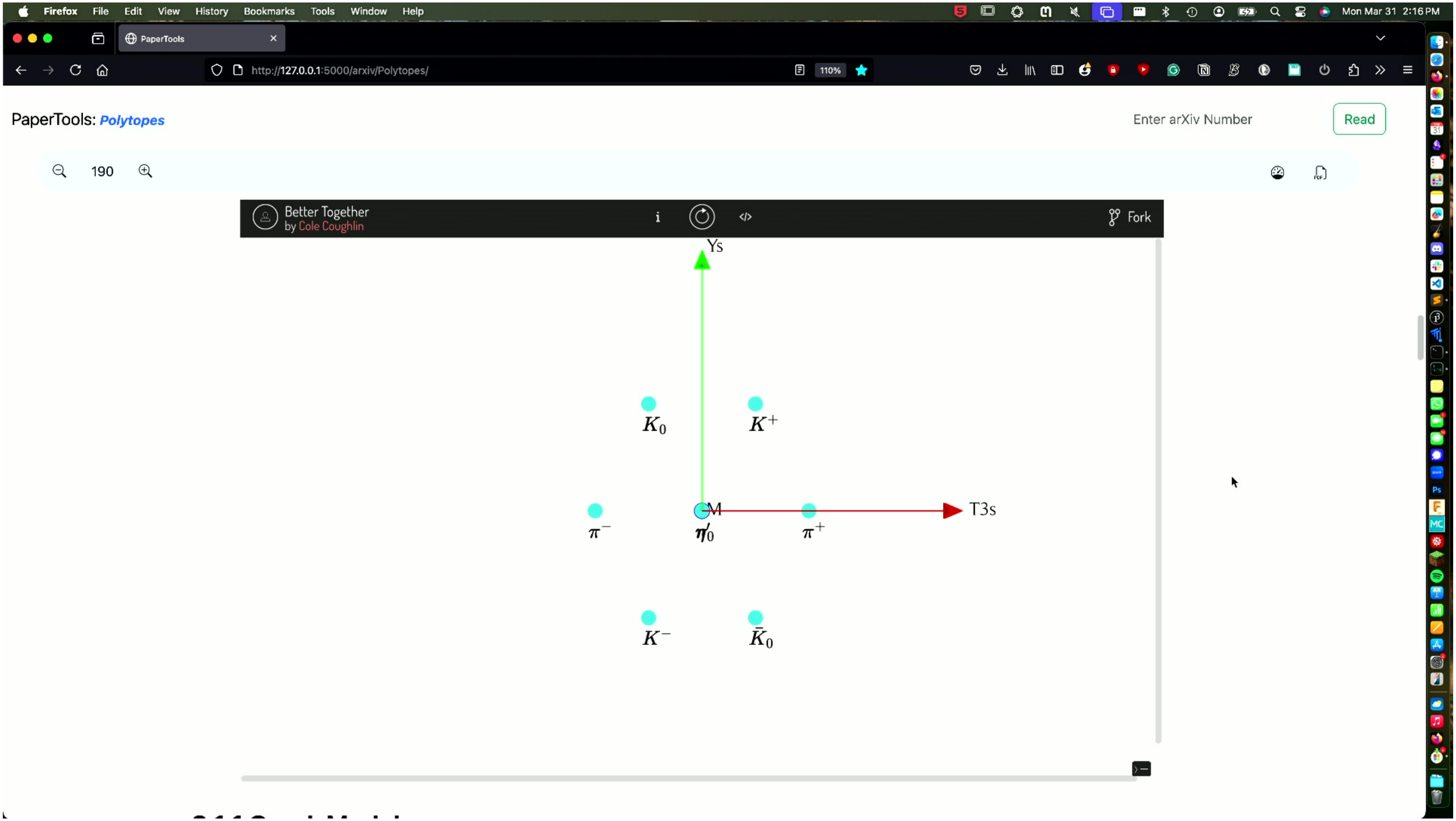
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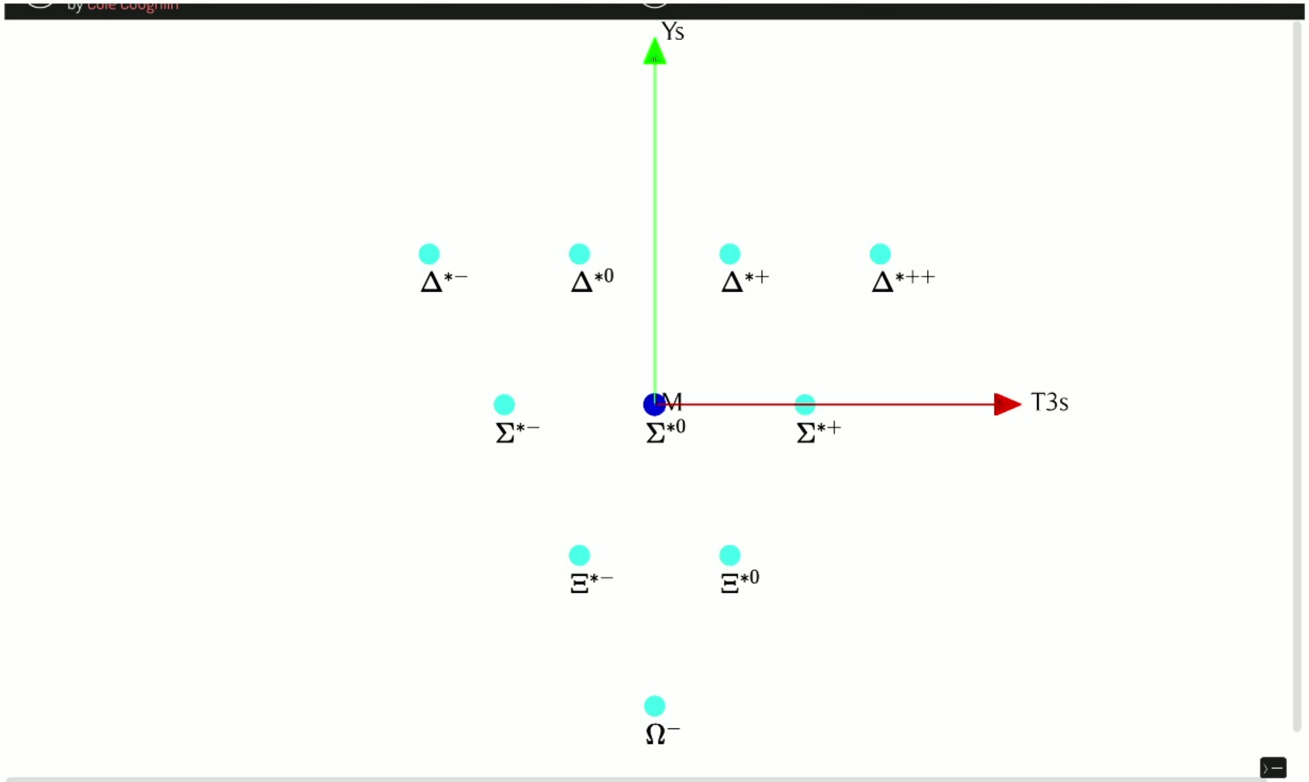

3.1.1 Quark Model

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3.1.1 Quark Model

The apparent $SU(3)_f$ symmetry of the hadrons immediately pointed towards substructure. The particles discovered up to this point could be neatly categorized as irreps coming from tensor products of the fundamental and antifundamental representation of $SU(3)$

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}, \tag{1}$$

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1}, \tag{2}$$

We now understand the flavour symmetry discovered by Gell-Mann as a result of chiral symmetry breaking in QCD. In the limit of the theory with 3 masless flavours of quark, there exists a global $SU(3)_L \times SU(3)_R$ symmetry of the lagrangian, but this is broken to the diagonal subgroup $SU(3)_f$ of Gell-Mann. The eight broken degrees of freedom are "eaten" by the mesons and give them mass.

3.2 Electroweak Theory

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subgroup $SU(3)_f$ of Gell-Mann. The eight broken degrees of freedom are "eaten" by the mesons and give them mass.

3.2 Electroweak Theory

Parity violation of the weak interaction discovered by C.S. Wu in 1956, proposed by T.D. Lee and C.N. Yang.

$$\psi_{f_L} \overset{U(1)}{\to} e^{i\theta_{f_L}}\psi_{f_L}, \quad \psi_{f_R} \overset{U(1)}{\to} e^{i\theta_{f_R}}\psi_{f_R}$$
$$\overline{\psi}_{f_L}\psi_{f_R} \overset{U(1)}{\to} e^{i(\theta_{f_R}-\theta_{f_L})}\overline{\psi}_{f_L}\psi_{f_R} \neq \overline{\psi}_{f_L}\psi_{f_R}, \quad \theta_{f_R} \neq \theta_{f_L}$$

The introduction of the Higgs allows us to build gauge invariant mass terms for our Dirac fermions which we observe in nature.

$$\overline{\psi}_{f_L} h\psi_{f_R} \overset{U(1)}{\to} e^{i(\theta_h+\theta_{f_R}-\theta_{f_L})}\overline{\psi}_{f_L} h\psi_{f_R} = \overline{\psi}_{f_L} h\psi_{f_R}, \quad \theta_h = \theta_{f_L} - \theta_{f_R}$$

3.2.1 Higgs Mechanism

Introduce a complex scalar field

$$h = \frac{1}{\ldots}\begin{pmatrix}\chi_1 + i\chi_2 \\ \ldots\end{pmatrix} \doteq \frac{1}{\ldots} e^{i\xi_i T^i}\begin{pmatrix}0 \\ \ldots\end{pmatrix} \quad (2)$$

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such that

$$\frac{v_h^2}{4}(W^3_\mu\ B_\mu)M^2\begin{pmatrix}W^3_\mu\\ B_\mu\end{pmatrix} = \frac{v_h^2}{4}(W^3_\mu\ B_\mu)\begin{pmatrix}\cos\theta_W & \sin\theta_W\\ -\sin\theta_W & \cos\theta_W\end{pmatrix}\widehat{M}^2\underbrace{\begin{pmatrix}\cos\theta_W & -\sin\theta_W\\ \sin\theta_W & \cos\theta_W\end{pmatrix}\begin{pmatrix}W^3_\mu\\ B_\mu\end{pmatrix}}_{\equiv\begin{pmatrix}Z_\mu\\ A_\mu\end{pmatrix}}$$
$$= \frac{v_h^2}{4}(Z_\mu\ A_\mu)\widehat{M}^2\begin{pmatrix}Z_\mu\\ A_\mu\end{pmatrix} \quad \text{where} \quad \widehat{M}^2 = \operatorname{diag}(\widehat{m}^2, 0). \tag{12}$$

With our redefined fields which are mass eigenstates

$$\begin{pmatrix}Z_\mu\\ A_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta_W & -\sin\theta_W\\ \sin\theta_W & \cos\theta_W\end{pmatrix}\begin{pmatrix}W^3_\mu\\ B_\mu.\end{pmatrix} \tag{13}$$

Leading us to the relation between the angle of rotation and the ratio of the couplings

$$\tan\theta_W = \frac{g_1}{g_2}, \tag{14}$$

Finally yielding

$$(D_\mu h)(D^\mu h^\dagger) = -\frac{1}{2}\left[(\partial_\mu H)^2 + M_W^2((W^1_\mu)^2 + (W^2_\mu)^2) + M_Z^2(Z_\mu)^2 + M_A^2(A_\mu)^2\right], \tag{15}$$

with masses $M_W^2 = g_2 v_h^2 / 4, M_Z^2 = (g_1^2 + g_2^2)v_h^2 / 4$ and $M_A = 0$.

3.3 Grand Unification

$$\bar{5} \to (\bar{3},1)_{\frac{1}{3}} \oplus (1,2)_{-\frac{1}{2}} \qquad d^c \text{ and } l$$

$$\mathbf{10} \to (3,2)_{\frac{1}{6}} \oplus (\bar{3},1)_{-\frac{2}{3}} \oplus (1,1)_1 \qquad q, u^c \text{ and } e^c$$

$$\mathbf{1} \to (1,1)_0 \qquad \nu^c$$

$$\mathbf{24} \to (8,1)_0 \oplus (1,3)_0 \oplus (1,1)_0 \oplus (3,2)_{-\frac{5}{6}} \oplus (\bar{3},2)_{\frac{5}{6}}$$

3.3.3 $SO(10)$

$$\mathrm{SO}(10) \supset \mathrm{SU}(5) \times \mathrm{U}(1)_\chi$$

$$45 \to 24_0 \oplus 10_{-4} \oplus \overline{10}_4 \oplus 1_0$$

$$16 \to 10_1 \oplus \bar{5}_{-3} \oplus 1_5$$

$$10 \to 5_{-2} \oplus \bar{5}_2.$$

$$\mathrm{SO}(10) \supset \mathrm{SU}(4) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$$

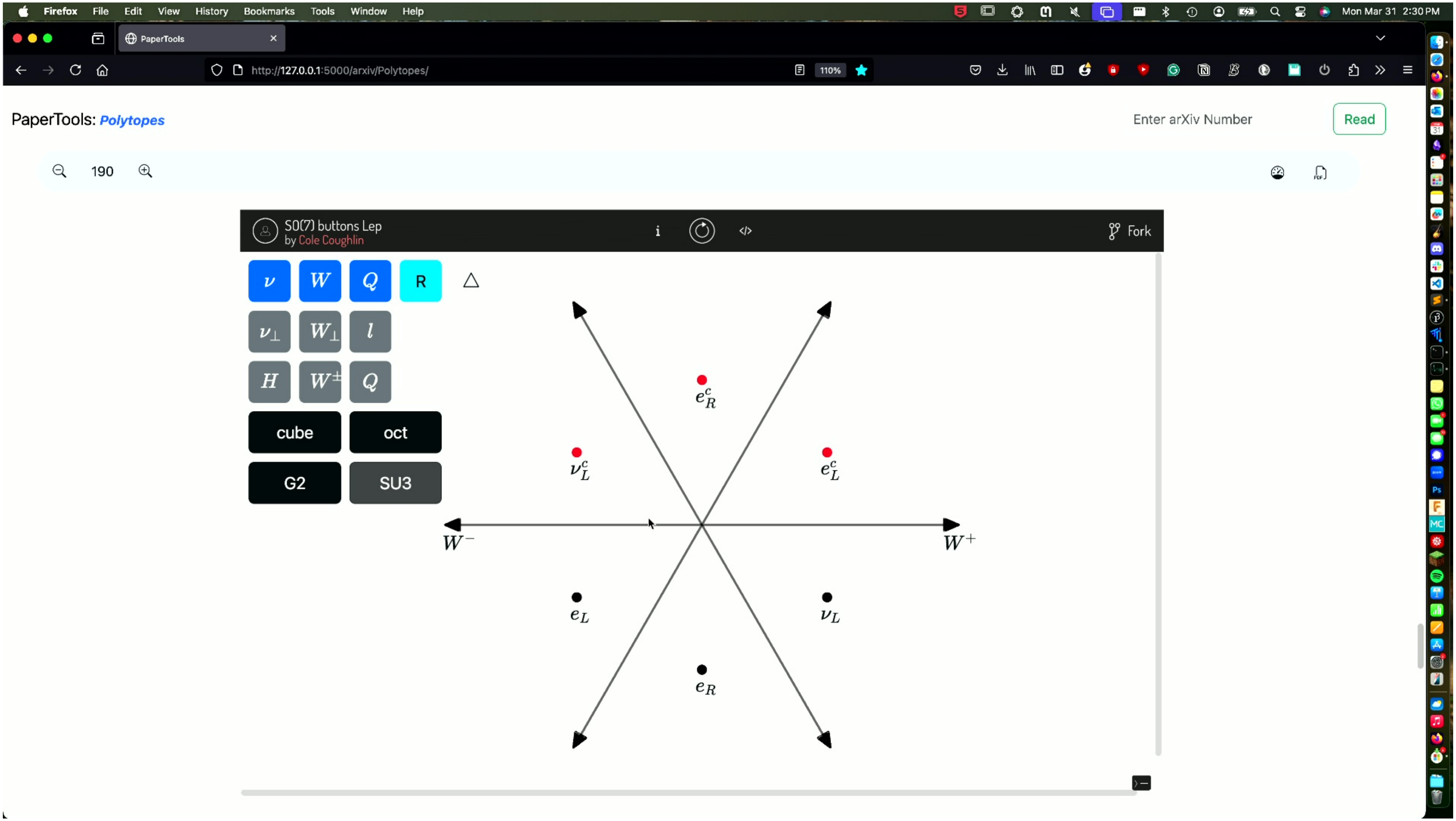
$$45 \to (15,1,1) \oplus (6,2,2) \oplus (1,3,1) \oplus (1,1,3)$$

$$16 \to (4,2,1) \oplus (\bar{4},1,2).$$

4 Weight Diagrams and the Standard Model

4.1 Electroweak Polytopes

4.1.1 $SU(2)_L \times U(1)_Y \subset SU(3) \subset G2$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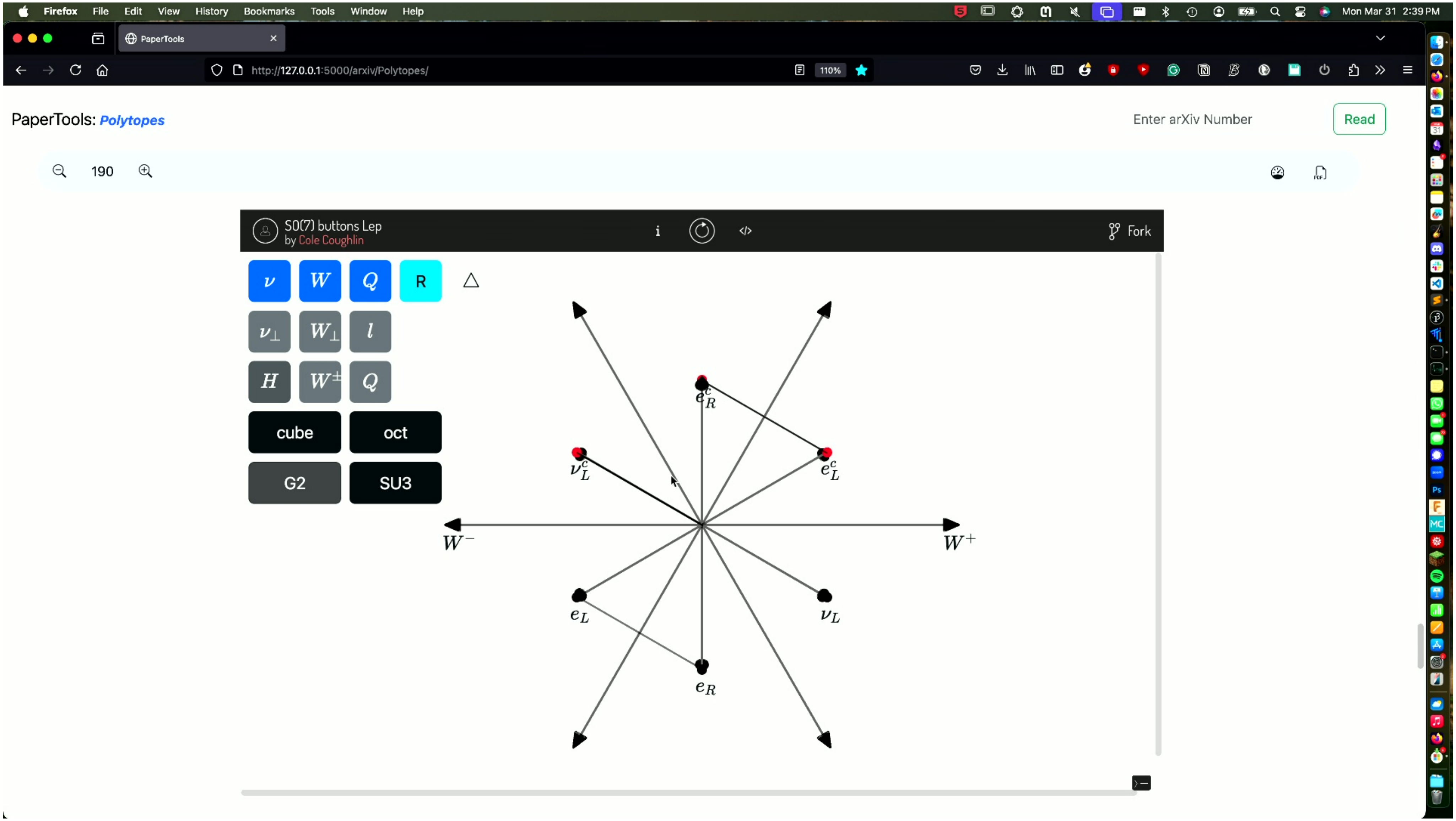
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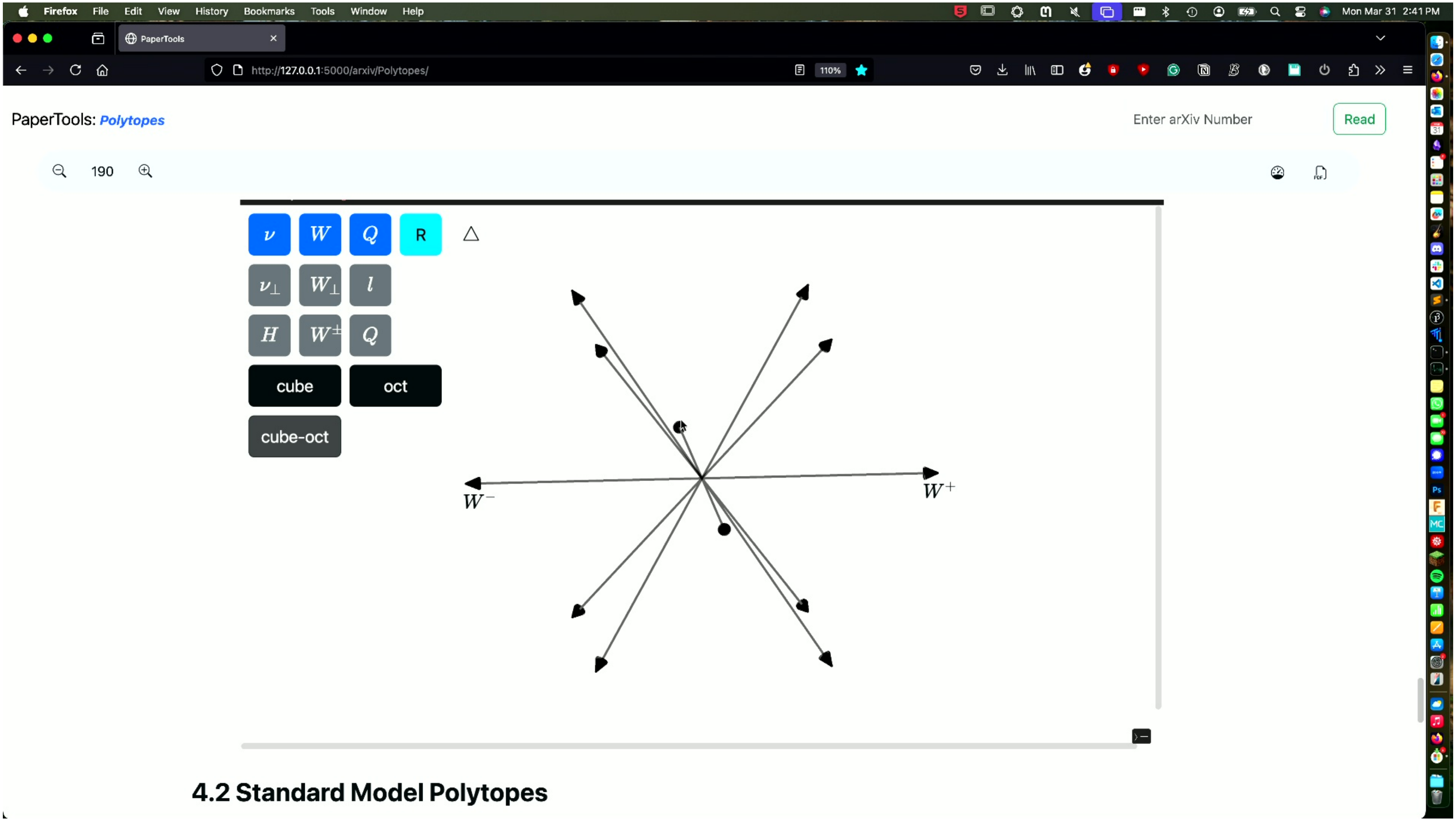
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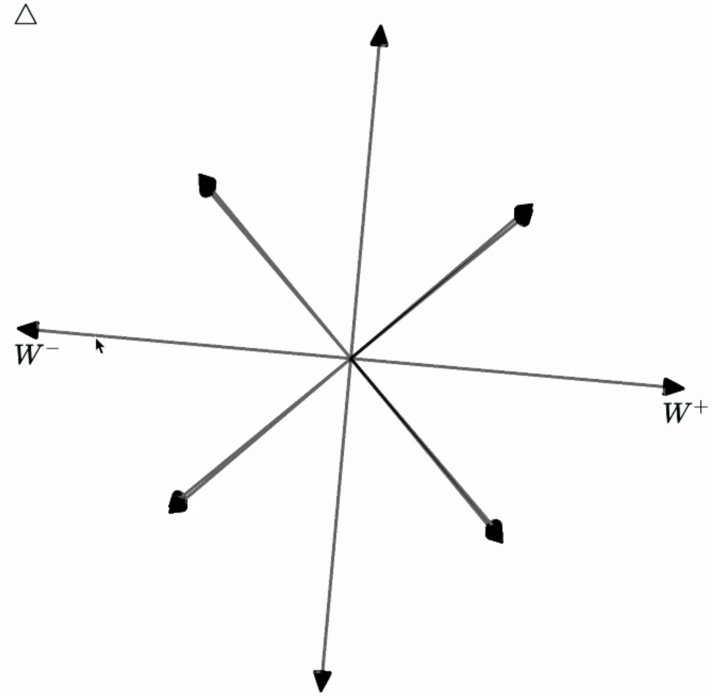

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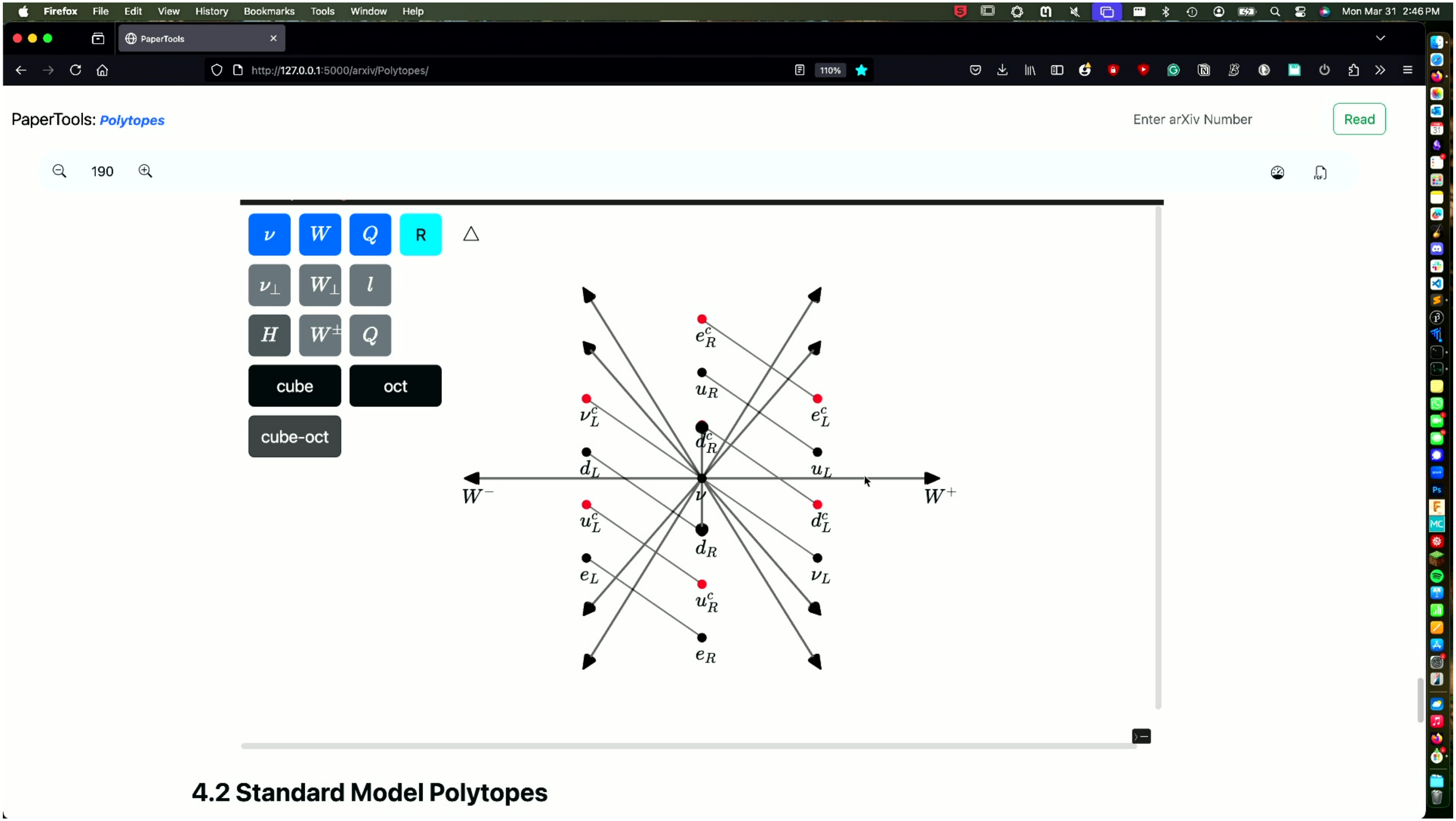
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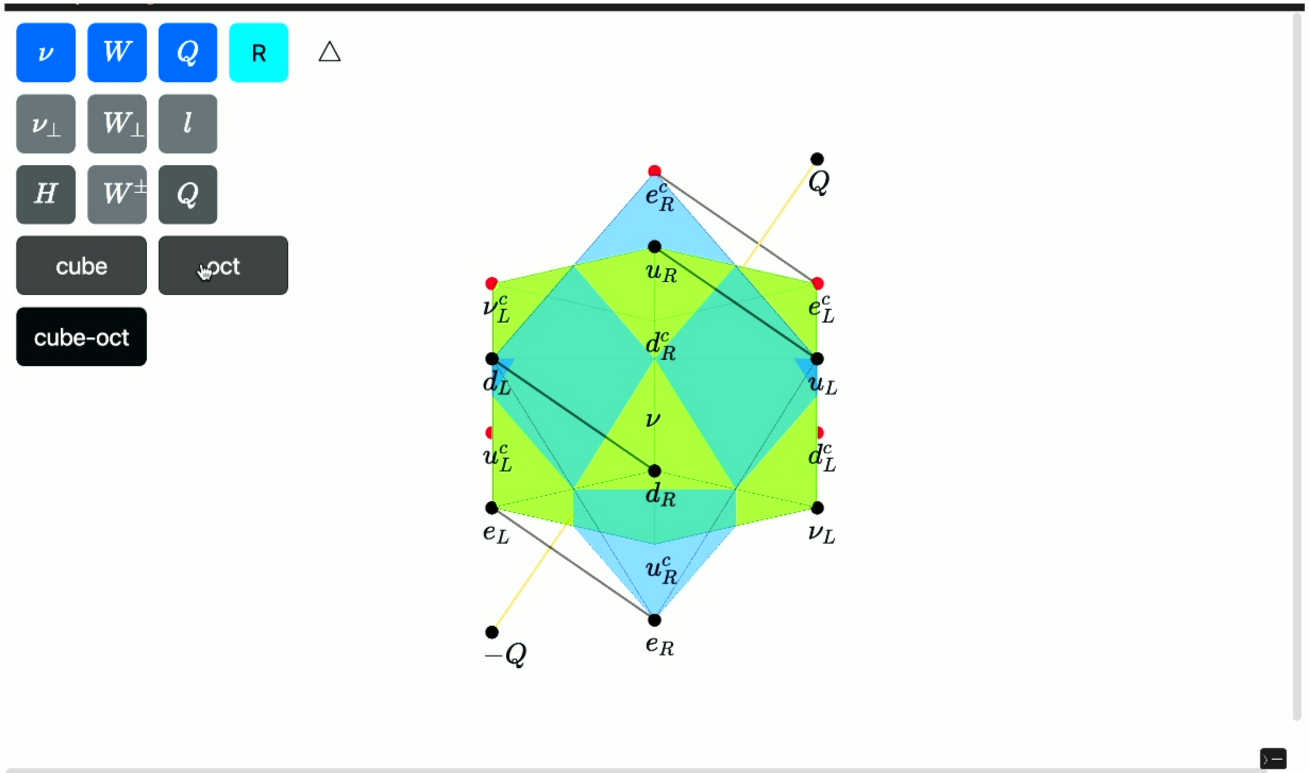

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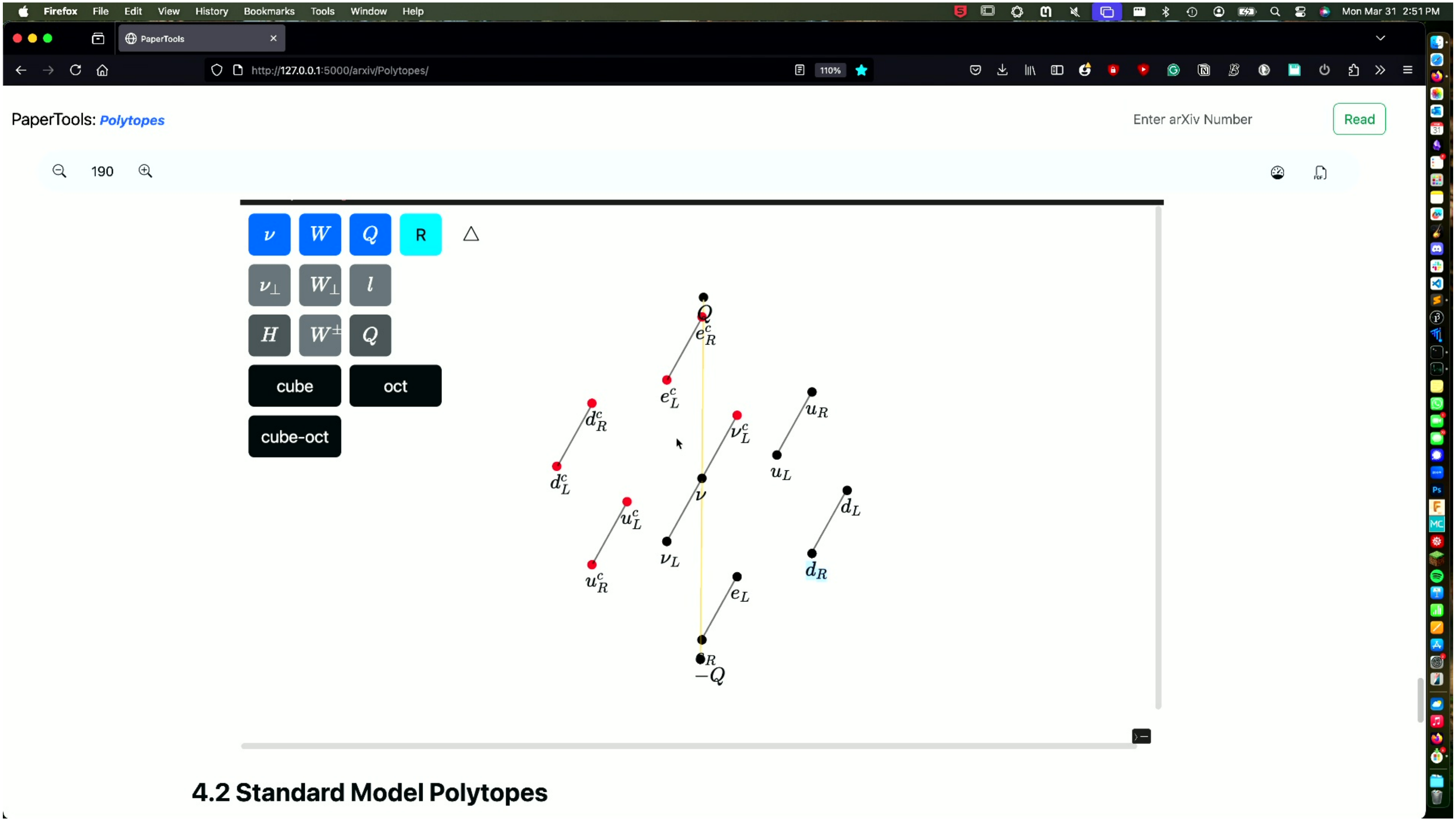
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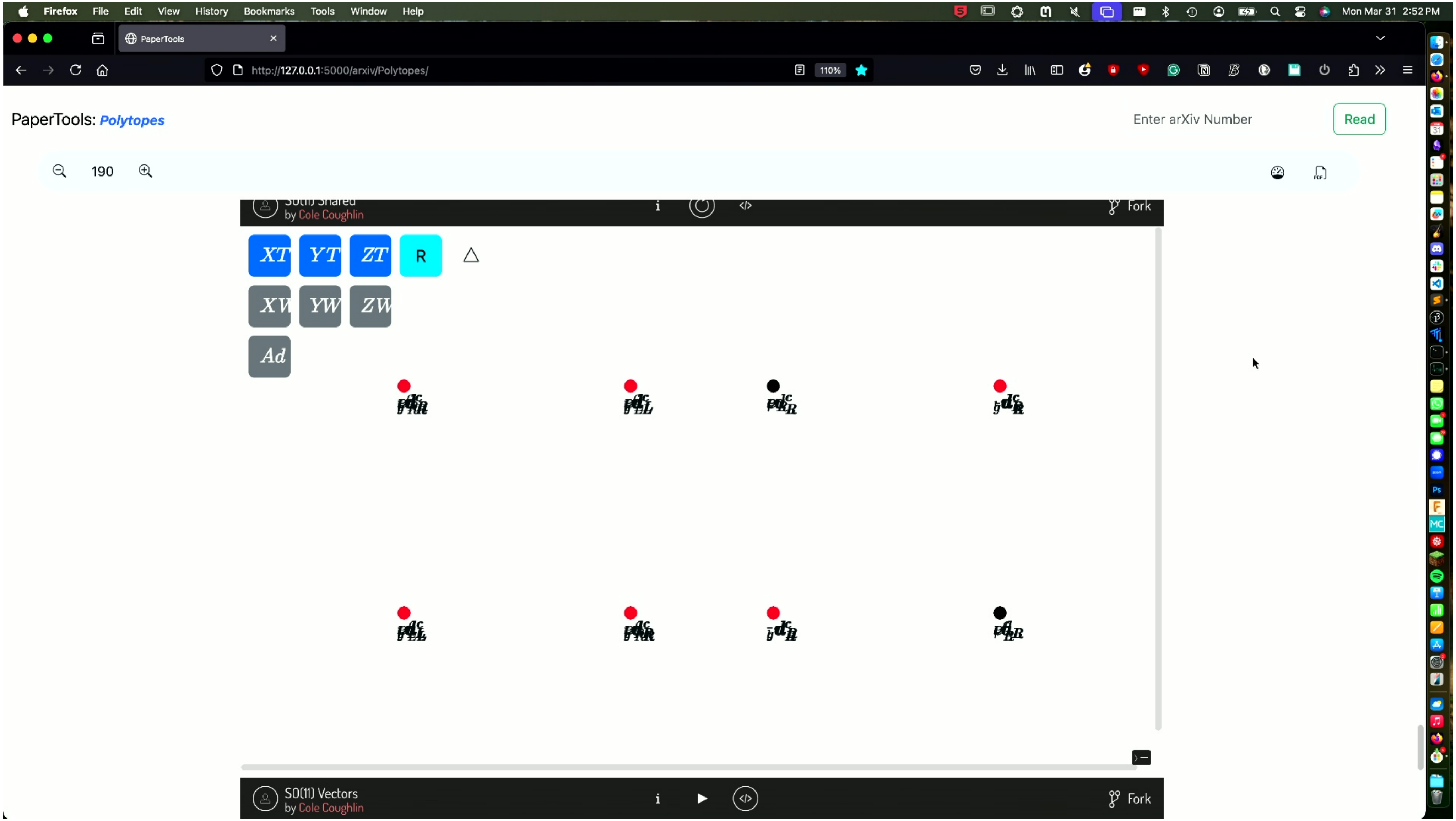
4.2 Standard Model Polytopes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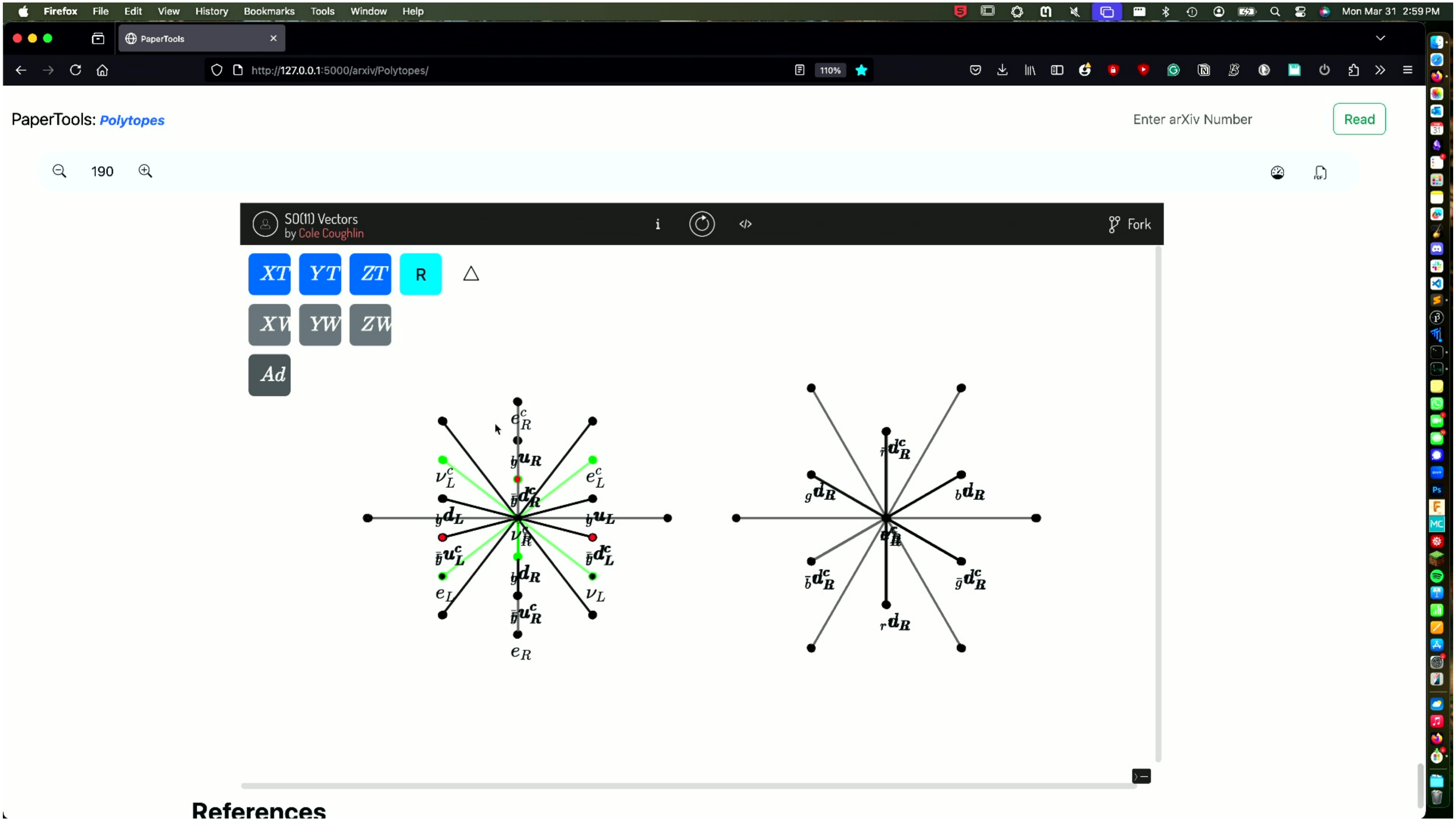
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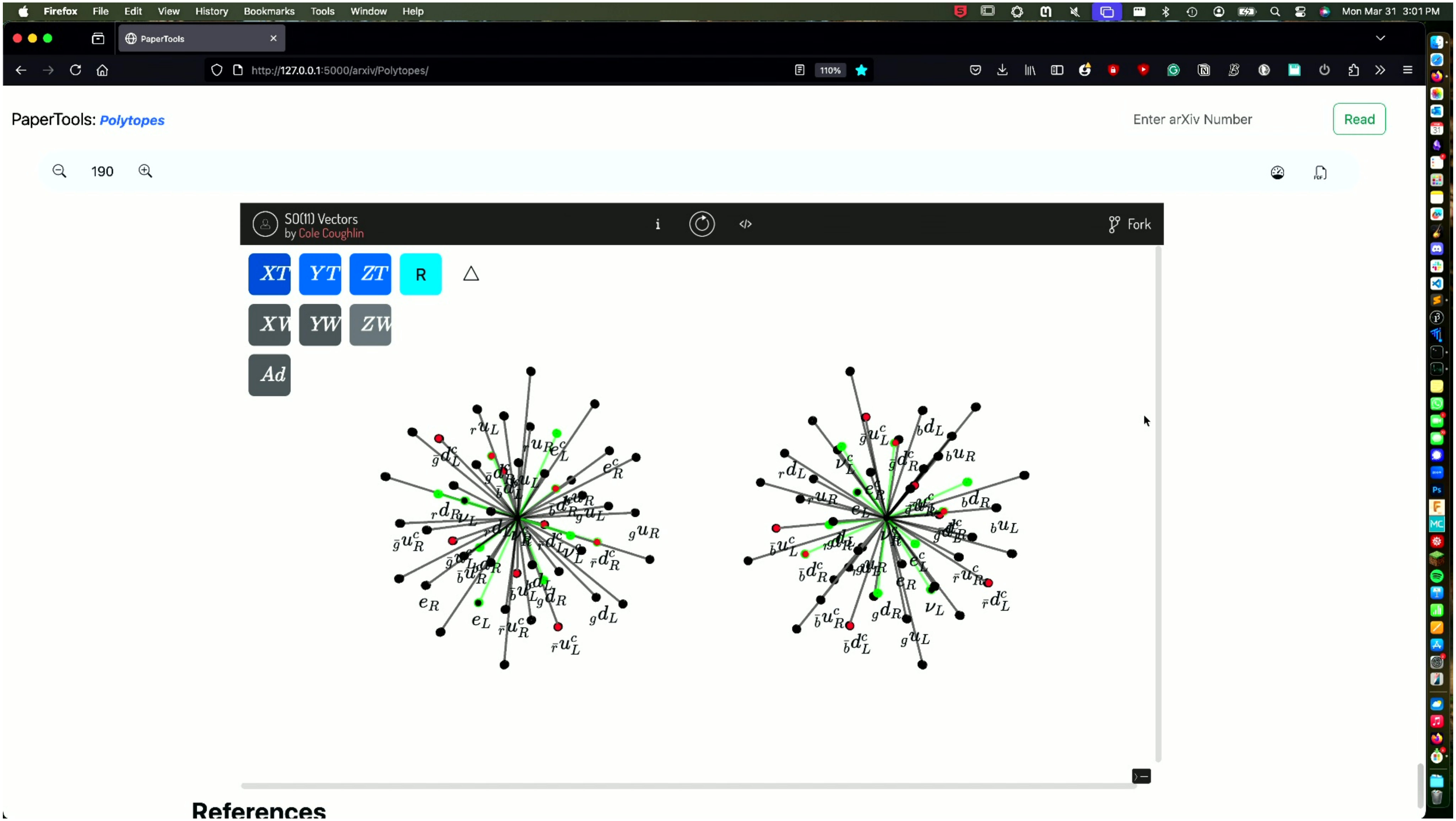

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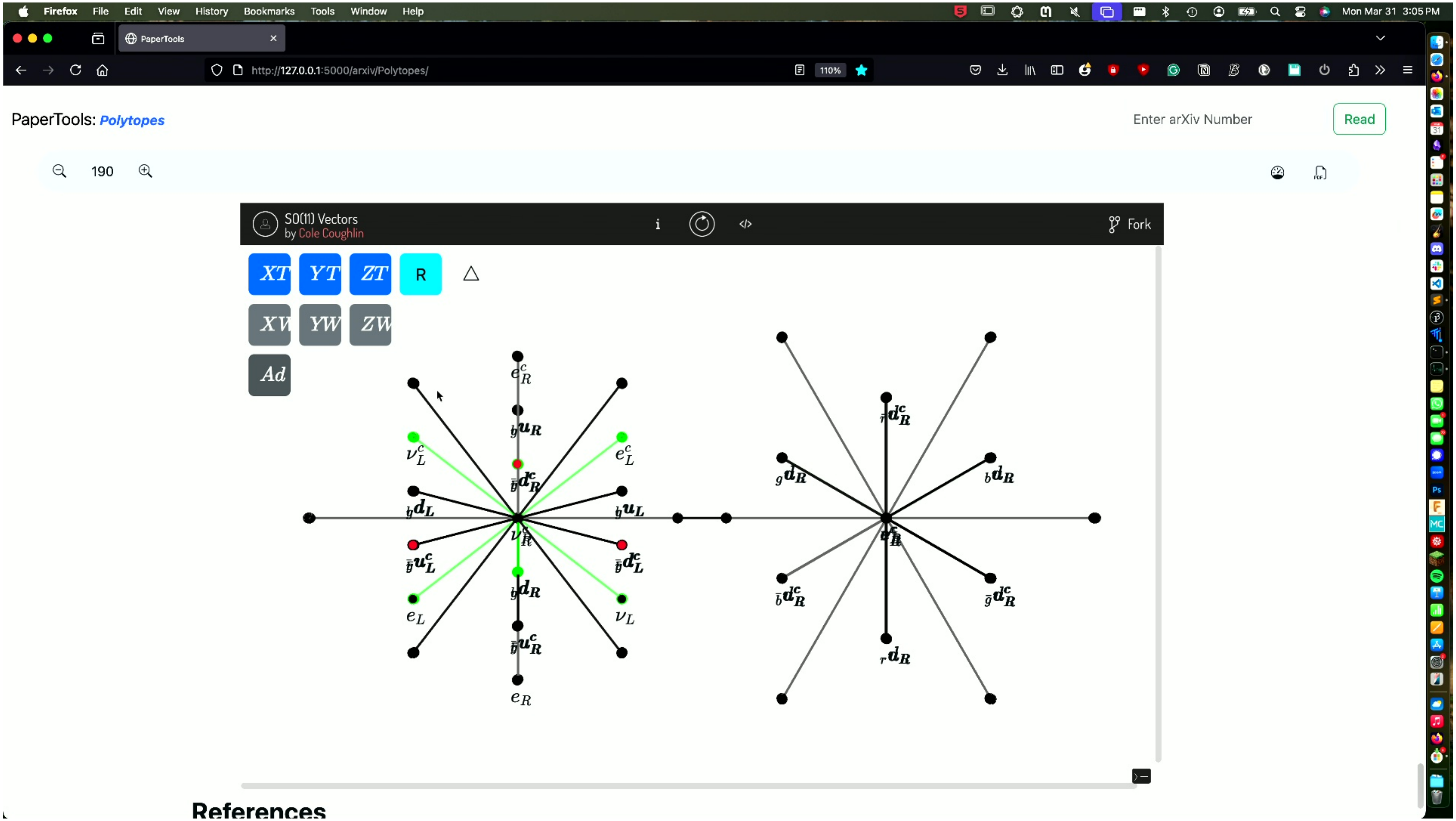
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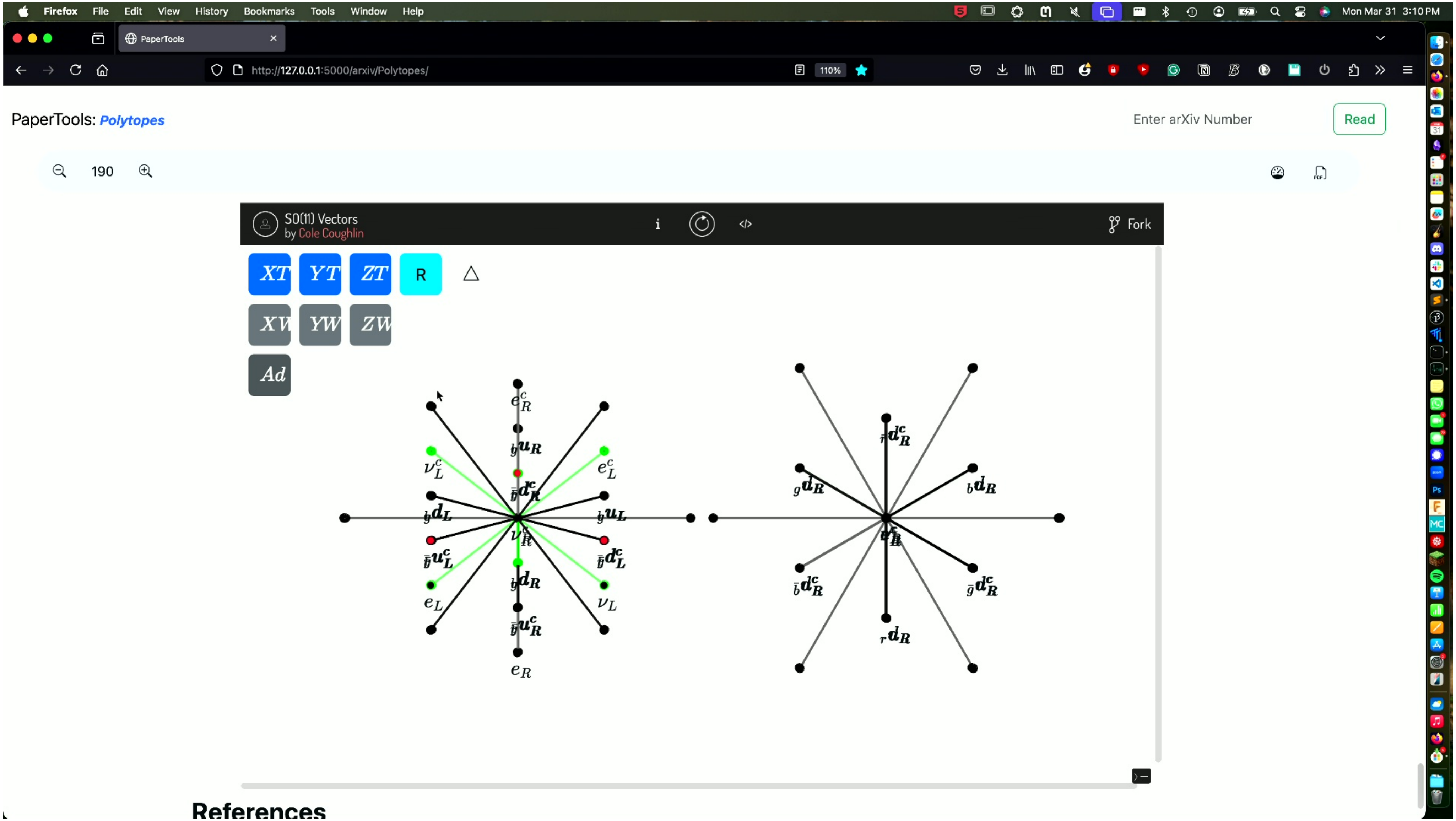
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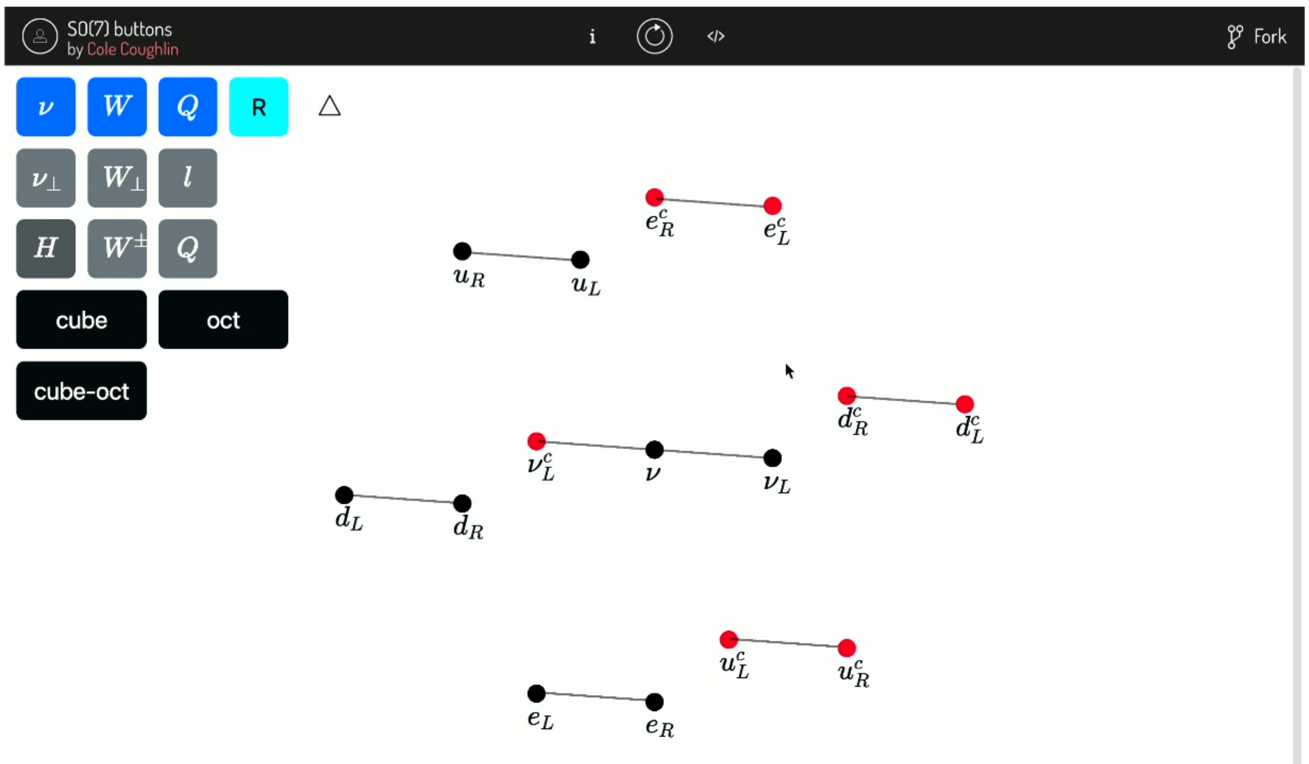

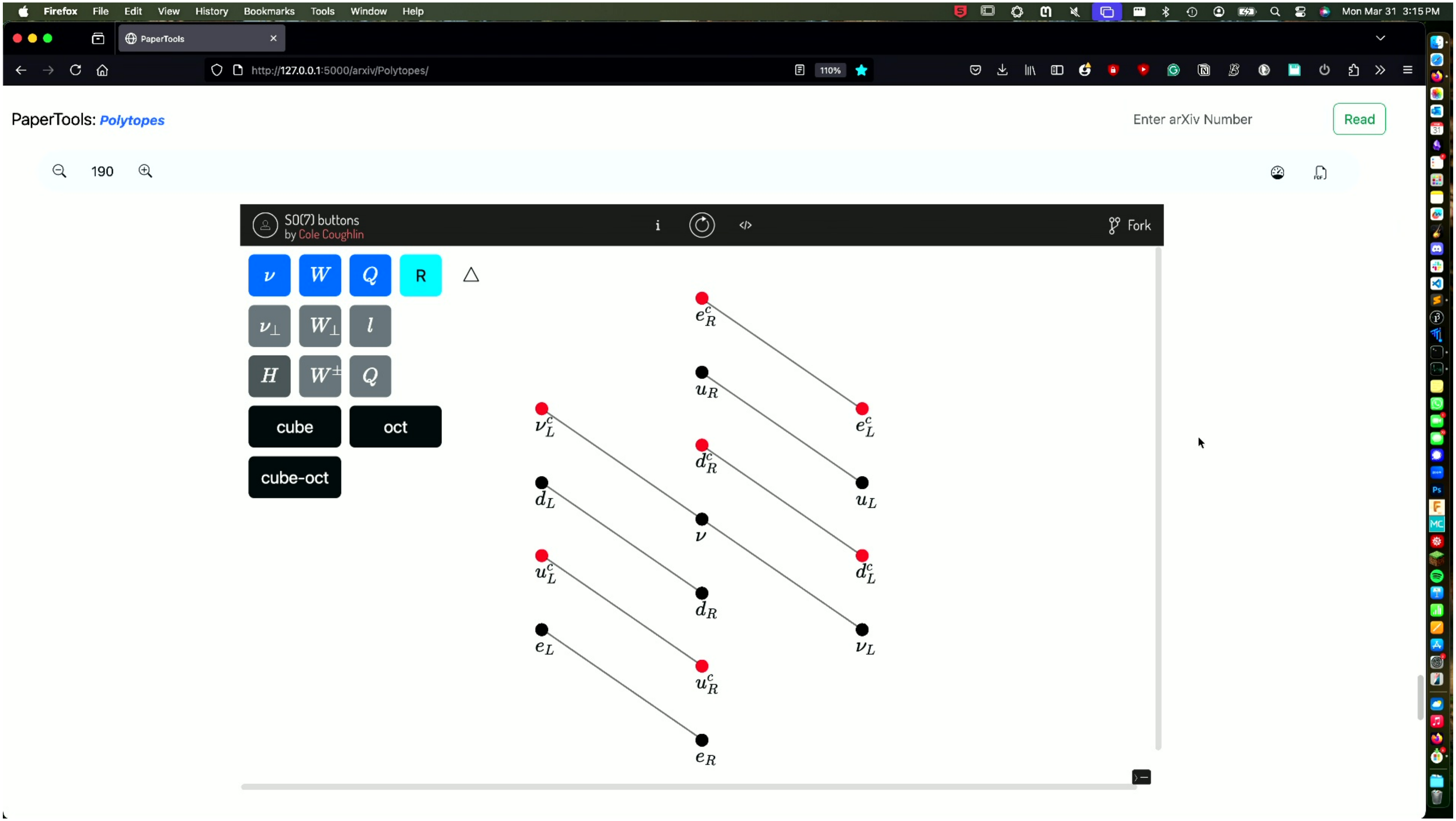
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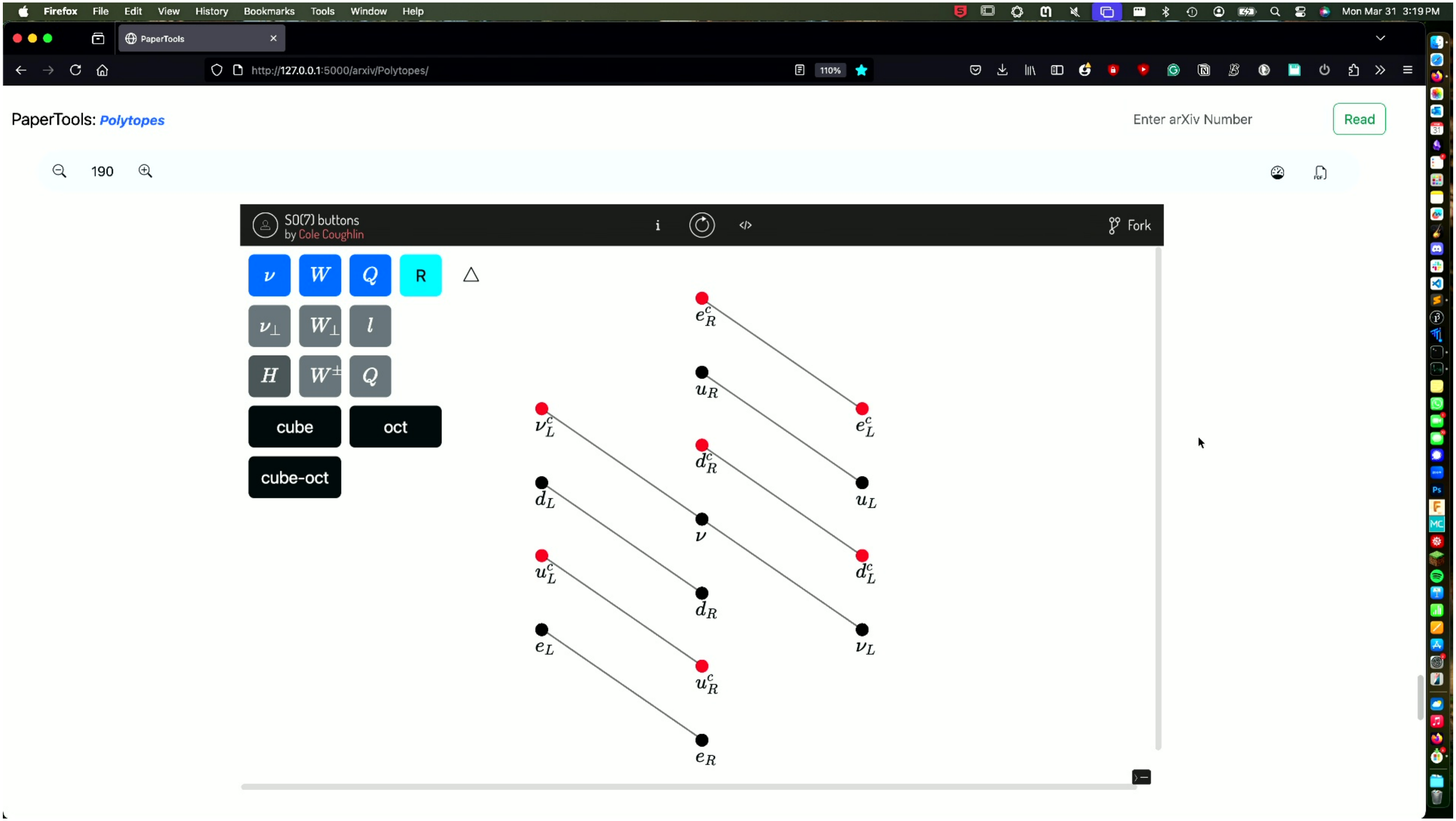

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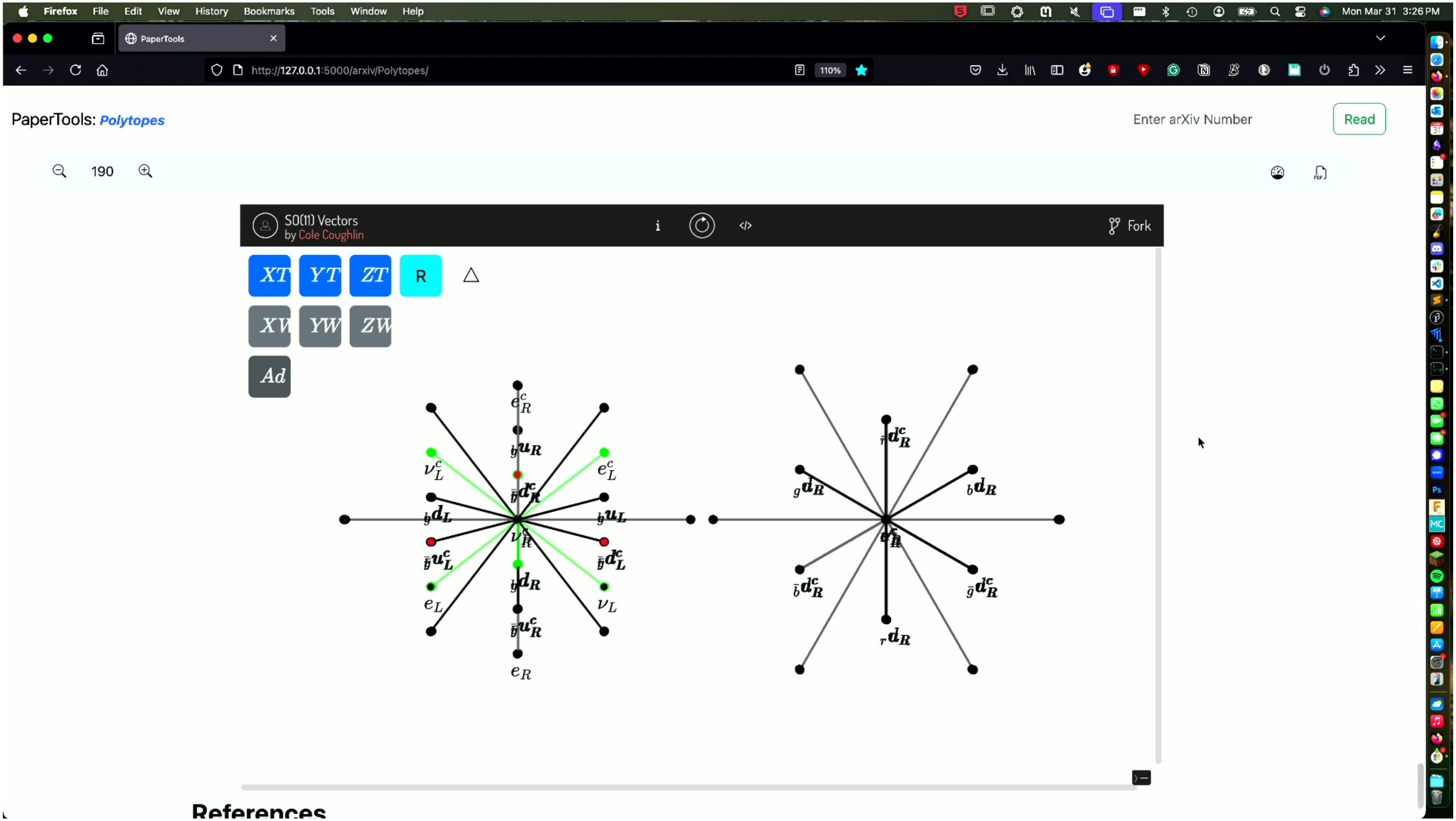
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