Title: Partially deterministic polytopes: a unifying outlook on various forms of nonclassicality

Speakers: Marwan Haddara

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Subject: Quantum Foundations

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Partially deterministic polytopes; Unified outlook on different forms of nonclassicality

Marwan Haddara

PhD student at Griffith University

Principal supervisor: Eric G. Cavalcanti

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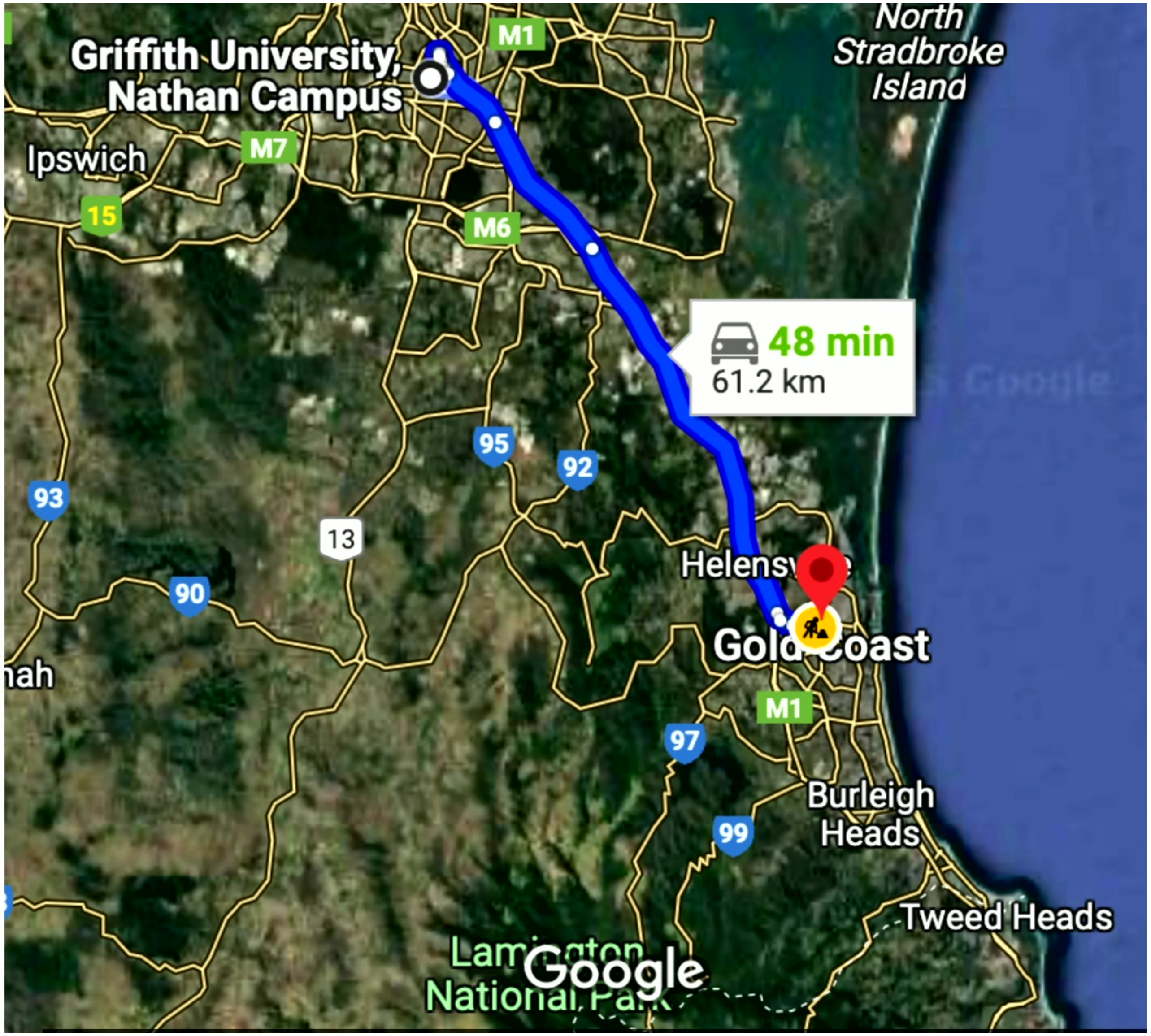
Talk based on: Marwan Haddara and Eric G. Cavalcanti, Local Friendliness Polytopes In Multipartite Scenarios, Phys. Rev. A **111**, 012206, (2025), Marwan Haddara, Howard M. Wiseman and Eric G. Cavalcanti, Manuscript in preparation (2024)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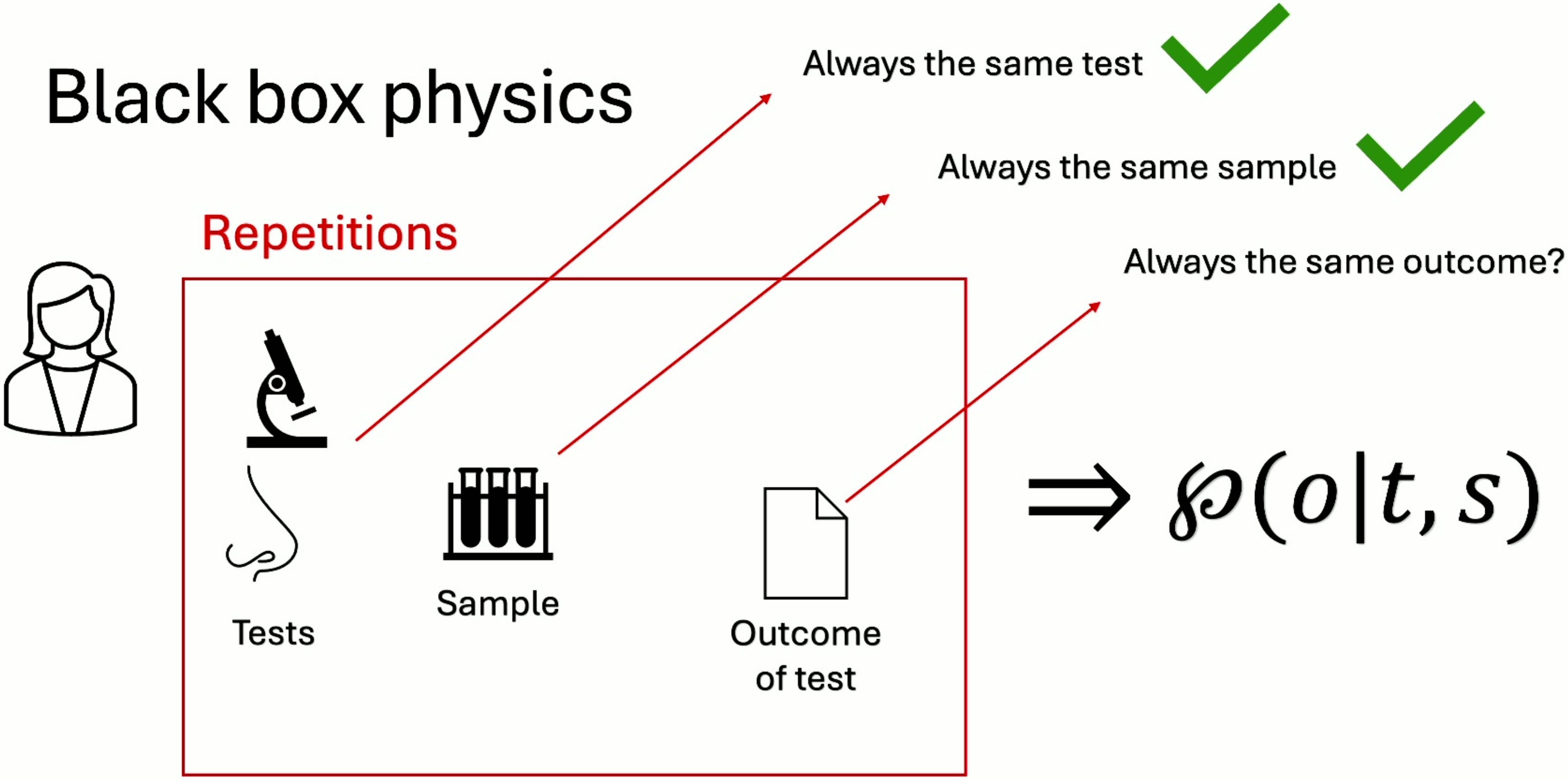
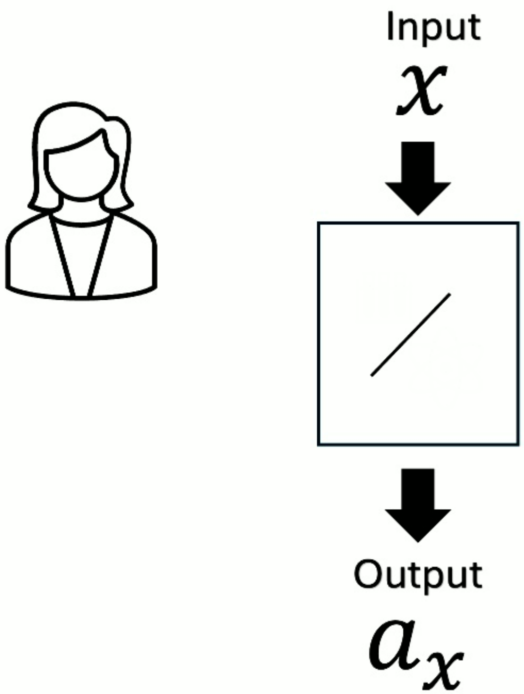
Contents

- Introduction to black box physics
- Partially deterministic polytopes
- Example applications

Black box physics



Black box physics



The behaviour

$$\{\wp(a|x)\} = \wp$$

Predictable iff

$$\wp(a|x) \in \{0,1\}\ \forall x$$

Can $\wp$ be always given a deterministic model?

I.e $\wp(a|x) = \sum_{\lambda} P(\lambda) D(a|x,\lambda)$

for some $P(\lambda), \lambda \in \Lambda, \mathrm{D}(\mathrm{a}|\mathrm{x},\lambda) \in \{\,0,1\,\}$

Does there always exist a quantum model?

I.e $\wp(a|x) = tr[M_{a|x}\rho]$

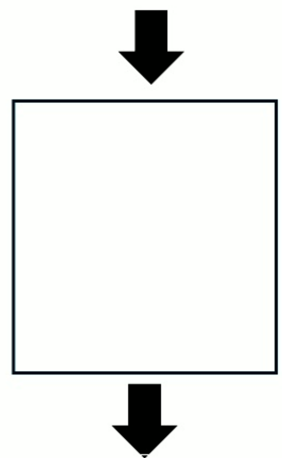
for some POVM's $M_{a|x}$ and state ρ?

Black box physics: correlation experiments

$\{\wp(a,b|x,y)\}$

y

(x,y)

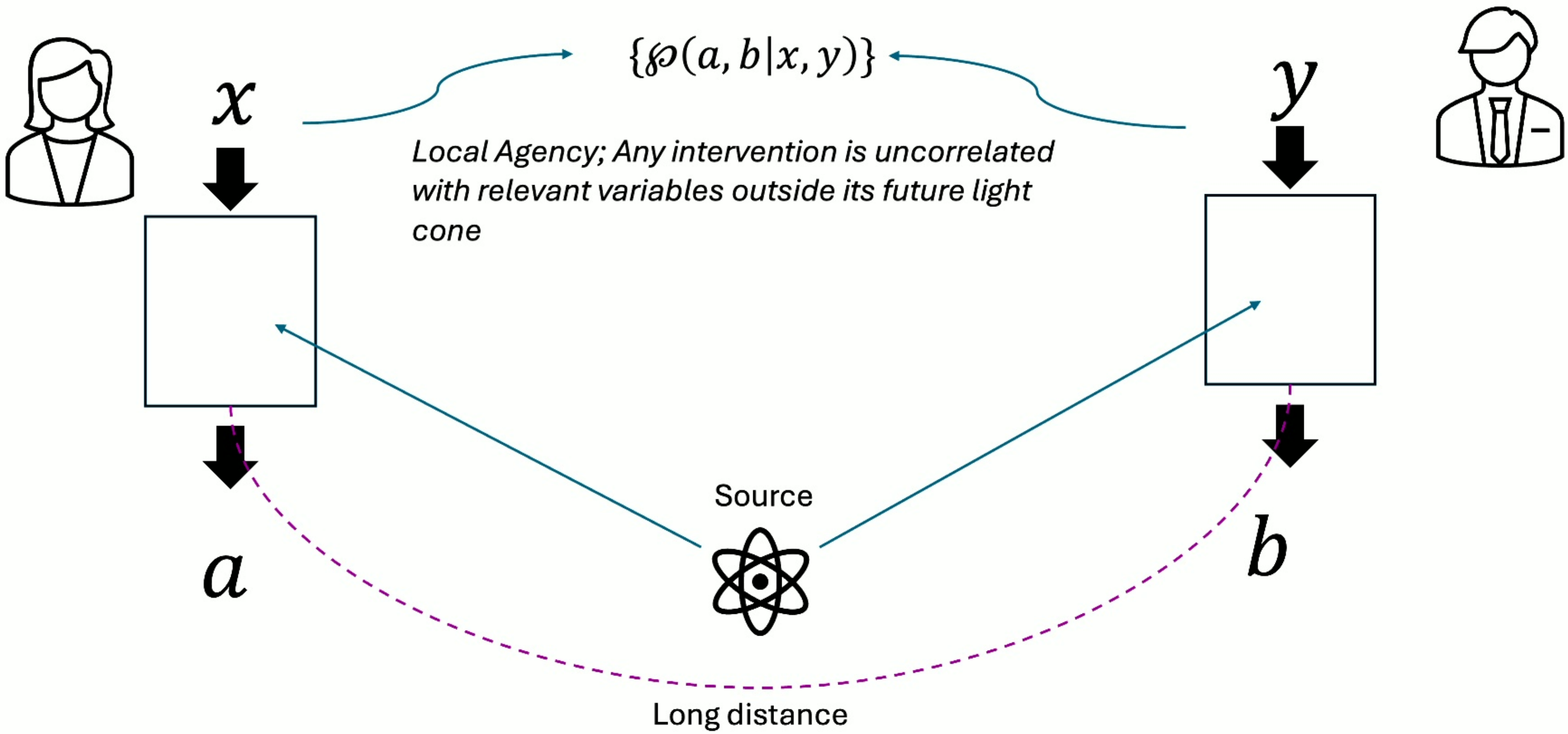


a

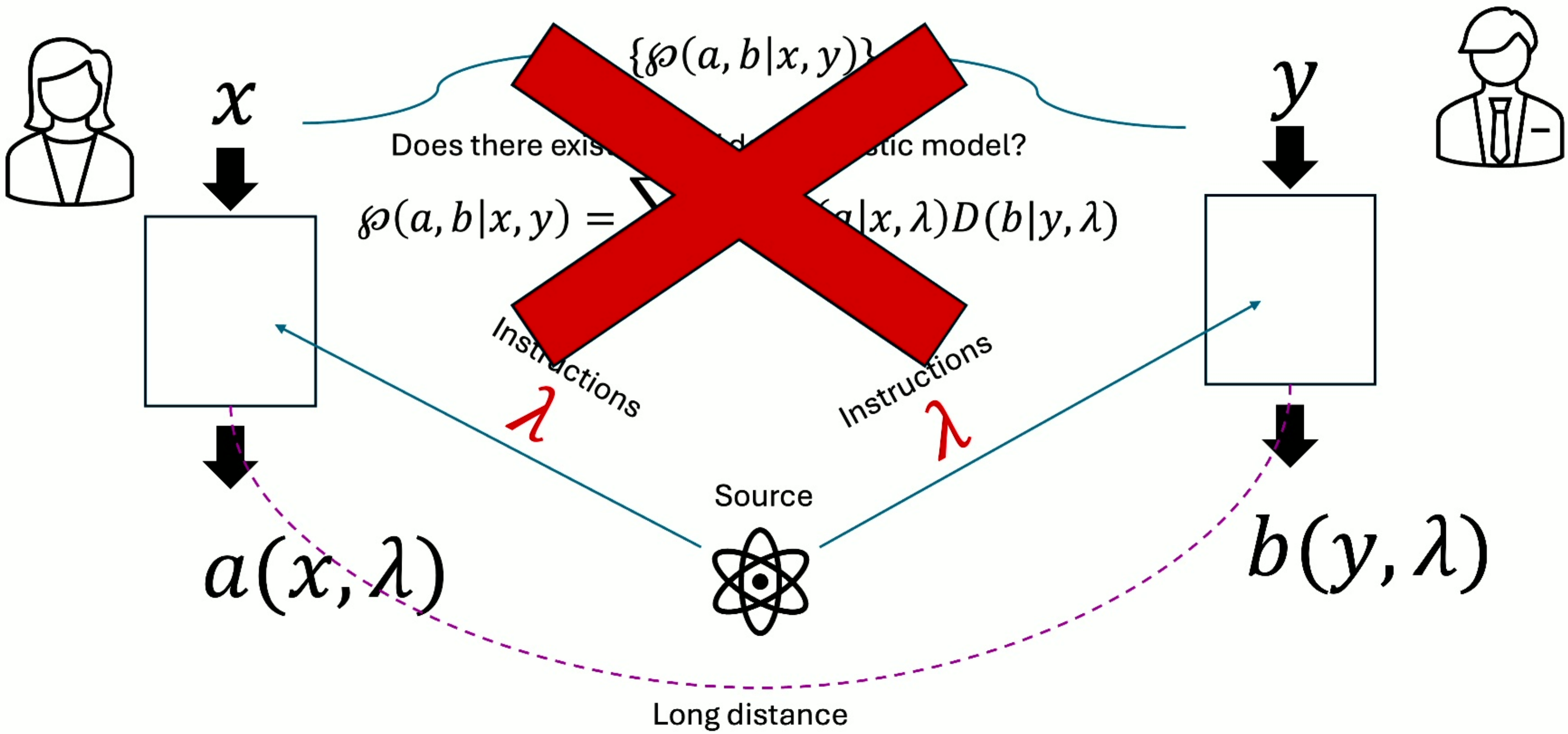
b

(a,b)

Black box physics: correlation experiments



Black box physics: correlation experiments

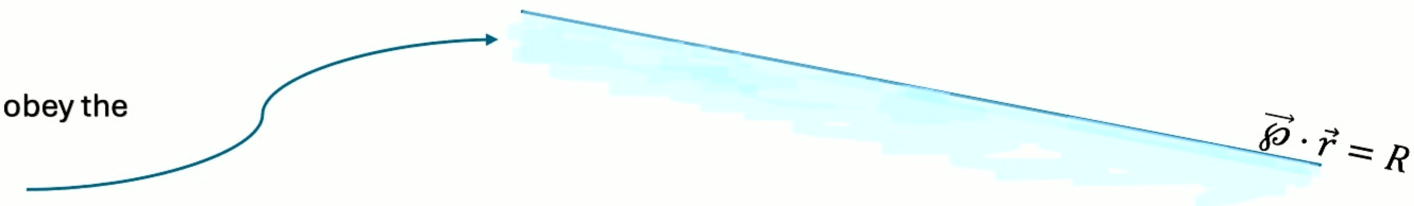


Bell’s theorem

$\vec{\wp}, \vec{r} \in \mathbb{R}^D, R \in \mathbb{R}$. Dimension D specified by parameters of the experiment. $\vec{r}$ is a fixed vector specifying coefficients of inequality.

$$\text{Local Agency} + \text{Determinism} \Rightarrow \vec{\wp} \cdot \vec{r} \leq R$$

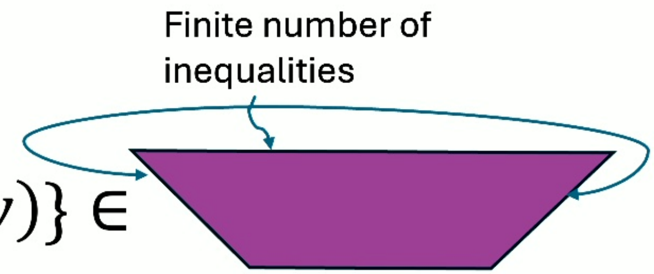
Region of behaviour $\wp$ which obey the inequality



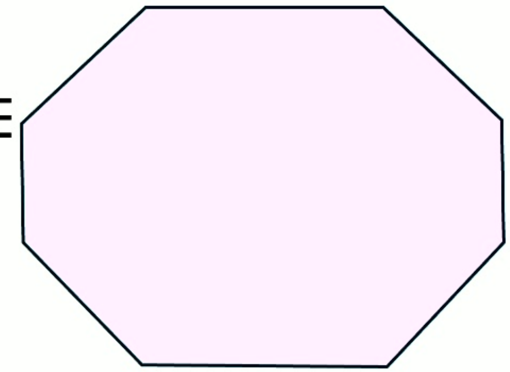
J. S. Bell, Physics Physique Fizika **1**, 195 (1964), Review; Nicolas Brunner, Daniel Cavalcanti, Stefano Pironio, Valerio Scarani, and Stephanie Wehner Rev. Mod. Phys. 86, 419 (2014)

Bell's theorem and beyond

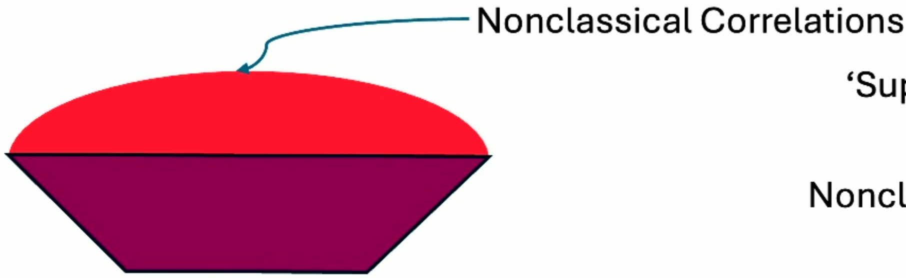
Local Agency + Determinism $\Rightarrow \{\wp(a,b|x,y)\} \in$

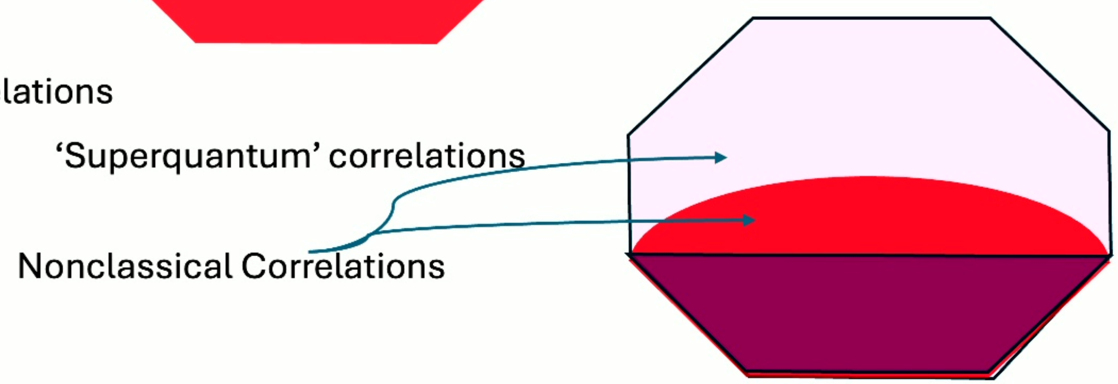


Local Agency + ~~Determinism~~ $\Rightarrow \{\wp(a,b|x,y)\} \in$



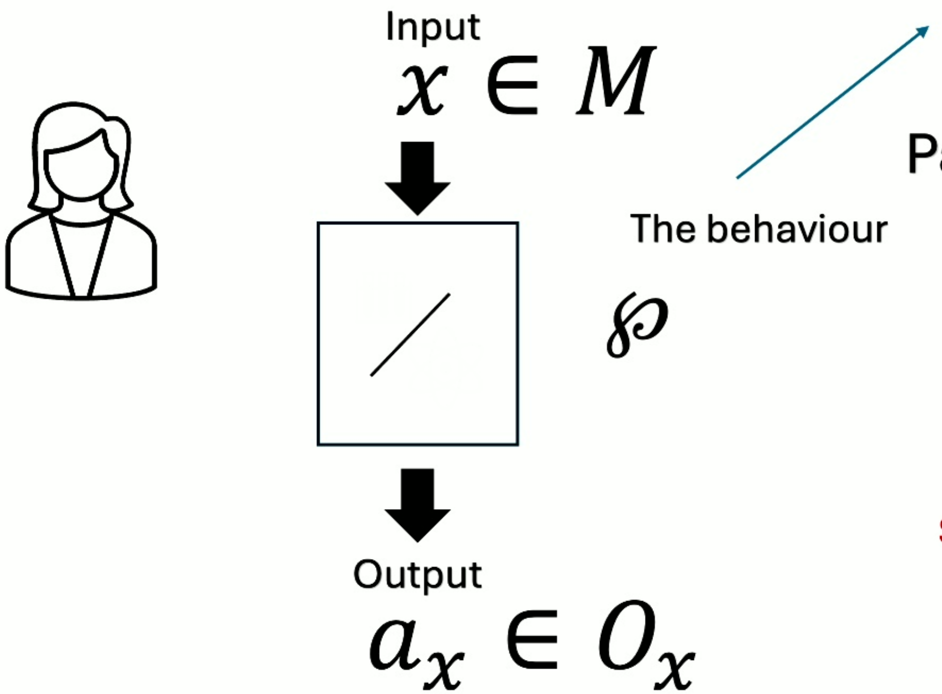
Quantum $\Rightarrow \{\wp(a,b|x,y)\} \in$





J. S. Bell, Physics Physique Fizika **1**, 195 (1964), Review; Nicolas Brunner, Daniel Cavalcanti, Stefano Pironio, Valerio Scarani, and Stephanie Wehner Rev. Mod. Phys. 86, 419 (2014)

New definitions!



Partially Predictable (wrt $M' \subset M$) iff

$$\wp(a|x) \in \{0,1\}\ \forall x \in M'$$

$$PP \subset PD$$

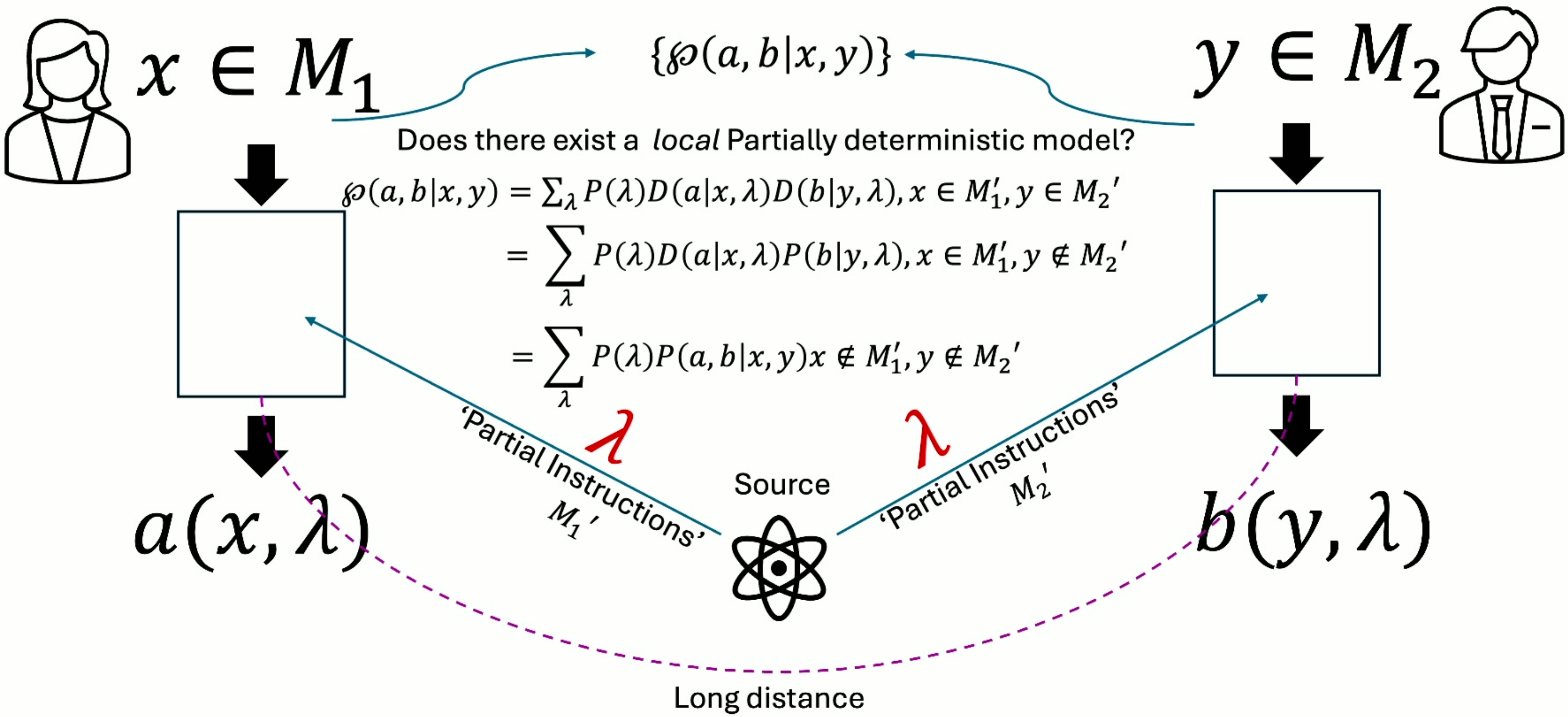
Partially Deterministic (wrt $M' \subset M$) iff

$$\wp(a|x) = \sum_{\lambda} P(\lambda) D(a|x,\lambda)$$

for some $P(\lambda), \lambda \in \Lambda, \mathrm{D}(\mathrm{a}|\mathrm{x},\lambda) \in \{\, 0,1 \,\}\ \forall x \in M'$

Single box case not very interesting because..

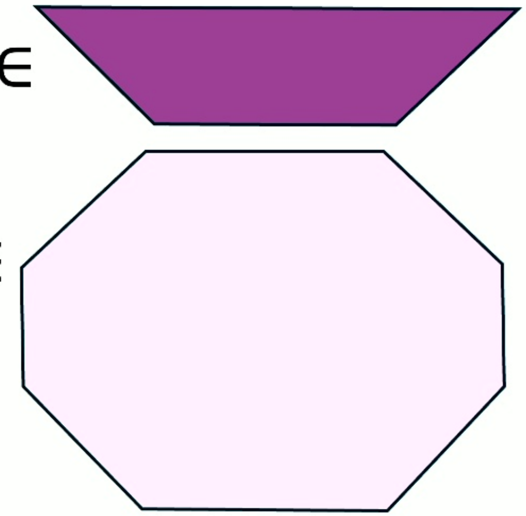
Partial determinism in correlation experiments



Partial determinism: example

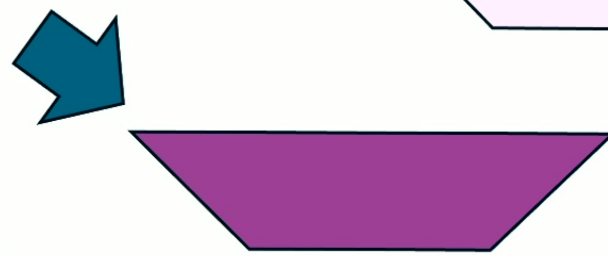
$a, b, x, y \in \{1,2\}$

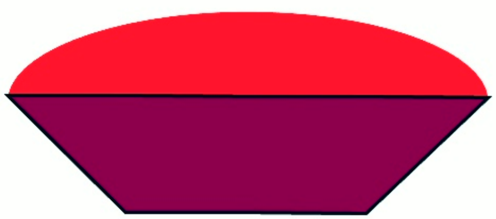
Local Agency + Determinism $\Rightarrow \{\wp(a, b|x, y)\} \in$

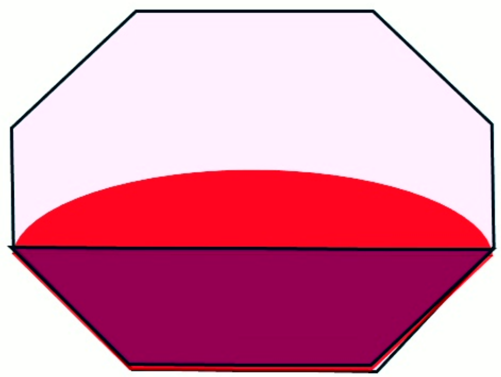


Local Agency + ~~Determinism~~ $\Rightarrow \{\wp(a, b|x, y)\} \in$

Local Agency + Partial Determinism (wrt. any nontrivial $M_1{}' \neq \emptyset$ or $M_2' \neq \emptyset$)

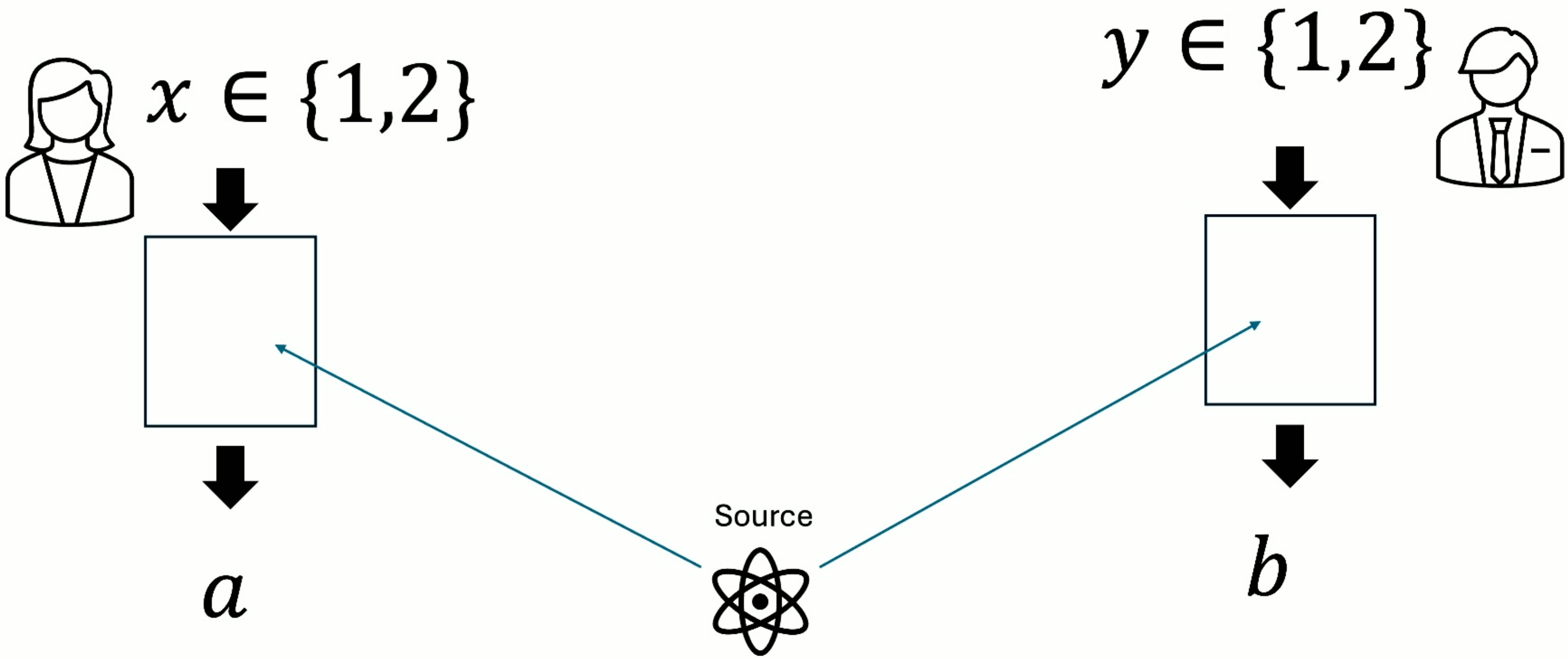




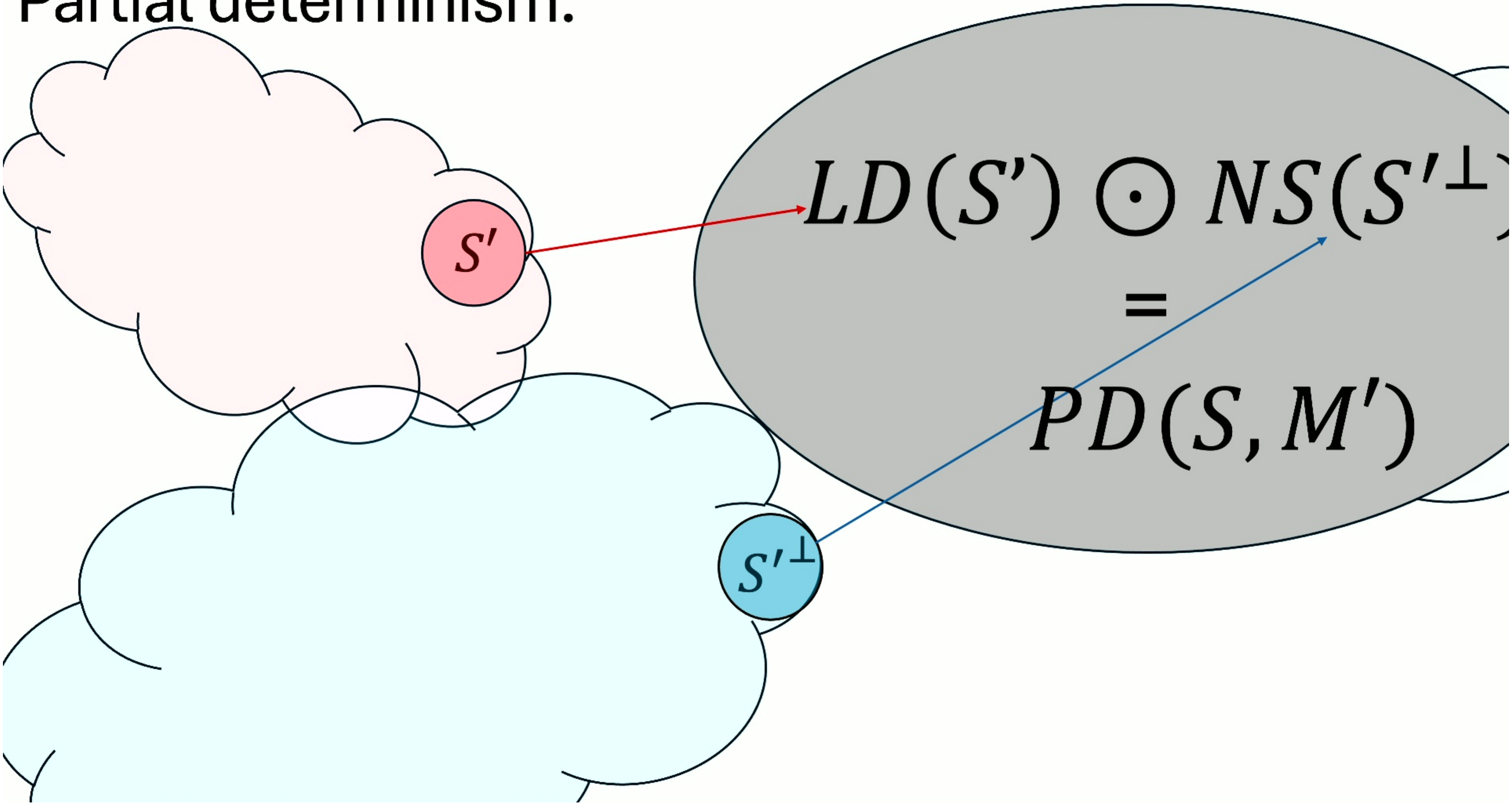


Partial determinism: example

$a, b, x, y \in \{1,2\}$

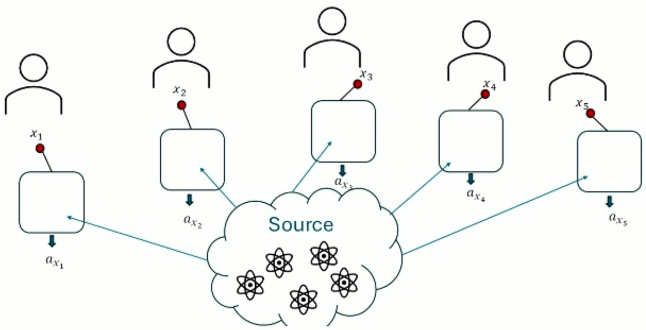


Partial determinism:



Partially deterministic polytopes

$S = I, M, O$



A correlation scenario S is defined by

1. A set I of parties
2. A collection M of all the input sets M_i of each party i
3. A collection O of all the output sets O_{x_i} corresponding to the input x_i of the ith party.

Impose determinism on some subset $M_i' \subset M_i$ of the measurements of the party i. Denote $M' \subset M$ the collection of the M_i'

PDP

$$\wp(\vec{a}\,|\vec{x}\,) = \sum_{l,k} P(l,k)\; D_l\left(\vec{a}_{V_{\vec{x}}} \mid \vec{x}_{V_{\vec{x}}}\right) \times P_k\left(\vec{a}_{(I_A \setminus V_{\vec{x}})} \middle| \vec{x}_{(I_A \setminus V_{\vec{x}})}\right)$$

Here $V_{\vec{x}} = \{v \in I \; s.t \; x_v \in M'_v \; given \; \vec{x}\}$

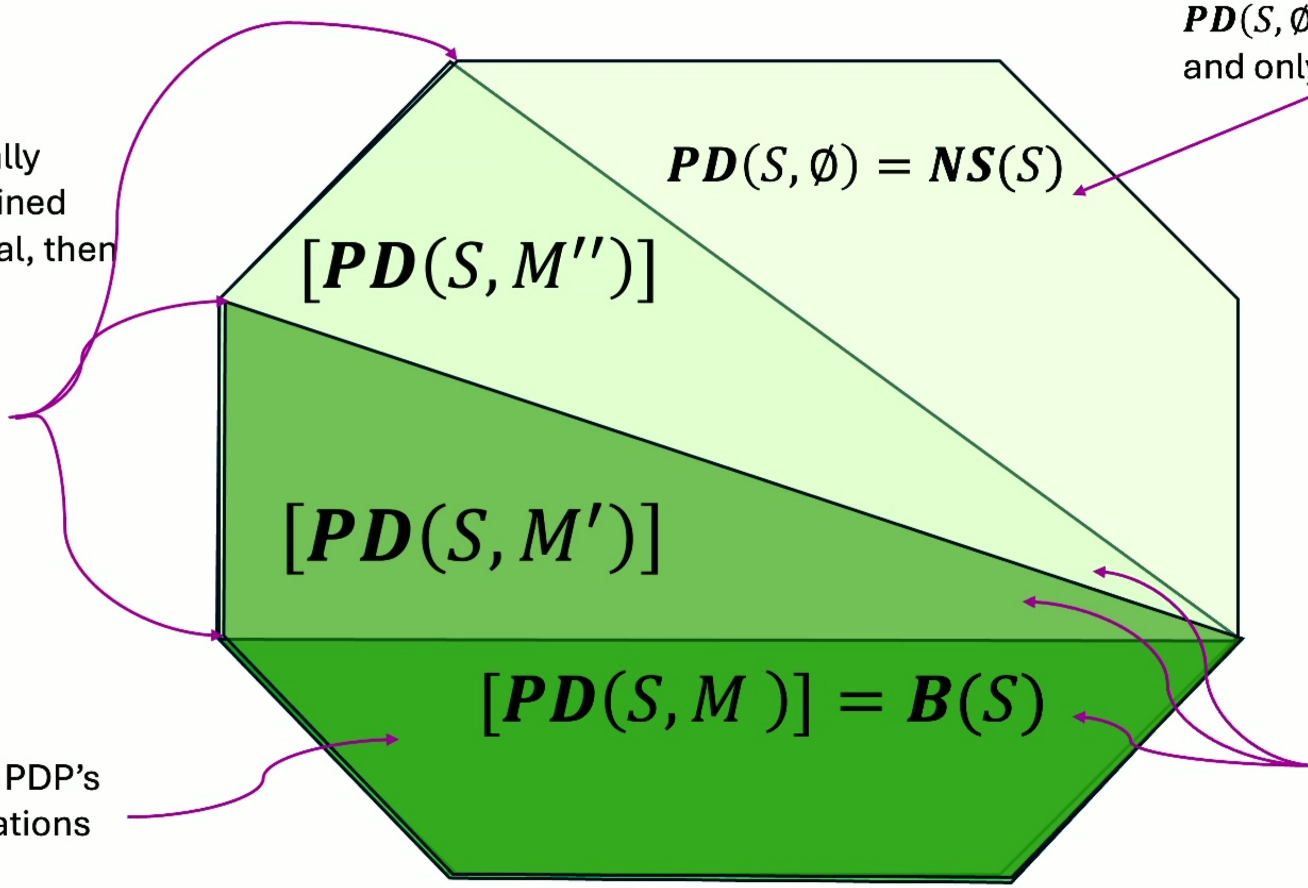
$\boldsymbol{PD}(S, M')$

Marwan Haddara and Eric G. Cavalcanti, Local Friendliness Polytopes In Multipartite Scenarios Phys. Rev. A **111**, 012206, (2025), :**M. Haddara, H.M. Wiseman and E.G. Cavalcanti, another manuscript in preparation (2024); E. Woodhead imperfections and self testing in prepare-and-measure quantum key distribution, Ph.D. thesis, Laboratoire d'Information Quantique Universite libre de Bruxelles (2014)

Some of our results

If $M_i' \subset M_i'' \; \forall i$ and the partially deterministic polytopes defined relative to them are not equal, then they obey a strict inclusion $\mathrm{EXT}(\boldsymbol{PD}(S, M'')) \subsetneq EXT(\boldsymbol{PD}(S, M'))$.

$\boldsymbol{PD}(S, \emptyset) = \boldsymbol{NS}(S)$ if and only if $M' = \emptyset$



Nontrivial cases where PDP's equal set of Bell correlations

Equivalence classes

Marwan Haddara and Eric G. Cavalcanti, Local Friendliness Polytopes In Multipartite Scenarios, arXiv:2407.20346, (2024); **M. Haddara, H.M. Wiseman and E.G. Cavalcanti, another manuscript in preparation (2024); E. Woodhead imperfections and self testing in prepare-and-measure quantum key distribution, Ph.D. thesis, Laboratoire d'Information Quantique Universite libre de Bruxelles (2014)

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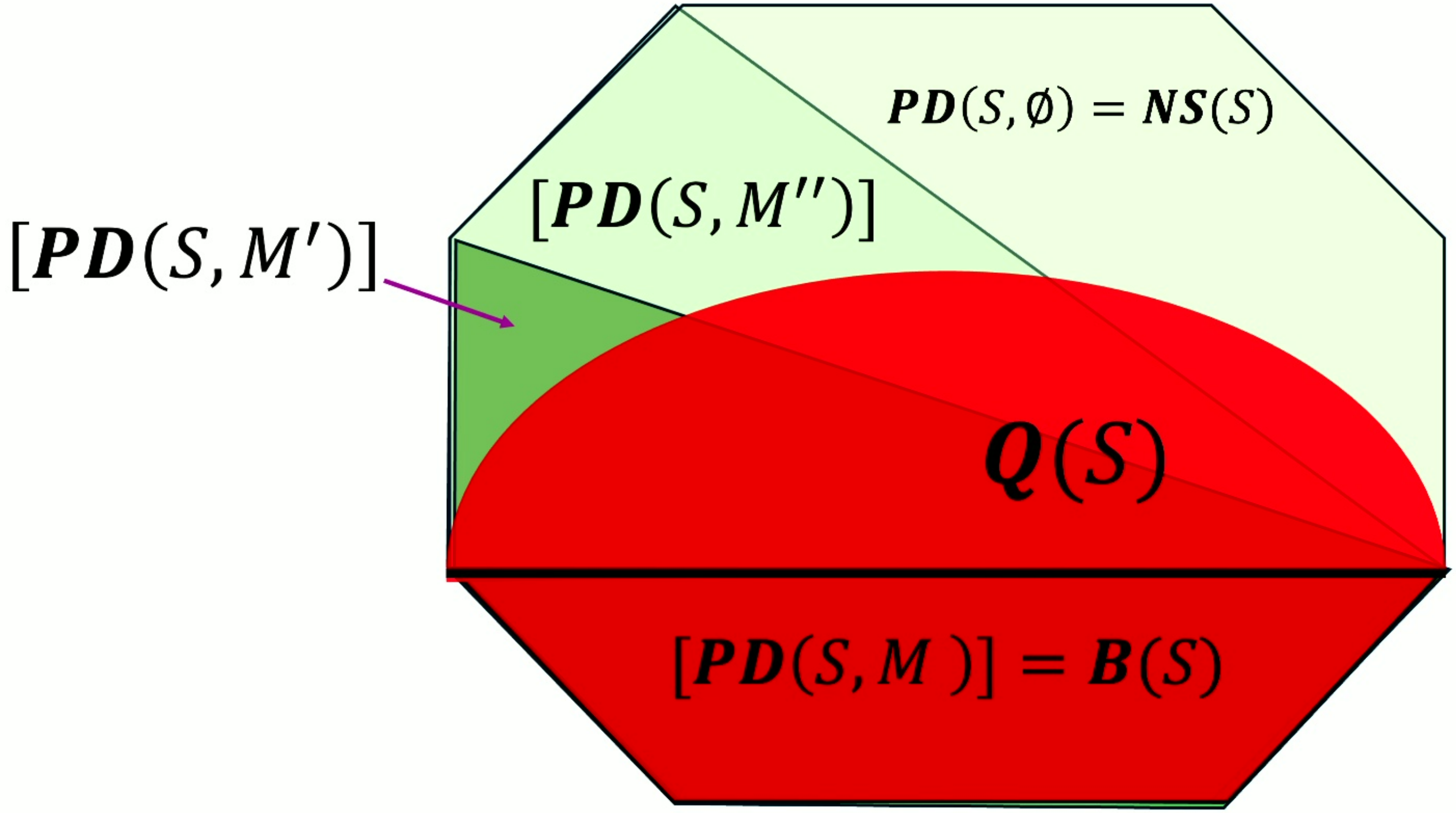

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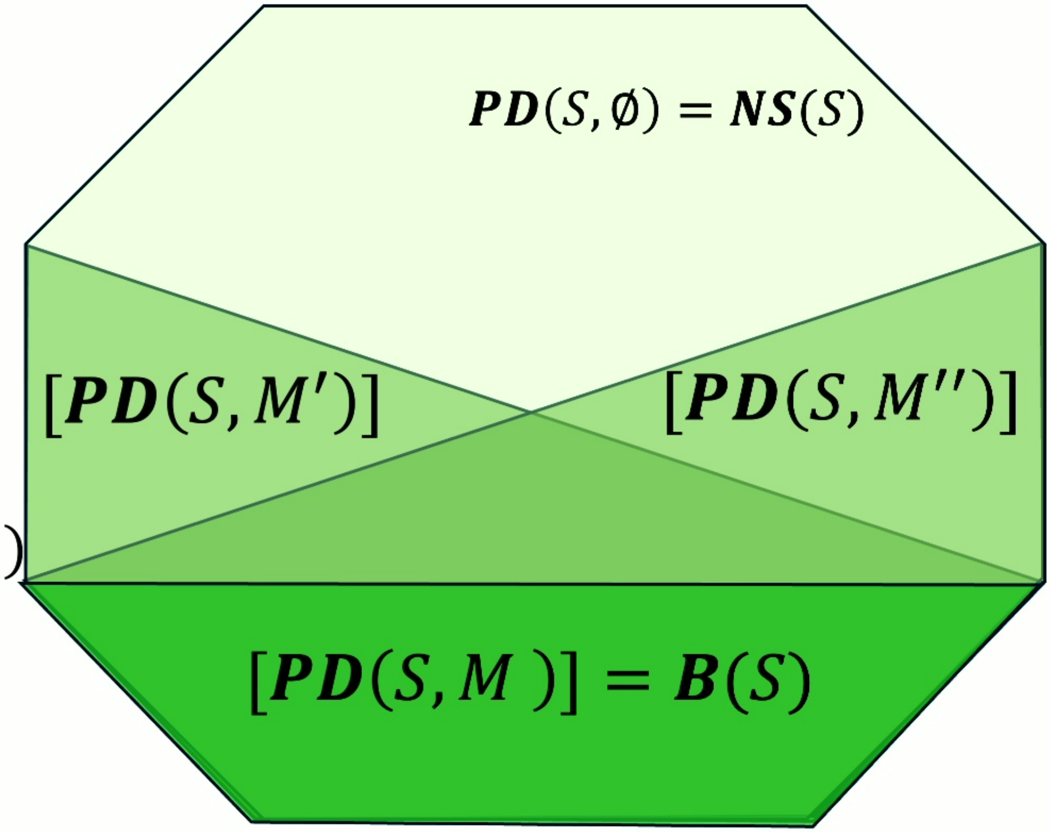
Some of our results

$M_i' \not\subset M_i''$ and $M_i'' \not\subset M_i'$
For some $i \in I$
and $\boldsymbol{PD}(S, M') \neq \boldsymbol{PD}(S, M'')$

$$\Rightarrow$$

Then $\boldsymbol{PD}(S, M') \not\subset \boldsymbol{PD}(S, M'')$
And $\boldsymbol{PD}(S, M'') \not\subset \boldsymbol{PD}(S, M')$



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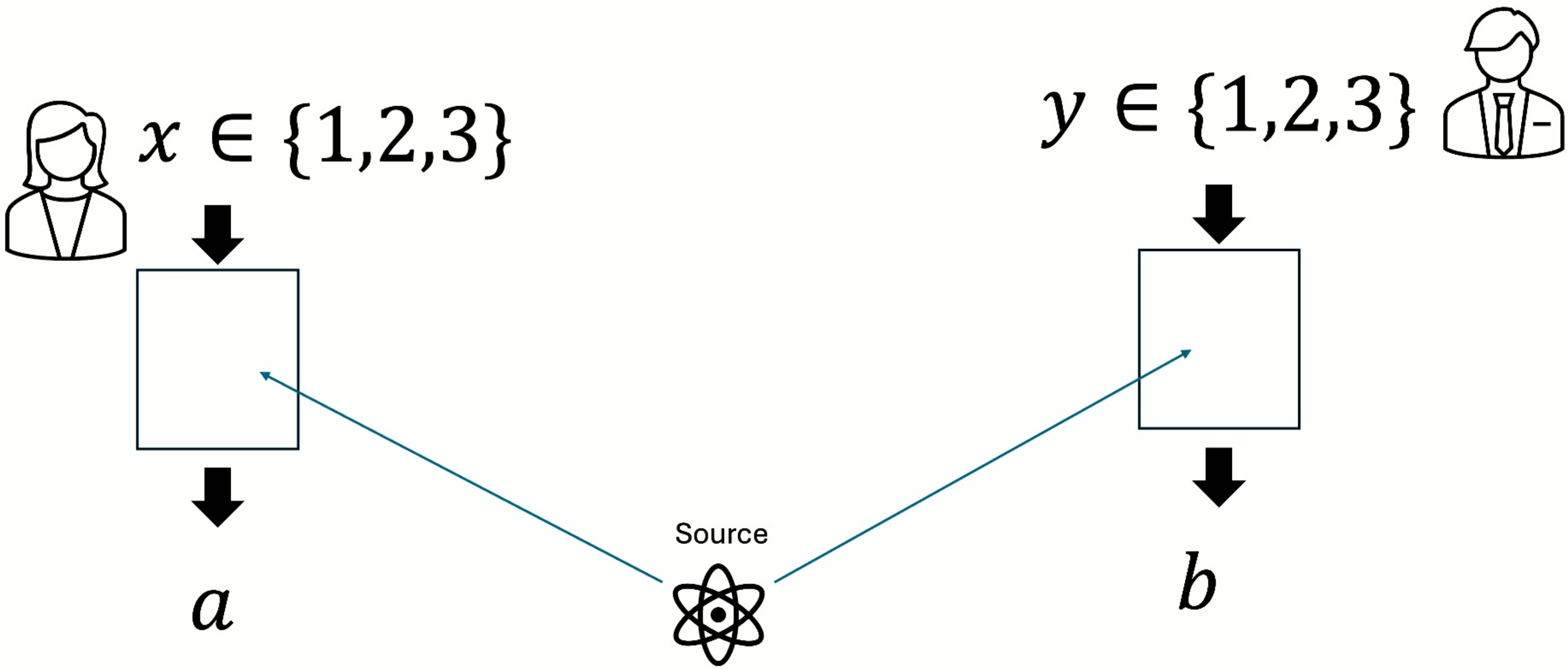
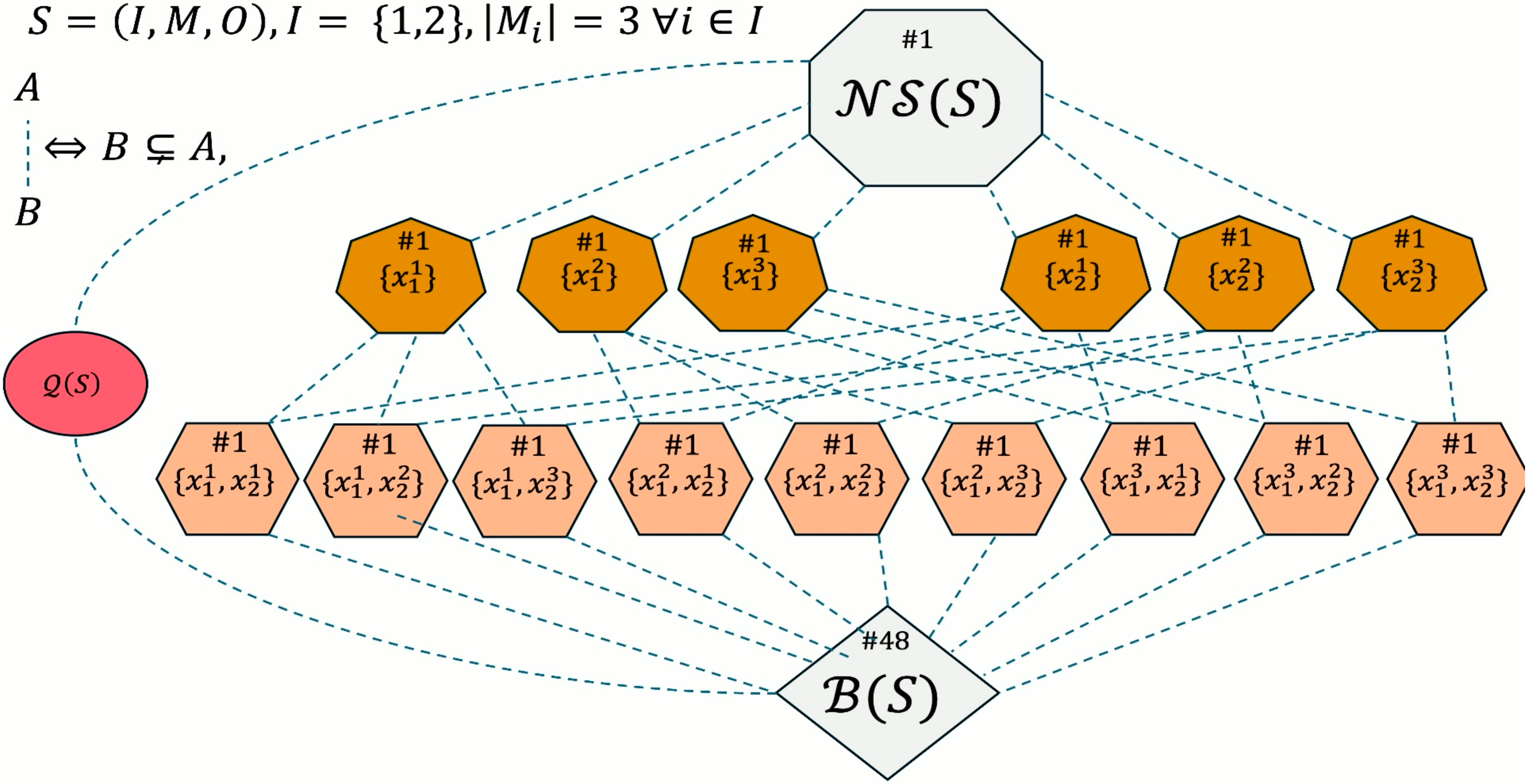
Example 1

$x, y \in \{1,2,3\}$



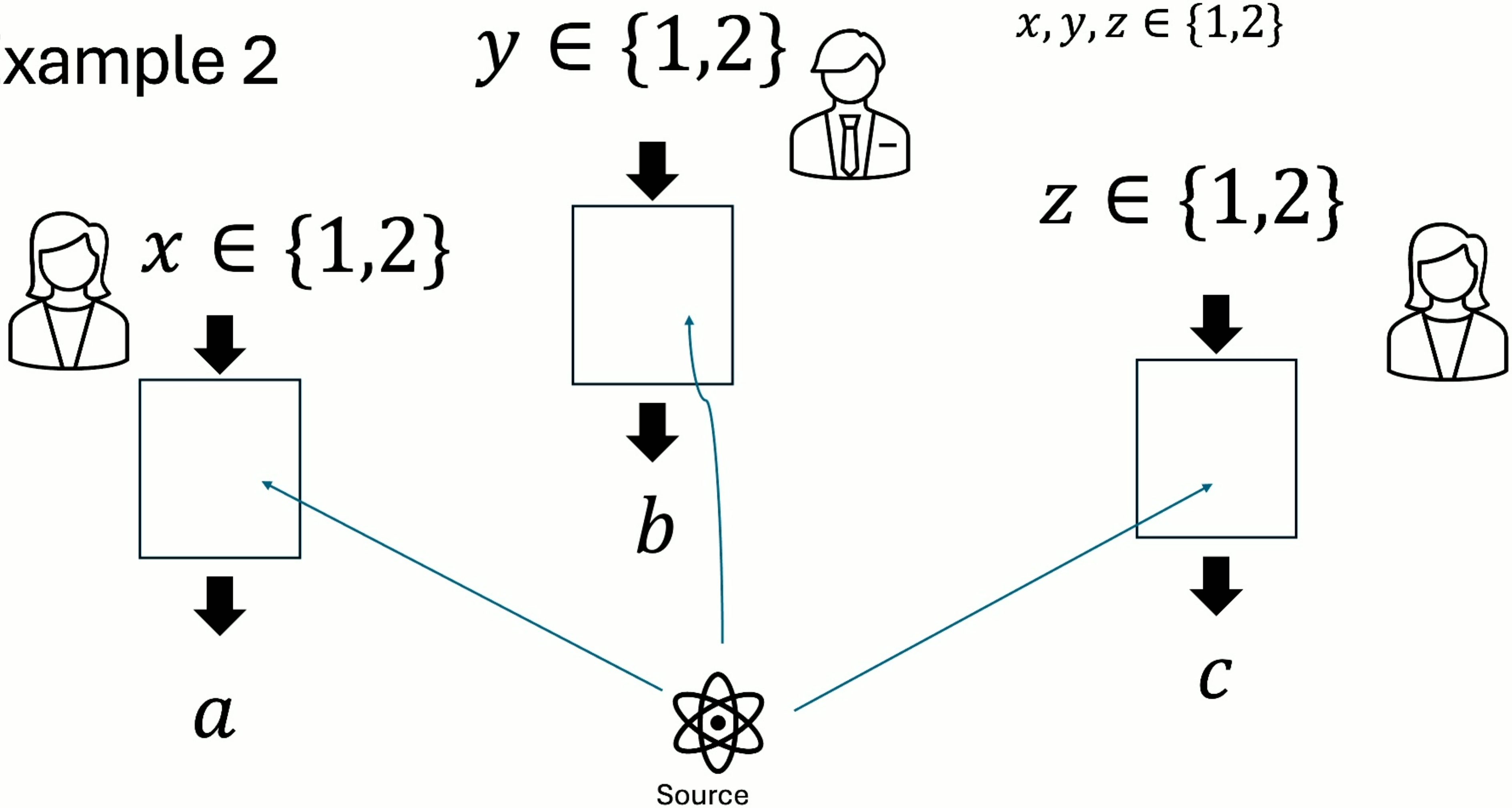
$S = (I, M, O), I = \{1,2\}, |M_i| = 3 \; \forall i \in I$

$A \;\vdots\; B \Leftrightarrow B \subsetneq A,$

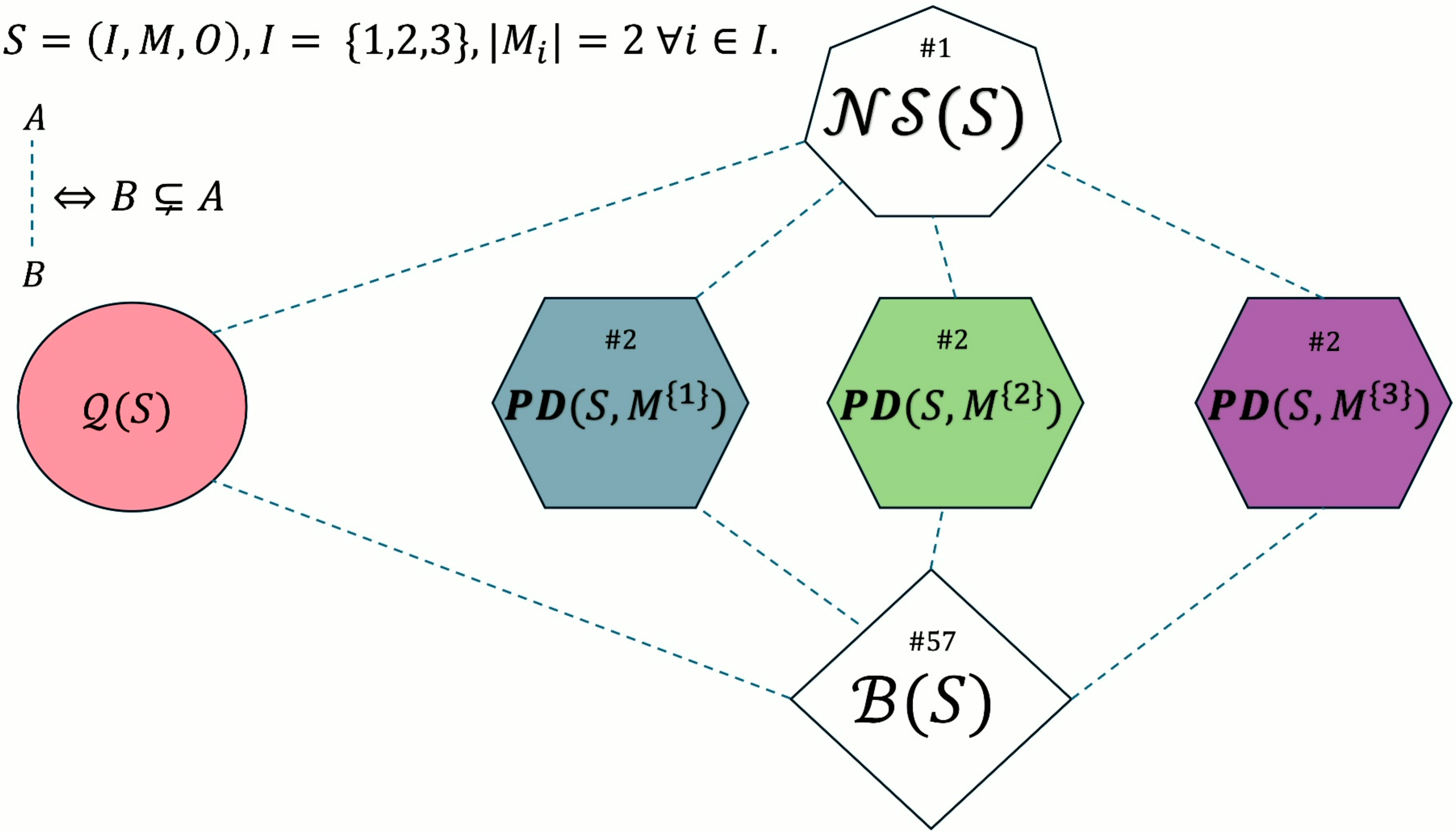


Example 2

$x, y, z \in \{1,2\}$



$S = (I, M, O), I = \{1,2,3\}, |M_i| = 2 \; \forall i \in I.$



Some of our results

Corollary of Fine's theorem*:

A behaviour is Local Deterministic if

$$\wp(\vec{a}|\vec{x}) = \sum_{\lambda}\prod_{i} D(a_i|x_i,\lambda)P(\lambda)$$

for some $\lambda \in \Lambda$, $\mathrm{P}(\lambda), D(a_i|x_i,\lambda) \in \{0,1\}\ \forall a_i, x_i$

A behaviour is Local Separable if

$$\wp(\vec{a}|\vec{x}) = \sum_{\lambda}\prod_{i} P(a_i|x_i,\lambda)P(\lambda)$$

for some $\lambda \in \Lambda$, $\mathrm{P}(\lambda)$

A behaviour is Local Factorizable, or Bell-local if

$$\wp(\vec{a}|\vec{x}) = \int_{\Lambda}\prod_{i} P(a_i|x_i,\lambda)p(\lambda)$$

for some Λ, a measure $\mathrm{p}(\lambda) \geq 0$, $\int_{\Lambda} p(\lambda) = 1$

$$LD(S) = LS(S) = B(S)$$

Marwan Haddara and Eric G. Cavalcanti, Local Friendliness Polytopes In Multipartite Scenarios, Phys. Rev. A **111**, 012206, (2025), **M. Haddara, H.M. Wiseman and E.G. Cavalcanti, another manuscript in preparation (2024); E. Woodhead imperfections and self testing in prepare-and-measure quantum key distribution, Ph.D. thesis, Laboratoire d'Information Quantique Universite libre de Bruxelles (2014), * A. Fine, Hidden Variables, Joint Probability, and the Bell Inequalities, Phys. Rev. Lett. 48, 291 (1982), A. Fine, Joint distributions, quantum correlations, and commuting observables, Journal of Mathematical Physics 23, 1306 (1982).

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Corollary of Fine's theorem*: Partial analogues**:

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$$\boldsymbol{PD}(S,M') = \boldsymbol{PS}(S,M') = \boldsymbol{PF}(S,M')$$

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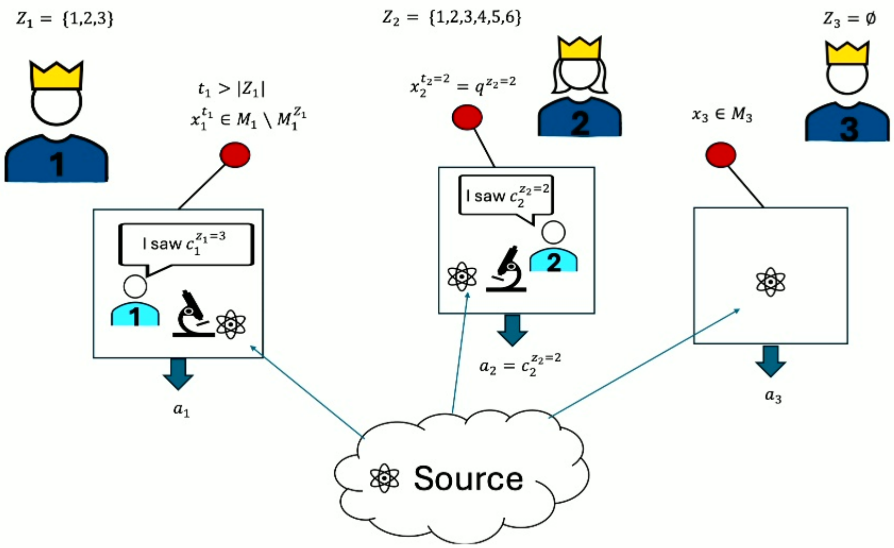
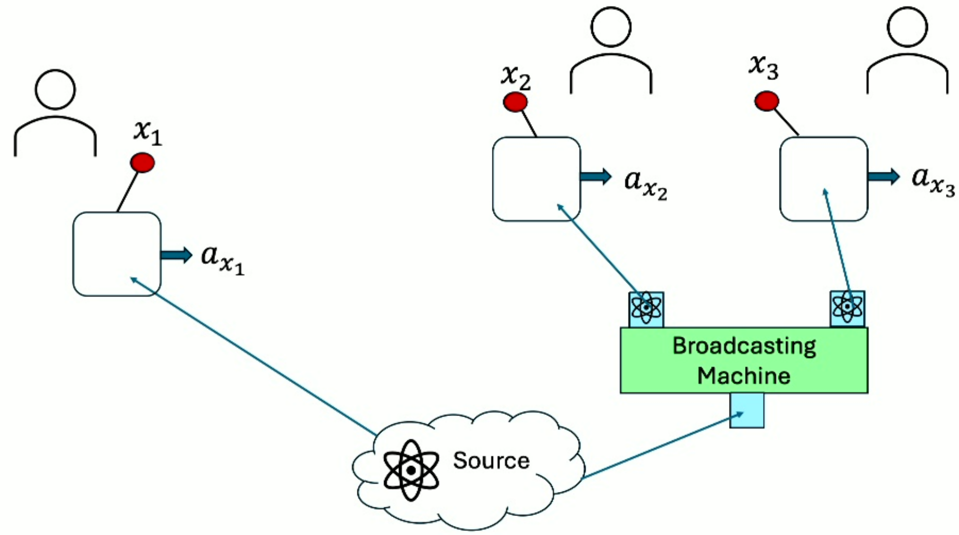
More exotic situations



Local Friendliness assumptions in classes of Wigner's friend-type scenarios lead to PDP's*



Broadcast locality** in classes of broadcasting scenarios lead to PDP's

*Kok-Wei Bong, Aníbal Utreras-Alarcón, Farzad Ghafari, Yeong-Cherng Liang, Nora Tischler, Eric G. Cavalcanti, Geoff J. Pryde and Howard M. Wiseman, Nature Physics 16, 1199–1205 (2020) , Utreras-Alarcón Aníbal, Cavalcanti Eric G. and Wiseman Howard M. 2024Allowing Wigner's friend to sequentially measure incompatible observablesProc. R. Soc. A.48020240040: Marwan Haddara and Eric G. Cavalcanti, Local Friendliness Polytopes In Multipartite Scenarios, Phys. Rev. A **111**, 012206, (2025),; M. Haddara, H.M. Wiseman and E.G. Cavalcanti, another manuscript in preparation (2024)
**J. Bowles, F. Hirsch, and D. Cavalcanti, Single-copy activation of Bell nonlocality via broadcasting of quantum states, Quantum 5, 499 (2021). Luis Villegas-Aguilar, Emanuele Polino, Farzad Ghafari, Marco Túlio Quintino, Kiarn T. Laverick, Ian R. Berkman, Sven Rogge, Lynden K. Shalm, Nora Tischler, Eric G. Cavalcanti, Sergei Slussarenko and Geoff J. Pryde , Nature Communications 15, 3112 (2024);

Physical situations where PDP's arise?

Foundations:

- Local Friendliness no-go theorem extending on Wigner's friend paradox
  i. Canonical scenarios
     Marwan Haddara and Eric G. Cavalcanti, Local Friendliness Polytopes In Multipartite Scenarios Phys. Rev. A 111, 012206, (2025),
  ii. Sequential scenarios
     One to one correspondence between PDP's and Sequential LF correlations

Quantum information

- Randomness certification
- Nonlocality activation (broadcast locality)
- Device independent quantum state Inseparability witnesses

Marwan Haddara and Eric G. Cavalcanti, Local Friendliness Polytopes In Multipartite Scenarios, Phys. Rev. A **111**, 012206, (202),); M. Haddara, H.M. Wiseman and E.G. Cavalcanti, another manuscript in preparation (2024); E. Woodhead imperfections and self testing in prepare-and-measure quantum key distribution, Ph.D. thesis, Laboratoire d'Information Quantique Universite libre de Bruxelles (2014) ;J. Bowles, F. Hirsch, and D. Cavalcanti, Single-copy activation of Bell nonlocality via broadcasting of quantum states, Quantum 5, 499 (2021). Luis Villegas-Aguilar, Emanuele Polino, Farzad Ghafari, Marco Túlio Quintino, Kiarn T. Laverick, Ian R. Berkman, Sven Rogge, Lynden K. Shalm, Nora Tischler, Eric G. Cavalcanti, Sergei Slussarenko and Geoff J. Pryde , Nature Communications 15, 3112 (2024); i) Kok-Wei Bong, Aníbal Utreras-Alarcón, Farzad Ghafari, Yeong-Cherng Liang, Nora Tischler, Eric G. Cavalcanti, Geoff J. Pryde and Howard M. Wiseman, Nature Physics 16, 1199–1205 (2020) ii) Utreras-Alarcón Aníbal, Cavalcanti Eric G. and Wiseman Howard M. 2024Allowing Wigner's friend to sequentially measure incompatible observablesProc. R. Soc. A.48020240040

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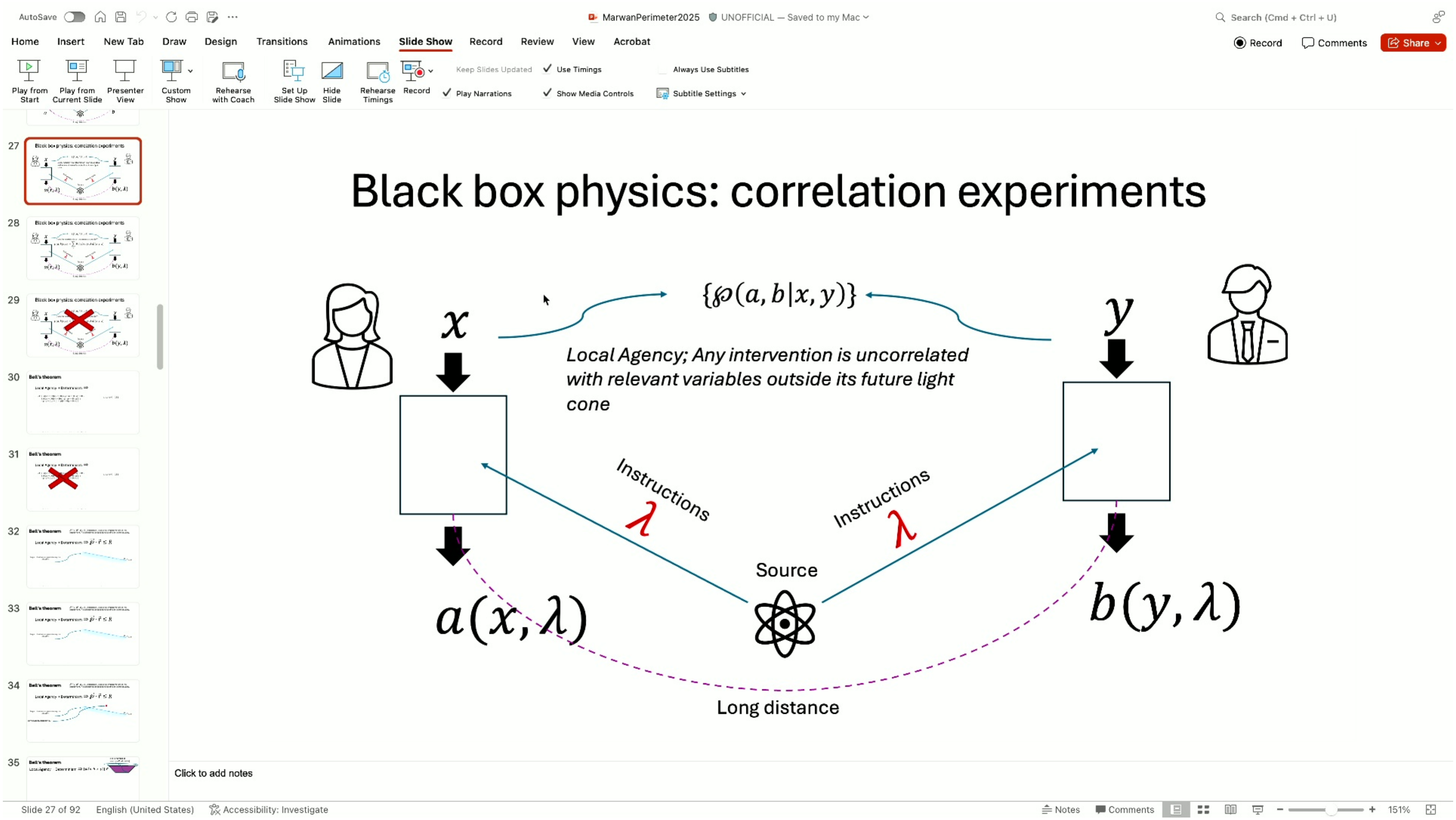
Black box physics: correlation experiments
$\{\wp(a,b|x,y)\}$
x
y
Local Agency; Any intervention is uncorrelated with relevant variables outside its future light cone
Instructions
λ
Instructions
λ
Source
$a(x,\lambda)$
$b(y,\lambda)$
Long distance
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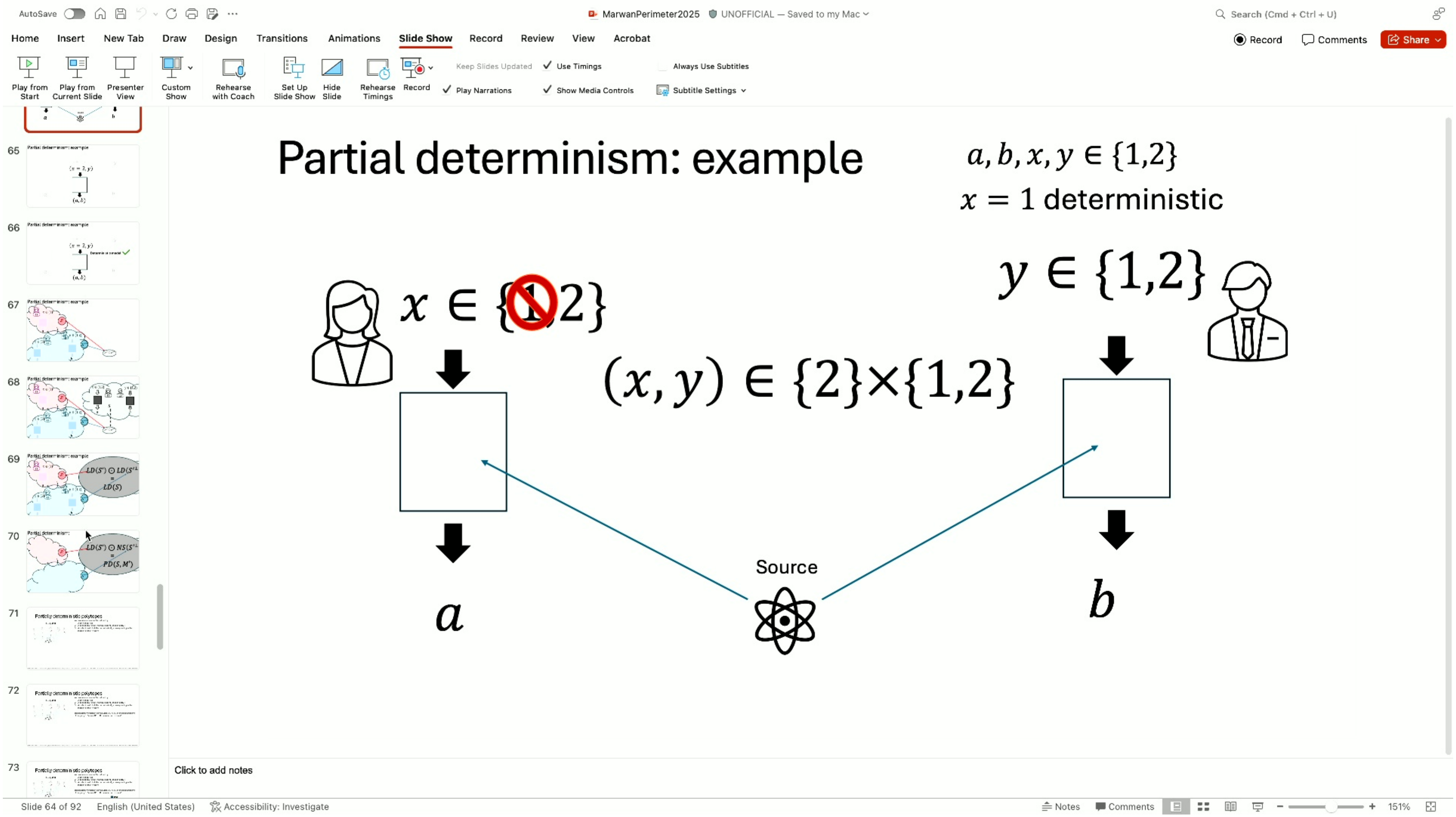
English (United States)


Partial determinism: example
a, b, x, y ∈ {1,2}
x = 1 deterministic
x ∈ {1,2}
y ∈ {1,2}
(x, y) ∈ {2}×{1,2}
Source
a
b
Click to add notes
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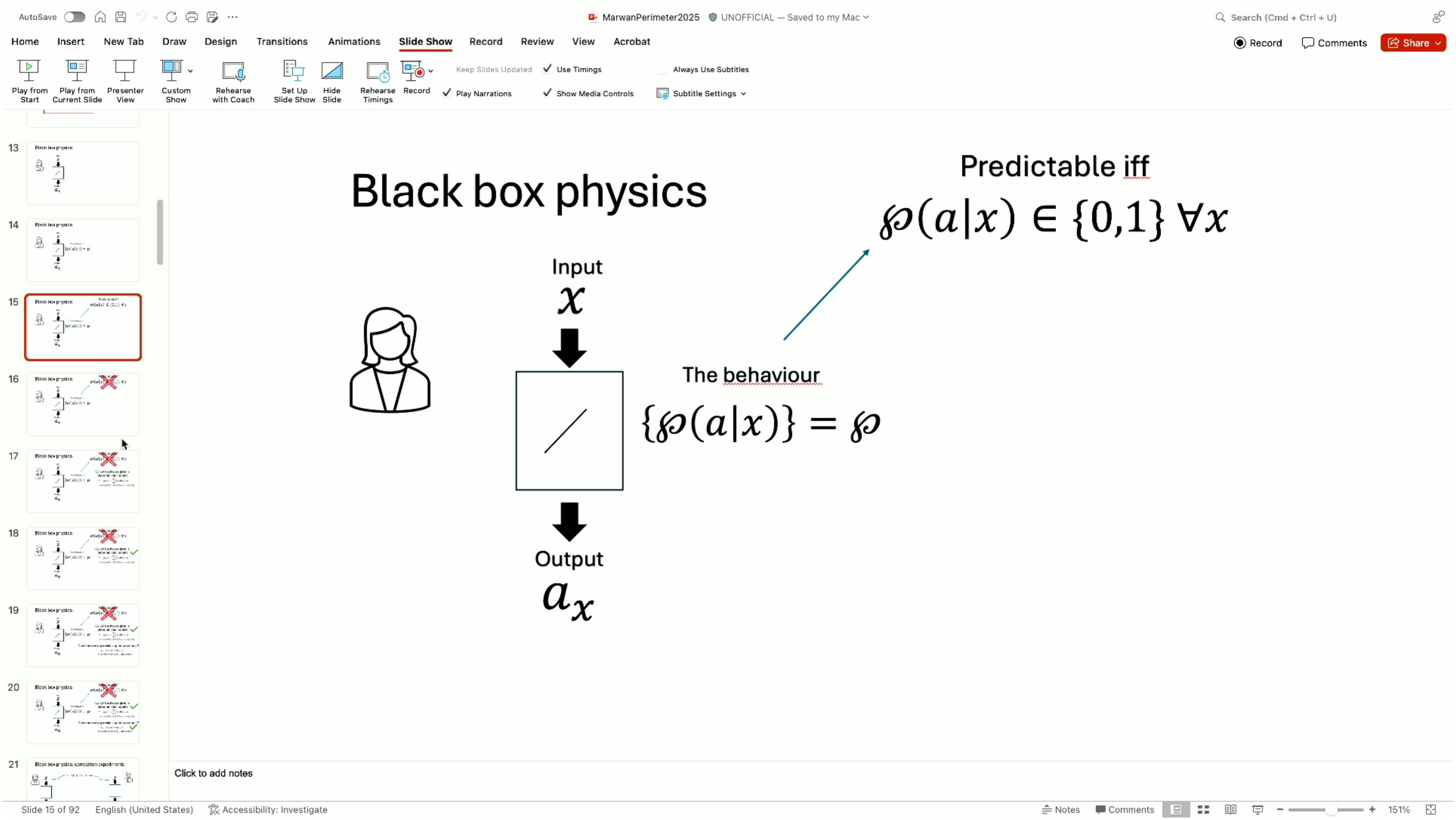
MarwanPerimeter2025
Black box physics
Input
x
Output
a_x
The behaviour
$\{\wp(a|x)\} = \wp$
Predictable iff
$\wp(a|x) \in \{0,1\}\ \forall x$
Click to add notes
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