Title: Hilbert Bundles and the Hydrodynamic Approach to Quantum Gravity

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Abstract:

Several papers from the mid to late 1990s suggest that Einstein's equations should be thought of as the hydrodynamic equations of a special class of quantum systems. A classical solution defines subsystems by dividing space-time up into causal diamonds and Einstein's equations are the hydrodynamics of a system that assigns density matrices to each diamond with the property ⟨K⋄⟩= ⟨(K⋄−⟨K⋄⟩)2⟩=A⋄. These define 4GN the empty diamond state, the analog of the quantum field theory vacuum, in the background geometry. The assignment of density matrices to each diamond enables one to define the analog of half sided modular flow along geodesics in the background manifold, as a unitary embedding of the Hilbert space of a given diamond into the next one in a nesting with Planck scale time steps. We conjecture that this can be enhanced to a full set of compatible unitary evolutions on a Hilbert bundle over the space of time-like geodesics, using a Quantum Principle of Relativity defined in the text. The compatibility of this formalism with the experimental success of quantum field theory (QFT) is discussed, as well as the theoretical limits in which QFT emerges.

Hilbert Bundles and Holographic Space-time

A Hydrodynamic Approach to Quantum Gravity

Tom Banks (Rutgers NHETC/UCSC emeritus) - Perimeter Institute Lecture March 20, 2025

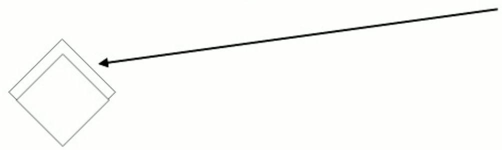
Evidence for the Fluctuation Law and a General Conjecture

- Verlinde/Zurek and de Boer et. al. proved the fluctuation law for RT diamonds in ALL AdS/CFT models with Einstein Hilbert duals, by two completely different methods.
- TB and Draper proved it for de Sitter space by a replica trick.
- It fails for large black holes in AdS but TB/Zurek explained this in terms of tensor network models: the universal law is valid in NODES of the network, but entropy of large holes is dominated by sound modes which propagate over many nodes and behave like a higher dimensional field theory. Part of a general lesson: AdS is special and locality at sub AdS radius scales needs a different explanation than the one given by tensor networks/QES/ECC etc.
- Conjecture: A solution of Einstein's equations defines the hydrodynamics of a quantum system, specifying the density matrices of an "infinite" number of subsystems in correspondence with causal diamonds, in the "empty diamond state" analogous to the QFT vacuum.

A Hilbert Bundle Over the Space of Time-like Geodesics

Farewell to "Background Independence": Build the Quantum System on the Hydro Background

- Nested causal diamonds along a time-like geodesic allow one to build embedding maps of smaller diamond Hilbert space into larger diamond Hilbert space.
- Analog of "one sided modular flow" in QFT. $e^{iL_0(\diamond_<)}e^{-iL_0(\diamond_>)}$ evolves system in the strip between the two



- Unitary embedding.
- Two UV cutoffs: 1+1 CFT lives on an interval, the stretched horizon. L_0 eigenvalues are discrete and cutoff in regime where Cardy formula is valid. Fuzzy cutoff on screen geometry: TB Fischler Kehayias, following Connes. Carlip and Solodukhin: volume of holoscreen encoded in c of CFT. Connes: volume "1" geometry encoded in Dirac operator D on the screen. $\Psi = \sum \psi_a(t, , z)\chi_a(\Omega)$ χ_a eigenspinors of D.

The Fuzzy Screen

- $D\chi_n = \lambda_n \chi_n$, λ_n come in discrete $\pm$ pairs. Expand solutions of D_f for fluctuating screen geometry in eigenspinors of D : $\Psi = \sum \psi_n \chi_n$. Let ψ_n be massless Dirac fields on the stretched horizon. C-S central charge constraint is UV cutoff on eigenspectrum of D: definition of fuzzy geometry that does not depend on Kahler structure.
- UV cutoff on individual ψ_n should be chosen so that Cardy formula still valid. Apart from that it's an experimental question. It's the size of causal diamond for which the semi-classical arguments of CS break down.

What's the CFT?

Fast Scrambling:Lindesay,Susskind,Hayden,Presskill,Susskind,Sekino

- Free Dirac fermions will not fast scramble information.
- Formally, Dirac bilinears $\Psi_a(t, z, \Omega)\gamma^m\Psi_b(t, z, \Omega)$ are 1+1 dimensional currents, which transform as a sum of p-forms on the holoscreen. If we ignore cutoffs, they commute at different values of Ω . Can always find one or more values of p such that p and d - 2 - p forms commute even at coinciding points on the screen. So we can find interactions that "formally" preserve both 1+1 conformal invariance and volume preserving mappings on the screen.
- Volume preserving mapping invariance doesn't respect distance and leads to fast scrambling.
- Conjecture that these are the right interactions, even in the presence of the required cutoffs.

General Correspondence between Matrix/tensor models and black holes

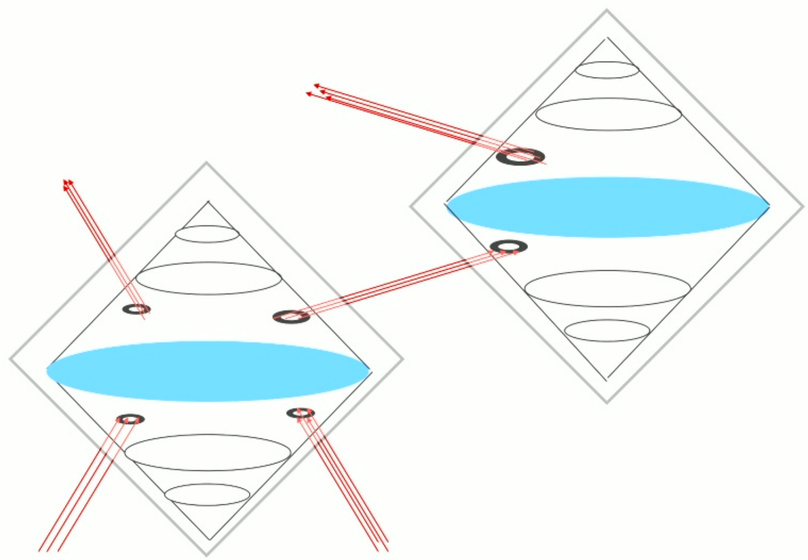
d-2 dimensions $\psi_{(i(1)\ldots i(d-2))}$ Counts components of spinor.

- $M_i^j = \psi_i A \psi^{\dagger jA}$ A runs over d - 3 indices. Some components of M_i^j can come from a small "volume" of just a few indices. Isolate that group of indices by setting $\psi_{(i(1)\ldots i(d-3)I)} | S_{init} \rangle = 0$. If the Hamiltonian has the form
- $H = N^{-1}\mathrm{Tr}P(M/N^{d-3})$ then it will take a time of order NlnN to equilibrate that subsystem with the bulk of the DOF: time to hit the singularity for a black hole or to merge into the horizon of dS.
- Entropy deficit $\sim k^{d-3}N$, implies energy of "localized object" scales like d - 3 power of its size (gravitational).
- The quantum states of an empty diamond and a black hole of the same size are locally the same. They differ w.r.t. to their entropy deficit in the degrees of freedom of larger diamonds that contain them. No firewalls.
- "Energy" defined by entropy deficits is conserved up to amounts of order 1/N. If too much "energy" comes into a single diamond, the number of localized excitations can change. This leads to an EFT description with "interaction vertices". The time scales for these interactions are much shorter than equilibration times in larger diamonds.
- If the above process happens for a large enough diamond, the whole system is initiated in a state far from equilibrium and rapidly returns to equilibrium: "black hole creation by particle collision". Happens parametrically when $E_{tot} \sim N^{d-3}$
- Long distance Newtonian interaction between two small blocks in large diamond, 2nd order perturbation theory switches on and off constrained variables.

How Does QFT Emerge and Why is This Consistent With Experiment?

Long known that most QFT states in a region back react to make black hole bigger than the region.

- Cohen, Kaplan, Nelson (98): Elimination of QFT states in a causal diamond that would create a black hole larger than the diamond does not affect agreement of QFT with any experiment. Very crude cutoff. Close to experimental bounds.
- TB-Draper (20), Blinov-Draper (21) other cutoffs put experimental clash further off.
- dS Entropy formula
  $ds^2 = -(1-(r_s^M/r)^{d-3} - r^2/R^2)dt^2 + dr^2/(1-(r_s^M/r)^{d-3} - r^2/R^2) + r^2 d\Omega_{d-2}^2$
- Inserting black hole lowers entropy by $2\pi RM$: Gibbons-Hawking Boltzmann factor.
- Fiol matrix model: suppressing off diagonal elements between small and large diagonal block reproduces this in d=4.



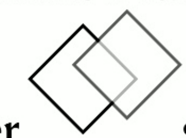
Black annuli are constrained variables. Arrows show propagation of constraints from larger diamonds (not shown fully) to smaller

If the c.c. < 0 and Size Approaches R_{AdS}

- The Embedding Maps Approach Those of the Tensor Network Renormalization Group of Evenbly and Vidal (2015) : TB and Fischler (2016)
- Just like Tensor Networks/Error Correcting Codes, Hilbert Bundles are a non-Isometric Embedding of a Local description into "the Hilbert Space of QG"
- In our case the phrase in quotes means "the Hilbert space along any fiber of the bundle as the proper time goes to infinity".
- The cosmological constant is an input parameter, which controls the relation between the infinite area and infinite proper time limits of diamonds. For negative c.c. we understand the limit because it corresponds to renormalization in QFT. It's controlled by conformal invariance, OPEs, etc.

The Quantum Principle of Relativity

The biggest lacuna in the formalism: easy to state, hard to implement/prove.

- Unitary embeddings are not unitary transformations on the full Hilbert space. This is the price we pay for locality/modular evolution. Physically, we need information about what is going on "outside our particular diamond at any fixed time"
- Overlap of two diamonds (on any two geodesics) contains a maximal area diamond.
- QPR: this is identified with a tensor factor on EACH diamond Hilbert space, and the two density matrices on that common factor must have the same entanglement spectra.
- Tensor networks invariant under  subgroups of hyperbolic translations implement this for AdS (hard to construct for interesting models). For non-negative c.c. we don't yet have a mechanism to implement the QPR.

Vanishing Cosmological Constant

- Perturbative String Theory: S matrix in Fock Space BUT
- Definitely fails in 4d (see especially recent work of Wald et. al.)
- de Sitter entropy goes to infinity in the limit
- Polchinski-Susskind "arena" diamond entropy goes to infinity in the limit.
- Connected correlators in CFT between a few Polchinski-Susskind operators and arbitrarily many operators that create excitations outside the arena but exchange gravitons through Witten diagrams.
- In BFSS matrix theory, o(1/N) energy for states with arbitrarily many small diagonal blocks and transverse momenta $o(N^{-1/2})$. Non-trivial S matrix elements to those states.
- WHAT IS THE NON-PERTURBATIVE HILBERT SPACE OF ASYMPTOTIC STATES?

Positive Cosmological Constant

- TB and Fischler, independently ca 2000 : the Hilbert space of future asymptotically dS space is finite dimensional (right) and the log of the dimension is the Gibbons Hawking entropy (wrong and Carlip and Solodukhin had already guessed the right answer).
- What does this mean for A) the dS isometry group? B)Mathematical Detectables C) real observations?
- Most of the isometry group maps one fiber of the Hilbert bundle to another. Information about one geodesic is rapidly thermalized, essentially at the classical level, from the point of view of another. Maximal subgroup of the isometry group that might be mathematically sensible is stabilizer of a geodesic (but see Collier et. al.)
- However, QM of detector CM, plus effects of other objects in dS means it's unrealistic for a detector to remain on a geodesic for longer than $R\ln(mR)$.
- The longest lived detectors in an asymptotic dS space are in $d = 4$ associated with local groups of galaxies and live till the group collapses into a black hole and the detector no longer has useful pointers. The detailed theory of these depends on idiosyncratic events and does not have an elegant mathematical formulation. At best one might hope to come up with some generic coarse grained statistical results.

Conclusions

- The Hilbert bundle formulation of QG translates hydrodynamic information gleaned from Einstein's equations into a detailed quantum picture of the system underlying any given solution of those equations. The translation involves quite a number of guesses, but they lead to interesting connections to things like SUSY, which we know to be important parts of the string theory approach to QG.
- Hilbert Bundles give a picture that meshes both with AQFT and the tensor network picture of AdS/CFT. It's biggest departure from both of these is the claim that localized excitations below the AdS radius, always have to be thought of as constrained states of the holographic variables. Once one realizes that the Polchinski-Susskind diamond is highly entangled with its exterior in any state where one has NOT inserted particles into it, this idea becomes more palatable from an AdS/CFT point of view.
- The biggest challenge of this approach is to find a way to make the implementation of the QPR transparent and mechanical. This is the principle that guarantees the existence of asymptotic symmetries if they exist, and is the only way to construct the evolution operator on the full Hilbert space from local data.
- The most sobering conclusion of this set of ideas is that we will not find an elegant theory that describes our own universe, and that much of the quantum information in our universe is not accessible to detection by local devices.