Title: Symmetry and Causality Constraints on Fermi Liquids

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Symmetry and causality constraints on Fermi liquids

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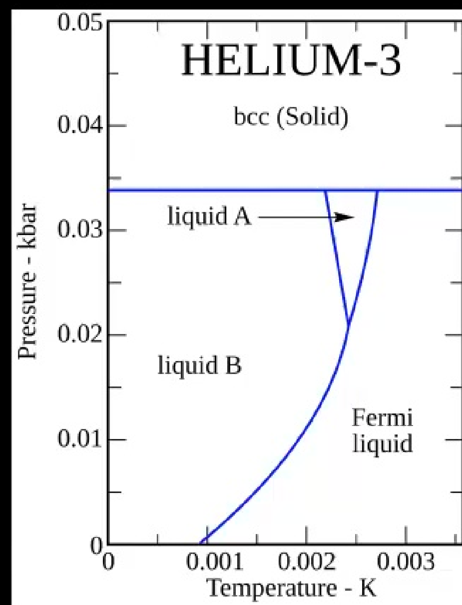
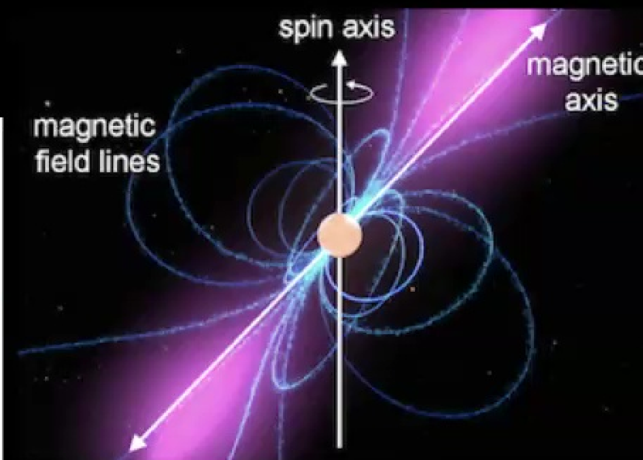
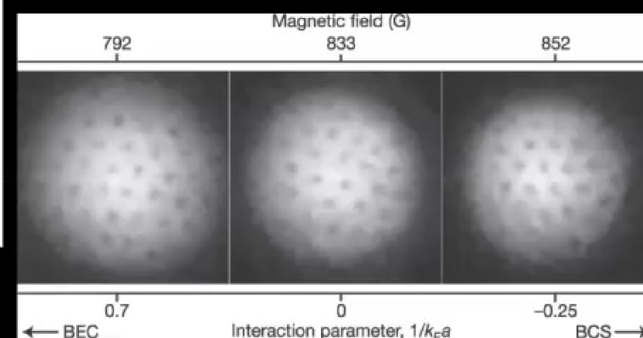
Chowdhury, Delacretaz, ***UM***, arXiv:2501.02073

Space-time symmetries in Fermi liquid phases

- He-3 (Galilean)
- Neutron stars, dense QCD (Lorentz)
- Unitary Fermi gas, non-relativistic anyon gas (Schroedinger)
- Interacting fermionic CFTs at finite density? (conformal)







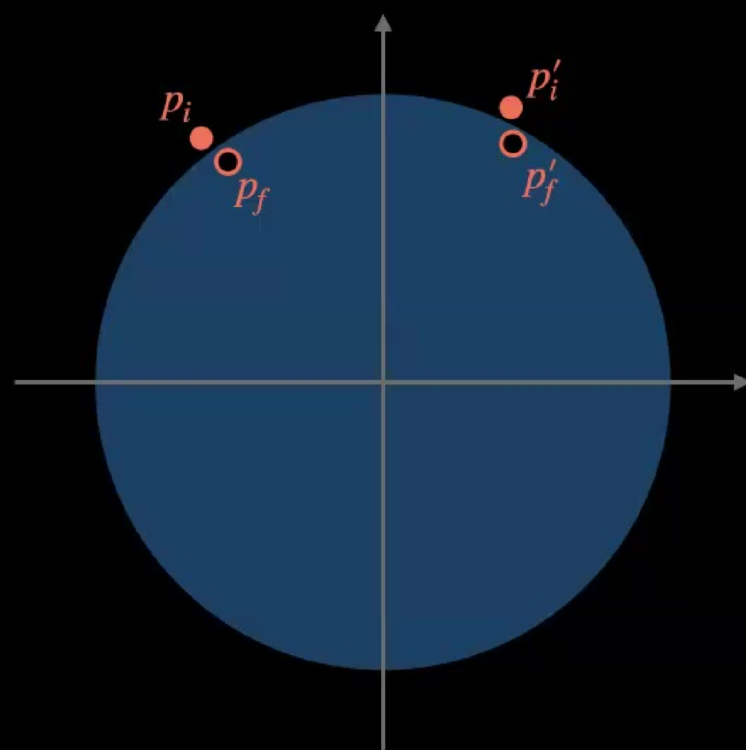
Zwierlein et al, Nature 05

History: Boosts constrain forward-scattering

- Effective mass relation:

$$m_* \equiv \frac{p_F}{v_F} = \begin{cases} (1+F_1)m & \textbf{Galilei} \\ (1+F_1)\mu & \textbf{Lorentz} \end{cases}$$

- F_1 measures the strength of current-current interactions



Landau, Baym+Chin

Constraints from thermodynamics

- Boost constraints from momentum susceptibility

$$\chi_{\pi\pi} = \begin{cases} m\rho & \textbf{Galilei} \\ \varepsilon + p = \mu\rho & \textbf{Lorentz} \end{cases}$$

- Scale constraints from charge susceptibility

$$\rho \propto \mu^{d/z}, \qquad \chi_{\rho\rho} = \frac{\partial\rho}{\partial\mu}, \qquad \Longrightarrow \qquad 1 + F_0 = z\frac{\mu}{p_F v_F}$$

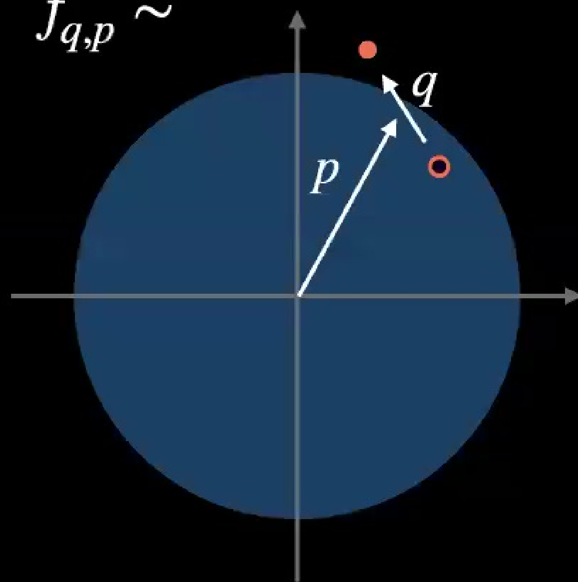
Are thermodynamic constraints sufficient?

- Need for a field theoretic approach (will use the coadjoint orbit action)
  - How do symmetries act on the space of states?
  - What constraints are obtained from requiring the action to be invariant under such transformations?

Beyond forward scattering?

$$H = \int_{xp} \epsilon_p f_{x,p}$$
$$+ \int_{xpp'} F^{(2,0)}_{pp'} \delta f_{x,p} \delta f_{x,p'} + F^{(2,1)}_{pp'} \left(\nabla_x \delta f_{x,p} \right) \delta f_{x,p'} + \ldots$$
$$+ \int_{xpp'p''} F^{(3,0)}_{pp'p''} \delta f_{x,p} \delta f_{x,p'} \delta f_{x,p''} + \ldots$$
$$+ \ldots$$

$f_{q,p} \sim$



Constraints on higher Landau paramaters?

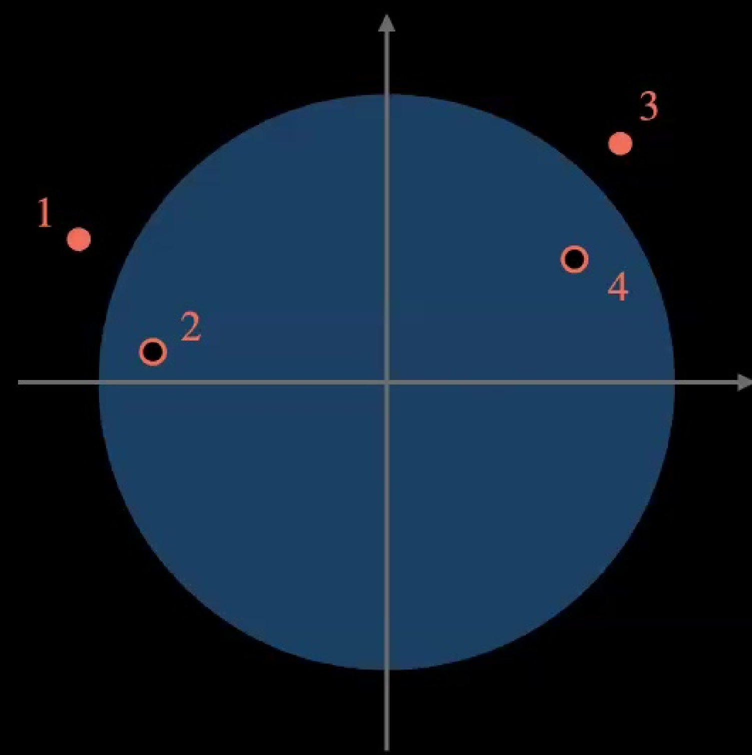
Delacretaz, Du, ***UM***, Son, PRR 23
UM, arXiv: 2307.02536

Outline

- Higher Landau parameters and irrelevant interactions
- Parameterizing the space of states using canonical transformations
- Symmetries as canonical transformations
- Constraints for boosts and dilatations
- (Micro)causality bounds on Landau parameters and consequences for conformal Fermi liquids

Interpreting the higher Landau functions

$$\int V_{1234}\, c_1^\dagger c_2 c_3^\dagger c_4 \qquad \sim$$

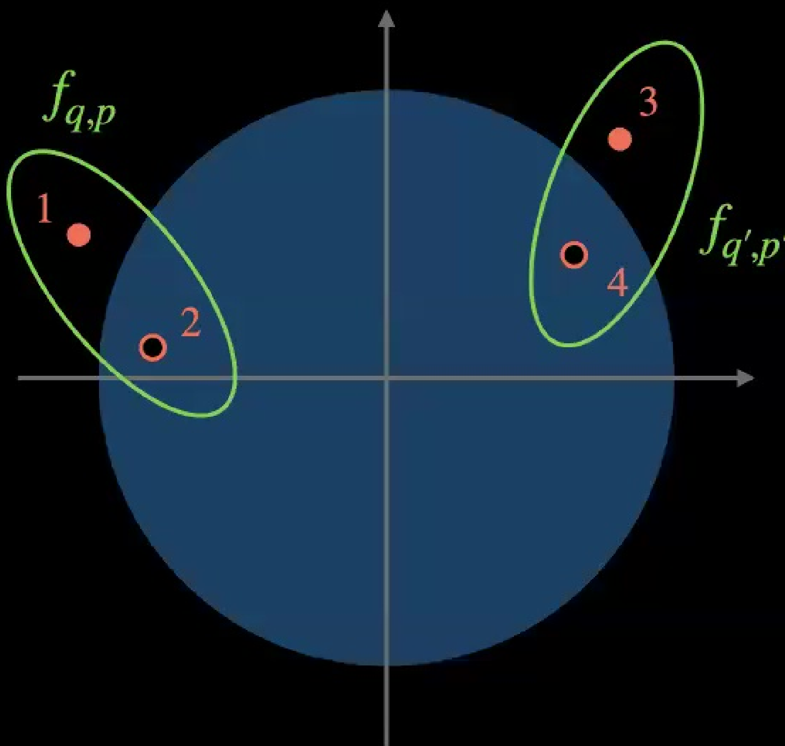


Interpreting the higher Landau functions

$$\int V_{xp,x'p'}\ f_{x,p} f_{x',p'}$$

↓ derivative expansion

$$\int F^{(2,0)}_{pp'}\ \delta f_{x,p} \delta f_{x,p'} + \dots$$



Interpreting the higher Landau functions

$$\delta f_{x,p} \sim \delta_{p_F}(p) + \delta'_{p_F}(p) + \ldots$$

localizes on the Fermi surface
under an expansion in fluctuations

$$F^{(2,0)}\big|_{p_F} \qquad \left[\partial_{|p|}F^{(2,0)}\right]_{p_F} \qquad F^{(2,1)}\big|_{p_F} \qquad \ldots$$

$F^{(2,0)}\big|_{p_F}$ → forward scattering

$\left[\partial_{|p|}F^{(2,0)}\right]_{p_F}$, $F^{(2,1)}\big|_{p_F}$ → non-forward two-body interactions

Higher Landau parameters (2D)

$$F^{(2,0)}\big|_{p_F} \sim \sum_l F_l e^{il(\theta-\theta')}$$ Landau parameters

$$\left[\partial_{|p|} F^{(2,0)}\right]_{p_F} \sim \sum_l \tilde{F}_l e^{il(\theta-\theta')}$$

higher Landau parameters

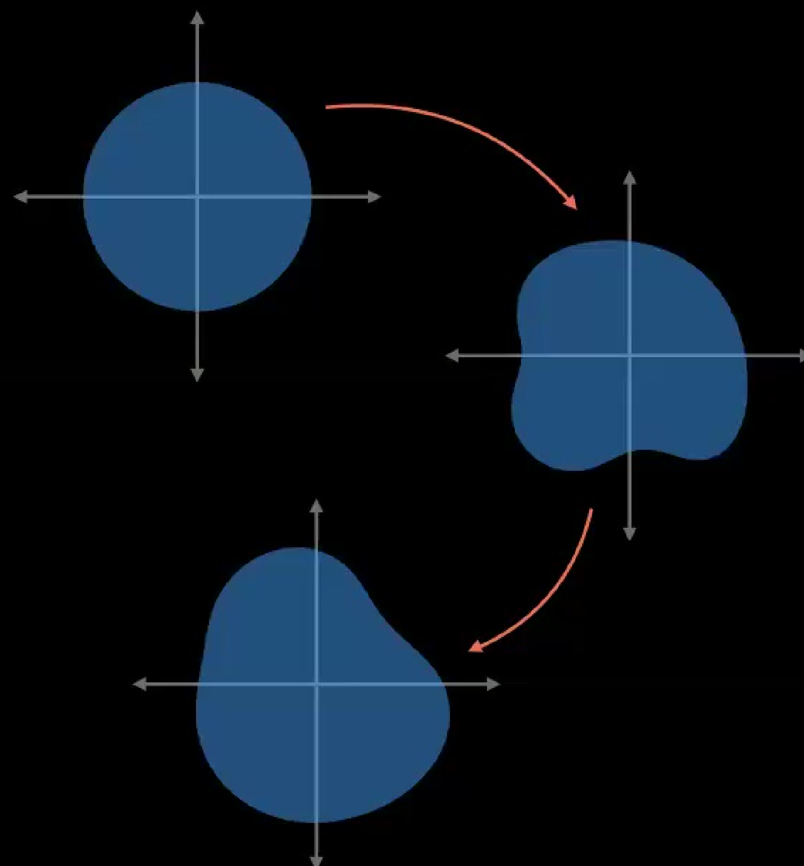
$$F^{(3,0)}\big|_{p_F} \sim \sum_{l,l'} G_{l,l'} e^{il(\theta-\theta')+il'(\theta-\theta'')}$$

The space of states

- Particle-hole coherent states are generated by canonical transformations

$$|\phi_{xp}\rangle = e^{i\int \phi_{xp}[\psi^{\dagger}\psi]_{xp}}\,|FS\rangle$$

- The space of states forms a coset G/H where G is the group of canonical transformations and H consists of transformations that leave the ground state invariant



Delacretaz, Du, ***UM***, Son, PRR 23
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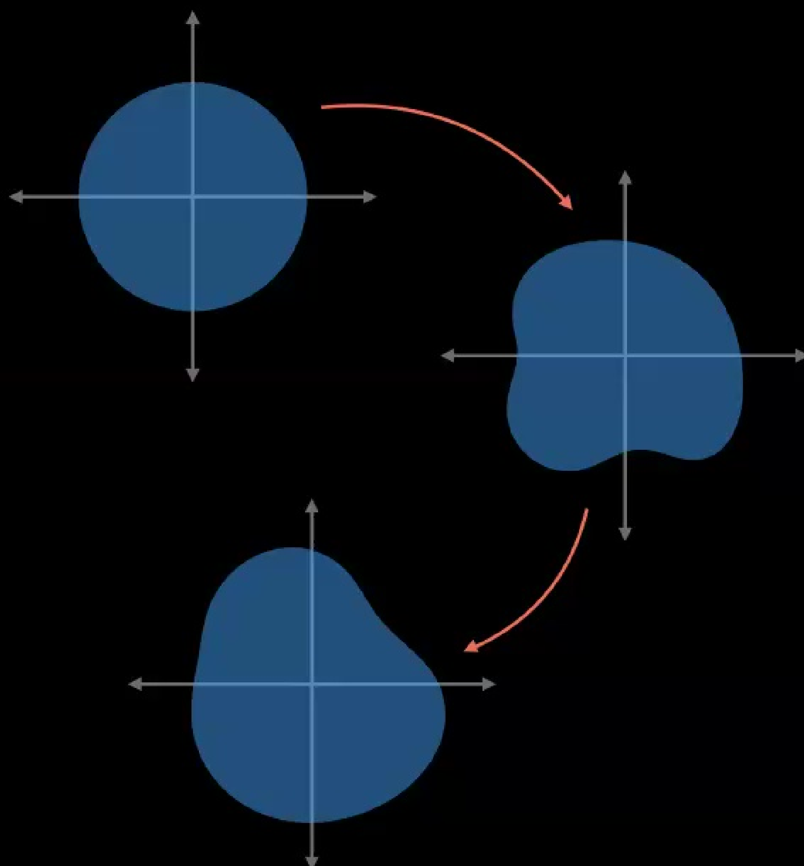
The space of states

- All states are related by canonical transformations, whose action on the distribution function is given by

$$f_{xp} \xrightarrow{F_{xp}} f + \{F, f\} + \frac{1}{2}\{F, \{F, f\}\} + \dots$$

- Space-time symmetries may be embedded into canonical transformations to deduce their action on the distribution function



Symmetries as canonical transformations

Symmetries that don't act on time:

Translations	$\{p_i\ ,\cdot\ \}$
Rotations	$\{\epsilon_{i_1\ldots i_{d-2}jk}x_j p_k\ ,\cdot\ \}$
Galilean boosts	$\{mx_i - tp_i\ ,\cdot\ \}$

Symmetries as canonical transformations

Symmetries that don't mix time and space:

Dilatations	$\{-x_i p_i \ , \cdot \ \} + zt\partial_t$
Non-relativistic special conformal transformations	$\left\{ \frac{mx^2}{2} - tx_i p_i \ , \cdot \ \right\} + t^2\partial_t$

Deriving the constraints

- Use transformation of f_{xp} to impose invariance of bosonized action $S[f_{xp}]$
- Expand f_{xp} in fluctuations $\phi_{x\theta}$ or $\delta p_F(x,\theta)$ (any any other parametrization)
- Set variation to zero order by order to obtain linear and non-linear constraints
- Check: Galilean effective mass constraint and scale invariance constraint can be derived in this way at leading order in fluctuations

Non-linear (quadratic) constraints

Galilei:

$$m v_F \tilde{F}_1 + m p_F \epsilon''_F - p_F = 0$$

$$3G_{1,l} + v_F(\tilde{F}_l + \tilde{F}_{l+1}) + \left[l F_l - (l+1) F_{l+1} \right] = 0$$

Dilatations:

$$v_F \tilde{F}_0 + p_F \epsilon''_F - (z-1) v_F = 0$$

$$3G_{l,0} + v_F(\tilde{F}_l + \tilde{F}_{-l}) + (z-2) v_F F_l = 0$$

An alternate derivation

- The constraints can also be derived from Ward identities by identifying conserved currents $j^\mu[f_{xp}]$ and $T^{\mu\nu}[f_{xp}]$
- Galilean invariance: $T^{0i} = mj^i$
- Scale (+conformal) invariance: $zT^0_0 + T^i_i = 0$
- Imposing only scale invariance at the level of the action (for both $z = 1{,}2$) gives the same result as the Ward identity for scale+conformal invariance
    - Scale invariance $\Longrightarrow$ conformal invariance even for $z = 2$?

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- Scale (+conformal) invariance: $zT^0_0 + T^i_i = 0$
- Imposing only scale invariance at the level of the action (for both $z = 1,2$) gives the same result as the Ward identity for scale+conformal invariance
    - Scale invariance $\Longrightarrow$ conformal invariance even for $z = 2$?

Lorentz invariance

- Lorentz boosts mix time and space - can't be implemented as canonical transformations
- Can still impose Lorentz invariance as a Ward identity $T^{0i} = T^{i0}$
- Linear constraint agrees with before + new quadratic constraint

$$\mu v_F \tilde{F}_1 + \mu p_F \epsilon''_F + p_F v_F^2 - p_F = 0$$

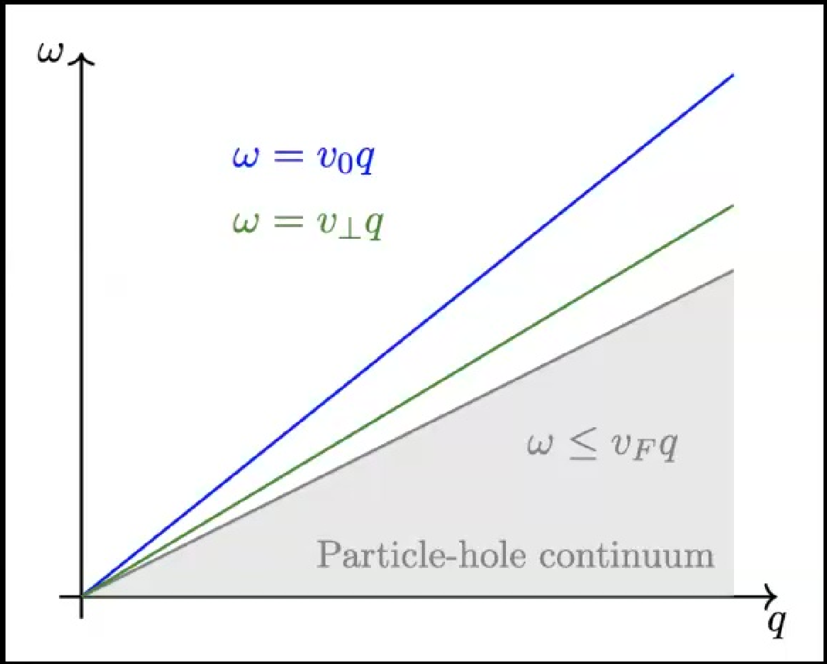
$$3\pi^2 \mu G_{1,l} + \pi^2 \mu v_F (\tilde{F}_l + \tilde{F}_{l+1}) + \pi^2 v_F (p_F v_F + (l+1)\mu) F_{l+1} + \pi^2 v_F (p_F v_F + l\mu) F_l + p_F v_F^2 F_l F_{l+1} = 0$$

Infinite towers of constraints

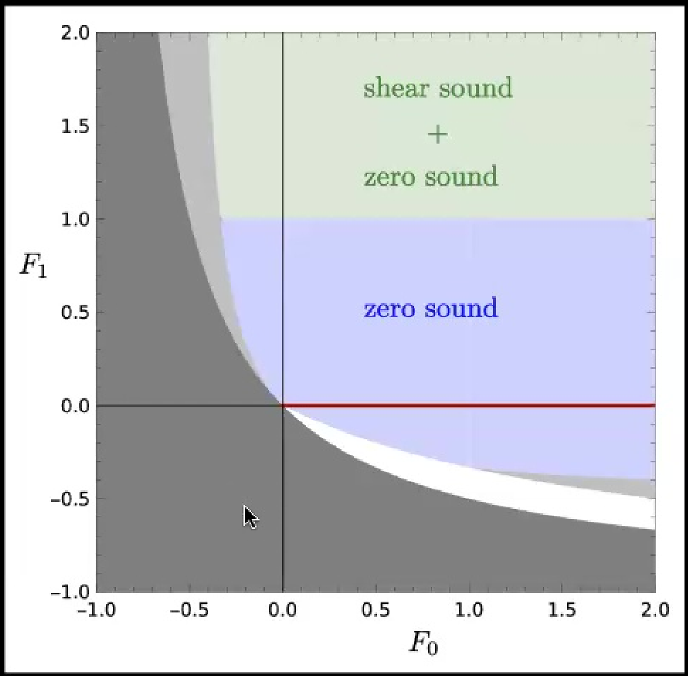
- The variation of the action under a symmetry transformation mixes orders in δp_F or ϕ
- Constraints relate higher Landau parameters like $G_{l,l'}$ and $\tilde{F}_l$ to lower ones like F_l — mixing happens at all orders
- We obtain an infinite tower of constraints between various higher Landau parameters
  - In particular, if e.g. $F_1 \neq 0$ in a boost invariant system, an infinite set of irrelevant interactions are forced to be nonzero. These will contribute to leading order in nonlinear response, e.g. $F^{(3,0)}|_{p_F}$ and $\left[\partial_{|p|}F^{(2,0)}\right]_{p_F}$ to $\langle \rho\rho\rho \rangle$

Collective modes and causality

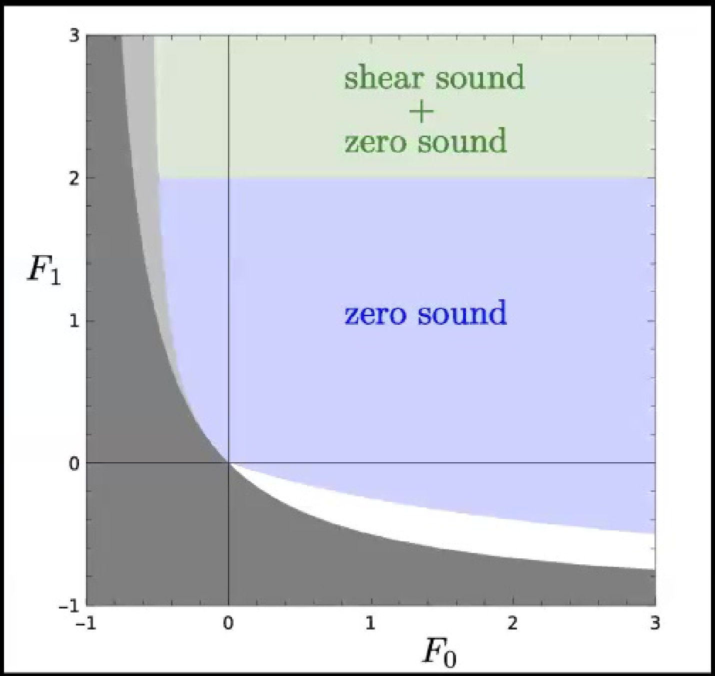
- Zero and shear sounds in Fermi liquids, corresponding to the presence of F_0 and F_1
- $v_0(F_0, F_1) \leq c, \quad v_\perp(F_1) \leq c, \quad v_F \leq c$ as a result of "microcausality"
- Relativistic conformal invariance fixes v_F in terms of F_0 and F_1, forbidding certain values of (F_0, F_1)



Causality constraints



$d = 2$



$d = 3$

Dark grey:
excluded by $v_F \leq c$
Light grey:
excluded by bounds on sound

Outlook

- A (mostly) systematic way to impose microscopic space-time symmetries on Fermi liquids
- Infinite towers of symmetry constraints on interactions, organized by (ir)relevance of interactions
  - Marginal interactions necessitate the presence of higher Landau parameters, which contribute to leading order in non-linear response
- Constraining the parameter space for conformal Fermi liquids using causality - new causality bounds on conformal couplings?