Title: Dressed Subsystems in Gravitational Theory

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Abstract:

Local subsystems play a fundamental role in theoretical physics, but they are usually defined in terms of background spacetime structures, in particular a Lorentzian metric. In gravity there are no such structures and the metric becomes dynamical. I will argue that nevertheless subsystems can be constructed in relation to other degrees of freedom, subject to the requirement that those variables reside within the subsystem itself. In operational terms an observer located within the system should be able to determine its edge by taking measurements within that region only. Within the context of classical field theory, I will demonstrate that this guarantees that the observables describing the system constitute a closed Poisson algebra. I'll discuss some examples, explain how surface charges generating subsystem symmetries, including the Bekenstein-Hawking area entropy, are incorporated, and make some remarks about how this can be extended to a fully quantum treatment within the framework of deformation quantization.

Dressed Subsystems in Gravity

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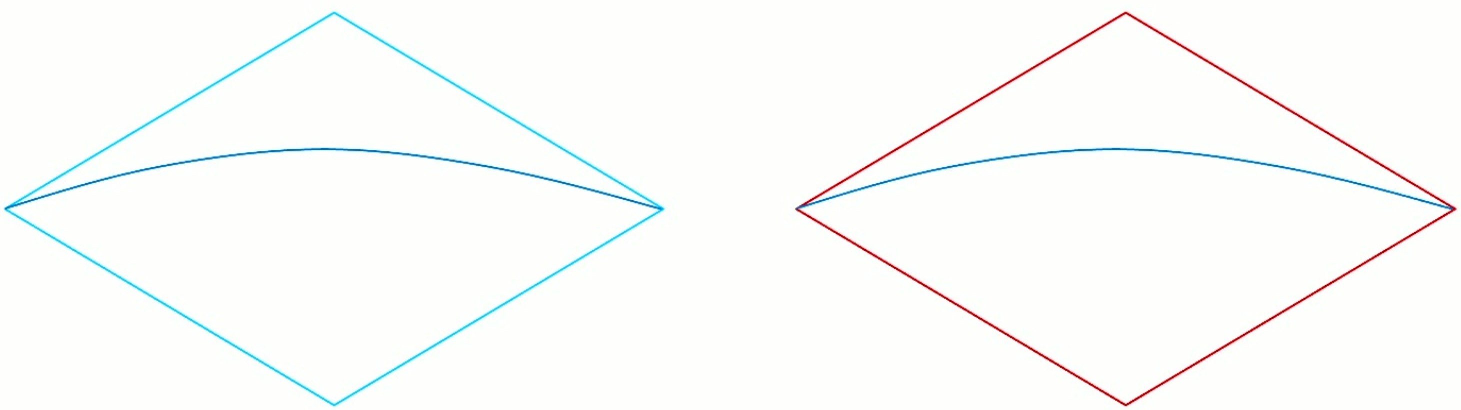
Introduction

The concept of a local subsystem is fundamental to the predictive power of physics. In quantum field theory, this takes a particularly sharp form:

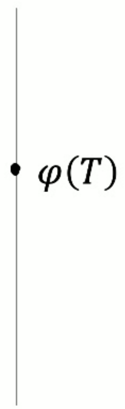
A subsystem is a *causally complete* spacetime region. Its degrees of freedom are encoded in an *algebra of observables* constructed from the dynamical fields. [Haag 1992]

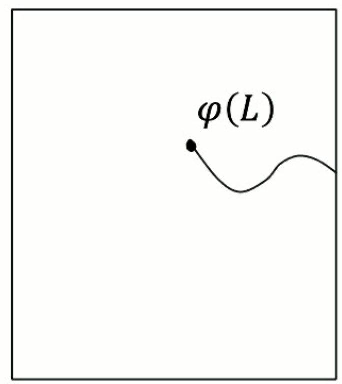
The algebras of spacelike separated subsystems commute.



We want to extend this concept to quantum gravity, at least within effective field theory. There are two problems:

- Causal structure in gravity is dynamical
- Diffeomorphism invariance precludes local observables



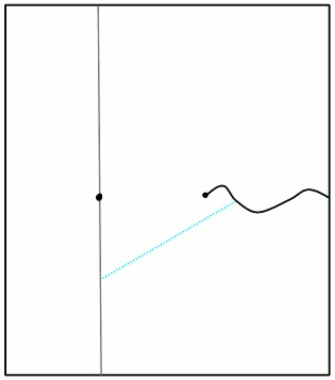


It's well-known that the resolution to the second is to consider *relational observables,* which are located with respect to dynamical degrees of freedom. [Bergmann+Komar 1960] Therefore it's natural to consider *relational subsystems*. [Kirklin et al. 2022, Chandrashekhar et al. 2022]

There are two objections in the literature:

- Dressing requires nontrivial structures to define locations. These may not exist in all states.

  This is alright; we only need a dressing prescription to be defined in a class of states where the structures exist.



- Relational observables that seem spacelike separated may not commute, unless their dressings are also spacelike separated. This has been achieved in simple examples [Marolf 2015], but [Donnelly+Giddings 2016] have argued that it generically obstructs defining independent subsystems.

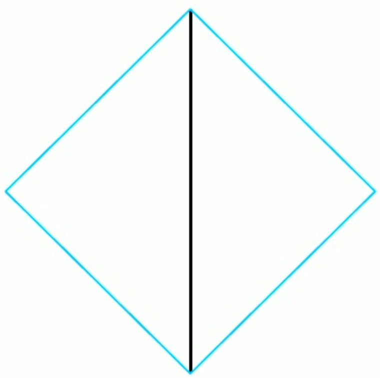
  This is premature! If we're careful, we can find a notion of subsystems that respects microcausality.

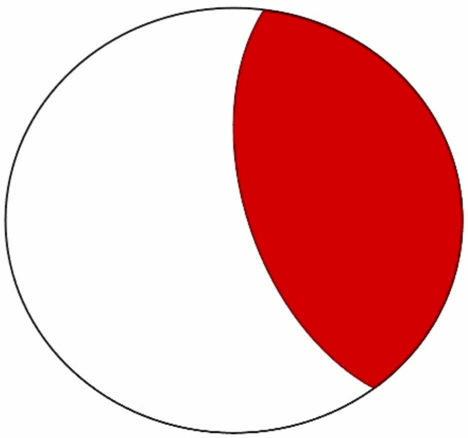
I claim that gravitational subsystems can be defined consistently iff the dressing is *internal*.

In operational terms, an observer inside the subsystem can determine its edge by taking measurements, without ever leaving the system. Again, the prescription need only be defined in a class of neighbouring states.

For illustration, consider

- The causal patch accessible to a worldline observer
- The domain of dependence of an extremal surface





For the purposes of this talk, I will work within classical field theory and study the structure of Poisson brackets using the covariant phase space formalism. This determines the leading order contribution to quantum commutators in an expansion in $\hbar$. We will see the following results:

- The observables localized in an internally dressed subsystem form a *closed Poisson algebra.*
- On the causal complement, they generate field-dependent gauge transformations.
- Observables belonging to spacelike separated subsystems Poisson commute.

The plan:

- Describe the framework needed
- Explain the main results
- Schematically discuss a few examples
- Implementing symmetry charges (B-H Area?)
- Path to including all orders of $\hbar$ in deformation quantization

Assumptions

We will consider field theories on a fixed manifold $\mathcal{M}$ satisfying a few criteria:

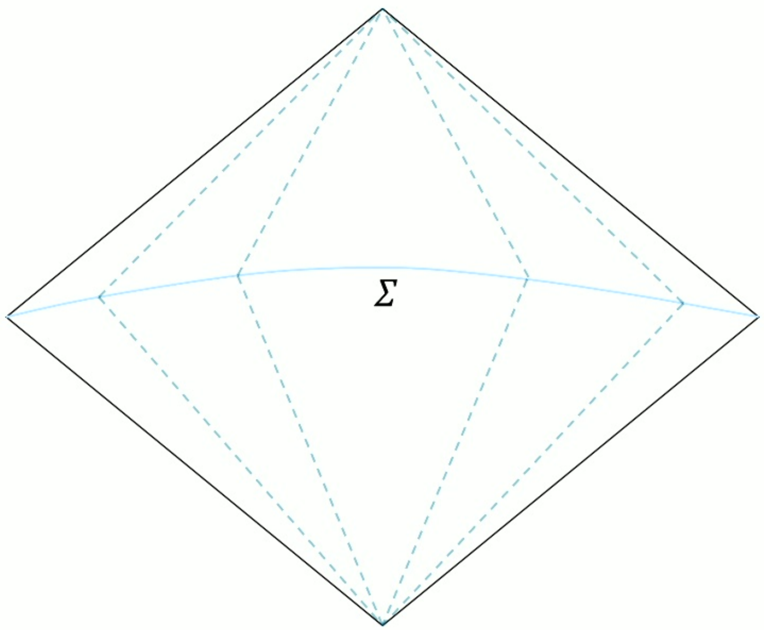
- The dynamical fields φ^a are *smooth* sections of some vector bundles
- Background fields are also smooth, but fixed
  - Will be dropped for generally covariant theories
- One of the fields is a Lorentzian metric g
  - Required to be globally hyperbolic
- The Lagrangian is a top form constructed *locally* out of the fields and their derivatives
- Varying the action with respect to φ^a yields the equations of motion E_a

The metric must determine the causal structure. A precise formulation of this is subtle; we will need to focus on the *linearized equations* around any particular solution. From experience we know that in physical examples, these are (Leray) hyperbolic with respect to *g in some choice of gauge*, so initial data on a partial Cauchy surface determines a unique solution in the domain of dependence.

- Yang-Mills
- Einstein gravity

This is a natural but somewhat awkward requirement. Instead we will make two gauge-independent postulates, that only refer to the *existence* of linearized solutions. These can be verified readily in the hyperbolic case.

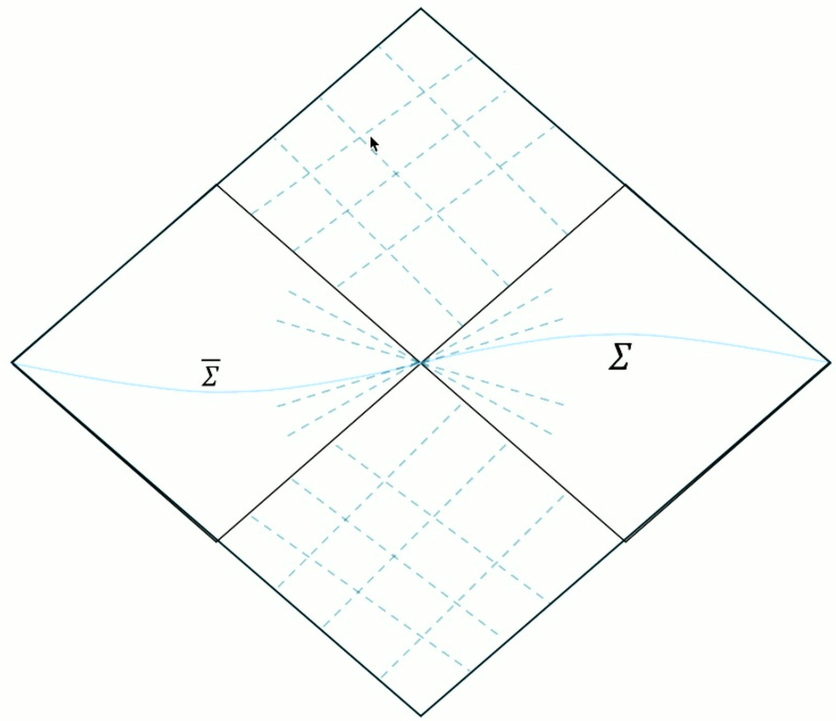
Assumption 1:



For any solution and partial Cauchy surface, there exists a definition of data for the linearization that is:

- Constructed out of the linearized fields and their derivatives up to some finite order
- Possibly subject to some local, differential constraints from the EoMs

Assumption 2:



Let a Cauchy slice be partitioned in two, and consider smooth linearized solutions on the domain of dependence of each, that agree to all derivatives at the intersecting surface.

Then there exists a linearized solution on the complete domain of dependence.

Field Space Geometry

The covariant phase space method treats the space of configurations as an infinite dimensional manifold, and the solution space as a submanifold. It's standard to take a heuristic attitude, which we will also do. Because we took the configurations to be smooth, the configuration space is a *Frechet* manifold. We can use exterior calculus on such manifolds, which is enough to carry out the usual construction of a symplectic form.

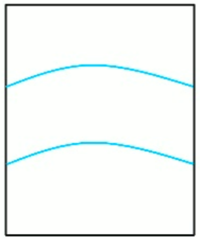
Warning!: The Frobenius theorem fails on Frechet spaces. This has a concrete physical interpretation in gravitational theories in terms of the absence of local observables. We will run into vector distributions that are involutive; remember that they are not necessarily integrable.

Covariant Phase Space

The symplectic form on solution space is obtained by integrating an object ω called the *symplectic current* on a Cauchy slice:

- ω is a locally constructed $D-1$ form on spacetime, and a two-form on solution space.
- There is an ambiguity corresponding to the addition of edge terms. This can be resolved by imposing boundary conditions and being careful about the action principle. [Harlow+Wu 2019] We will assume that some such prescription has been applied; it doesn't matter which.
- After this, the symplectic form is conserved on-shell; the choice of Cauchy slice doesn't matter.

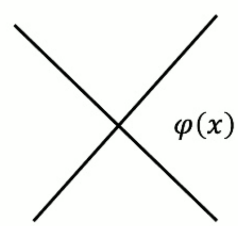
$$\Omega = \int_{\mathfrak{C}} \omega$$



Poisson Brackets

We can use this symplectic form to define Poisson brackets in the usual way, but there are subtleties. An observable R generates a flow χ (which is a vector tangent to solution space) if $\delta R = \Omega(\cdot, \chi)$.

- If Ω has null directions, R must be invariant under them. These are the gauge transformations.
- *Not* all observables generate flows. For example, the field at a point in scalar field theory does not generate a smooth flow.



We will call observables that generate a flow *regular observables*. Other observables are called *singular*.

- The Poisson bracket of a regular observable R with any other observable O is defined by $\{O, R\} = \delta O(\chi)$.
- If $R_3 = \{R_1, R_2\}$, then it generates $\chi_3 = [\chi_2, \chi_1]$.

There is a natural quantum interpretation that works in most cases. The regular observables stand for operators in the quantum theory that act on the set of *Hadamard states,* as these go to smooth solutions in the classical limit. The singular observables stand for *quadratic forms* on this set, which have expectation values but do not actually act as operators. In some cases there can be quantum anomalies.

What is a Gauge Symmetry?

We have defined gauge transformations as null directions of the symplectic form.

There is also a notion given to us by [Noether 1918], which defines gauge transformations as symmetries of the action containing free parameters that vanish sufficiently fast towards global boundaries.

- Noetherian symmetries are *local*.
- They are also *computable*, as a consequence of Noether's second theorem.

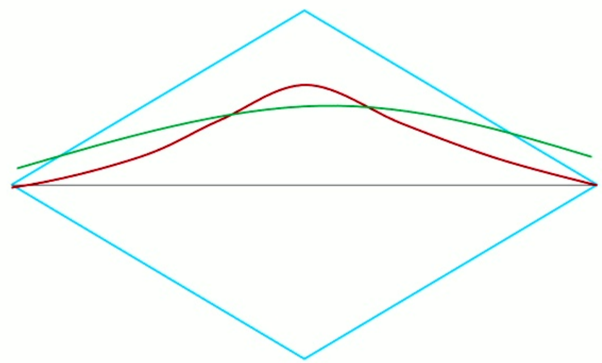
Is there a relationship between these concepts? In one direction, certainly; Noetherian symmetries are symplectic gauge transformations.

Locally Gauge Flows

The main technical engine of this work is a theorem which goes in the opposite direction:

Consider a partial Cauchy surface Σ, and suppose that a flow X satisfies $\Omega_\Sigma(\eta, X) = 0$

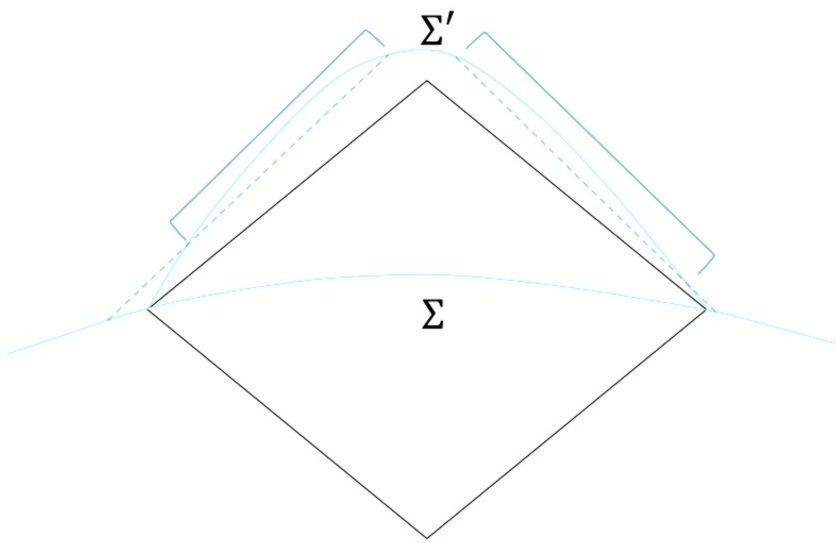
for all flows η that vanish smoothly towards (finite) edges of Σ.



$$\Omega_\Sigma = \int_\Sigma \omega$$

Then X is equal to an infinitesimal Noetherian gauge symmetry throughout the entire domain of dependence of Σ. This means that it is the restriction of a symplectic gauge transformation to the domain of dependence. (There is a caveat with linearization instabilities).

The causal properties of the theory are essential for this to work. To illustrate this, consider the following example in scalar field theory, where there are *no* gauge transformations:



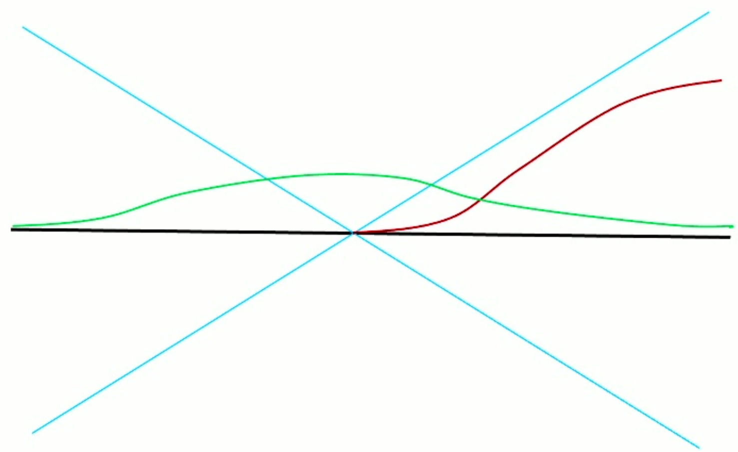
Consider a flow that vanishes on Σ but is nonvanishing on the rest of the Cauchy slice. This will propagate forward to be nontrivial on Σ', which is a partially timelike surface homologous to Σ .

This flow satisfies the criteria of X for the previous theorem. Since the symplectic current is conserved, we know that this is true on Σ' as well as Σ. However, the flow does not vanish on Σ', so the theorem fails on that surface.

Subsystems in Gauge Theory

This is enough to start proving results about subsystems in gauge theory. Consider a partial Cauchy surface Σ and its complement $\bar{\Sigma}$.

- Define the vector distribution $\mathcal{S}'(\Sigma)$ on solution space to consist of flows that restrict to gauge transformations on $\bar{\Sigma}$ (likewise $\mathcal{S}'(\bar{\Sigma})$).

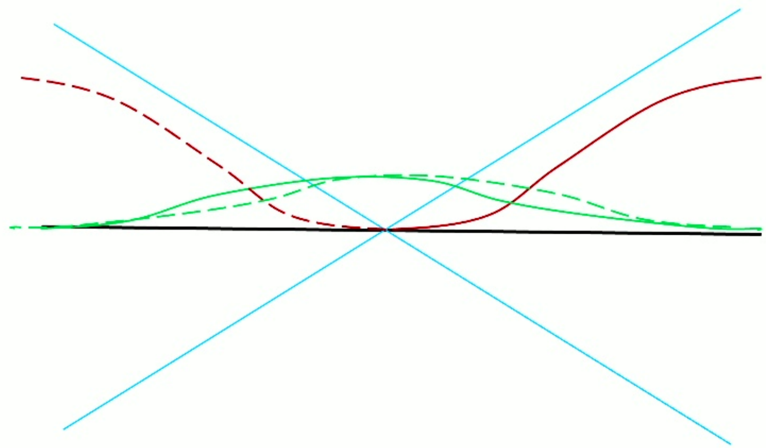


- These are exactly the flows that can be split into gauge transformations and flows vanishing at the edge.

These distributions have two critical properties:

- They are involutive, as shown by a simple computation.
- They are *symplectic complements* of each other; for $\chi \in \mathcal{S}'(\Sigma)$, $\Omega(\chi, \zeta) = 0$ iff $\zeta \in \mathcal{S}'(\bar{\Sigma})$ and vice versa.

The "if" direction is easy; the "only if" follows from the gauge theorem.



The observables supported on Σ are *precisely* the functionals that are invariant under $\mathcal{S}'(\bar{\Sigma})$. This implies a sequence of statements:

- Because $\mathcal{S}'(\bar{\Sigma})$ are also gauge throughout the DoD, the same statement applies to the observables in the DoD. In other words, the observables in the DoD and on Σ are the same. (More detail requires causality).
- A regular observable in this region must generate a flow belonging to $\mathcal{S}'(\Sigma)$, in order to symplectically annihilate the flows in $\mathcal{S}'(\bar{\Sigma})$. Conversely, the observables that generate flows in $\mathcal{S}'(\Sigma)$ (everywhere in solution space) are *exactly* those that are in the region.

- The Poisson bracket of two subregion regular observables always generates a flow in $\mathcal{S}'(\Sigma)$, and must be a subregion observable itself! The subsystem has a *closed algebra.*
- Finally, regular observables in one region commute with all observables in the complement.

The recipe is as follows:

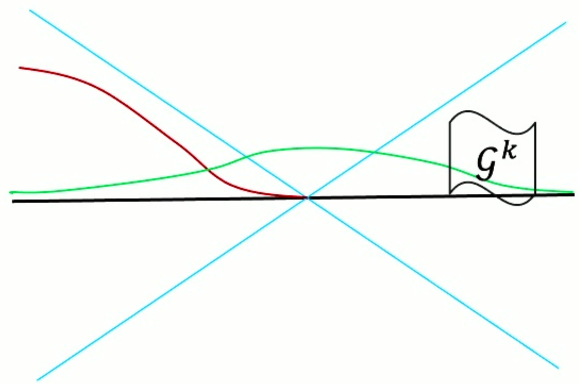
- First gauge fix so that Σ is always part of a fixed Cauchy surface (this is very weak, and just for convenience).
- Then, choose a set of functionals $\mathcal{G}^k$. These must
  - Form a valid gauge fixing scheme, so that any solution is connected to a solution where $\mathcal{G}^k = 0$ by a gauge path.
  - Depend *only* on the fields in the DoD of Σ, so that infinitesimal perturbations outside of this region do not affect them.
  - Completely eliminate diffeomorphisms that move $\partial\Sigma$.

Again, these need only be defined in a class of states. For example, in an open set of solution space where there is a *single* extremal surface, its location can be gauge fixed to $\partial\Sigma$, in which case $\mathcal{G}^k$ consists of the expansions on $\partial\Sigma$.

This is enough to get observables that are constructed only out of fields in the DoD. We'll see some examples later. Note that adding *more* gauge fixing conditions, beyond those used to fix the edge, does not increase the number of subregion observables. However, it may rewrite them in a simpler form.

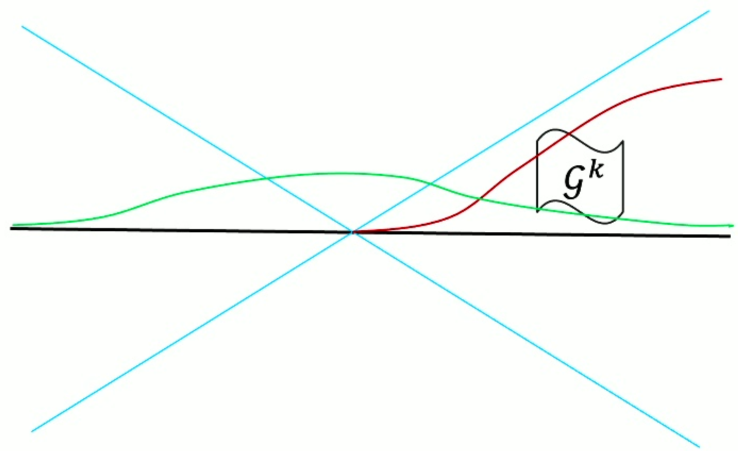
Dressed Subsystem Algebras

We want to extend the results we found for subsystems in gauge theories. The distributions $\mathcal{S}'(\Sigma)$ and $\mathcal{S}'(\bar{\Sigma})$ restrict to the constraint surface defined by $\mathcal{G}^k = 0$. They are still involutive, and are symplectic complements of each other.



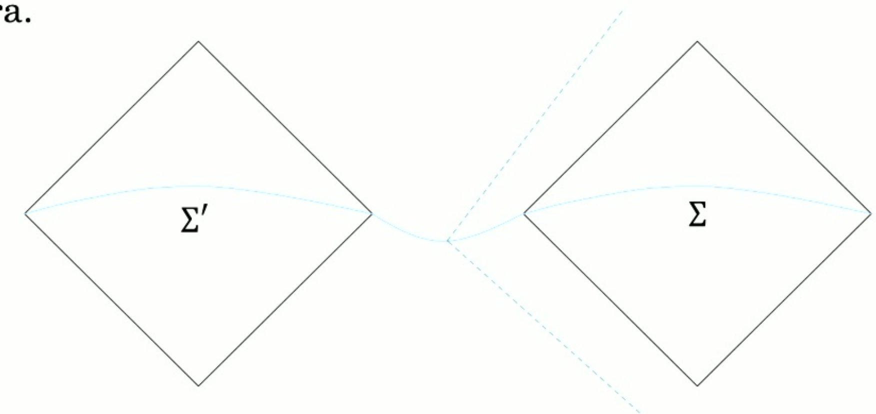
The flows in (restricted) $\mathcal{S}'(\bar{\Sigma})$ can be decomposed as before into the sum of a flow vanishing in the DoD of Σ and a gauge transformation. Since $\mathcal{G}^k$ are supported in the DoD, the former does not affect them, so the latter must be a *residual* gauge transformation.

Thus, gauge fixed observables supported in the DoD of Σ are again *exactly* those that are unaffected by $\mathcal{S}'(\bar{\Sigma})$. The opposite is not true, because after splitting a flow in $\mathcal{S}'(\Sigma)$ the gauge transformation is not necessarily *residual*. The two parts may separately affect $\mathcal{G}^k$, such that their contributions cancel.



Microcausality

This excludes the result about commutativity of complementary regions. This no longer holds because the observables in the causal complement need not be invariant under $\mathcal{S}'(\Sigma)$. They are also not guaranteed to form an algebra.

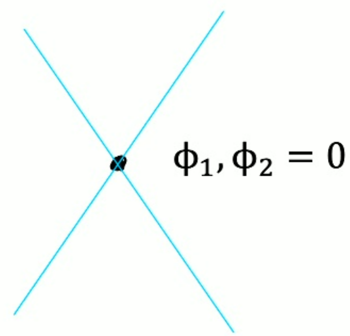


However, there is a way around this. Suppose there are two internally dressed subsystems that are, in an open region of solution space, spacelike separated. The flows in $\mathcal{S}'(\Sigma)$ are gauge in the causal complement, so they can be "turned off" before reaching the DoD of Σ'. These algebras commute.

Example: Scalar Models

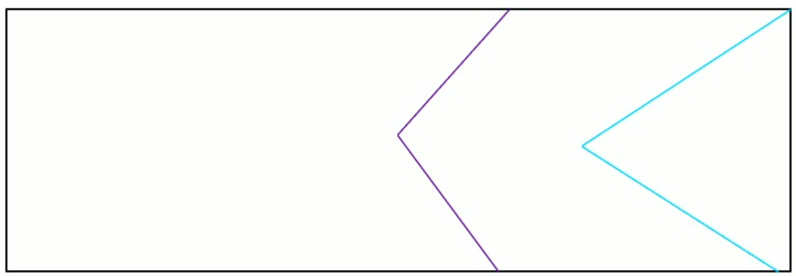
Consider 4 scalar fields that form coordinates for some region of spacetime in an open set of solution space. The codimension-2 edge of a subsystem can be located as a fixed function of these coordinates.

There are variations on this theme that are more physically sensible, using curvature scalars [Bergmann+Komar 1960] or dust fields [Brown+Kuchar 1994].



Example: Causal and Extremal Wedges

Now consider a two-sided aAdS spacetime, and a boundary spatial subregion. The causal wedge is the intersection of the causal future and past. Gauge-fixing the light sheets to fixed locations exhibits this as an internally dressed subsystem. The causal complement is not internally dressed, and the two do not commute.



However, the entanglement wedge bounded by the HRT surface is specified by a condition supported purely on the codimension-two edge. This means the causal complement is *also* internally dressed. They form commuting algebras.

Surface Charges

Charges that generate surface symmetries of a subsystem, while commuting with the exterior, have some applications [Donnelly+Freidel 2016]. They are locally constructed on the entangling surface, and are *not* regular observables. They can still be interpreted as generating a distributional flow by dualization. There is not a unique charge associated with a given symmetry; the exact choice depends on the singularity structure of the distribution.

Surface Charge : Electromagnetism

Consider a "gauge transformation" supported on a Cauchy surface in electromagnetism.

If the transformation is defined as $\delta A = \theta d\lambda$, where θ is a step function, the generator of this flow is the smeared electric flux $\int_{\partial\Sigma} \lambda E$.

On the other hand, if it is defined as $\delta A = d(\theta\lambda)$, then the generator is *zero*; this is a "true singular gauge transformation".

The only difference between the two flows is a delta function contribution at the interface.

The latter flow is an “on-shell” flow; the distribution satisfies the equations of motion.

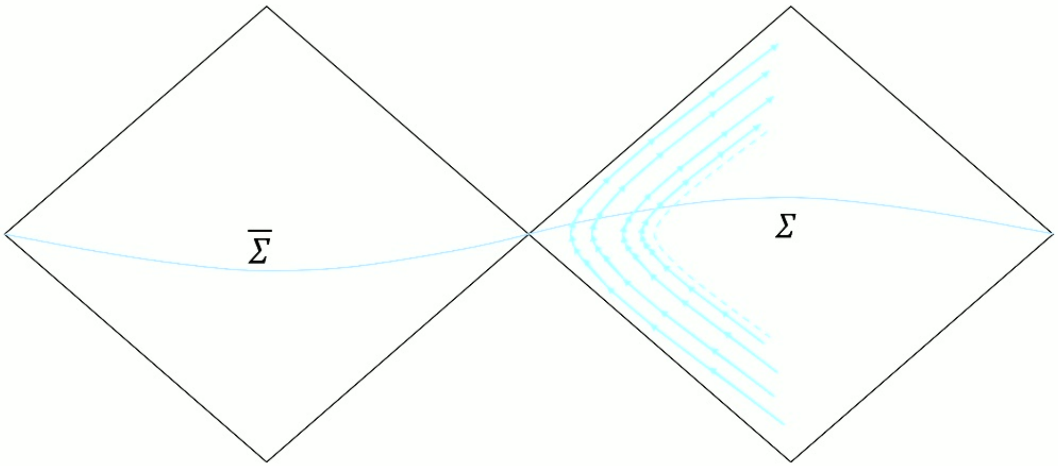
The former is not. It is only on-shell when the surface is *extremal,* as only then is the area invariant at linear order.

Surface Charge : Kink Transform

A similar story applies for the “one-sided boost” around a codimension-two surface.

If the infinitesimal flow is defined as $\delta g = \theta \mathcal{L}_\xi g$, its infinitesimal generator is (minus) the Bekenstein-Hawking area term.

If instead it is $\delta g = \mathcal{L}_{\theta\xi} g$, then the generator is zero.



Quantization

The natural framework to extend this to higher orders in $\hbar$ is deformation quantization. This is nontrivial to apply to field theories, due to the issue of renormalization.