Title: Particle Acceleration in Magnetically-Dominated Turbulence

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Collection/Series: Magnetic Fields Around Compact Objects Workshop

Subject: Strong Gravity

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Abstract:

Building on recent advancements in understanding particle transport in magnetized media, we present a first-principles scaling law for the formation of non-thermal tails in particle spectra within mildly and strongly magnetized turbulent plasmas. This scaling is validated using results from kinetic Particle-In-Cell simulations, which show excellent agreement with our theoretical predictions. Finally, we discuss the astrophysical implications of these findings, particularly for the proton spectra in the coronae of supermassive black holes.

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Non-Thermal Power Laws in Magnetized Turbulent Plasmas

Rostom Mbarek

In collaboration with
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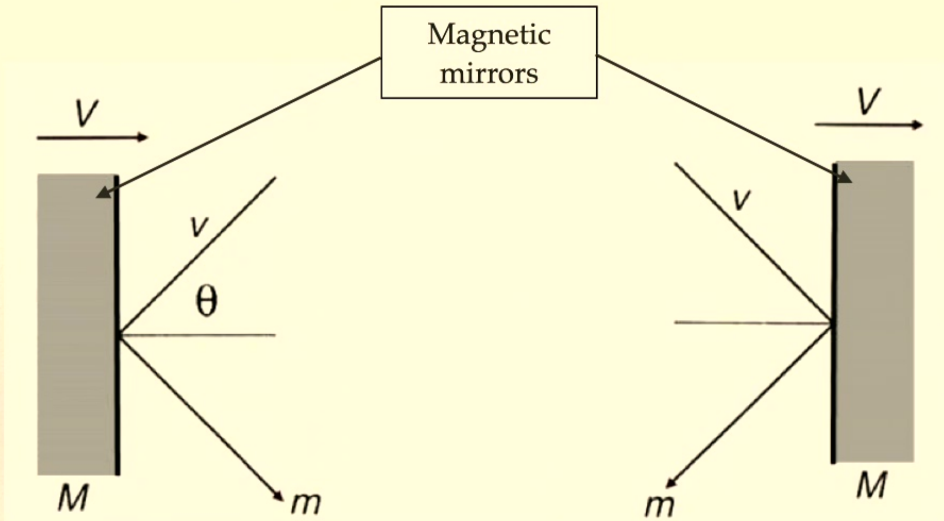
All about understanding Particle Acceleration

Fermi acceleration



head-on collision results in an energy gain

tail-on collision results in an energy loss

head-on collisions are more likely than tail-on collisions, so we end up with a net energy gain

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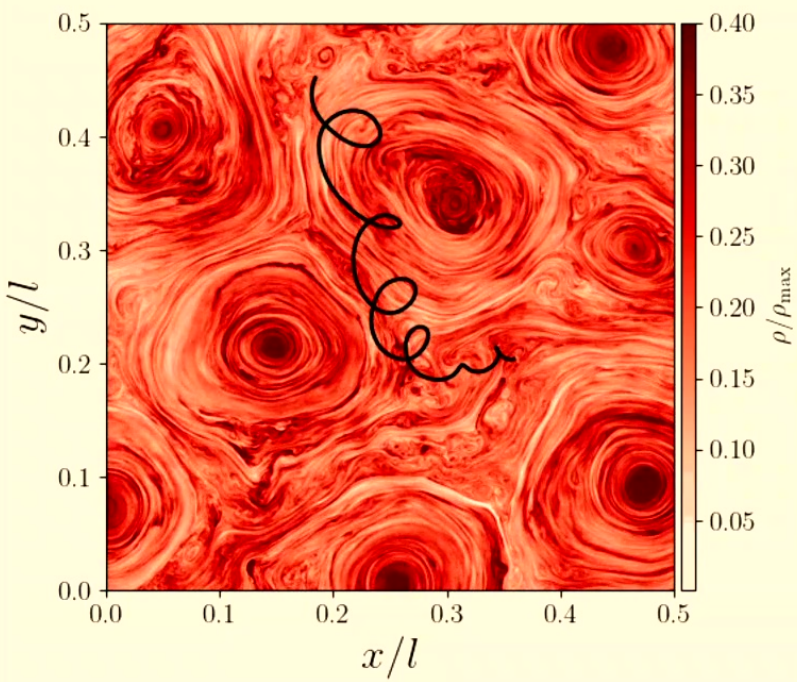
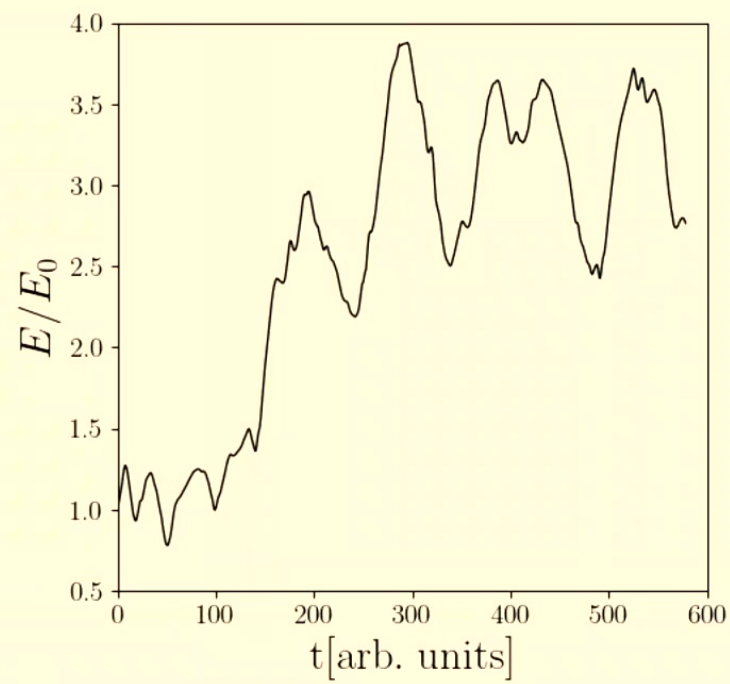
Fermi Acceleration: Particle trajectory and Energy gain in PIC simulations



Particle Trajectory



Particle energy gain through the Fermi process

Generally dealt with using a Fokker-Planck formalism

$$\frac{\delta f_0}{\delta t} - \nabla_i D_{ij} \nabla_j f_0 + (\vec{u} \cdot \nabla) f_0 - \frac{\nabla \cdot \vec{u}}{3} p \frac{\delta f_0}{\delta p} = \frac{1}{p^2} \frac{\delta}{\delta p} p^2 D_{pp} \frac{\delta f_0}{\delta p}$$

spatial diffusion (D_{ij}) — momentum diffusion (D_{pp})

However: The validity and appropriate form of the Fokker–Planck equation for turbulence is not yet known (e.g. Zhdankin et al. (2020), Isliker et al. (2020))

Alternative: focus on the power law

Essence of the Fermi mechanism and Bell (1978) for general power laws:
$\mathscr{G}$: average energy gain per collision
$\mathscr{P}$: probability of remaining in the accelerating region after one collision

After k collisions, $N = N_0 \mathscr{P}^k$ particles with energies $E = E_0 \mathscr{G}^k$.

Eliminating k: $$\frac{N(>E)}{N_0} = \left(\frac{E}{E_0}\right)^{\ln \mathscr{P}/\ln \mathscr{G}}$$

$$\frac{dn}{d\gamma} \propto \gamma^{-1+\frac{\ln \mathscr{P}}{\ln \mathscr{G}}}$$

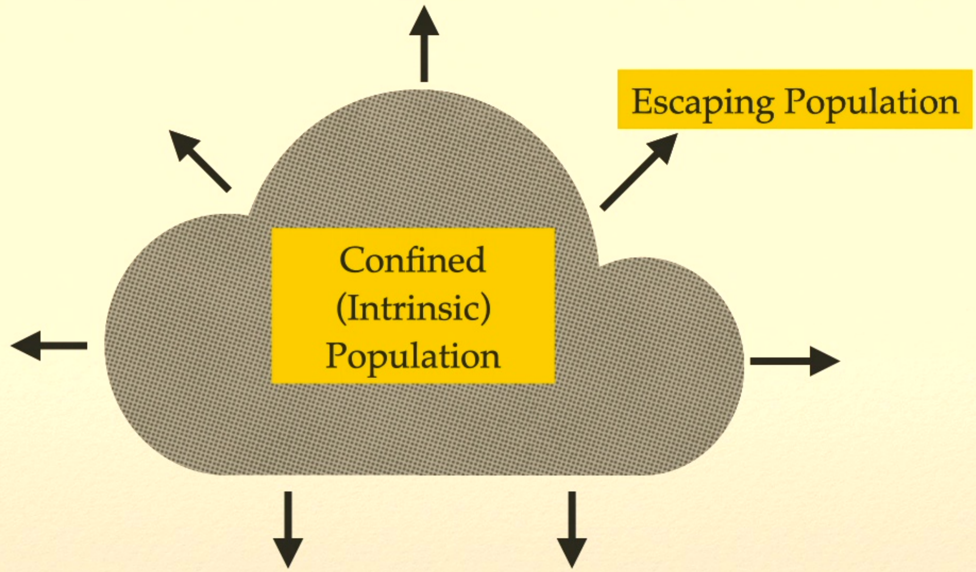
If magnetized turbulent plasma, $\sigma n(\sigma) \simeq \gamma n(\gamma)$ for $\gamma > \sigma$, we get

$$EN(>E) = \sigma mc^2 N_0 \mathscr{P}^p$$

$$\frac{dn}{d\gamma} \propto \gamma^{-2+\frac{\ln \mathscr{P}}{\ln \mathscr{G}}}$$

Need to figure out $\mathscr{P}$ and $\mathscr{G}$

Some Definitions



$\mathscr{R}$: Larmor radius of particles

ℓ_c: coherence length of the magnetic field

$d_e = \sqrt{\frac{m_e c^2}{4\pi \bar{n}_e e^2}}$: skin depth of the plasma

$$\sigma = B^2/(4\pi n m c^2)$$

$$\delta B/B \simeq 1$$

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Confined Population: Probability $\mathscr{P}_{\rm conf}$

$$\frac{dn}{d\gamma} \propto \gamma^{-2+\frac{\ln \mathscr{P}_{\rm conf}}{\ln \mathscr{G}}}$$

The evolution of the confined particles can be described with a Poisson process

$$\frac{dn_{\rm conf}}{dt} = -\, n_{\rm conf}/t_A \text{ with } t_A = \ell_{\rm c}/v_A$$

Solution to this process

$$n_{\rm conf} \propto e^{-t/t_{\rm A}}$$

For higher-energy particles $\gamma \to \gamma_{\rm max}$

$$n_{\rm conf} \propto e^{-t_{\rm acc}/t_{\rm A}}$$

$$\mathscr{P}_{\rm conf} = e^{-t_{\rm acc}/t_{\rm A}}$$

Confined Population: Energy Gain $\mathcal{G}$

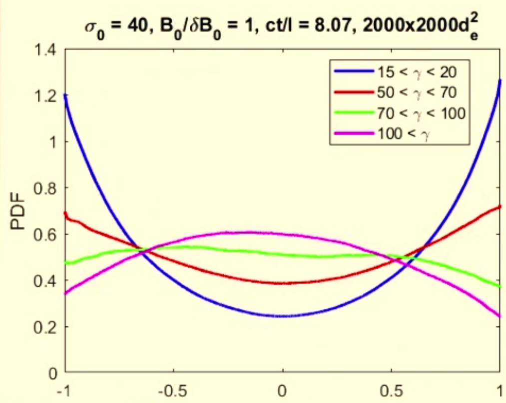
$$\frac{dn}{d\gamma} \propto \gamma^{-2-\frac{(t_{acc}/t_A)}{\ln \mathcal{G}}}$$

For plasma with $\sigma_p \geq 0.1$ and $\delta B/B \gtrsim 1$,
Vega et al. (2024) pointed out:

$$\Delta\gamma/\gamma \simeq 2\beta_b \cos\theta$$

$\beta_b = \sqrt{\sigma_p/(1+\sigma_p)}$ is the bulk velocity of the plasma

θ is the pitch angle of the particles



Vega et al. (2024)

Distribution is rather uniform with $\langle \cos\theta \rangle \simeq 2/\pi$

$$\mathcal{G} \simeq 1 + \frac{4}{\pi}\sqrt{\sigma_p/(1+\sigma_p)}$$

Confined Population: Acceleration Time

$$\frac{dn}{d\gamma} \propto \gamma^{-2-\frac{(t_{acc}/t_A)}{\ln \mathcal{G}}} \text{ with } \mathcal{G} \simeq 1 + \frac{4}{\pi}\sqrt{\sigma_p/(1+\sigma_p)}$$

$$t_{acc} = \gamma^2 / D_{\gamma\gamma}$$ → Energy Diffusion Coefficient

Definition of $D_{\gamma\gamma}$

$$D_{\gamma\gamma} = (\Delta\gamma)^2/(2\Delta t)$$

we have

$$\Delta\gamma \simeq 2\gamma\beta_b \cos\theta$$

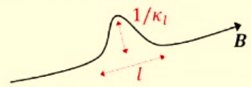
and $\Delta t = t_{sc}$ for this Fermi process

Need to figure out the scattering time t_{sc}

Interlude: The scattering time

$ct_{sc} \simeq \ell_c^{1-r} \mathscr{R}^r$ → Coefficient r dependent on properties of system; → Larmor radius

Lemoine (2023); Kempski et al. (2023)



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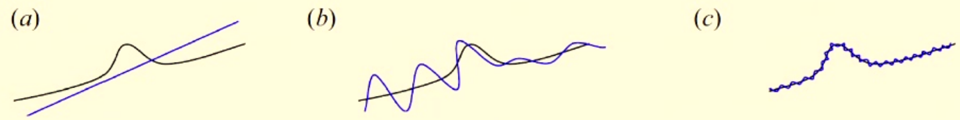
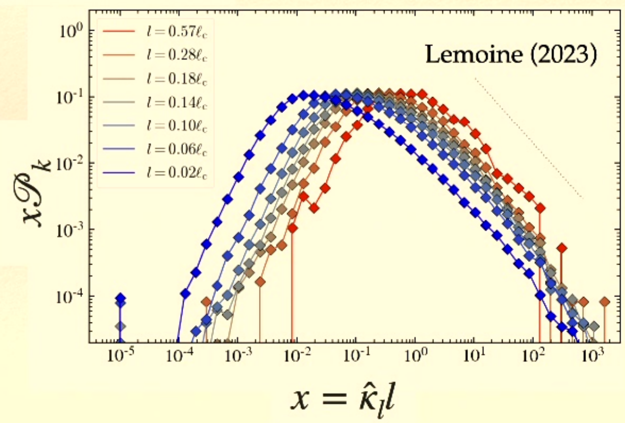


$\mathscr{R} \gg l$ | $\mathscr{R} \sim l,\ \kappa_l \mathscr{R} \gtrsim 1$ | $\mathscr{R} \ll l,\ \kappa_l \mathscr{R} \ll 1$

+ Probability of interaction depends on filling factor at each scale:

$$ct_{sc} \simeq \frac{l}{\int_1^\infty dx \mathscr{P}_k(x)} \text{ for } l \sim \mathscr{R}$$

+ The slope α sets the probability distribution with $\mathscr{P}_\kappa \propto (\hat{\kappa}_l l)^{-\alpha}$ for $l\hat{\kappa}_l \gtrsim 1$



Probability distribution of the magnetic field curvature $\mathscr{P}_\kappa$ in the system

Slope of non-thermal confined population in magnetized turbulence

Let's go back to the Acceleration Time

$$t_{\rm acc} \simeq \frac{\pi}{2c}\left[\frac{1+\sigma_p}{\sigma_p}\right]\ell_c^{1-r}\mathcal{R}^r$$

⟶ Extracted directly from the $\mathcal{P}_k$ distribution

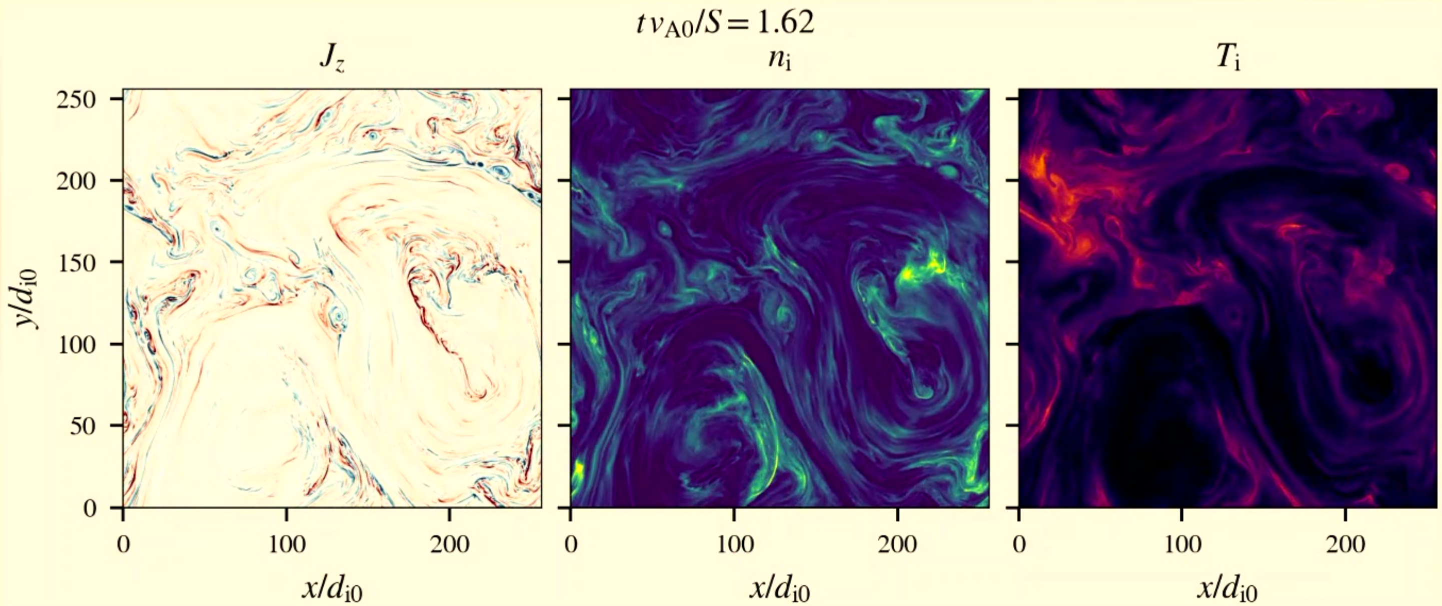
$$\frac{dn}{d\gamma} \propto \gamma^{-s}$$

$$s \simeq 2 + \frac{\pi}{2}\sqrt{\frac{1+\sigma_p}{\sigma_p}\left[\frac{\gamma\frac{m_p}{m_e}d_e}{\sqrt{\sigma_e}\ell_c}\right]^r\frac{1}{\ln\mathcal{G}}}$$

Since $\mathcal{R} = \gamma\frac{m_i}{m_e}d_e/\sqrt{\sigma}$

Does this work with PIC?

- Simulation Properties
  - 2D driven ion-electron turbulence with mass ratio $m_p/m_e = 5$, $d_e = 1.5$
  - $\delta B/B = 1$
  - $t_{\rm esc} = L/4c$: particles are taken out when their displacement reaches L/4



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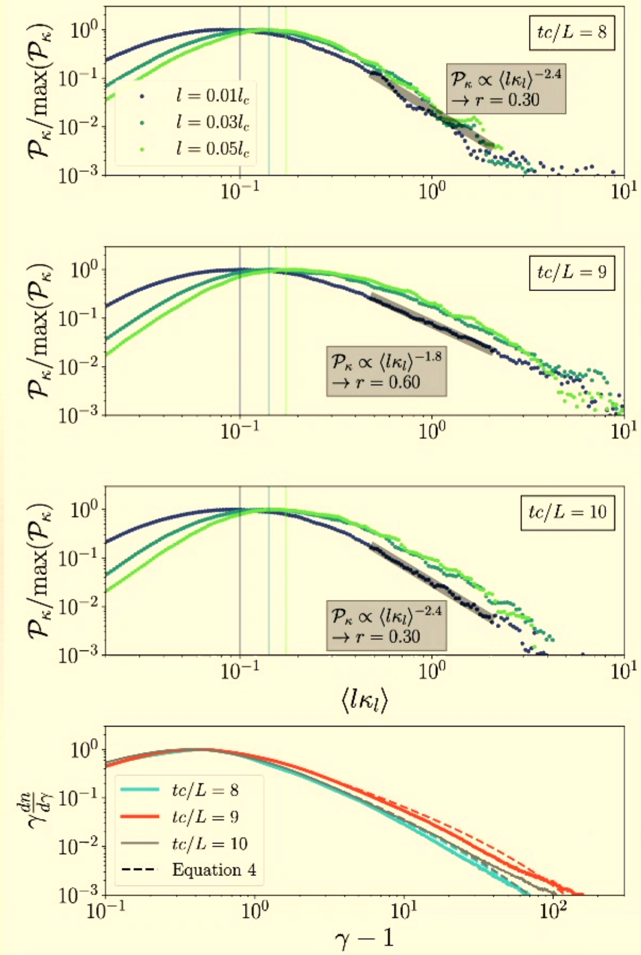

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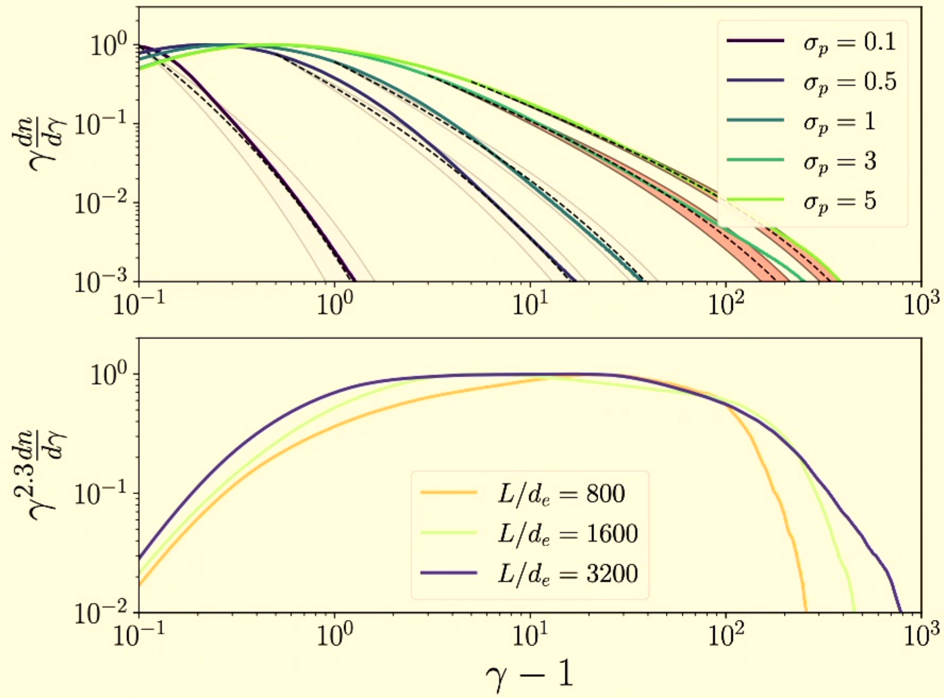
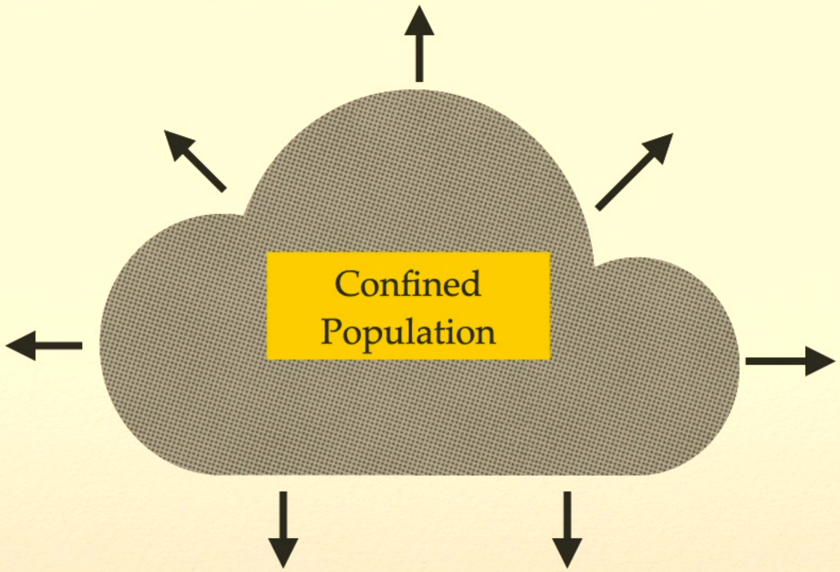
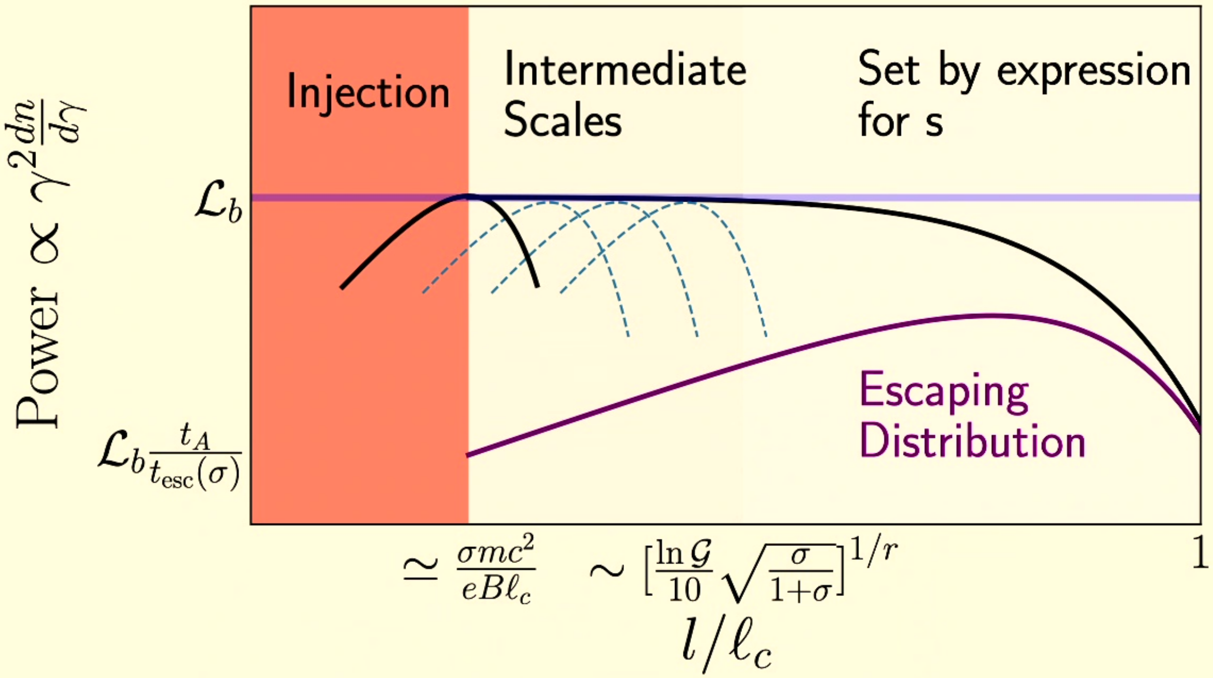


Now, the escaping population



Summary Plot



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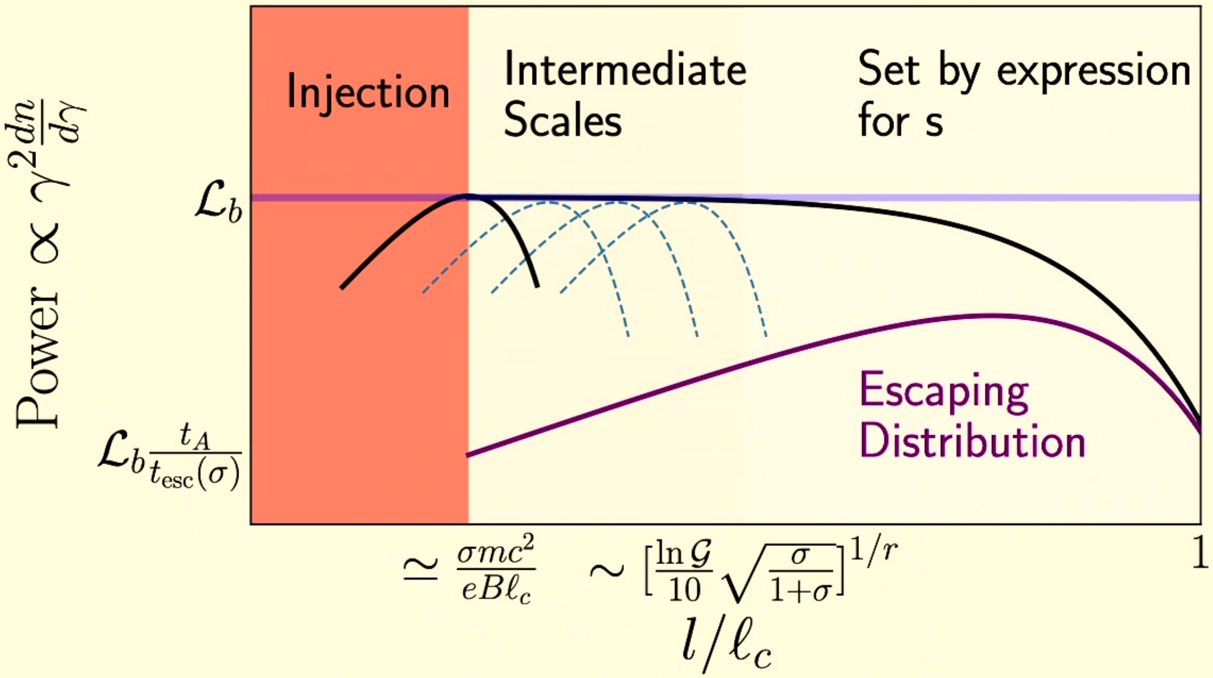
Summary Plot



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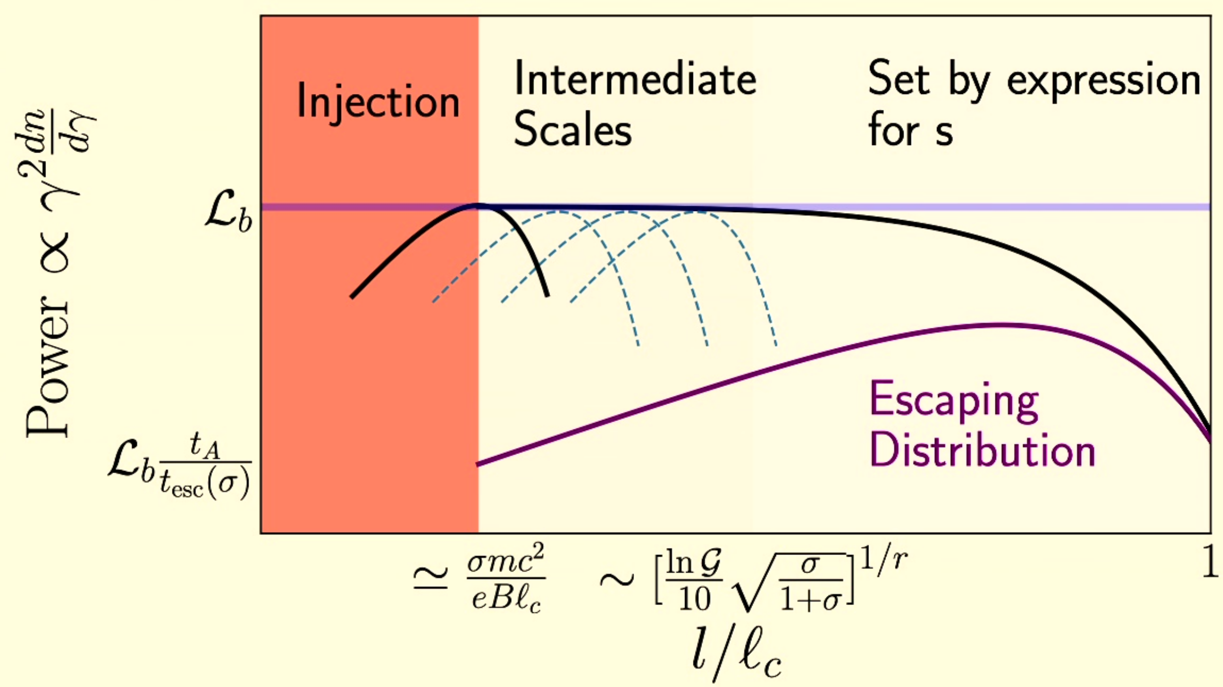

$$s \simeq 2 + \frac{\pi}{2}\sqrt{\frac{1+\sigma_p}{\sigma_p}}\left[\frac{\gamma \frac{m_p}{m_e} d_e}{\sqrt{\sigma_e}\ell_c}\right]^r \frac{1}{\ln \mathcal{G}}$$

Implications

- **Astrophysical:** Directly relating non-thermal emission from Jets, coronae, magnetospheres … to plasma properties in the source
- **For Simulations:**
  - Expected non-thermal emission in MHD simulations
  - Injection of non-thermal test-particle (hadrons) in turbulent regions for hadronic cooling