Title: Using continuous variable systems to unearth quantum nonlocality and to perform quantum information processing

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Abstract:

Continuous variable (CV) systems play an important role in quantum foundations and quantum information processing, especially because of their quantum optical realization. Gaussian states of CV systems, on the one hand have non-trivial quantum properties, and on the other hand, can be realized in the laboratory. There are non-Gaussian states which can be obtained from Gaussian states via physically realizable operations and these too can be useful in enhancing non-classicality and improving the performance of quantum information processing protocols. In the talk, I will introduce CV systems and their Gaussian and non-Gaussian states. The violation of Bell-type inequalities using CV systems will be discussed. Quantum key distribution protocols and quantum teleportation schemes using Gaussian and non-Gaussian states will also be taken up.

Using continuous variable systems to unearth quantum nonlocality and to perform quantum information processing

Perimeter Institute, Waterloo, CA, March 17, 2025

Arvind

March 17, 2025

Indian Institute of Science Education and Research, Mohali

Two approaches to QIP

System observables used for information encoding have discrete eigenvalues: discrete variable QIP, Qubits, Qudits

System observables with continuous eigenvalues: continuous variable (CV) QIP

- Harmonic oscillator
- Physical realization: quantized electromagnetic field

Advantages

- Can propagate over long distances with little decoherence
- Can be implemented using the existing hardware

Representation in Hilbert space

N-mode CV system $\equiv$ N harmonic oscillators

- Representation $(\hat{q}_1, \hat{p}_1, \ldots, \hat{q}_i, \hat{p}_i, \ldots \hat{q}_N, \hat{p}_N)$
- Commutation relation ($\hbar = 1$): $[q_i, p_j] = i\delta_{ij}$
- Anhilation and creation operators:

$$\hat{a}_i = \frac{1}{\sqrt{2}}(\hat{q}_i + i\hat{p}_i), \quad \hat{a}_i^\dagger = \frac{1}{\sqrt{2}}(\hat{q}_i - i\hat{p}_i)$$

- Fock states are eigenstates of number operator $\hat{N}_i = \hat{a}_i{}^\dagger \hat{a}_i$:

$$\hat{N}_i|n_i\rangle = n_i|n_i\rangle$$

- Fock states, $\{|n_i\rangle\}_{n_i=0}^{\infty}$, form an orthonormal basis of the infinite-dimensional Hilbert space.

Phase space representation

- It is convenient to represent infinite dimensional system in phase space.
- N-mode CV system can be represented in $2N$-dimensional phase space
- $2N$ variables: $\xi = (q_1, p_1, \ldots, q_i, p_i, \ldots, q_N, p_N)^T$
- State representation: density operator $\Rightarrow$ quasiprobability distributions
- Wigner distribution of a single mode system with density operator $\hat{\rho}$:

$$W(q,p) = \int \frac{dq'}{2\pi} \exp(iq'p) \langle q - \frac{q'}{2} | \hat{\rho} | q + \frac{q'}{2} \rangle.$$

- Used for classifying Gaussian and non-Gaussian states

Gaussian states: Gaussian Wigner Distribution

- Completely specified by first and second-order moments
- Displacement vector and covariance matrix for single mode system

$$\boldsymbol{d} = \begin{pmatrix} \langle \hat{q} \rangle \\ \langle \hat{p} \rangle \end{pmatrix}, V = \begin{pmatrix} \langle \hat{q}^2 \rangle - \langle \hat{q} \rangle^2 & \frac{1}{2} \langle \hat{q}\hat{p} + \hat{p}\hat{q} \rangle - \langle \hat{q} \rangle \langle \hat{p} \rangle \\ \frac{1}{2} \langle \hat{q}\hat{p} + \hat{p}\hat{q} \rangle - \langle \hat{q} \rangle \langle \hat{p} \rangle & \langle \hat{p}^2 \rangle - \langle \hat{p} \rangle^2 \end{pmatrix},$$

- Thermal state

$$\boldsymbol{d} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, V = \begin{pmatrix} \langle \hat{N} \rangle_{\rho_{\text{th}}} + \frac{1}{2} & 0 \\ 0 & \langle \hat{N} \rangle_{\rho_{\text{th}}} + \frac{1}{2} \end{pmatrix}$$

Gaussian operations

Transform Gaussian states into Gaussian states

- Displacement operator implemented by linear Hamiltonian
- Symplectic operations or linear canonical transformations implemented by quadratic Hamiltonian

Three basic symplectic operations

- Phase change operation
- Squeezing operation
- Beam splitter

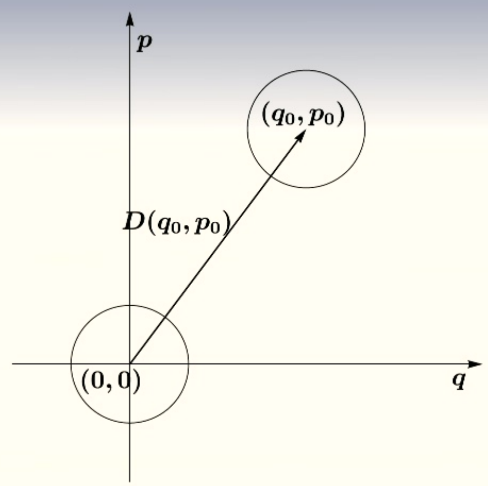
Displacement operator

- **Shifts the mean of a quantum state**
- $D(q_0, p_0) = \exp[i\underbrace{(p_0\hat{q} - q_0\hat{p}]}_{\text{Linear}})$
- $D(\alpha) = \exp(\underbrace{\alpha\hat{a}^\dagger - \alpha^*\hat{a}}_{\text{Linear}}),$

  $\alpha = \frac{q_0 + ip_0}{\sqrt{2}}$

- Coherent state as eigenstate of annihilation operator $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$
- Expansion in Fock basis:

$$|\alpha\rangle = D(\alpha)|0\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$



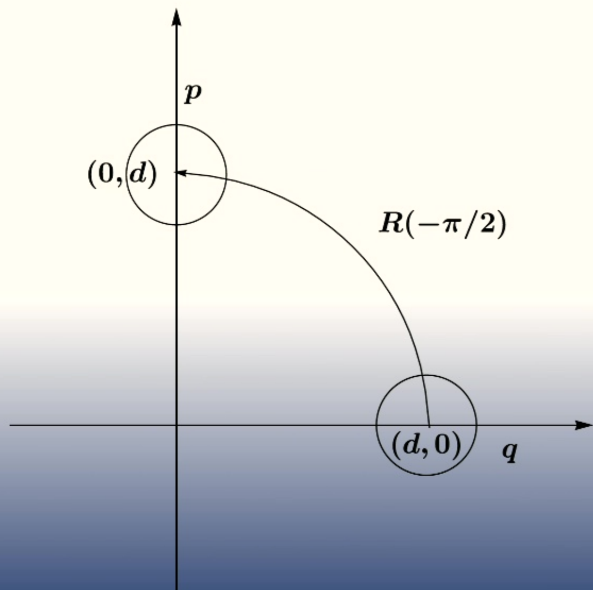
Displacement operator transforms vacuum state into coherent state

Mean: $\langle\hat{q}\rangle = q_0, \quad \langle\hat{p}\rangle = p_0$

Variance: $(\Delta q)^2 = (\Delta p)^2 = \frac{1}{2}$

Phase change operation
$$\exp(-i\phi \underbrace{\hat{a}^\dagger \hat{a}}_{\text{Quadratic}})$$
produces a rotation in the phase space

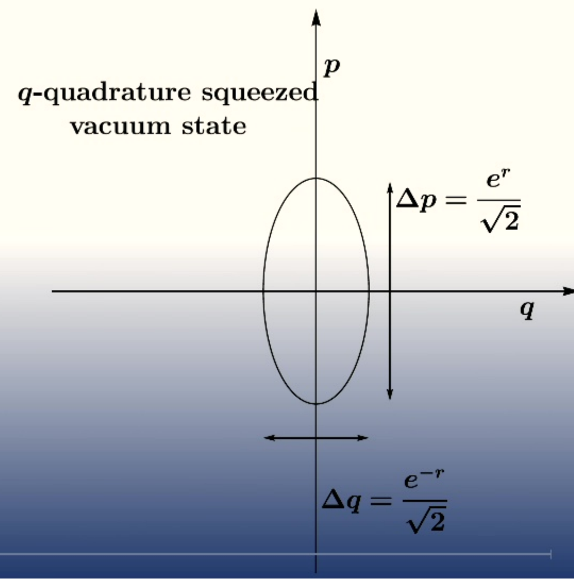
$$\begin{pmatrix}\hat{q}\\ \hat{p}\end{pmatrix} \to \underbrace{\begin{pmatrix}\cos\phi & \sin\phi\\ -\sin\phi & \cos\phi\end{pmatrix}}_{\text{Phase change transformation}} \begin{pmatrix}\hat{q}\\ \hat{p}\end{pmatrix}$$



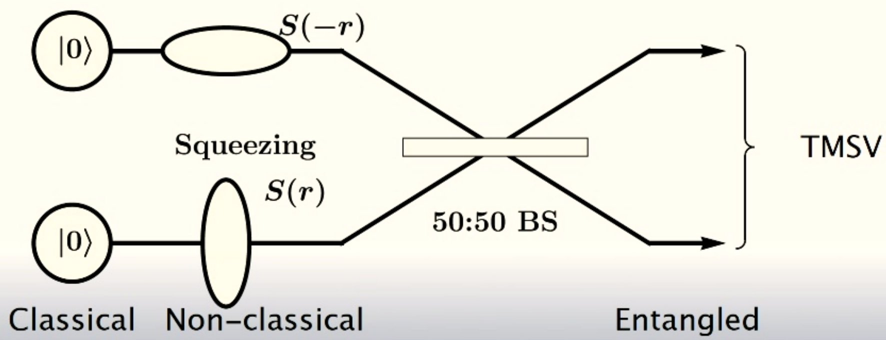
Squeezing operation reduces variance of one of the quadratures

$$\exp[r \underbrace{(a^2 - \hat{a}^{\dagger 2})}_{\text{Quadratic}}/2]|0\rangle = \sum_{n=0}^{\infty} c_n |2n\rangle.$$

$$\begin{pmatrix}\hat{q}\\ \hat{p}\end{pmatrix} \to \underbrace{\begin{pmatrix}e^{-r} & 0\\ 0 & e^{r}\end{pmatrix}}_{\text{Squeezing}} \begin{pmatrix}\hat{q}\\ \hat{p}\end{pmatrix}$$

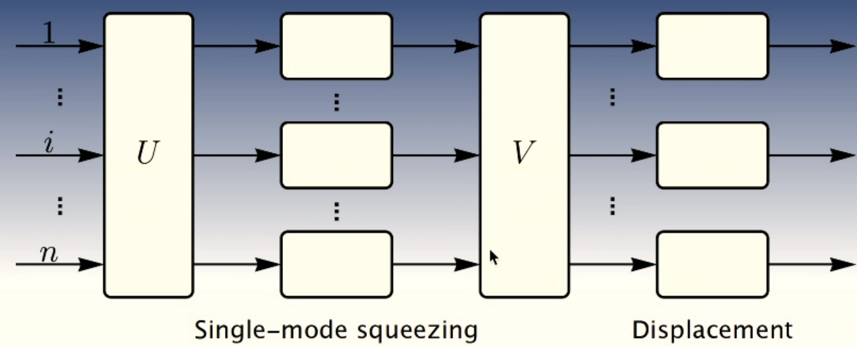


- **Beam splitter** $\exp[\theta(\hat{a}_1^\dagger \hat{a}_2 - \hat{a}_1 \hat{a}_2^\dagger)]$, Transmissivity $\tau = \cos^2\theta$
- Can generate entangled state from separable nonclassical state
- $$\underbrace{|1\rangle}_{\text{Nonclassical}} \underbrace{|0\rangle}_{\text{Classical}} \xrightarrow{\text{50:50 BS}} \underbrace{\frac{1}{\sqrt{2}}(|1\rangle|0\rangle - |0\rangle|1\rangle)}_{\text{Singlet (entangled)}}$$
- Generation of two mode squeezed vacuum (TMSV) state



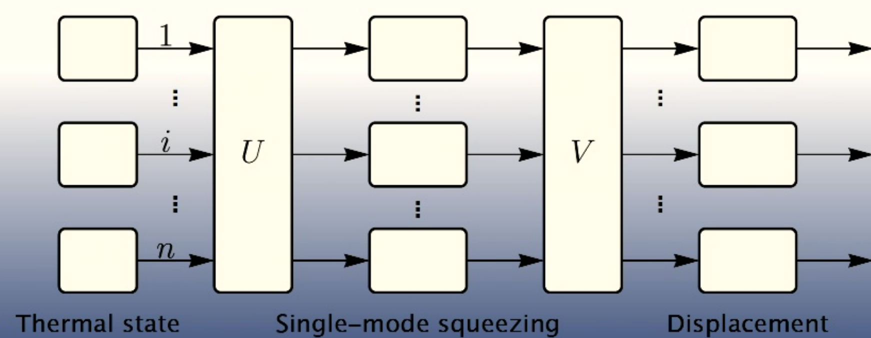
$$|\Psi\rangle \propto \sum_{n=0}^{\infty}(-\tanh r)^n |n\rangle|n\rangle$$

- General Gaussian operation of n-mode system



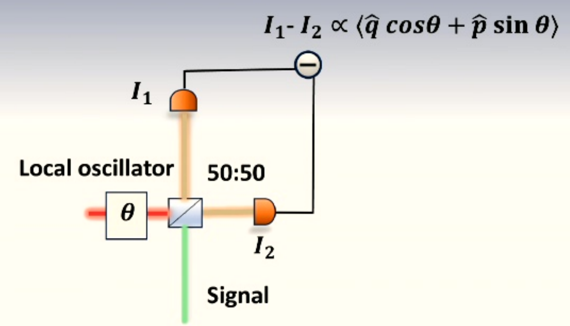
U and V are networks of beam splitters and phase shifters

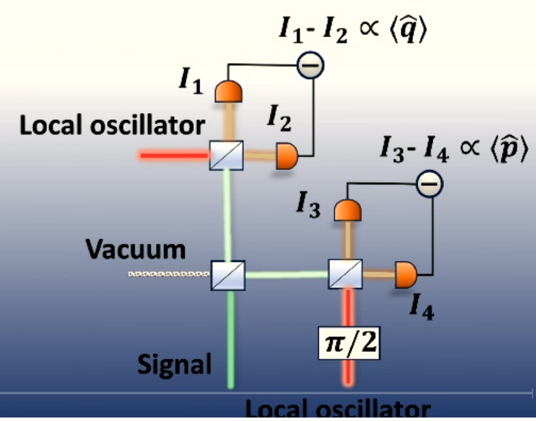
- General Gaussian state of n-mode system



Gaussian measurements

- The probability distribution of the measurement outcome of a Gaussian state is Gaussian
- **Homodyne measurement:** Measurement of the rotated quadrature $\hat{q}\cos\theta + \hat{p}\sin\theta$
- Special cases: $\hat{q}$ $(|q\rangle\langle q|)$, $\hat{p}$ $(|p\rangle\langle p|)$
- **Heterodyne measurement:** Joint measurement of $\hat{q}$ and $\hat{p}$ -quadratures $(|\alpha\rangle\langle\alpha|)$
- Experimental implementation of double homodyne





Estimation of single mode Gaussian states

- Compare the relative performance of four Gaussian measurement schemes
- **Homodyne measurement**
- **Heterodyne measurement**
- **Sequential measurements of conjugate observables:** Weak measurement of $\hat{q}$-quadrature followed by homodyne measurement of $\hat{p}$-quadrature
- **Arthurs-Kelly measurement scheme:** Both $\hat{q}$ and $\hat{p}$-quadratures are measured simultaneously; however, one of the quadratures can be measured more precisely at the cost of reducing the precision of the other quadrature

Estimation of the Wigner distribution of single-mode Gaussian states: A comparative study

Chandan Kumar, and Arvind

Phys. Rev. A 105, 042419 (2022).

Ensemble of displaced squeezed vacuum state

- Mean and covariance matrix

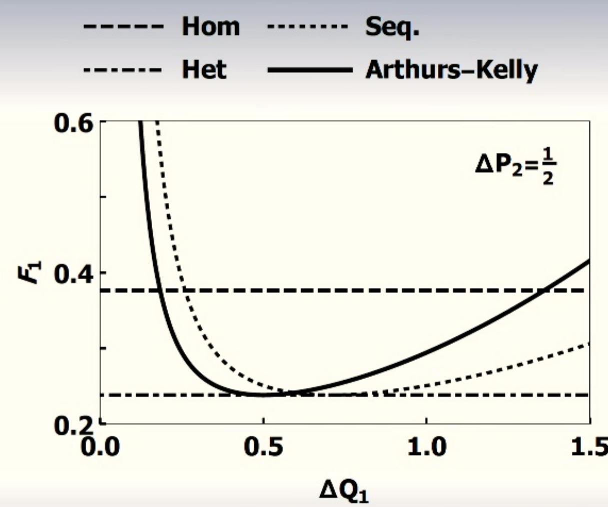
$$\boldsymbol{d} = \begin{pmatrix} q_0 \\ p_0 \end{pmatrix}, V = \frac{1}{2}\begin{pmatrix} \exp(-2r) & 0 \\ 0 & \exp(2r) \end{pmatrix}$$

- Measure for mean estimation

$$F_1 = \langle (q^A - q^M)^2 \rangle + \langle (p^A - p^M)^2 \rangle$$

A: Actual, M: Measured

- **Result:** Heterodyne performs better than homodyne in mean estimation.



Ensemble of 20 identically prepared displaced squeezed vacuum states $(r = 1)$

Photon counting (non-Gaussian) measurements

Perfect photon number resolving detector

- Unique response for every input photon number state

$$\text{Photon number projectors: } \{|n\rangle\langle n|\}_{n=0}^{\infty}, \ \sum_{n=0}^{\infty} |n\rangle\langle n| = I$$

Non-unique response for input photon number states

- **Detects up to k photons:** $\{|1\rangle\langle 1|, |2\rangle\langle 2|, \ldots, |k\rangle\langle k|, I - \sum_{n=1}^{k} |n\rangle\langle n|\}$
- **Single photon detector:** $\{|1\rangle\langle 1|, I - |1\rangle\langle 1|\}$
- **On-off detector:** $\{\underbrace{|0\rangle\langle 0|}_{\text{OFF}}, \underbrace{I - |0\rangle\langle 0|}_{\text{ON}}\}$

Glauber–Sudarshan P representation

- $$\hat{\rho} = \int_{\mathbb{C}} d^2\alpha P(\alpha) |\alpha\rangle\langle\alpha|$$

Classical state: statistical mixture of coherent states

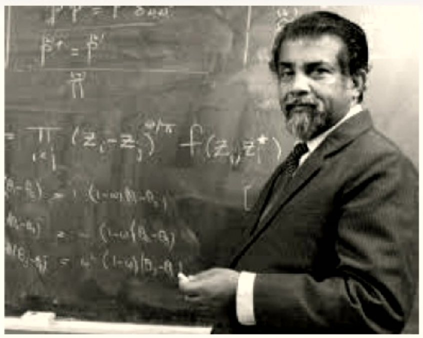
- $P(\alpha)$ function is positive and not too singular
- Coherent states $\rho = |\alpha_0\rangle\langle\alpha_0| \Rightarrow P(\alpha) = \delta^2(\alpha - \alpha_0)$
- $\rho = \sum_{n=0}^{\infty} \frac{\langle n\rangle^n}{(\langle n\rangle+1)^n} |n\rangle\langle n| \Rightarrow P(\alpha) = \frac{1}{\pi\langle n\rangle} \exp\left(-\frac{|\alpha|^2}{\langle n\rangle}\right)$

Nonclassical state: Fock state

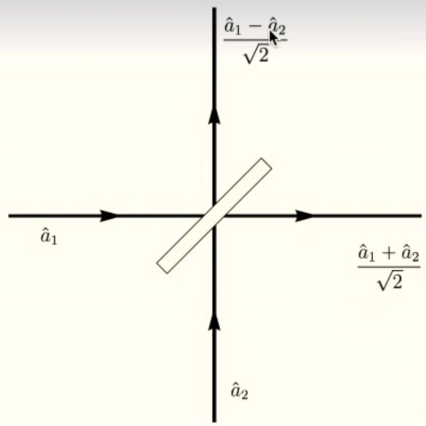
$$\rho = |n\rangle\langle n| \Rightarrow P(\alpha) = e^{|\alpha|^2} \frac{1}{n!} \frac{\partial^{2n}}{\partial\alpha^n \partial\alpha^{*n}} \delta^2\alpha$$

General picture

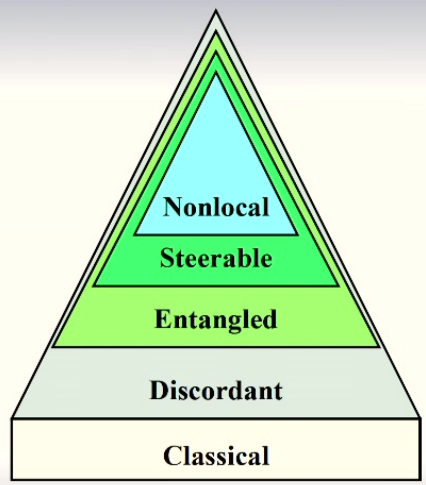
- Phase space nonclassicality vs. nonclassicality based on correlations



Phase space nonclassicality based on Sudarshan-Glauber representation



Generate intermode correlations using passive optics



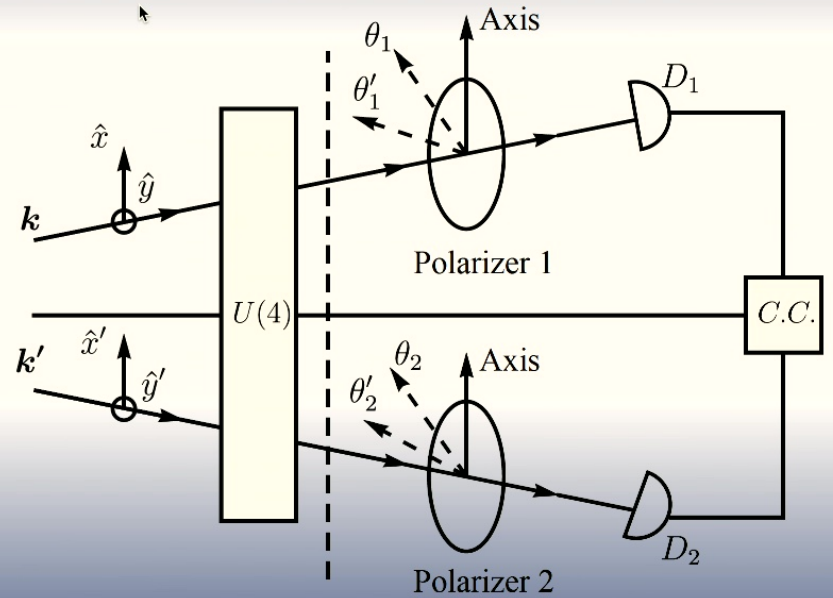
Hierarchy of quantum correlations present in mixed states.

- Nonclassicality in phase space: well defined.
- Nonclassicality based on correlations: still open problem.

Bell Nonlocality

- Based on the Clauser-Horne 1974 Bell test inequality

- Four modes: two propagation direction $\boldsymbol{k}$ and $\boldsymbol{k}'$ each with two polarizations
- Separable squeezed thermal state is mixed by passive $U(4)$ operation



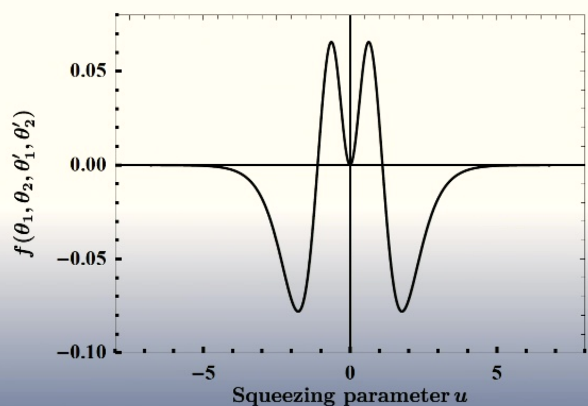
Arvind and N. Mukunda, Physics Letters A, 259, 6 (1999).

Chandan, Gaurav and Arvind, Phys. Rev. A 103, 042224 (2021).

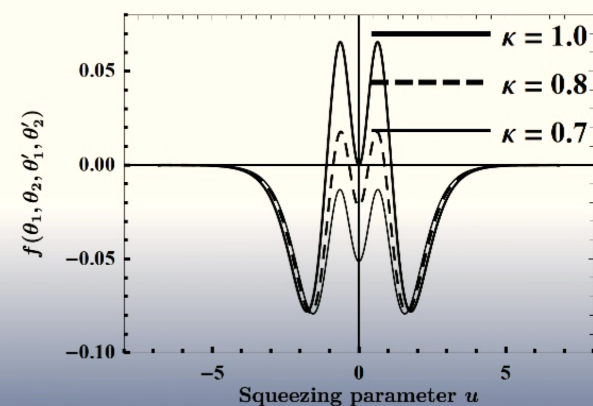
Bell inequality violation

$$-P(\ \ ,\ \ ) \leq \underbrace{P(\theta_1,\theta_2) - P(\theta_1,\theta_2') + P(\theta_1',\theta_2) + P(\theta_1',\theta_2') - P(\theta_1',\ \ ) - P(\ \ ,\theta_2)}_{f(\theta_1,\theta_2,\theta_1',\theta_2') = \text{Average of Bell operator}} \leq 0$$

- $P(\theta_1,\theta_2) =$ Coincidence count rate with polarizers set at θ_1 and θ_2
- Positive value of $f(\theta_1,\theta_2,\theta_1',\theta_2')$ indicates violation $\implies$ state has non-local quantum correlations

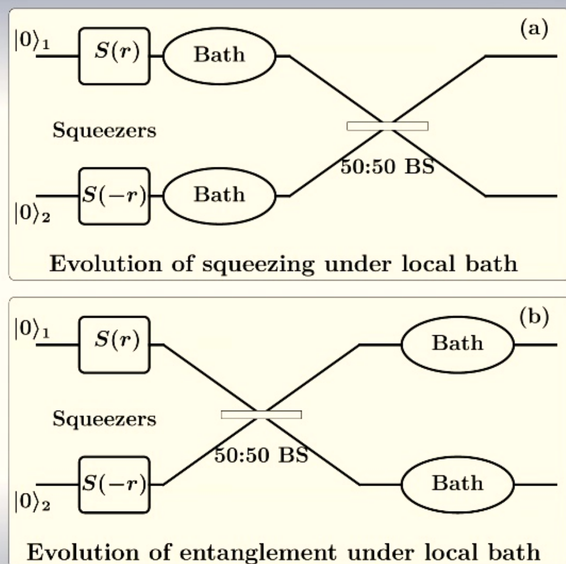


Four-mode squeezed vacuum state



Four-mode squeezed thermal state for different thermal parameters κ

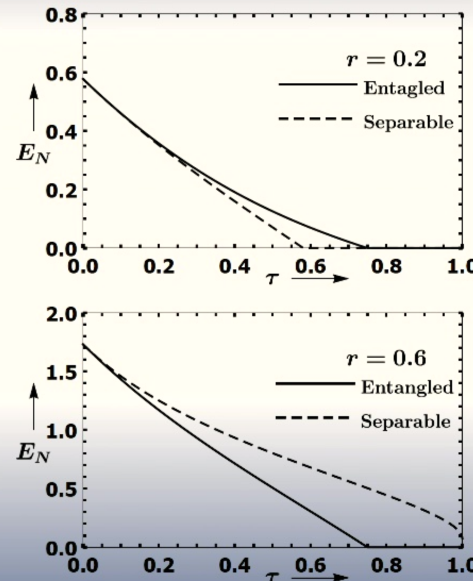
Robustness of squeezing and entanglement resources: local thermal baths



Both modes interact with identical local thermal baths:

resources decay at the same rate

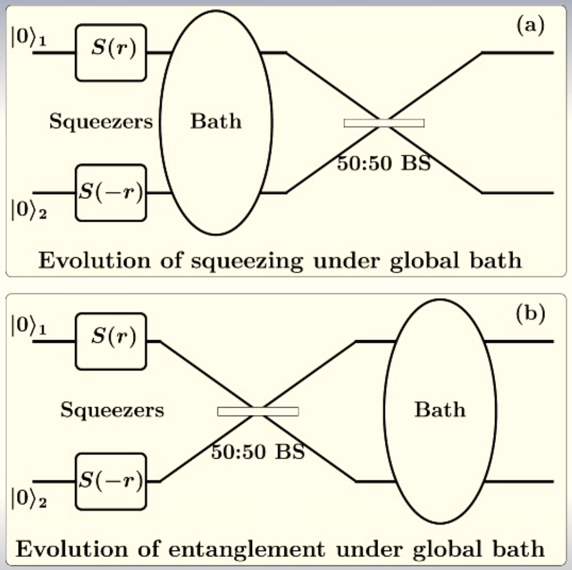
- One mode interacts with bath

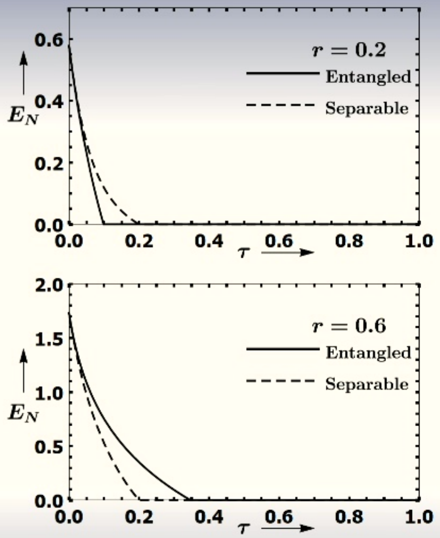


Logarithmic negativity E_N as a function of dimensionless time $\tau (= 1 - e^{-2\gamma t})$.

Rishabh, Chandan, Geetu and Arvind, Phys. Rev. A 105, 042405 (2022).

Global thermal baths

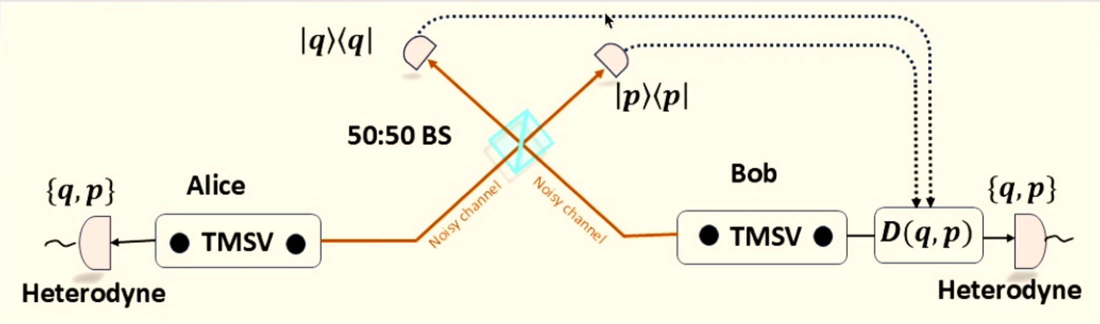




Logarithmic negativity vs time.

- Depending on the nature of dissipative environments and the initial squeezing, one can select the more robust form of resource out of squeezing and entanglement to store quantumness.

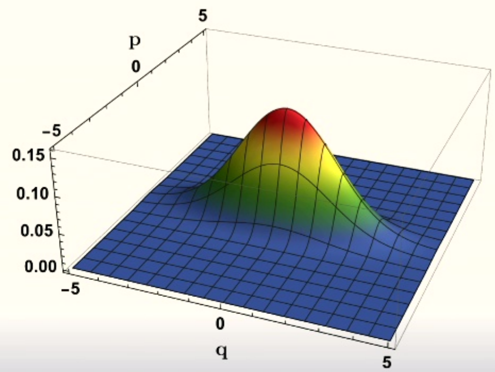
Measurement device independent quantum key distribution



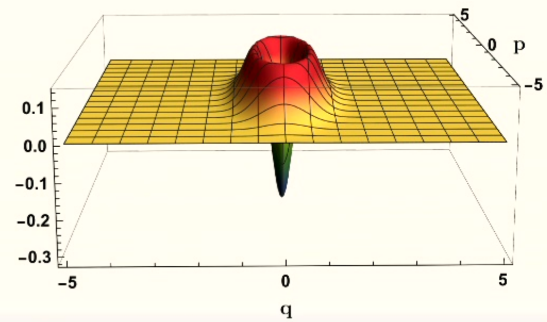
- Based on entanglement swapping
- $\Rightarrow$ Immune to measurement attacks

Optical state engineering using non-Gaussian operations

- Transform Gaussian states into non-Gaussian states
- Photon subtraction (PS) $\hat{a}$, photon addition (PA) $\hat{a}^\dagger$



Gaussian: squeezed vacuum state



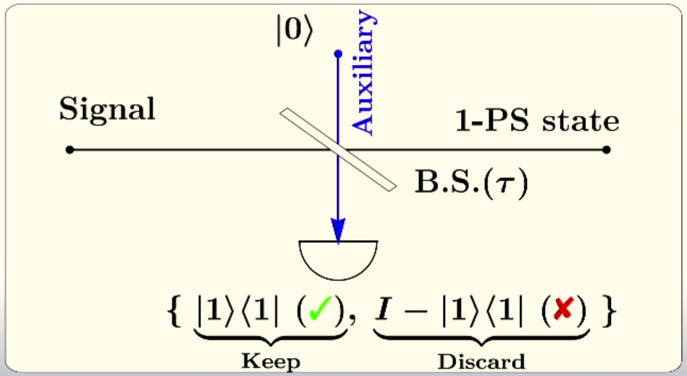
Non-Gaussian: photon subtracted squeezed vacuum state

- Essential for CV entanglement distillation, CV quantum computation
- Can enhance the performance of CV quantum protocols

Photon counting based implementation

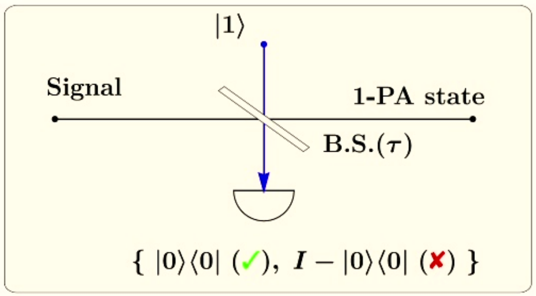
- $\hat{a}$ and $\hat{a}^\dagger$ are nonunitary
- Experimental implementation requires consideration of auxiliary mode

- Beam splitter and single photon detector
- Probabilistic process

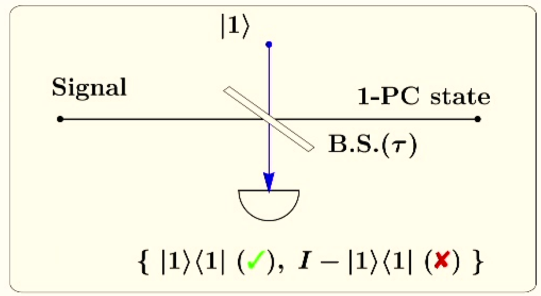


Single photon subtraction

Photon addition and photon catalysis

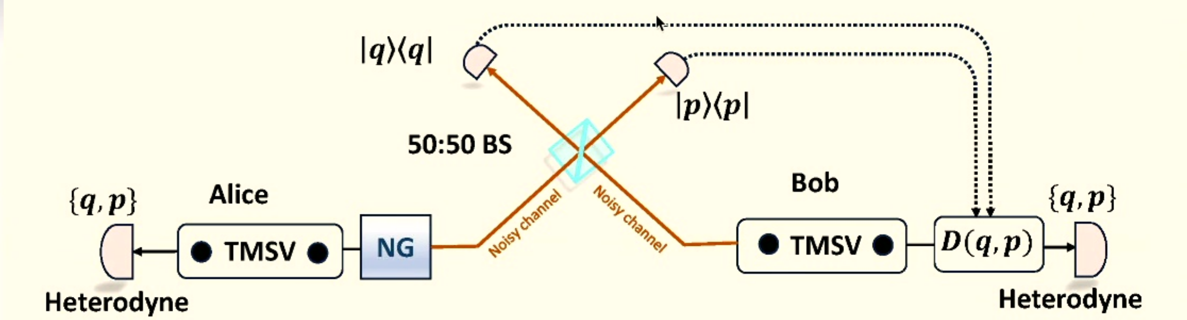


Single photon addition

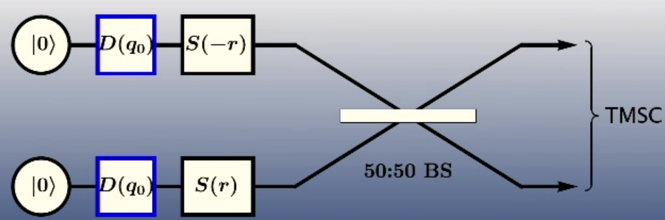


Single photon catalysis

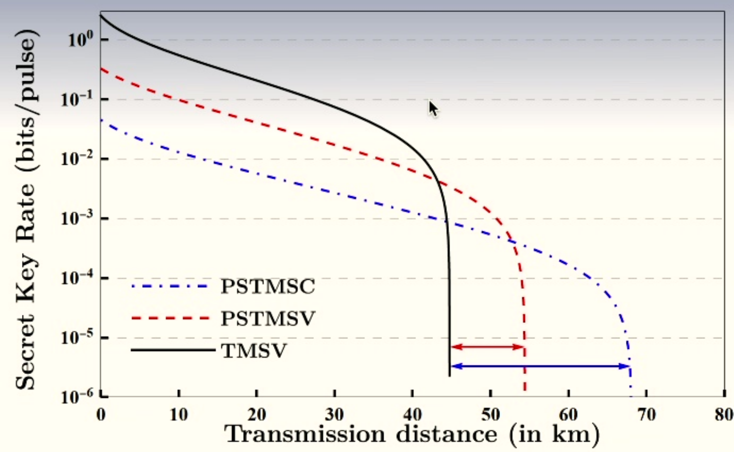
Measurement device independent quantum key distribution



- PSTMSV can enhance transmission distance[a]
- Introduce displacement $D(q_0) \Rightarrow$ Two mode squeezed coherent state



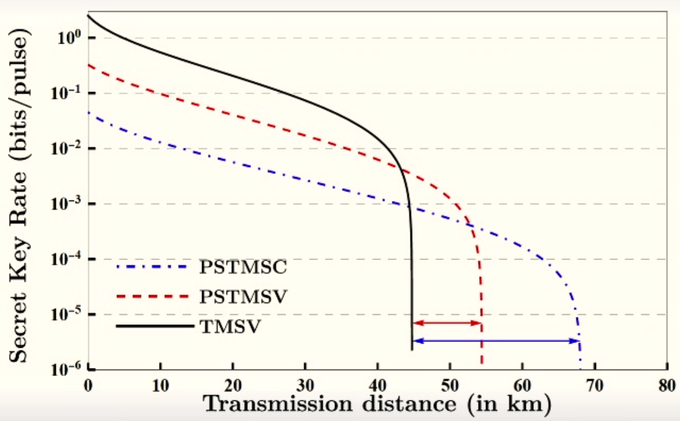
Positive effect of photon subtraction on displaced state

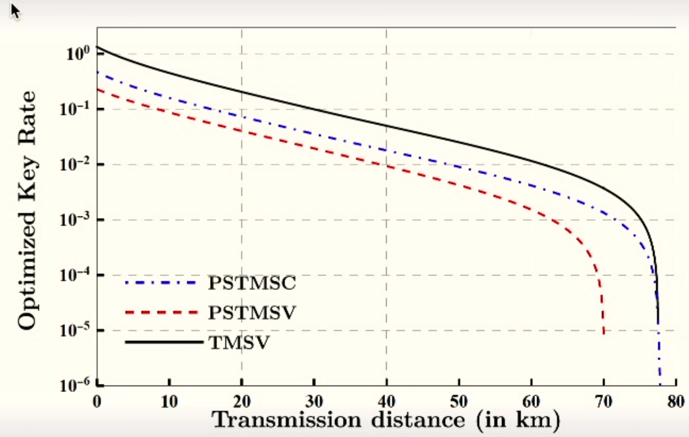


- Photon subtraction on displaced state can further enhance transmission distance[1]
- Result at fixed squeezing ($r = 2.3$), displacement ($d = 2$), and transmissivity ($\tau = 0.9$).

Parameter optimization

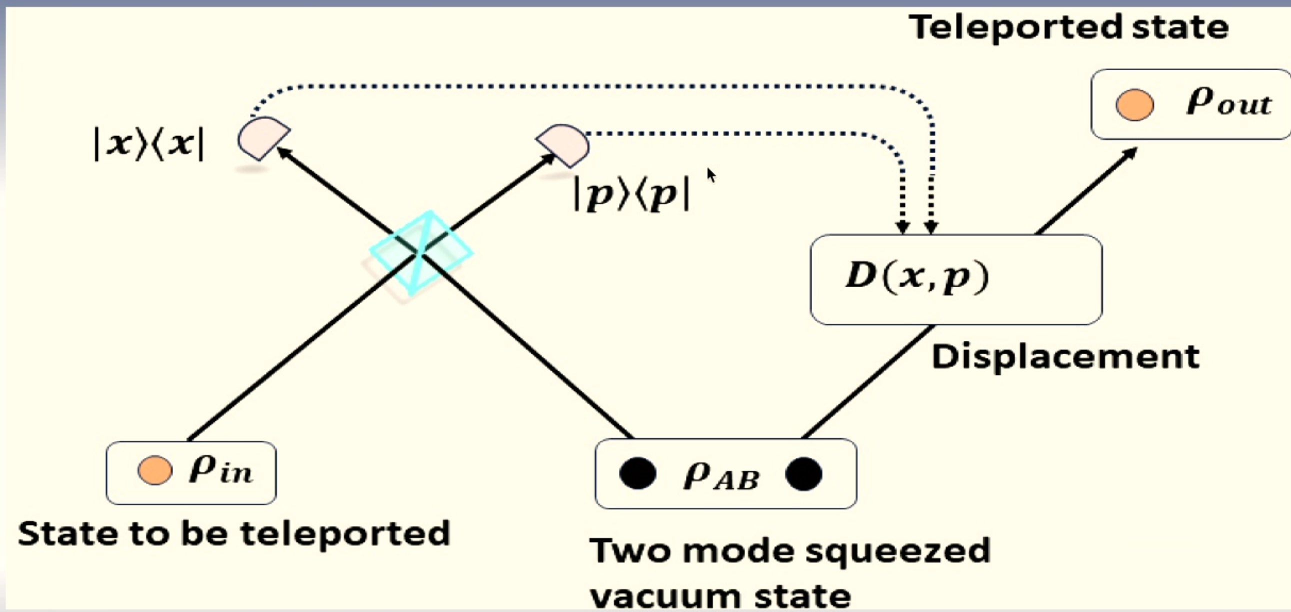
- Optimize secret key rate with squeezing, displacement and transmissivity.





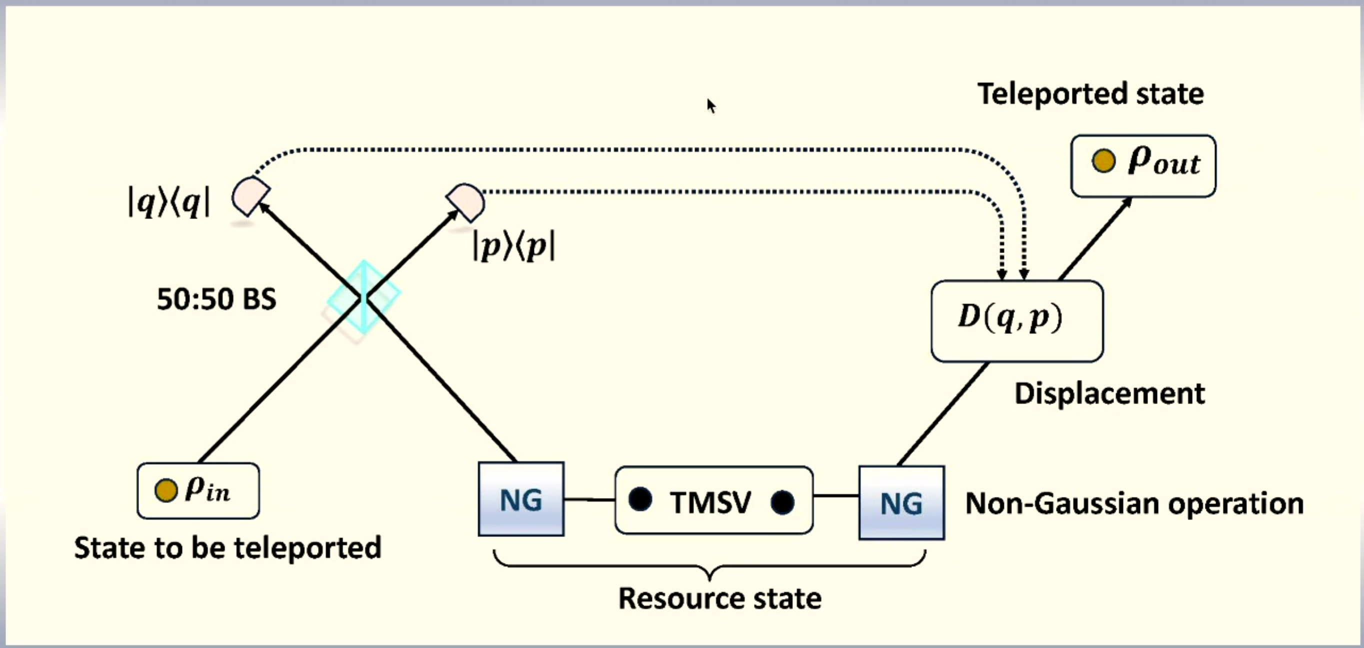
- No enhancement in transmission distance. Advantage was an artifact of working at high squeezing[1]
- PSTMSV result is wrong[2]

Teleportation using TMSV state



$$F = \frac{1+\lambda}{2}, \lambda = \tanh r, \quad F > 1/2 \Rightarrow \lambda > 0, \; F = 1 \Rightarrow \lambda = 1$$

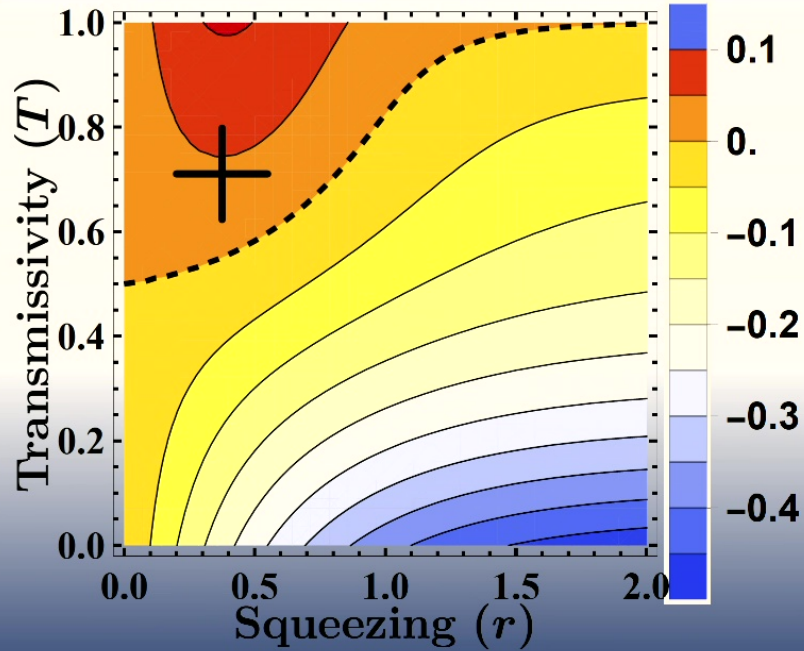
Teleportation with non-Gaussian state



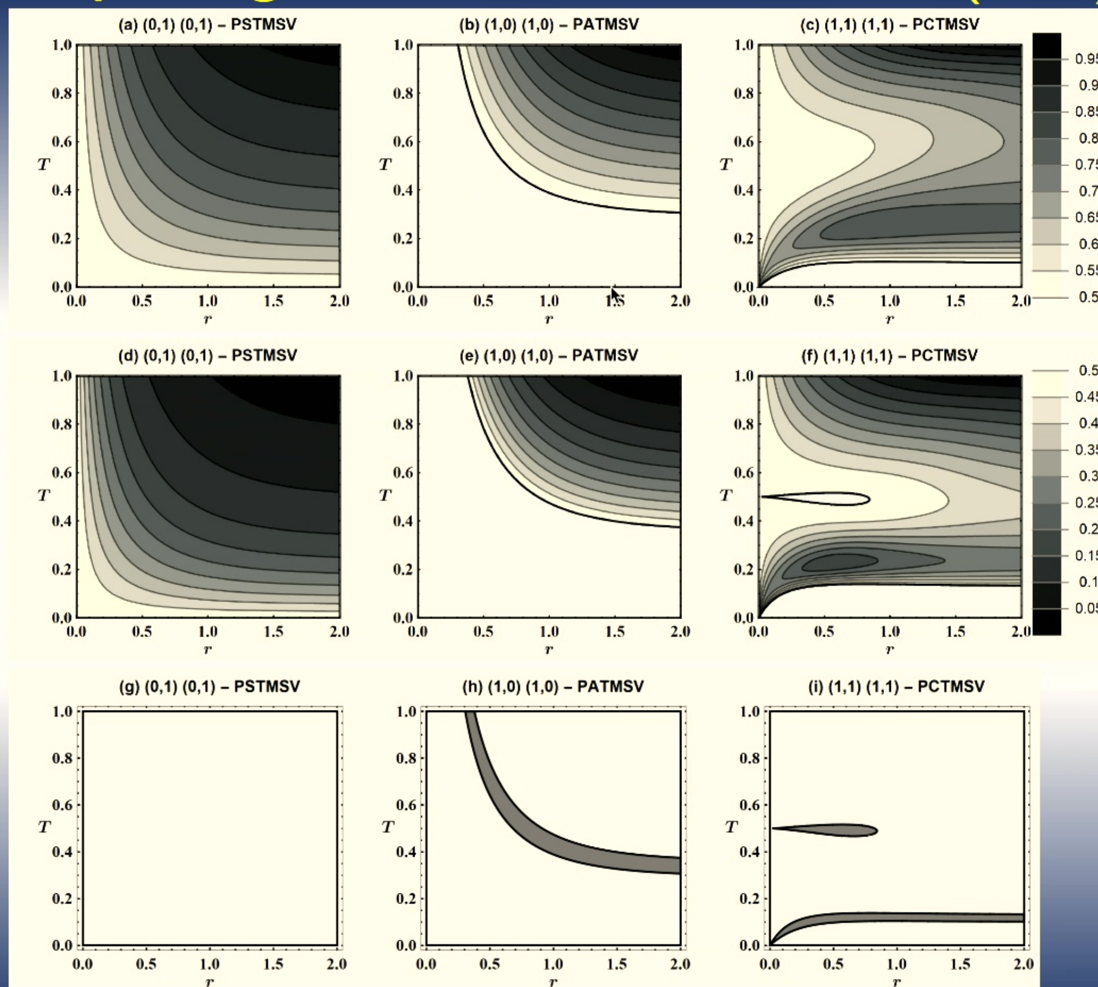
Photon Sub. $\Rightarrow$ Fidelity Enhancement

$$F^{\mathsf{PS}} = \frac{(\lambda T+1)^3(2-\lambda T(2-\lambda T))}{4(\lambda^2T^2+1)}, \quad F^{\mathsf{TMSV}} = \frac{1+\lambda}{2}, \quad \lambda = \tanh r.$$

- Photon subtraction does enhance the fidelity in the region enclosed by the dashed curve
- Fidelity enhancement $\Delta F^{\mathsf{PS}} = F^{\mathsf{PS}} - F^{\mathsf{TMSV}}$. Dashed line $\Delta F^{\mathsf{PS}} = 0$.
- arXiv:2407.06037



Is squeezing essential? arXiv:2502.17182 (2025)



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New Gaussian Bound entangled states

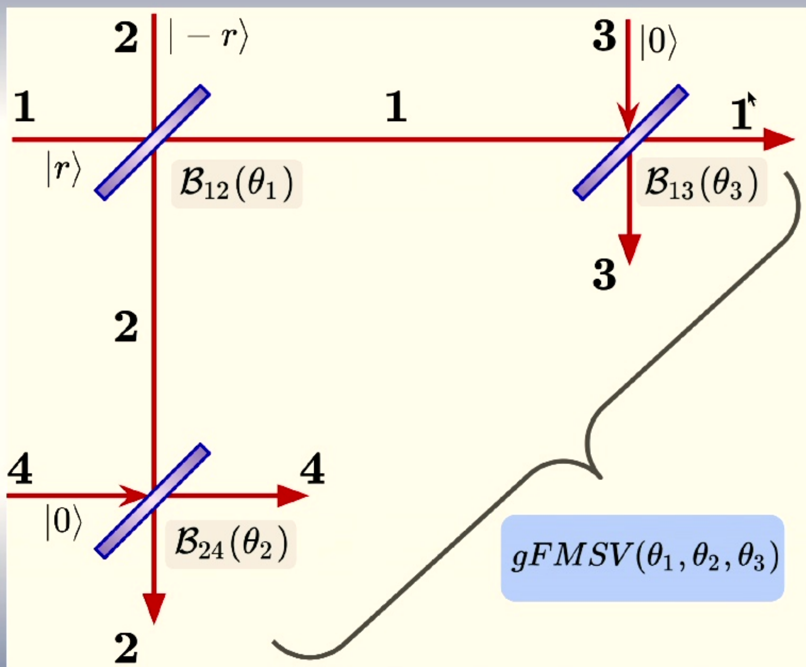
State Space: Entangled States, Separable States

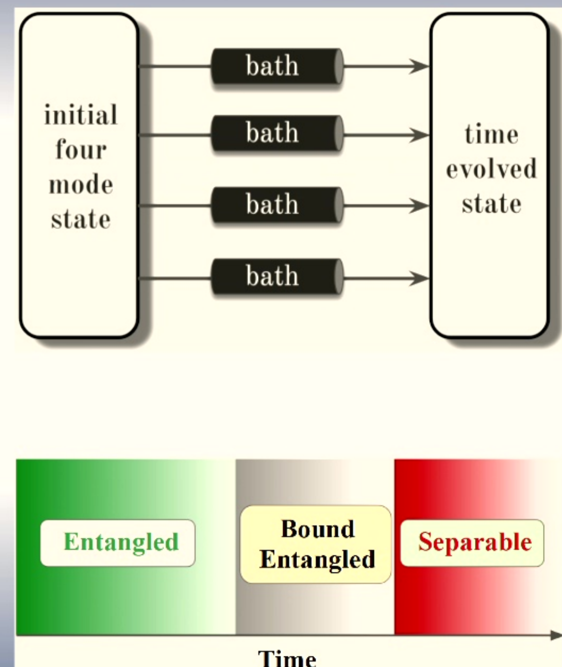
NPT Entangled states(Distillable), PPT Entangled State(Bound Entangled)

Gaussian Bound Entangled states are rare and only a few examples are known.

New Gaussian Bound Entangled States of 4 mode CV systems

gFMSV States





Results

Noisy Modes	τ^*		
	$N = 2$	$N = 4$	$N = 10$
$\{1\}, \{2\}, \{3\}, \{4\}$	-	-	-
$\{1, 2\}, \{3, 4\}$	0.82	0.60	0.32
$\{1, 4\}, \{2, 3\}$	-	-	-
$\{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$	0.71	0.48	0.24
$\{1, 2, 3, 4\}$	0.38	0.20	0.09

Noisy Modes	N	τ_{BE}	τ^*
3*$\{1, 3\}, \{2, 4\}$	2	0.71	0.82
	4	0.48	0.60
	10	0.24	0.32

Thank you